Report on *Attention Please!*

Yuet Lyu (D10323010)

1 What is the question?

How does the manipulation of attention towards an item affects the Decision Maker (DM)'s demand?

- Compared with the baseline strategy, How will the attention allocated differently with the manipulated strategy where the target item is focused in the initial period?
- How will the different attention allocations affect DM's demands?

本文探討對決策者（DM）注意力的操控 如何影響DM 的需求：

- 如果 DM 的策略中，某「目標商品」在初始時刻得到關注，那 DM 的獲得的訊息會有怎樣的變化？
- 這種訊息獲得的改變，怎樣影響 DM 的需求？

2 Why should we care?

在這個注意力稀缺而選擇過剩的時代，人們不再像傳統的經濟學理論中預測的那樣：從有限的商品集合中選擇可以最大化效用的商品；而是：因精力、時間有限，在商品集中做選擇的時候，只考慮有資訊的子集。

所以，經濟學家關心注意力 怎樣影響、以及透過何種機制影響人們的需求。

- **A real world application**: 商家對某商品的廣告、店家在菜單上對「推薦菜品」的標識、網購平台上推薦商品的排序（按照閱讀習慣的第一個），都是本文中提到的「目標商品」。商家對消費者注意力的操縱時時刻刻都在改變消費者的需求與決策時間，正如本文描述的過程一樣。

3 What is the author's answer?

- Manipulation of attention increases demand and decreases the time to decision in favor of the target item, even if the target item is worse than the other items and the outside option.
- 操控 DM 的注意力使其在開始時期關注目標商品，會增加其對目標商品的需求；縮短決策時間。

4 How did the author get there?

Different from past literature, this paper identifies a mechanism through which grabbing attention increases demand without influencing preferences or changing the information available to the DM:

（作者先介紹簡化的 2 商品模型，再推廣至多商品模型，但直觀不變。爲便於同學理解，以下詳細介紹 2 商品模型，且嘗試用中文解釋各定義、式子、定理的直觀說明。）

4.1 Assumptions

1. Stationary & *independence of irrelevant alternatives* (IIA):
   - Allow the learning about the target item to be independent of the learning among the remaining items so that the model could reduce the problem to one with two items.
   - 上述假設保證了對目標商品的訊息獲取不受到其他商品的影響，使模型可以被簡化爲兩個商品。
2. Presence of an outside option:
   - Ensure DM's stopping rule to be a form of satisficing behavior.
   - Without outside option, the DM could choose by a process of elimination rather than approval, and choose the last remaining item with little knowledge of its value.
   - 如果 DM 必須選擇商品集中的一個，那麼她可能使用排除法，選擇一個不了解的商品。一個適中的「外部選項」促使 DM 使用「滿意即可策略」，避免上述情形發生。

4.2 Model: Setting

In each period $t = 0, 1, \cdots$, a decision-maker (DM):

- faces two items $j \in \{1, 2\}$, with values $v^j \in \{-1, 1\}$ (可能是好商品，或壞商品)
- focus on one item $\iota_t \in \{1, 2\}$, and see a signal $x_t \in \{-1, 1\}$ about value about ι_t;
  - $\Pr(x_t = 1 | v^{\iota_t} = 1) = \lambda = \Pr(x_t = -1 | v^{\iota_t} = -1)$ where $\lambda > 1/2$.
  - signal x_t 與商品的 value v^{ι_t} 相等的機率大於 50%，這代表 signal 是有參考價值的。

Then, we write down the log-likelihood ratio (LLR) p_t^j comparing $v^j = 1$ to $v^j = -1$, after adding up all signals from period 1 to period t:

$$p_t^j = \sum_{s<t:\iota_s=j} \log \frac{\Pr(x_s|v^j = 1)}{\Pr(x_s|v^i = -1)} = \log \frac{\lambda}{1-\lambda} \sum_{s<t:\iota_s=j} x_s \tag{1}$$

- p_t^j 越大，代表在第 t 期，看到 1-t 期所有 signal 的 DM，越相信 j 商品的價值爲 1（是好商品）。

And mapping the log-likelihood ratio pair $\mathbf{p} = (p_t^1, p_t^2)$ to the final choice, one of the two items$\{1, 2\}$, as our *attention strategy*, $\alpha : \mathbb{R}^2 \to \{1, 2\}$, which specifies focus $\iota_t = \alpha(\mathbf{p}_t)$.

- 關注策略 (attention strategy)：DM 在對價值不同的belief 下，選擇關注的商品，可以被函數 α 描述。

4.3 Model: Manipulation of Attention

This paper compares *baseline strategy* β with a *manipulated strategy* μ constructed from β.

$$\mu(\mathbf{p}, t) = \begin{cases} 1 & \text{if } t = 0, \\ \beta(\mathbf{p}) & \text{if } t > 0. \end{cases}$$

And denote *cumulative focus* on item 1 as $k_t = |\{s < t : \beta(\mathbf{p_s}) = 1\}|$ and $\hat{k_t} = |\{s < t : \mu(\mathbf{p_s}) = 1\}|$.

- Manipulated Strategy μ 和 Baseline Strategy β 間的不同：在初始階段是否一定關注「目標商品」。
- $k_t(\hat{k_t})$：在 0-t 期間，給商品 1 的關注次數。$k_t(\hat{k_t})$ 越大，代表 DM 越關注商品 1。

Then we have the **Proposition 1**: For each $t \geq 1$ and all pairs of values $\mathbf{v}$, $\hat{k_t}$ (weakly) first-order stochastically dominates k_t.

- 使用 Manipulated Strategy μ 的 DM 比 Baseline β 的 DM 更加關注商品 1。(Manipulation 有效！)

4.4 Model: From Attention to Demand

1. One-shot choice:
   - DM uses satisficing rules/滿意即可策略 (Thresholds: $\underline{p} < 0 < \overline{p}$)
     - Chooses j when collected enough evidence that j is of high value, with stopping regions $F^j = \{\mathbf{p} : p^j \geq \overline{p}\}$. (見好就收！)
     - Chooses neither (chooses the outside option) when collected enough evidence that both items are of low value, with stopping regions $F^{oo} = \{\mathbf{p} : p^j \leq \underline{p} \text{for} j = 1, 2\}$. (太差，算了。)
   - Learning stops in period $\tau = \min\{t : \mathbf{p}_t \in F\}$, $\tau^j = \tau$ if item j is chosen and $\tau^j = \infty$ otherwise.
   - Define *interim demand* as the probability that the DM stops and choose j with values $\mathbf{v}$: ($P_\alpha^{\mathbf{v}}$ is the joint stochastic process of LLRs and focus of attention)

   $$D^j(\mathbf{v}; \alpha) = P_\alpha^{\mathbf{v}}(\mathbf{p}_\tau \in F^j) \tag{2}$$

   - τ 爲決策時間，$D^j(\mathbf{v}; \alpha)$ 爲給定價值時，在策略 α 下選擇 j 的機率（對 j 的需求）。

   Then we have the **Proposition 2**: Suppose that the baseline attention strategy β is non-wasteful. For all pairs of values $v \in \{0, 1\}^2$, manipulating attention toward item 1 in the first period:

   - (weakly) increases the demand for item 1 and decreases the demand for item 2: $D^1(\mathbf{v}; \mu) \geq D^1(\mathbf{v}; \beta)$; $D^2(\mathbf{v}; \mu) \leq D^2(\mathbf{v}; \beta)$;
   - accelerates the choice of item 1 and decelerates the choice of item 2: $\tau^1(\beta)$ first-order stochastically dominates $\tau^1(\mu)$ and $\tau^2(\mu)$ first-order stochastically dominates $\tau^2(\beta)$.
   - 操控 DM 注意目標商品，可以增加其對目標商品的需求，降低對另一個的需求；加快其對目標商品的決策時間，減慢對另一個的決策時間。

2. Repeated choice:

   Manipulation of attention affects choices in bandit problems similar to the one-shot case.

4.5 Model: extension to general

This paper then extends the setting to allow for more than two items, general signal structures and stochastic attention strategies. Then, they have **Proposition 3** generalizing Proposition 1 and **Proposition 4** generalizing Proposition 2 where the intuition is similar to the previous model.

Then, this paper drives **Theorem 1 (Attention Theorem)** with *coupling construction* in Section 4, and also provides a **counterexample** of Proposition 4 when the attention strategy does not satisfy IIA. Also, they provide **the fastest strategy** in Section 4.3. In addition, this paper also describes the choice by elimination and approval when there is no outside option in Section 4.4 and explains the general multi-armed bandits problem in Section 5.

Notably, **Proposition 7** conclude the manipulation effect $e^i(\mathbf{p})$ in a general case:

The manipulation effect $e^i(\mathbf{p})$ is

- positive (i.e., nonzero) whenever manipulation affects attention (i.e. when $p^i \leq p^{-i}$),
- decreasing in p^{-i} on $\{p^i[+], \cdots, \overline{p}\}$,
- increasing in p^i on $\{\underline{p}, p[+], \cdots, p^{-i}[-]\}$,
- nonvanishing as $p \to 0$ and $p \to 1$ (in the region where the effect is positive).
- 對注意力的操縱給需求帶來的影響，在不同的 belief 下的變化。請注意，無論如何變化，注意力的操縱對需求的影響是恒正的，意味著無論如何，更多的注意力總是帶來更高的需求。

5 What are your comments or thoughts?

It is beautiful but really complicated!!! I spent so much time reading this paper, but I still did not get all the author’s information.

My interests lie in experimental economics, so I was thinking maybe we could test this theory paper in the lab. Through the eye-tracker, we can track the attention of subjects and verify both the result and the mechanism of attention manipulation.

6 Notation Table

j	items
v^j	value of items
ι_t	focused items in t-period
x_t	signals in t^{th}-period
λ	the probability that the signal equals value
p_t^j	log-likelihood ratio comparing $v_j = 1$ to $v_j = -1$ given signals from the beginning to period t.
$\mathbf{p}_t$	pair (p_t^1, p_t^2) in t^{th}-period
$\alpha(\cdot)$	attention strategy functions
β	baseline attention strategy functions
μ	manipulated attention strategy functions
$k_t(\hat{k}_t)$	cumulative focus on item 1 under $\beta(\mu)$
$\underline{p}, \overline{p}$	lower and upper thresholds
F^j	stop regions of $\mathbf{p}$: choosing j
F^{oo}	stop regions of $\mathbf{p}$: choosing the outside option
τ, τ_j	period when learning stops, when choosing item j
$P_\alpha^{\mathbf{v}}$	the joint stochastic process of LLRs and focus
$D^j(\mathbf{v}; \alpha)$	interim demand
$e^i(\mathbf{p})$	manipulation effect