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Optimal Capital Income Taxes in the Infinite Horizon Model with Progressive Income Taxes

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In infinitely lived, representative-agent models with linear income taxes, the influential studies by Chamley (1986) and Judd (1985) have shown that the optimal capital tax is zero in the long run. Our paper uses a similar model with progressive taxes and the results are as follow. First, the long-run optimal capital income tax is positive with progressive income taxes. Second, the welfare gain from moving current tax rates toward positive optimal income taxes with progressive tax rates is larger than moving toward a zero capital income tax with linear income taxes. Our findings lend support to positive capital income taxes under a system of progressive income taxes adopted in developed countries since the late 19^{th} century.

Keywords: infinite horizon model, optimal capital income taxation, progressive taxes
JEL classification: E1, E6, H2

1 Introduction

One of the most striking results in optimal tax theory is the famous finding by Chamley (1986) and Judd (1985). They employed the neoclassical growth model with a government that finances an exogenous stream of

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government purchases. The production factors are raw labor and physical capital on which the government levies flat-rate factor income taxes. The problem is to determine the optimal sequences for the two taxes. Although working in slightly different setting, they established the same conclusion: the optimal capital tax should be zero in the long run.

The neoclassical growth model was augmented with human capital by Lucas (1990) and Jones, Manuelli, and Rossi (1993, 1997). Moreover, the model was extended to open economies by Correia (1996) and Gross (2014). Indeed, several other studies revisited the same issue by relaxing key assumptions. They all found a zero optimal capital tax in the long run. In particular, Atkeson, Chari, and Kehoe (1999) proved that the zero long-run capital tax result is robust to other economic environments.

All these models restrict attention to linear income taxes. However, income taxes and capital income taxes are progressive in most developed countries. For example, corporate profits in the US are first taxed at a rate up to 40%. Then, when these profits are distributed as dividends, the income is taxed again at progressive rates from 10% to 35% at the household level. In fact, most developed countries have adopted comprehensive income tax systems with graduated taxes that date back as early as the second half of the 19$^{\text{th}}$ century.[1] In fact, progressive income taxes have been incorporated into models of a representative agent. For example, Guo and Lansing (1998) and Dromel and Pintus (2008) studied the effect of tax progressivity on dynamic stability. Li and Sarte (2004) envisaged the effect of tax progressivity on long-run economic growth. Cassou and Lansing (2004) and Erosa and Koreshkova (2007) quantified the effect of shifting from a graduated-rate tax system to a flat tax rate. Santoro and Wei (2012) analyzed the impact of progressive dividend taxation on investment decisions. None of these papers analyze optimal capital taxes.

The purpose of this paper is to study the model of a representative agent with progressive income taxes and analyze optimal capital income taxes. To be specific, we incorporate progressive income taxes into the model of Chamley (1986). We will find that if the income tax rate schedule is sufficiently progressive, a positive capital tax rate is optimal in the long run. Our result is not only in sharp contrast to the zero long-run capital tax Chamley-

[1] According to Saez (2013), the German states such as Prussia and Saxony introduced the modern income tax during the second half of the 19$^{\text{th}}$ century, Japan in 1887, the UK in 1909, the US in 1913 and France in 1914.

Judd result, but is also consistent with positive tax rates under the progressive income taxation adopted in developed countries.

The result is understood as follows. In the long run, the Ramsey planner chooses capital such that the time-preference rate equals the planner's post-tax *marginal gain of capital* (hence, MGK), which includes three terms: (i) the post-tax return to capital, (ii) the gain in utility from increases in capital tax revenues, and (iii) the loss in utility from decreases in post-tax returns to capital due to progressive taxes. Moreover, in the long run the household chooses capital such that the time-preference rate equals the household's post-tax MGK, which includes only the post-tax return to capital. For the Ramsey planner's choice to be consistent with the household's choice, the planner's post-tax MGK needs to equal the household's post-tax MGK.

In the case of linear capital income taxes, the gain in term (iii) is zero. In this case, if the capital tax rate is positive, the gain in utility from increases in capital tax revenues in term (ii) would be positive and therefore, the planner's post-tax MGK would be larger than the household's post-tax MGK, implying an *under-accumulation* of capital from the social perspective. It is thus optimal to decrease the capital tax rate to zero.

By contrast, in the case of progressive capital income taxes, the gain term (iii) is not zero. If the capital income tax schedule is sufficiently progressive, the elasticity of the marginal capital tax rate with respect to capital income is not too large. In this situation, if the capital tax rate is zero, the gain in term (ii) would be smaller than the gain in term (iii). Then, the planner's post-tax MGK is smaller than the household's MGK. As a result, the level of capital chosen by the planner is smaller than the level chosen by the household and there is an *over-accumulation* of capital from the social perspective. An increase in capital income taxes enlarges the effect in term (ii) and decreases the effect in term (iii). Therefore, it is optimal to tax capital income.

Finally, we compare the welfare between employing an optimal progressive tax and employing an optimal linear tax. We carry out the exercise in a quantitative version of our model calibrated to the system of progressive income taxes in the US. We find that the optimal capital tax rate is positive and increasing in the degree of income tax progressivity. In particular, the welfare gain of a tax reform from the current tax code toward the optimal positive income taxes under a progressive-rate tax system is larger than that toward an optimal zero capital tax under a linear-rate tax system. The result justifies the choice of a positive tax.

Our model complements three existing articles that studied models of a representative agent with linear income taxes and obtained a positive optimal capital tax rate.[2] First, Lansing (1999) studied the Judd model. He found a long-run positive capital income tax rate if capitalists' utility is logarithmic and there is no government debt. Next, Chen and Lu (2013) analyzed the human capital model and revisited the Chamley model. They uncovered a positive capital income tax rate in the long run if the technology for human capital accumulation is the same as that in Lucas (1988). Finally, Straub and Werning (2020) revisited both the Judd model and the Chamley model. For the Judd model, they obtained a positive long-run capital tax rate if the intertemporal elasticity of substitution is below one but a zero capital tax when the elasticity is higher. For the Chamley model, they found a positive long-run capital tax rate if the upper bound on the capital tax rate binds forever and a zero capital tax if otherwise. Our model differs from these three papers in that positive capital taxation does not require any assumptions concerning the logarithmic utility, the zero debt issue, the intertemporal elasticity of substitution, and the binding upper bound on capital taxes; instead, positive capital taxation only requires that the income tax rate schedule is sufficiently progressive.[3]

Our model also complements Saez (2013) that studied progressive wealth taxation for income redistribution in a model of infinitely-lived agents with heterogeneous wealth. Saez specified a two-bracket wealth tax with an exemption and a linear tax rate. He found that positive wealth income taxation is optimal and that wealth income taxation has a drastic impact on the long-run wealth distribution.[4] However, because of heterogeneous wealth, there

[2] There were models of a representative agent with linear income tax rates that obtained positive optimal capital tax rates based upon inherent distortions wherein the capital taxation serves to internalize the distortions. See Guo and Lansing (1999) and Chen (2007) who incorporated market imperfections and productive public capital, and Chamley (2001) who considered credit constraints. There is another strand of the literature that obtain positive optimal capital income taxes in overlapping- generations (OLG) models. We should note that optimal capital taxes are generally positive in the long-run in an OLG model, simply because capital accumulation is due to life-cycle savings for retirement. See Atkinson and Sandmo (1980), Garriga (2001), Erosa and Gervais (2002) and Conesa, Kitao, and Krueger (2009).

[3] There is another strand that studies first-best capital income taxes in model with market failures. See, for example, Liu and Turnovsky (2005).

[4] For models with a tension between equity and the efficiency of capital accumulation, see also Benabou (2002), Farhi and Werning (2012) and Farhi et al. (2012).

is a tension between equity and the efficiency of wealth accumulation and a positive wealth income tax is the result of the equity concern. Despite of this, the fraction of individuals subject to wealth income taxation in his model vanishes to zero in the long run in analogy to the zero long-run capital tax result of Chamley and Judd with linear taxes. Our model is different from the model in Saez (2013) in that although the efficiency of capital accumulation is the only tension, we still obtain a positive capital income tax rate in the long run once a sufficiently progressive tax schedule is allowed for.

We organize this paper as follows. In Section 2, we set up a model with progressive factor income tax schedules and analyze households' optimizations. In Section 3, we study the optimal factor income tax incidence in the Ramsey second-best problem. Finally, concluding remarks are offered in Section 4.

2 The model

Our basic model is otherwise the same as the Chamley (1986) model with the exception of a progressive-rate tax system for factor income. There are infinitely lived and identical households and identical firms. Households supply labor and capital to firms, earn labor and capital income and decide consumption and savings. There is a government which taxes capital and labor income in order to pay for wasteful expenditure that is given.

2.1 The household's problem

In each period, given a fixed time endowment normalized to unity, the representative household allocates the time endowment between work and leisure. The household maximizes the following discounted utilities over sequences of consumption and leisure hours.

$$\underset{\{c_t, k_{t+1}, b_{t+1}, l_t\}}{\text{Max}} \sum_{t=0}^{\infty} \beta^t u\left(c_t, 1 - l_t\right), \tag{1}$$

where c_t is consumption, l_t is hours worked and $\beta \in (0, 1)$ is the discount factor. The felicity function $u(c_t, 1-l_t)$ is assumed to be twice continuously differentiable and increasing and concave in c_t and $1 - l_t$.

In each period, the representative household faces the following flow budget constraint.

$$c_t + k_{t+1} + b_{t+1} = (w_t l_t - T\,(w_t l_t)) + [k_t + r_t k_t - T_k\,(r_t k_t)] \\ + R_t b_t + \pi_t, \quad k_0 \text{ and } b_0 \text{ given}, \tag{2}$$

in which k_t is physical capital at the beginning of period t, b_t refers to one-period, real government bonds carried into period t and π_t is the profit remitted from firms in period t. The wage rate is w_t, the gross return to bonds is R_t and r_t is the rental rate of capital net of the depreciation rate. Capital depreciation expenses are tax-deductible in (2) in order to be consistent with the US tax code. The results are not changed if capital depreciation expenses are not tax-deductible.

In (2), labor and capital income are taxed at these tax schedules $T_l(w_t l_t)$ and $T_k(r_t k_t)$, respectively. Differing from the linear income tax rate studied by Chamley (1986) and Judd (1985), our income taxes are progressive and depend on the income level. We assume that these tax schedules are continuously differentiable with respect to income. In particular, we assume an income tax system with strictly progressive taxes. To be specific, we assume $T_l'(w_t l_t) > 0$, $T_l''(w_t l_t) > 0$, $T_k'(r_t k_t) > 0$ and $T_k''(r_t k_t) > 0$.[5] In the case of linear taxes, $T_l''(w_t l_t) = 0$ and $T_k''(r_t k_t) = 0$ and the model degenerates to the Chamley (1986) model.[6]

The representative household's dynamic programming problem is to choose a sequence of $\{c_t, k_{t+1}, l_t, b_{t+1}\}_{t=1}^{\infty}$ in order to maximize its lifetime utilities (1) subject to the constraint (2). When making choices, the household takes prices w_t, r_t and R_t as determined by the market. It also takes

[5]It is well understood that, without restrictions on non-linear taxes that the government can implement, the government can pick labor taxes and capital taxes such that they act exactly like lump sum taxes, which implement the first-best allocation. Specifically, if the tax rates are not strictly progressive, one can choose positive tax rates $T_{lt} > 0$ and $T_{kt} > 0$ along with regressive tax rates $T_{lt}'' < 0$ and $T_{kt}'' < 0$ in order to meet the conditions in household's optimization (cf. (3a)–(3c) below) $T_{lt}' = 0$ and $T_{kt}' = 0$ so that the government problem yields the first best outcome. Our restrictions to progressive tax rates $T_{lt}'' > 0$ and $T_{kt}'' > 0$ rule out such a situation to arise.

[6]When the government compares between flat (linear) and progressive tax systems, the progressive tax system is chosen if the representative household's welfare under the optimal income taxes in the progressive tax system is larger than that in the linear tax regime. Our quantitative analysis in subsection 3.2 confirms this result.

tax schedules $T_l(w_t l_t)$ and $T_k(r_t k_t)$, as given by the government. Yet, as the tax schedules are progressive, the household knows that its choices of hours worked and savings affect not only tax bases but also marginal taxes. The first-order conditions give:

$$u_1(c_t, 1-l_t) = \beta u_1(c_{t+1}, 1-l_{t+1})\left[1 + \left(1 - T_k'(r_{t+1}k_{t+1})\right) r_{t+1}\right], \tag{3a}$$

$$u_2(c_t, 1-l_t) = u_1(c_t, 1-l_t)\left(1 - T_l'(w_t l_t)\right) w_t, \tag{3b}$$

$$R_{t+1} = 1 + \left(1 + T_k'(r_{t+1}k_{t+1})\right) r_{t+1}, \tag{3c}$$

along with the transversality conditions $\lim_{t\to\infty} \beta^t u_1(c_t, 1-l_t)k_{t+1} = 0$ and $\lim_{t\to\infty} \beta^t u_1(c_t, 1-l_t)b_{t+1} = 0$ which ensure that there is no "Ponzi scheme." Eq. (3a) is the standard consumption-Euler equation that is traded off against consumption in period t and $t+1$. Eq. (3b) refers to the tradeoffs between leisure and consumption in period t, and (3c) is the no-arbitrage condition for trade in bonds and capital that ensures the same rate of return for these two assets.

2.2 The firm's problem

The representative firm rents capital and hires labor and produces a single final good yt given by:

$$y_t = f(k_t, l_t). \tag{4}$$

The function $f(\cdot)$ is assumed to be twice continuously differentiable and is strictly increasing and concave in capital and labor. Taking factor prices as given, the firm chooses capital and labor in order to maximize profits. The optimal conditions are standard and are as follows:

$$r_t = f_1(k_t, l_t) - \delta, \tag{5a}$$

$$w_t = f_2(k_t, l_t), \tag{5b}$$

where δ is the depreciation rate.

2.3 The government

The government finances an exogenous stream of expenditure by taxing factor income and issuing debt. Denote g_t as the exogenous government expenditure which increases neither the firms' productivity nor the households' utilities. The government's flow budget constraint is

$$T_t\left(w_t l_t\right)+T_k\left(r_t k_t\right)+b_{t+1}=g_t+R_t b_t. \tag{6}$$

3 The Ramsey planner's problem

The Ramsey planner's problem is to determine the optimal sequence of the factor income taxes. In analyzing the Ramsey planner's problem, like Chamley (1986), we rule out the lump-sum taxation that would be first-best and assume that the income taxes in the initial period are given by their historical values, since taxing initial private assets is equivalent to a lump sum tax.[7] We allow for the government to issue debt and thus the government does not have to run a balanced budget in each period. In order to avoid time inconsistency, we assume that the government can take a full commitment of policies announced at $t=0$.

We start the planner's problem by defining a *competitive equilibrium* which is a feasible allocation $\{c_t, k_{t+1}, l_t, g_t\}_{t=0}^{\infty}$ a price system $\{w_t, r_t, R_t\}_{t=0}^{\infty}$ and a government policy $\{g_t, T_{kt}, T_{lt}, b_{t+1}\}_{t=1}^{\infty}$ such that (i) given the price system and the government policy, the allocation solves both the firm's and the household's problem and satisfies the resource constraint; and (ii) given the allocation and the price system, the government policy satisfies the sequence of government budget constraints. Different government policies would yield different competitive equilibria. Given k_0 and b_0, the Ramsey planner's problem is to choose a competitive equilibrium that maximizes the welfare of the household.

The existing studies use the primal approach to the Ramsey planner's problem which eliminates all prices and taxes, so the government is thought of as directly choosing a feasible allocation. To use the primal approach, we need to express taxes and prices in terms of the allocation and substitute these expressions into the household's present-value budget constraint in order to obtain the so-called "implementability condition." However, we cannot eliminate all terms involving taxes in these expressions, the reason being that, as capital income or labor income increases, in addition to original taxes, there are marginal taxes. Thus, we cannot use the primal approach.

[7] Consumption taxes (Coleman, 2000) and dividend taxes with immediate capital expenditure (investment) deductions (Abel, 2007) can mimic initial wealth expropriation. Both are disallowed.

Following Chamley (1986), we formulate the Ramsey problem as if the government chooses the after-tax rental rate of capital and the after-tax wage rate. Then, capital, labor and consumption chosen by the Ramsey planner would affect the after-tax rental rate of capital and the after-tax wage rate, which then implies the optimal capital tax and the optimal labor tax.

A competitive equilibrium consists the resource constraint in the economy, the government budget constraint and the best responses of firms and households. First, the resource constraint is:[8]

$$c_t + k_{t+1} - (1-\delta)k_t + g_t = f\,(k_t, l_t)\,. \tag{7a}$$

Next, the government's flow budget constraint is:[9]

$$\begin{aligned} g_t + R_t b_t - b_{t+1} = f\,(k_t, l_t) - \delta k_t - w_t l_t + T_l\,(w_t l_t) \\ - r_t k_t + T_k\,(r_t k_t)\,. \end{aligned} \tag{7b}$$

Finally, the best responses of households are (3a)–(3b).

Thus, the Lagrange equation of the Ramsey planner's optimization problem is:

$$\begin{aligned} L = \sum_{t=0}^{\infty} \beta^t \Big\{ & u\,(c_t, 1-l_t) + \theta_t\big[f\,(k_t, l_t) - c_t - k_{t+1} + (1-\delta)k_t - g_t\big] \\ & + \Psi_t\big[f\,(k_t, l_t) - \delta k_t - w_t l_t + T_l\,(w_t l_t) - r_t k_t \\ & + T_k\,(r_t k_t) - g_t + b_{t+1} - R_t b_t\big] \\ & + \mu_{1t}\big[\beta u_1\,(c_{t+1}, 1-l_{t+1})\,R_{t+1} - u_1\,(c_t, 1-l_t)\big] \\ & + \mu_{2t}\big[u_1\,(c_t, 1-l_t)\big(1 - T_l'\,(w_t l_t)\big)\,w_t - u_2\,(c_t, 1-l_t)\big]\Big\}, \end{aligned} \tag{8}$$

where $R_t = 1 + (1 - T_k'(r_t k_t))r_t$. θ_t, Ψ_t, μ_{1t} and μ_{2t} are the Lagrange multipliers associated with constraints (7a), (7b), (3a) and (3b), respectively.

The Ramsey planner chooses the optimal paths of c_t, k_{t+1}, b_{t+1}, and l_t.

[8] The constraint is obtained by substituting (6) into the household's flow budget constraint (2).

[9] The constraint is derived by substituting (5a) and (5b) into the government's flow budget constraint (6).

In particular, the first-order condition with respect to k_{t+1} is:

$$\theta_t = \beta \bigg[\theta_{t+1} \left(f_{1t+1} - \delta + 1 \right) + \Psi_{t+1} \left(f_{1t+1} - \delta - \tilde{r}_{t+1} \right) \\ - \Psi_{t+1} \frac{dR_{t+1}}{dk_{t+1}} b_{t+1} + \mu_{1t} u_{1t+1} \frac{dR_{t+1}}{dk_{t+1}} \bigg], \tag{9}$$

where $(dR_{t+1}/dk_{t+1}) = -T_k''(r_{t+1}k_{t+1})r_{t+1}^2$ and $\tilde{r}_{t+1} \equiv (1 - T_k'(r_{t+1}k_{t+1}))$ r_{t+1}^2 denotes the post-tax return to capital.[10]

Similar to Ljungqvist and Sargent (2012, p. 622), condition (9) may be interpreted as follows. The shadow price of capital $\theta_t > 0$ is the social marginal value of a marginal increment of capital investment in period t. The capital investment creates the following effects in the next period. Firstly, it increases the quantity of goods available by the amount $f_{1t+1} - \delta + 1$, which has a social marginal value θ_{t+1}. Secondly, there is an increase in capital income tax revenues equal to $f_{1t+1} - \delta - \tilde{r}_{t+1}$, which enables the government to reduce its debt or other taxes and the reduction in the 'excess burden' is equal to $\Psi_{t+1}(f_{1t+1} - \delta - \tilde{r}_{t+1})$. These two effects exist in both cases of flat and progressive taxes. In addition, there are other effects arising from progressive taxes due to the change of the post-tax return to capital. A higher capital tax decreases the post-tax return to capital equal to $T_{kt+1}'' r_{t+1}^2$. This reduces the interest that government needs to pay and thus, further relaxes the 'excess burden' of the government by $\Psi_{t+1} T_{kt+1}'' r_{t+1}^2 b_{t+1}$. However, this decrease in the post-tax return to capital reduces the return of saving made by households and the loss in terms of utility equal to $u_{1t+1} T_{kt+1}'' r_{t+1}^2$, which has a social marginal value μ_{1t}.[11] In the optimum, the discounted sum of these effects in period $t + 1$ is equal to the social marginal value of the initial capital investment in period t.

3.1 The steady-state optimal capital income tax

With the use of (5a), the first-order condition concerning the Ramsey planner's optimal choice of capital in (9) in the steady state is

$$\theta = \beta \left\{ [\theta(r + 1) + \Psi(r - \tilde{r})] + (\Psi b - \mu_1 u_1) T_k'' r^2 \right\}.$$

[10] The other first-order conditions of the Ramsey planner's problem are relegated to the Appendix.

[11] In the Appendix, we have shown that $\mu_1 > 0$ and $\theta > 0$ in the steady state.

This expression indicates that in the steady state, the social marginal value of the initial investment is equal to the discounted effects in the two brackets. The effects in the first brackets always appear, while the effects in the second brackets emerge only when the income tax is progressive. In the steady state, the shadow price of the government budget constraint is equal to the marginal utility of consumption, $\Psi = u_1$ (cf. the Appendix). We can rewrite the expression above as

$$\frac{1}{\beta} - 1 = \tilde{r} + \frac{\theta + u_1}{\theta}(r - \tilde{r}) + \frac{b - \mu_1}{\theta} u_1 T_k'' r^2. \tag{10}$$

Eq. (10) equates the time-preference rate, $(1/\beta) - 1$ to the post-tax social marginal gain of capital, denoted by MGK^s, which includes three terms: (i) the post-tax return to capital, $\tilde{r}$, (ii) the gain in utility from increases in income tax revenues adjusted by the social shadow price of capital, $(\theta + u_1/\theta)T_k' r$ and (iii) the change in utility from decreases in post-tax returns to capital arising from progressive taxes adjusted by the social shadow price of capital, $(b - \mu_1/\theta)u_1 T_k'' r^2$.

In the steady state, the household's optimal choice of capital is the modified golden rule condition in (3a), and yields:

$$\frac{1}{\beta} - 1 = \tilde{r}, \tag{11}$$

which requires the time-preference rate to be equal to the post-tax private marginal gain of capital, denoted as MGK^p, which includes only post-tax returns to capital.

In order for the Ramsey planner's choice of capital in (10) to be consistent with the household's choices in (11), it is clear that MGK^s should be equal to MGK^p, which requires:

$$(\theta + u_1)(r - \tilde{r}) + (b - \mu_1)u_1 T_k'' r^2 = 0. \tag{12}$$

Thus, the sum of the gain in utility from increases in income tax revenues and the change in utility from decreases in post-tax returns to capital arising from progressive taxes is zero.

Case 1: The linear income tax

In this case, the income tax schedule is flat as was the case in Chamley (1986) and Judd (1985). Thus, $T_k'' = 0$ for all t and there is no change in utility

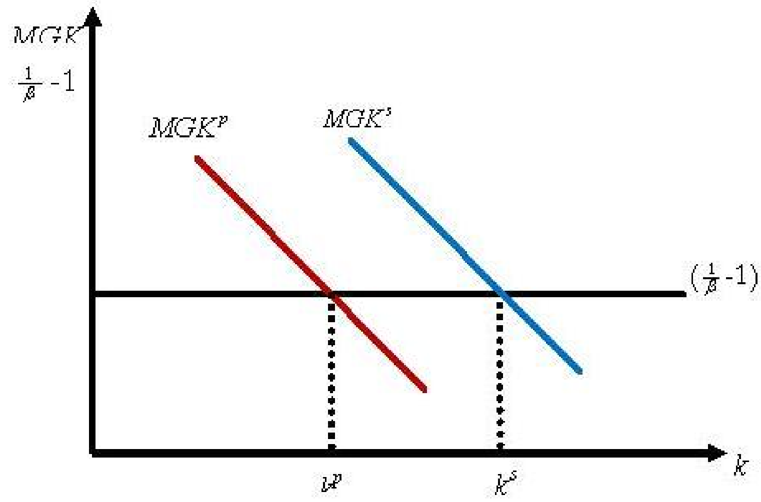

Figure 1: Positive Linear Taxes

due to decreases in post-tax returns to capital arising from progressive taxes. Then, the gain in utility from increases in income tax revenues should be zero and condition (12) degenerates to:

$$(\theta + u_1)(r - \tilde{r}) = 0, \tag{13}$$

which yields the result of a zero capital income tax, $r = \tilde{r}$.

To understand the reason, suppose that the marginal capital income tax is positive; thus, $r - \tilde{r} = T_k' r > 0$. Then, $\mathrm{MGK}^s = \tilde{r} + (\theta + u_1/\theta)T_k' r$, while $\mathrm{MGK}^p = \tilde{r}$. A positive capital tax implies that the MGK^p is *smaller than* the MGK^s. See Figure 1. With a given time-preference rate, the modified golden rule condition and the relative position of MGK^p and MGK^s in Figure 1, there is an under-accumulation of capital from the social perspective when the marginal capital income tax is positive. As a result, the efficiency is improved if the marginal capital income tax is reduced from the positive level, as this decreases the post-tax social marginal gain of capital and increases its counterpart from the private perspective. A zero marginal capital income tax is optimal as MGK^s then meets MGK^p.[12]

[12] It is still optimal if we tax a lump sum on capital income in the long run, i.e. $T(rk) =$ constant.

Case 2: The progressive income tax

When the capital income tax is progressive, $T_k'' > 0$. Using the definition of $\tilde{r} \equiv (1 - T_k'(rk))r$, we rewrite (12) as

$$(\theta + u_1)T_k' + (b - \mu_1)u_1 T_k'' r = 0. \tag{14a}$$

Obviously, if $b > \mu_1$, the equation cannot hold and thus the second best allocation does not exist when the capital income tax is progressive. The following condition requires the government debt be not too large in order to ensure the second best allocation is feasible.

Condition D (Government debt) $b < \mu_1$.[13]

To analyze the optimal capital income tax, let the capital income tax revenue be denoted by $T_k(rk) \equiv \tau_k(rk)rk$. First, the average tax rate is simply $(T_k(rk)/rk) = \tau_k$, while the tax rate applied to the last dollar earned is $(\partial T_k(rk)/\partial(rk)) \equiv T_k' = \tau_k + \tau_k' rk$, the marginal tax rate. The tax schedule is progressive if the marginal tax rate is larger than the average tax rate: $T_k' > (T_k(rk)/rk)$. Thus, $\tau_k' > 0$. Next, since $T'' = \tau_k'' rk + 2\tau_k'$, then $\tau_k'' = (T'' - 2\tau_k'/rk)$. Substituting these expressions into (14a), we further rewrite (14a) as

$$(\theta + u_1)\tau_k r + \Omega = 0, \tag{14b}$$

where $\Omega \equiv (\theta + u_1)\tau_k' rk + (b - \mu_1)u_1(2\tau_k' r + \tau_k'' rk)$. If $\Omega < 0$, then the optimal capital income tax rate is no longer zero but is positive.

To see when $\Omega < 0$, we let $\xi_{\tau_k} \equiv -[(rk/\tau_k')(\partial\tau_k'/\partial(rk))] = -(\tau_k'' rk/\tau_k') > 0$ denote the elasticity of the marginal capital income tax rate with respect to capital income. For simplicity, ξ_{τ_k} is referred to as the marginal capital tax rate elasticity. An elasticity is normally a constant. The following condition requires the marginal capital tax rate elasticity be not too large in order to ensure $\Omega < 0$.

Condition E (Marginal Capital Tax Rate Elasticity) $\xi_{\tau_k} < 2r - [(\theta + u_1) rk/(\mu_1 - b)u_1]$

[13] Straub and Werning (2020) revisited the long-run Ramsey capital taxation in the Chamley model and found that a positive long-run tax on capital income is guaranteed if debt is high enough. Here, we have the opposite result, debt cannot be too high.

Under Condition D and E, $\Omega < 0$. Then, $\tau_k > 0$ and thus, the capital tax rate is positive.

To illustrate that Condition E is easily met, we follow Li and Sarte (2004) and use the following capital income tax rate schedule:[14] $\tau_k(r_t k_t) = \eta_k(r_t k_t/\overline{rk})^{\phi_k}$, $\eta_k \geq 0$ and $0 \leq \phi_k < 1$, where $\overline{rk}$ stands for the steady-state level of post-depreciation capital income. In this tax rate schedule, η_k controls for the limiting value of the average tax rate and ϕ_k determines the degree of income tax rate progressivity.[15] For $\phi_k = 0$, the tax rate schedule is flat and, the average tax rate equals the marginal tax rate, i.e. $\tau_k = \eta_k$. For $\phi_k > 0$, the tax rate schedule is progressive and the average tax rate τ_k is smaller than the marginal tax rate $(1+\phi_k)\tau_k$. The progressive tax rate schedule gives a constant marginal capital tax rate elasticity equal to $\xi_{\tau_k} = 1 - \phi_k$. Then, Condition E is met if $\phi_k > 1 + [(\theta + u_1)rk/(\mu_1 - b)u_1] - 2r$. Thus, Condition E basically requires that the capital income tax rate schedule be sufficiently progressive.

Now, we can state our main result.

Proposition 1. In a system of progressive income taxes, if the capital income tax rate schedule is sufficiently progressive, the optimal tax rate on capital income is positive in the long run.

To understand the reason, suppose that the capital tax rate is $\tau_{k0} = 0$. Then, the left-hand side of (14b) reduces to Ω. If the tax rate schedule is sufficiently progressive as is required in Condition E, then $\Omega < 0$. With $(\theta+u_1)\tau_k' rk > 0$ this negative term suggests that $(b-\mu_1)u_1(2\tau_k' r+\tau_k'' rk) < 0$ and thus there is a loss in utility from lower post-tax returns to capital.

A negative Ω at $\tau_{k0} = 0$ indicates that the MGK^s is *smaller than* the MGK^p. See Figure 2. That is, if the capital tax rate is zero, a progressive capital tax rate would make the Ramsey planner choose a level of capital that is smaller than the level chosen by the household. There is an *over-accumulation* of capital from the social perspective. Hence, it is optimal to tax capital income in order to reduce the level of capital chosen by the household.

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[14] The tax rate schedule was based on the form proposed by Guo and Lansing (1998). Another form of nonlinear taxes was the one proposed by Gouveia and Strauss (1994), which has been employed in the quantitative public finance literature by Conesa and Krueger (2006) and Conesa, Kitao, and Krueger (2009).

[15] We should note that although in the steady state the capital tax rate boils down to η_k, the tax rate applied to the last dollar earned in the steady state is $\eta_k(1 + \phi_k)$.

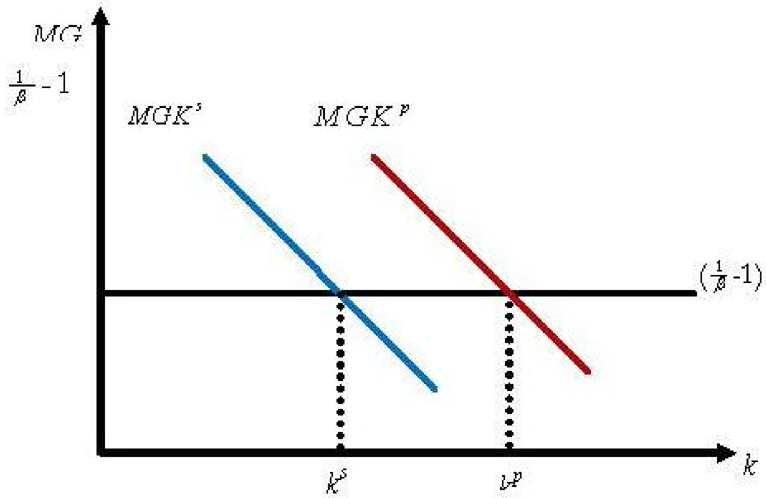

Figure 2: Progressive Taxes When $\tau_{k0} = 0$

Intuitively, if the capital income tax rate schedule is sufficiently progressive, a zero capital tax rate gives the gain in utility from increases in capital income tax revenues as being smaller than the loss in utility from lower post-tax returns to capital. An increase in the capital tax rate from zero raises the gain in utility from increases in capital tax revenues and lowers the loss in utility from lower post-tax returns to capital. The optimal capital tax rate is set at the level when the gain would completely offset the loss.

We should mention that if government expenditure is not a waste but a lump sum transfer to the household, the optimal capital tax is still positive. The reason is that the effect of capital taxes works through the post-tax marginal gain of capital but the effect of the lump-sum transfer is neutral.

We have noted in the Introduction that Lansing (1999), Chen and Lu (2013) and Straub and Werning (2020) have obtained a positive capital tax. Our result adds value to these studies in that a positive capital income tax rate is obtained without requiring any assumptions concerning the logarithmic utility, the zero debt issue, the intertemporal elasticity of substitution, and the binding upper bound on capital taxes. Our positive capital tax is obtained only if the income tax rate schedule is sufficiently progressive.

Finally, in a heterogeneous-agent model, Piketty and Saez (2013) derived optimal inheritance tax formulas in terms of a "sufficient statistics" including tax elasticity and other parameters.[16] They found a larger optimal inheritance tax if the elasticity of aggregate bequest flows with respect to the net-of-bequest-tax rate is smaller. Our capital income tax rate formula is $\tau_k(rk) = \tau_k'(rk)\{[(\mu_1 - b)u_1/(\theta + u_1)r](2r - \xi_{\tau_k}) - k\}$ and thus, our tax rate depends negatively on the marginal capital tax rate elasticity ξ_{τ_k}. If the capital income tax rate schedule is more progressive, the marginal capital tax rate elasticity is smaller and then our optimal capital tax rate is larger. From this perspective, we view our result as complementary to the Piketty and Saez (2013) result. Moreover, their paper and our paper both find an optimal capital tax rate that is increasing in capital income and is thus consistent with graduated-rate tax system in practice. Our model adds values to Piketty and Saez (2013) in that they obtain a positive capital tax rate in a model with heterogeneous agents wherein there is a tension between equity and the efficiency of capital accumulation. By contrast, we obtain a positive capital tax rate, even though our model has only homogeneous agents and thus the efficiency of capital accumulation is the only concern.

3.2 Quantitative analysis

To offer quantitative analysis, we calibrate our model to match the US annual data. First, we follow Conesa, Kitao, and Krueger (2009) to adopt the Cobb-Douglas production function, $y = f(k, l) = Ak^{\alpha}l^{1-\alpha}$ and the CES utility function $u(c, 1 - l) = (1/1 - \sigma)[c^{v}(1 - l)^{1-v}]^{1-\sigma}$.[17] We also go along with these authors and take $A = 1$, $\sigma = 4$, $l = 1/3$, $\alpha = 0.36$, $k/y = 2.7$, $I/y = 0.255$. The values of $k/y = 2.7$ and $I/y = 0.255$ give a value of an annual capital depreciation rate of $\delta = 9.44\%$.

Next, we take the form of tax rate schedules that was used by Li and Sarte (2004) and mentioned in subsection 3.1 above: $\tau_i(x_{it}) = \eta_i(x_{it}/\bar{x}_i)^{\phi_i}$, $i = k, l$, where x_{it} is factor i's income in period t and $\bar{x}_i$ is its steady-state level. Thus, x_{kt} is $r_t k_t$ and x_{lt} is $w_t l_t$. While these authors set $\phi_k = \phi_l =$

[16]Piketty and Saez (2013) studied inheritance tax structure in a heterogeneous-agent model with a discrete set of generations and linear income tax structure. They derived long-run optimal inheritance tax formulas in terms of "sufficient statistics" including tax elasticity and distributional parameters. Their optimal tax is zero if the long-run elasticity of aggregate bequest flows with respect to the net-of-bequest-tax rate is infinite nesting the zero capital tax Chamley-Judd result as a limiting case.

[17]The utility function is consistent with steady-state growth in a deterministic version of the real business cycle model (c.f., King and Rebelo (1999)).

$\phi = 0.75$, we will start with $\phi = 0.5$ in the baseline parameterization. The tax rate schedule is thus progressive for both capital and labor income taxes.[18] Our analytical results above indicate that the optimal capital tax rate depends on the degree of income tax progressivity. We will carry out the sensitivity analysis to see how the optimal capital tax rate depends on the degree of income tax progressivity. With the tax series from McDaniel (2007),[19] the average tax rates on capital income and labor income in the US during 1960–2007 were around 0.3 and 0.2, respectively. Thus, we choose initial average income tax rates equal to $\tau_k = 30\%$ and $\tau_l = 20\%$. This pins down the parameter values $\eta_k = 0.3$ and $\eta_l = 0.2$.

By using the foregoing parameter values, we utilize (5a) to compute the initial steady-state rental rate of capital equal to 0.1637. We then use the initial steady-state values of r_0 and l_0 to compute the initial steady-state capital equal to $k_0 = 1.5736$, and the initial steady-state output equal to y_0. We employ (7a) to compute $c_0/y_0 = 0.5855$ which, with the value of y_0, gives $c_0 = 0.3412$. Finally, we calibrate the discount factor $\beta = 0.9791$ from (3a) and the preference parameter $v = 0.3952$ from (3b). Thus, the allocation in the initial steady state is $(c_0, l_0, k_0) = (0.3412, 0.3333, 1.5736)$.

To avoid violating Condition D, we set $b_0 = 0$. We then calibrate $g_0 = 0.0930$ so as to balance the government budget (6) in the initial steady state, which leads to $g/y = 0.1595$ close to 0.17 in Conesa, Kitao, and Krueger (2009).

We are now ready to quantify the incidence of the Ramsey optimal factor income tax. In the exercise, the government expenditure is fixed at its initial level of $g_0 = 0.0930$. Our quantitative results provide a tax rate schedule with average rates of optimal factor income taxes $(\tau_k, \tau_l) = (27.56\%, 7.28\%)$ associated with the new steady state $(c^*, l^*, k^*) = (0.4067, 0.3820, 1.8560)$. See Table 1. Thus, the optimal capital tax rate is positive in the long run. The optimal income tax rate schedule suggests the following tax reform: a small decrease in the average capital tax rate by 2.44 percentage points from the current $\tau_k = 30\%$ level with a large decrease in the average

[18] Conesa, Kitao, and Krueger (2009) used a non-linear labor income tax and a linear capital income tax to ensure computational feasibility. In our paper, we adopt a more general strategy and employ non-linear tax schedules for both capital and labor income.

[19] McDaniel (2007) calculated a series of average tax rates on consumption, investment, labor and capital using national account statistics in 15 OECD countries. The data has been used by Rogerson (2008) and others.

Table 1: Optimal Tax Incidence

	τ_k	τ_l	c	l	k	Welfare gain* (%)
Baseline	0.3000	0.2000	0.3412	0.3333	1.5736	0.00
$\phi = 0.85$	0.3293	0.0004	0.4561	0.4254	1.8873	8.93
$\phi = 0.75$	0.3240	0.0150	0.4426	0.4137	1.8698	8.05
$\phi = 0.5$	0.2756	0.0728	0.4067	0.3820	1.8560	6.11
$\phi = 0.25$	0.0567	0.2099	0.3536	0.3361	1.8562	2.90
$\phi = 0.23$	0.0196	0.2246	0.3488	0.3321	1.8582	2.54
$\phi = 0$	0	0.2397	0.3436	0.3291	1.7926	1.81

Note: Baseline parameter values: $A = 1$, $\sigma = 4$, $\alpha = 0.36$, $\delta = 9.44\%$, $\eta_k = 0.3$, $\eta_l = 0.2$, $\beta = 0.9791$, $v = 0.3952$ and $g = 0.0930$. * The welfare gain is in terms of consumption equivalence.

labor tax rate by 12.72 percentage points from the current $\tau_l = 20\%$ level. The results reveal that moving away from the current income tax code in the US toward the optimal income tax would increase consumption, labor supply and capital accumulation. The reform would have a welfare gain of 6.11% in terms of changes in consumption equivalence. As compared to those results in the existing literature, the welfare gain is large. For example, a similar welfare gain of a factor income tax reform in terms of changes in consumption equivalence is 5.5% in Lucas (1990) in which case human capital accumulation is exogenous, 3.4% in Conesa, Kitao, and Krueger (2009) in which labor supplies are elastic, and 1.7% in Conesa and Krueger (2006).

To understand how the optimal capital income tax rate depends on the degree of income tax rate progressivity, we change the degree of income tax rate progressivity.[20] First, we increase the degree of tax rate progressivity. When the degree is increased from 0.5 to 0.75, the average rate of the optimal capital income tax is increased from 27.56% to 32.40% while the average rate of the optimal labor income tax is decreased from 7.28% to 1.50%. Our results indicate that the largest degree of income tax rate progressivity is 0.85, when the average optimal capital tax rate is 32.93% and the optimal

[20] When the degree of the tax progressivity is changed below, we recalibrate the values of β and v bases on the representative household's first-order conditions (3a) and (3b) so as to be consistent with the model and then calculate optimal factor income tax rates.

labor income tax rate is 0.4%. Next, we decrease the degree of income tax rate progressivity. When the degree is decreased from 0.5 to 0.25, the average rate of optimal capital income taxes is decreased from 27.56% to 5.67% with the average optimal labor income tax rate increasing from 7.28% to 20.99%. The smallest degree of income tax rate progressivity is 0.23, when capital income taxes remain positive.

It is clear from the table that, under a linear tax, the optimal capital tax is zero and the optimal labor tax is higher than those in progressive taxes. The welfare gain of a tax reform to this linear optimal tax mix is lower than those of progressive taxes. As a result, the progressive tax schedule in our model is justifiable.

Our results thus indicate that if the income tax rate schedule is sufficiently progressive, it is optimal to tax capital income with the average tax rate increasing in the degree of income tax rate progressivity. Moreover, with a larger degree of income tax rate progressivity, a tax reform from the current income tax code to the optimal tax gives a larger welfare gain.

Is the welfare gain of a tax reform toward the optimal income tax under progressive income tax rate schedule larger than that toward the optimal income tax under linear income tax rate schedules? When the income tax schedule is linear, the degree of income tax rate progressivity is decreased to zero and this is the case studied by Chamley (1986). In this case, the optimal capital tax is zero and a tax reform from the current income tax code to the optimal tax gives a welfare gain of 1.81% in terms of changes in consumption equivalence. See the bottom row in Table 1. Such a welfare gain is smaller than those in cases with positive optimal capital taxes when the degree of income tax progressivity is larger than or equal to 0.23.

Is the required threshold degree of income tax rate progressivity 0.23 too high? Recently, using the data from the Internal Revenue Service, the U.S. Census Bureau, and the Bureau of Economic Analysis, Mathews (2014) constructed the degree of federal income tax rate progressivity in the US over the period 1929-2009. He constructed annual tax concentration curves and income concentration curves with respect to income, which are like the well-known Lorenz curve. Based on the measure proposed by Suits (1977), the degree of income tax progressivity is calculated as the ratio of the area between the income concentration curve and the tax concentration to the area below the income concentration curve. While the constructed degrees of income tax rate progressivity vary over the years, the median degree is 0.416 in the period under study. Earlier, Li and Sarte (2004) used Individual Income Tax Returns publications of the Internal Revenue Service and pinned down the degree of the income tax rate progressivity. They found the degree of the income tax rate progressivity at 0.75. These two values indicate that the degree of income tax rate progressivity in the US is above the threshold value 0.23. This thus indicates that it is optimal to tax capital income.

4 Conclusion

Most countries in the world adopt a progressive income tax rate system and tax capital income. This article shows that in a representative-agent model with a progressive income tax rate system, the optimal capital income tax is positive in the long run. The result is in a sharp contrast to the zero long-run capital income taxation of Chamley and Judd in a model with a linear income tax rate system. The result also provides a rationale for the positive capital income taxes under a system of graduated marginal income tax rates adopted in most developed countries since the second half of the 19^{th} century.

The result emerges because a progressive income tax rate creates tensions from the social perspective between the gain in utility due to increases in capital income tax revenues and the loss in utility owing to decreases in post-tax returns to capital. We show that with a sufficiently progressive income tax rate schedule, if the capital tax rate is zero, the gain in utility from increases in capital income tax revenues would be smaller than the loss in utility from decreases in post-tax returns to capital. As a result, the level of capital chosen by the Ramsey planner is smaller than the level chosen by the household. There is thus an over-accumulation of capital from the social perspective. An increase in the capital tax rate would increase the gain in utility from increases in capital income tax revenues and decrease the loss in utility from decreases in post-tax returns to capital. Therefore, it is optimal to tax capital income.

By calibrating our model to the US economy, we find a large welfare gain of a tax reform toward the optimal income tax under a progressive income tax system. Moreover, the welfare gain of a tax reform toward the optimal income tax under a progressive income tax system is larger than that of a zero capital tax under a linear income tax rate system.

Mathematical Appendix

1. Derivation of the ramsey planner's problem

Let the capital income tax revenue be denoted by $T_k(rk) \equiv \tau_k(rk)rk$, the first-order conditions for c_t, k_{t+1}, b_{t+1}, and l_t for the Ramsey planner's problem in (8) are

$$u_{1t} + (\mu_{1t-1}R_t - \mu_{1t})\, u_{11t} + \mu_{2t}\left(u_{11t}\left(1 - T'_{lt}\right) w_t - u_{21t}\right) - \theta_t = 0, \tag{A1}$$

$$\beta\Big[\theta_{t+1}\,(f_{1t+1} - \delta + 1) + \Psi_{t+1}\,(f_{1t+1} - \delta - \tilde{r}_{t+1}) + (\Psi_{t+1}b_{t+1} - \mu_{1t}u_{1t+1})\, T''_{kt+1}r^2_{t+1}\Big] - \theta_t = 0, \tag{A2}$$

$$\Psi_t - \beta\Psi_{t+1}R_{t+1} = 0, \tag{A3}$$

$$-u_{2t} + \theta_t f_{2t} + \Psi_t\left[f_{2t} - \left(1 - T'_{lt}\right) w_t\right] - (\mu_{1t-1}R_t - \mu_{1t})\, u_{12t} - \mu_{2t}\left[u_{12t}\left(1 - T'_{lt}\right) w_t + u_{1t}T''_{lt}w_t^2 - u_{22t}\right] = 0. \tag{A4}$$

Moreover, we rewrite household's best responses as follows.

$$u_{1t} = \beta u_{1t+1}\left[1 + \left(1 - T'_{kt+1}\right) r_{t+1}\right], \tag{A5}$$

$$u_{2t} = u_{1t}\left(1 - T'_{lt}\right) w_t. \tag{A6}$$

Notice that $R_{t+1} = 1 + (1 - T'_{kt+1})r_{t+1}$. Thus, (A3) and (A5) imply that the shadow price of the government budget equals the marginal utility of consumption as follows.

$$\Psi_t = u_{1t}.$$

In the steady state, these conditions and the constraints give

$$\theta = u_1 + \mu_1 u_{11}(R - 1) + \mu_2\left(u_{11}\left(1 - T'_l\right) f_2 - u_{21}\right), \tag{A7}$$

$$\theta\left(\frac{1}{\beta} - 1 - \tilde{r}\right) = (\theta + \Psi)(r - \tilde{r}) + (\Psi b - \mu_1 u_1)T''_k r^2, \tag{A8}$$

$$\beta R = 1, \tag{A9}$$

$$u_2 = \theta f_2 + \Psi T'_l f_2 - \mu_1 u_{12}(R - 1) - \mu_2\left[u_{12}\left(1 - T'_l\right) f_2 + u_1 T''_l f_2^2 - u_{22}\right]. \tag{A10}$$

$$\Psi = u_1, \tag{A11}$$

$$c = f(k, l) - \delta k - g, \tag{A12}$$

$$T_l + T_k = g + (R - 1)b, \tag{A13}$$

$$u_2 = u_1 \left(1 - T_l'\right) f_2, \tag{A14}$$

where (A7), (A8), (A9), (A10) and (A11) come from (A1), (A2), (A3), (A4), (A6), respectively. (A12) comes from the market clearance condition, (A13) from the government budget constraint and (A14) from the representative household's first-order condition in the steady state.

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累進稅制下無窮生命模型的最適資本所得稅

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在線性所得稅制的無窮生命代表性個人模型, Chamley (1986) 和 Judd (1985) 已經證明長期的最適資本稅爲零。本篇研究一個除了累進稅制外其餘設定都相同的模型, 結果如下。首先, 累進稅制下的長期最適資本所得稅爲正。其次, 累進稅制下由當前稅率改制爲正最適所得稅率的福利利得大於線性稅制下改制爲零資本所得稅率的福利利得。我們的發現支持自19世紀後期之後, 採行累進稅制的先進國家課徵資本稅。

關鍵詞: 無窮生命模型, 最適資本所得稅, 累進稅制
JEL 分類代號: E1, E6, H2