Optimal Contracting with Subjective Evaluation Revisited

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Abstract

This paper revisits the property of the optimal contract of a static principal-agent relationship with subjective performance evaluations (MacLeod, 2003), but assuming that the principal and the agent receive correlated signals. It is shown that, when the agent is risk-averse and the principal's information is relatively more precise than the agent's, the optimal contract requires the agent's compensation to depend only on the principal's signal. When the agent is risk-neutral, we further fully characterize the optimal contract. It either requires the agent's compensation to depend only on the principal's signal, or the principal's wage cost to depend only on the agent's signal. That is, the truthful revelation constraints prevent full utilization of information. In either case, although one party's pay (or cost) depends only on the other party's signal, his own signal is nonetheless used as an instrument to prevent the other party from strategically misreporting.

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1 Introduction

The structure of the optimal labor contract, when the performance measure of the agent is non-verifiable, has been an area long attracting the attention of contract theory scholars. One issue which attracts particular interest is the case when the principal and the agent both observe signals that are correlated to the agent's performance.

When the principal's and the agent's signals are identical, the "relational contract" literature has shown that there exists a simple repeated game equilibrium strategy which implements efficient effort level (e.g., Bull, 1987; Levin, 2003).[1] More interesting, and perhaps more realistic, case is when they receive different, but correlated signals. In this case the optimal contract not only has to decide which signal is to be incorporated into the contract, but also how. Is the optimal contract dependent only on either the principal's, or only the agent's signal, or both? In the latter case, how to compromise the two signals if they conflict? Moreover, since the signals observed by both parties are soft in the sense that they are non-verifiable, how to ensure that they report the true value of the signals they receive?

A recent contribution by MacLeod (2003) adopts a static model to characterize the optimal contract under the case of " subjective performance measures" mentioned above. In his model, although the signals received by the principal and the agent are not verifiable, they can each send a verifiable message (indicating the signal each receives) so that the contract is a function of the message sent. The optimal contract must not only satisfy the usual incentive compatibility and individually rational constraints, but also the truthful revelation constraint in which both parties report the true values of the signals they receive. He shows that, since the messages are reported after the output is realized, truthful revelation entails money burning, in the sense that there must be some contingencies under which the principal's wage payment is strictly

[1] For excellent survey of the literature on the relationship contract, please see Malcomson (20)

greater than the wage received by the agent. The most important result in his paper is that, in the optimal contract, the agent's compensation does not depend on his own information. That is, the agent's compensation is totally dependent on how the principal reports the agent's performance to be. This result mainly comes from two crucial assumptions in the model. The first is that the principal is risk neutral while the agent is risk averse. The second is that the principal's signal is a sufficient statistics of that of the agent. Because of this, on the one hand, a compensation which depends on both principal's and agent's signals are no informational efficient than one depending on the principal's only, and only introduces additional risks to the agent's compensation on the other. Therefore, as the agent is risk-averse, a contract which depends on the messages of both the principal and the agent will be dominated by one which depends only on the principal's signal averaged over the agent's possible signals.[2] The first assumption is one usually adopted in the literature. The second, however, is a strong one.

In this paper, we reinvestigate the issue of the structure of the optimal contract, under the more general informational structure in which the principal and the agent receive different, but correlated, signals. Under the assumption of binary signals, we show that MacLeod's (2003) main result holds true if the principal's signal is a sufficient statistics of that of the agent. That is, although the proof for this result has been incomplete in MacLeod (2003), his result is nonetheless valid in the binary signal case. In the general case when their signals are correlated, however, the agent's compensation does not always only depend on the principal's signal, even in the two-signal case. Specifically, we show that the structure of the optimal contract falls into two regimes. In the first, which parallels MacLeod's (2003) result, the agent's compensation depends on the principal's signal only, while the latter's wage cost depends on the signals of

[2] The proof for this result (Proposition 3 in MacLeod, 2003) is actually invalid. While the contract constructed in the proof of Proposition 3 satisfies the incentive compatibility and individually rational constraint, it does not satisfy the truthful revelation constraint for the principal. Moreover, we have found a counter-example against this result (see the discussion after Proposition 3). Therefore, the result in general is incorrect.

both parties. The second regime is symmetric to that of the first: the agent's compensation depends on the signals of both parties, while the principal's wage cost depends only on the agent's signal. The truthful revelation constraint is always biding: In either regime it is always the case that one party's information is not fully incorporated into the contract, and there is always one party whose benefit (or cost) is solely determined by the other's signal.

Our characterization of the optimal contract also implies that one of the determinants of which regime prevails is the relative precision of the principal's and the agent's signals. If the principal's (agent's) information is relatively more precise, then regime 1 (2) prevails.

1.1 Related Literature

Besides MacLeod (2003), there are several recent papers which also investigate the structure of the optimal contract when performance evaluation is subjective. Levin (2003) assumes that both the principal and the agent are risk-neutral, and they have an infinitely repeated relationship. In each period they observe a common, but non-contractible signal, regarding the agent's performance. He shows that the optimal contract can take a simple stationary form. Moreover, under the moral hazard case, the agent's wage in optimal contract only takes two values: high wage for high-range outcomes and low wage for low-range outcomes. His model differs from ours in that, first, the agent is risk- neutral while we allow risk-aversion. Second, in his model the principal and the agent receive identical signal, while in our model we allow for different, but correlated signals. Third, his model is a dynamic principal-agent relationship while ours is static.

Fuchs (2007) also proposes an infinite-horizon setting with risk-neutral principal and agent. His model differs from that of Levin (2003) mainly in that, in every stage, only the principal observes the value of output. Both effort level and output are binary. Wage is contingent on the message that the principal sends to the agent and the realization of a public randomization device. He shows that the optimal contract can be implemented by a "review contract" in which the agent is paid a constant wage without bonus, the principal never sends any message, and the incentive is solely provided by the threat of termination when, under the principal's eyes, the agent's performance over a specific length of period is sufficiently unfavorable.

In a model similar to that of Fuchs (2007), but allowing for continuous effort levels and for agent also receiving a signal related to output, Maestri (2012) compares the efficiency-wage equilibrium (which is exactly the optimal contract derived in Fuchs, 2007) and the bonus-payment equilibrium, in which the agent is motivated by a bonus payment, but can quit after disagreement between his self-appraisal and the principal's evaluation. He shows that the former can approximate efficient outcome as discount factor goes to one, while the payoffs of the latter are bounded away from the efficiency frontier.

Chan and Zheng (2011) consider a finite-horizon model in which both the principal and the agent are risk-neutral. Moreover, both the effort levels and the output are binary. There is a contractible performance measure, but the principal and the agent each receives a private binary signal correlated with output. They mainly show that the optimal contract is not stationary: given the same number of good evaluations, the agent is better rewarded under increasingly improving performance than deteriorating performance. Their model is very similar to ours, but the optimization problem is different: In their paper, the optimization problem is to minimize money burning, while in this study. the optimization problem is to minimize the cost.

2 The Model

A risk-neutral principal delegates a project to an agent who is risk-averse or risk-neutral. The agent chooses an effort level $e \in [0,1]$, which is his private information, and whose value determines the distribution of the output. The agent's utility function is $U(c,e) = u(c) - V(e)$, where $u(c)$ is the utility gained from compensation $c > 0$ and $V(e)$ is the disutility incurred by effort. Assume conventionally that $u' > 0$, $u'' \leq 0$, $V' > 0$, $V'' > 0$, and assume technically that $\lim_{e \downarrow 0} V'(e) = 0$, $\lim_{e \uparrow 1} V'(e) = \infty$. The agent's reservation utility is denoted by $\underline{U}$. In order to economize the use of notations, we set $\underline{U} = u(0) = V(0) = 0$.

Following MacLeod (2003), performance measure of the agent is assumed to be *subjective* in the sense as follows. The output is not observable, but the principal and the agent each observes a signal of the output, $t \in \{0,1\}$ and $s \in \{0,1\}$, respectively. The realization of the signal, (t,s) , depends on the value of the output, which in turn (as assumed above) is a random variable of the effort level, e. For convenience, we then assume that (t,s) is a function of e.[3] Specifically, let $\alpha_{ts}(e)$ be the joint probability of (t,s) being realized when the effort level is e. Also, $\alpha_{ts}(e)$ is strictly positive and continuously differentiable for all $t,s \in \{0,1\}$ and for all $e \in [0,1]$.

We assume that signals of the parties are positively correlated in the following sense:

Assumption 1. $\alpha_{11}(e)\alpha_{00}(e) > \alpha_{10}(e)\alpha_{01}(e)$ *for all* $e \in (0,1)$.

We will call $t = 1$ or $s = 1$ the good signal, and $t = 0$ or $s = 0$ the bad. Since a higher effort level tends to yield a better outcome of the project, we also assume

Assumption 2. $\alpha'_{11}(e) + \alpha'_{10}(e) > 0$ *and* $\alpha'_{11}(e) + \alpha'_{01}(e) > 0$ *for all* $e \in (0,1)$.

[3] In MacLeod (2003), the output could be either success or failure, and e stands for the probability of the success being realized. In sum, the outcome of the output is binary in MacLeod (2003), while the outcome in this paper is more general, e.g. it can be continuously distributed over a region, though the signals are binary.

Assumption 2 essentially says that, when the agent's effort level increases, both the principal and the agent will be more likely to receive the high signal. We also assume that, given the other party's signal, the (conditional) distribution of each party's signal satisfies the usual Monotone Likelihood Ratio Condition (MLRC):

Assumption 3. *For all* $e \in (0,1)$, $\alpha'_{1s}(e)/\alpha_{1s}(e) \geq \alpha'_{0s}(e)/\alpha_{0s}(e)$ *for* $s \in \{0,1\}$ *and* $\alpha'_{t1}(e)/\alpha_{t1}(e) \geq \alpha'_{t0}(e)/\alpha_{t0}(e)$ *for* $t \in \{0,1\}$.

It is instructive to compare the informational structure specified here to that in MacLeod(2003). In his model, the principal receives a signal, conditional on output. The agent's signal, however, is conditional on the principal's signal, rather than output. Specifically, every signal the principal receives induces a (different) probability distribution for the signals that the agent might receive. Under this specification, the principal's signal is actually a sufficient statistic of that of agent. In other words, given signal that the principal observes, the signal that the agent observes has no additional informational value. This specification plays a crucial role in his characterization of the optional contract.

Note that Assumption 3 implies that, $\alpha'_{11}(e)/\alpha_{11}(e) \geq \alpha'_{ts}(e)/\alpha_{ts}(e) \geq \alpha'_{00}(e)/\alpha_{00}(e)$, for $t, s \in \{0,1\}$, which in turn implies the following fact:

Fact 1. *For all* $e \in (0,1)$, $\alpha'_{11}(e) > 0 > \alpha'_{00}(e)$.

Proof. Suppose, on the contrary, that $\alpha'_{00}(e) \geq 0$. Then from the inequality immediately above we know that $\alpha'_{10}(e) \geq 0, \alpha'_{01}(e) \geq 0$, and $\alpha'_{11}(e) \geq 0$. Since $\sum_{t,s\in\{0,1\}} \alpha_{ts}(e) = 1$ for all $e \in (0,1)$, we have $\sum_{t,s\in\{0,1\}} \alpha'_{ts}(e) = 0$ for all $e \in (0,1)$. This implies $\alpha'_{ts}(e) = 0$ for all $t, s \in \{0,1\}$, which contradicts Assumption 2. Similar argument yields $\alpha'_{11}(e) > 0$. □

Since neither output nor effort is observable, and since signals are positively correlated with the effort level, the contract between the principal and the agent must depend on the signals to provide incentives. That is, the payment to the agent is a function of t and s. As argued in MacLeod (2003) and Fuchs (2007), in the case when performance measure is subjective, there might involve money burning in the optimal labor contract. That is, the principal's payment might not be equal to the agent's wage in all contingencies. In particular, there must be at least one contingency in which the principal's payment is strictly greater than the agent's wage. Therefore, the form of the contract is

$$\{w_{ts}, c_{ts}\}_{t,s\in\{0,1\}},$$

where w_{ts} and c_{ts} are respectively the principal's payment and the agent's wage (or compensation) when the principal's signal is t and that of the agent is s. We assume limited liability so that $w_{ts} \geq c_{ts} \geq 0$ for all t and s.

As usually assumed in the literature, the principal designs a take-it-or-leave-it contract to the agent to maximize his profit. Since the principal is risk-neutral, we can decompose the principal's problem into two steps, with the first step being a cost-minimization problem (Grossman and Hart, 1983):

$$C^*(e) \equiv \min_{\{w_{ts},c_{ts}\}_{t,s\in\{0,1\}}} C = \sum_{t,s\in\{0,1\}} w_{ts}\alpha_{ts}(e), \tag{P}$$

subject to

$$\sum_{t,s\in\{0,1\}} u(c_{ts})\alpha_{ts}(e) - V(e) \geq \underline{U}, \tag{IR}$$

$$e \in \arg\max_{\tilde{e}\in[0,1]} \sum_{t,s\in\{0,1\}} u(c_{ts})\alpha_{ts}(\tilde{e}) - V(\tilde{e}), \tag{IC}$$

$$w_{ts} \geq c_{ts} \geq 0, \forall t, s \in \{0,1\}, \tag{LL}$$

where (IR) and (IC) are respectively the usual individual rationality and incentive compatibility constraints, and (LL) is the limited liability constraint. Since the values of the signals are private information, two additional constraints must be satisfied to include the principal and agent to report their signals truthfully:

$$\sum_{s\in\{0,1\}} w_{ts}\alpha_{ts}(e) \le \sum_{s\in\{0,1\}} w_{\widetilde{t}s}\alpha_{ts}(e), \forall t, \widetilde{t} \in \{0,1\}, \tag{TRP}$$

$$\sum_{t\in\{0,1\}} u(c_{ts})\alpha_{ts}(e) \ge \sum_{t\in\{0,1\}} u(c_{t\widetilde{s}})\alpha_{ts}(e), \forall s, \widetilde{s} \in \{0,1\}, \tag{TRA}$$

where (TRP) and (TRA) are the truthful revelation constraints for the principal and the agent, respectively.

For $e \in (0,1)$, (IC) implies the following first-order condition:[4]

$$\begin{aligned} V'(e) =& \sum_{t,s\in\{0,1\}} u(c_{ts})\alpha'_{ts}(e) \\ =&[u(c_{11}) - u(c_{10})]\alpha'_{11}(e) + [u(c_{00}) - u(c_{01})]\alpha'_{00}(e) + [u(c_{10}) - u(c_{01})][\alpha'_{11}(e) + \alpha'_{10}(e)], \end{aligned} \tag{ICF}$$

where the second equality comes from the fact that $\sum_{t,s\in\{0,1\}} \alpha'_{ts}(e) = 0$. By setting $\underline{U} = u(0) = V(0) = 0$, we ensure that the optimal contract which implements any effort level $e \in (0,1)$ must make (IR) slack. This is because, given $e > 0$, (ICF) implies that $c_{ts} > 0$ for some t, s. If (IR) were binding, the agent can increase his utility by switching to $e = 0$. In that case his expected utility becomes $\mathrm{E}[u(c_{ts})] - V(0) > 0 = \underline{U}$.

The two truthful revelation constraints have strong implications on the relative value between w_{ts}'s, and between c_{ts}'s.

Lemma 1. *(i) Either both $w_{10} = w_{00}$ and $w_{01} = w_{11}$ or both $w_{10} > w_{00}$ and $w_{01} > w_{11}$; (ii) Either both $c_{00} = c_{01}$ and $c_{11} = c_{10}$ or both $c_{00} > c_{01}$ and $c_{11} > c_{10}$.*

[4] Our assumptions that $V'(e) = \infty$ rules out implementing $e = 1$. Also, it is trivial for the principal to implement $e = 0$.

Proof. Constraint (TRP) implies that

$$w_{11}\alpha_{11}(e) + w_{10}\alpha_{10}(e) \leq w_{01}\alpha_{11}(e) + w_{00}\alpha_{10}(e) \quad \text{and}$$
$$w_{01}\alpha_{01}(e) + w_{00}\alpha_{00}(e) \leq w_{11}\alpha_{01}(e) + w_{10}\alpha_{00}(e),$$

which can be rewritten as

$$(w_{10} - w_{00})\frac{\alpha_{10}(e)}{\alpha_{11}(e)} \leq w_{01} - w_{11} \leq (w_{10} - w_{00})\frac{\alpha_{00}(e)}{\alpha_{01}(e)}. \tag{1}$$

The two inequalities of (1) imply (i) of Lemma 1 because $\alpha_{10}(e)/\alpha_{11}(e) < \alpha_{00}(e)/\alpha_{01}(e)$ by Assumption 1.

Constraint (TRA) implies that

$$u(c_{11})\alpha_{11}(e) + u(c_{01})\alpha_{01}(e) \geq u(c_{10})\alpha_{11}(e) + u(c_{00})\alpha_{01}(e),$$
$$u(c_{10})\alpha_{10}(e) + u(c_{00})\alpha_{00}(e) \geq u(c_{11})\alpha_{10}(e) + u(c_{01})\alpha_{00}(e),$$

which can be rewritten as

$$(u(c_{00}) - u(c_{01}))\frac{\alpha_{01}(e)}{\alpha_{11}(e)} \leq u(c_{11}) - u(c_{10}) \leq (u(c_{00}) - u(c_{01}))\frac{\alpha_{00}(e)}{\alpha_{10}(e)}. \tag{2}$$

The two inequalities of (2) imply (ii) of Lemma 1 because $\alpha_{01}(e)/\alpha_{11}(e) < \alpha_{00}(e)/\alpha_{10}(e)$ by Assumption 1.

□

Lemma 1 indicates that the principal's payment is either independent of his own signal, or pays higher when his report contradicts with the agent's. In particular, when the agent reports good signal ($s = 1$), the principal pays *higher* when he reports bad signal ($t = 0$) than when good ($t = 1$). That means when the agent reports good signal, the principal truthful revelation constraint must be binding in order to prevent him from mis-reporting. Similarly, the agent's compensation is either independent of his own report, or is strictly lower when his report contradicts with the principal. Lemma 1 immediately implies that money burning must occur in some contingency:

Proposition 1. *There exists a contingency, (t,s), such that* $w_{ts} > c_{ts}$*.*

Proof. Suppose to contrary, that, $w_{ts} = c_{ts}$ for all $t, s \in \{0, 1\}$. Lemma 1 then implies that $c_{01} = w_{01} \geq w_{11} = c_{11} \geq c_{10} = w_{10} \geq w_{00} = c_{00} \geq c_{01}$, which in turn yields that $c_{00} = c_{01} = c_{10} = c_{11}$. Then, (ICF) gives $V'(e) = \sum_{t,s\in\{0,1\}} u(c_{ts})\alpha'_{ts}(e) = u(c_{00}) \sum_{t,s\in\{0,1\}} \alpha'_{ts}(e) = 0$, contradicting our assumption that $V'(e) > 0$. □

The fact that agent's wage cannot equal to the principal's payment is also shown in MacLeod (2003). His reasoning is that, if w_{ts} and c_{ts} are equal in all contingencies, then at the stage when the principal and agent report their signals to determine the payoff of each, they are playing a constant-sum game. Since a constant game has a unique value, the agent's expected payoff is the same regardless of effort levels. This implies that only the lowest possible effort level can be implemented in any contract that satisfies (TRA) and (TRP). Therefore, in order to implement higher effort levels, the constant-sum nature must be broken, so that the principal's payment does not equal the agent's wage under at least one contingency.

This argument is

There are several problems with this argument. First, since the agent is risk averse, the game he played with the principal is not of constant-sum. Second, even though it is constant-sum, different effort levels will result in different (or at least different expectations of) constant-sum games. Therefore, it is still possible to implement an effort level higher than the minimum, as it implies a higher value of constant to split between them. As we have shown in Proposition 1, the crucial ingredient for this result is actually the statistical correlation between their signals. If the parties' signals are uncorrelated, i.e., if $\alpha_{11}(e)\alpha_{00}(e) = \alpha_{10}(e)\alpha_{01}(e)$, the argument based on 1 and 2 in the proof of Lemma 1 is invalid, and the optimal contract might involve no money burning. A numerical example is given as follows. Suppose $u(c) = c$, $V' = 1$, $(\alpha_{00}, \alpha_{01}, \alpha_{10}, \alpha_{11}) = (0.25, 0.25, 0.25, 0.25)$, and $(\alpha'_{00}, \alpha'_{01}, \alpha'_{10}, \alpha'_{11}) = (-0.101, 0.09, -0.1, 0.111)$. Then the unique optimal solution is $(w_{00}, w_{01}, w_{10}, w_{11}) = (c_{00}, c_{01}, c_{10}, c_{11}) = (100, 0, 0, 100)$.[5]

3 The Characterization of the Optimal Contract

In this section, we solve for the optimal contract which implements any given $e \in (0, 1)$. For simplicity, we will denote $u_{ij} \equiv u(c_{ij})$, for $i, j = 1, 2$. The following proposition fully characterizes the optimal contract and shows that its form depends on the marginal cost of rewarding the agent under the worst signals (i.e., $t = 0$ and $s = 0$). Lemmas 2 to 5 in the appendix shows that the minimum cost of implementing any effort level $c^*(e)$, can be written as a function of a single variable $u_{00} \equiv u(c_{00})$, i.e., $c^*(e) = c^*(u_{00})$. The marginal cost to the principal, denoted as $MC\,(u_{00})$, determines the form of the optimal contract.

Proposition 2. *The optimal contract falls into one of the three types:*

Type 1. If $MC(0) \geq 0$, *then* $w_{01} > w_{11} = c_{11} = w_{10} = c_{10} > w_{00} = c_{00} = c_{01} = 0$;

Type 2. If $MC(\bar{u}) \leq 0$, *then* $w_{01} = w_{11} = c_{11} > w_{10} = c_{10} = w_{00} = c_{00} > c_{01} = 0$;

Type 3. If $MC(0) < 0 < MC(\bar{u})$, *then* $w_{01} > w_{11} = c_{11} > w_{10} = c_{10} > w_{00} = c_{00} > c_{01} = 0$;

where

$$MC^*(u_{00}) = \frac{1}{\alpha'_{11} + \alpha'_{10}} \left[\begin{array}{c} (\alpha_{11} + \alpha_{01}) \left(\frac{\alpha_{00}}{\alpha_{10}} \alpha'_{10} - \alpha'_{00} \right) \tilde{u}'(u_{11}) \\ - \left(\alpha_{10} + \frac{\alpha_{10}}{\alpha_{11}} \alpha_{01} \right) \left(\alpha'_{00} + \frac{\alpha_{00}}{\alpha_{10}} \alpha'_{11} \right) \tilde{u}'(u_{10}) \end{array} \right] + (\alpha_{00} - \frac{\alpha_{10}}{\alpha_{11}} \alpha_{01}) \tilde{u}'(u_{00}),$$

[5] Note that the specification of signals in MacLeod (2003) does not preclude the principal's and the agent's signals from being uncorrelated.

$\tilde{u}(\cdot)$ is the inverse function of $u(\cdot)$, $u_{11} = \frac{V' + \left(\frac{\alpha_{00}}{\alpha_{10}}\alpha'_{10} - \alpha'_{00}\right)u_{00}}{\alpha'_{11} + \alpha'_{10}}$, $u_{10} = \frac{V' - \left(\alpha'_{00} + \frac{\alpha_{00}}{\alpha_{10}}\alpha'_{11}\right)u_{00}}{\alpha'_{11} + \alpha'_{10}}$, and $\bar{u} = \frac{V'}{\frac{\alpha_{00}}{\alpha_{10}}\alpha'_{11} - \alpha'_{01}} > 0$.

Proof. See Appendix A. □

For type 1 contract, $c_{11} = c_{10}$ and $c_{00} = c_{01}$. That is, the agent's compensation does not depend on his own report. This is exactly the main result (Proposition 3) in MacLeod (2003). As mentioned in footnote 1, his proof is also invalid. The contract constructed in his proof has not been shown to satisfy the principal's truthful revelation constraint. Moreover, his argument critically depends on the assumption that the principal's signal is a sufficient statistic of the agent's. In Section 3.1, we will show that even keeping this assumption, Proposition 3 in MacLeod (2003) is correct for the binary-signal case, but no longer holds truth for the 3-signal case. Proposition 2 also shows that, under the more general informational structure, there exists two other types of optimal contracts. For type 2 contract, since $w_{01} = w_{11}$ and $w_{10} = w_{00}$, the principal's signal does not influence her own pay. This is the opposite to Regime 1. Type 3 contract is the case when the signals of both the principal and the agent affect their own utilities.

When the agent is risk-neutral, $\tilde{u}'(\cdot)$, as well as the marginal cost $MC(\cdot)$, is constant. In that case, $MC(0) = MC(\bar{u})$. By Proposition 2, type 3 contract is never optimal, and the optimal contract form depends only on the sign of $MC(0)$. We immediately have the following corollary.

Corollary 1. *Assume that the agent is risk neutral. The optimal contract is of type 1 (2) if and only if $MC(0) \geq (\leq) 0$.*

Corollary 1 also implies that, when the agent is risk-neutral, either his signal does not affect his own consumption, or the principal's signal does not affect her own payment. In other words, when the principal and the agent are both risk neutral, one party (whether it is the principal or the agent depends on the sign of $MC(0)$) always has the authority over the other's payoff.[6]

The intuition behind Proposition 2 is as follows. In the absence of the truthful-revelation constraints (TRp) and (TRa), the optimal contract would be $w_{ts} = c_{ts}$ for all $t, s \in \{0, 1\}$, and that $c_{11} > c_{10} > c_{00}$ and $c_{11} > c_{01} > c_{00}$.[7] In other words, both the principal's and the agent's signals (if truthfully reported) are fully utilized to provide incentives for the agent. In this case, however, both parties have incentives to misreport: the principal will report $t = 0$ when she observes $t = 1$, and the agent will report $s = 1$ when he observes $s = 0$.

In order to exploit the agent's information while at the same time prevents him from inflating his signal, there must be at least one contingency under which the agent's wage is lower when he reports $s = 1$ than $s = 0$. However, since there is only two signals, this must occur only in exactly one contingency, otherwise reporting $s = 0$ will be the agent's dominant strategy. Because of the correlation of signals, $s = 1$ will be more likely to be a misreport from the agent when $t = 1$ than when $t = 0$. Therefore, it should be the case that $c_{11} > c_{10}$ and $c_{00} > c_{01}$. Since rewarding the agent in this fashion is adversarial to incentive provision (because $\alpha'_{00} < 0$), this implies a conflict between the efficiency of incentive provision and the truthfulness of the agent's report. Similarly, in order to prevent the principal from mis-reporting, there must be at least one contingency under which she pays more when she reports $t = 0$ than $t = 1$. Because of the correlation of signal, $t = 0$ will be more likely to be a misreport when $s = 1$. Therefore, $w_{01} > w_{11}$. This is exactly the case for the Type 3 contract.

Note that unlike the case for the agent, inducing the principal to truthfully report does

[6] It should be emphasized here that, even if the principal's signal is a sufficient statistic of the agent's, MacLeod's (2003) Proposition 3 is true only when there are two levels of output. When there are one more than two levels of output, the agent's wage can still depend on his own signal. See the counter-example following our proposition 3.

[7] By the MLRP, better outcome corresponds to higher wage.

not impare incentive provision. Note that when w_{ts}'s and c_{ts}'s are disentangled, the role of $\{w_{ts}\}$ is not to provide incentive ($\{c_{ts}\}$ does the job). Rather, w_{ts}'s are chosen to minimize the principal's cost, subject only to her trustful-revelation constraint and the requirement that $w_{ts} \geq c_{ts}$, the former of which results in money-burning under certain contingency (namely, $w_{01} > c_{01}$). The amount of money burnt, $w_{01} - c_{01}$, therefore measures the cost of truthful revelation constraint for the principal.

Type 3 contract, however, is not the only solution. There are also two possible corner solutions. Recall that $MC(0)$ is the marginal cost for the principal to increase the agent's utility of c_{00}, when $c_{00} = 0$. That means, although the principal can further decrease cost by decreasing the value of c_{00}, she cannot do it because c_{00} is already 0. Furthermore, since c_{01} cannot be greater than c_{00}, we have a corner solution where $c_{00} = c_{01}$. In this case, $c_{t1} = c_{t0}$ so that the agent's report does not affect his own consumption. As already mentioned, exploiting the agent's information forces the principal to reward the agent when the worst signals are received, and the magnitude of this reward (i.e., u_{00}) determines the degree of the exploitation (i.e., $u_{11} - u_{10}$) through (TRA). Proposition 2 essentially says that the form of the optimal contract depends on how worthy to put money on the worst signals. Putting money on the worst signals has three effects: i) reducing the efficiency of incentive provision, ii) increasing the degree of exploiting the agent's information, and iii) affecting the cost of inducing the principal to report truthfully—the amount of money burnt—through (TRP). These three effects together determine the marginal cost of u_{00}.

Because the marginal cost of u_{00}, $MC(u_{00})$, is increasing as proven in Proposition 2, when the marginal cost of u_{00} is high such that $MC(0) \geq 0$, it is not worthy to put any money on the worst signals. As a result, the optimal contract entails that $c_{00} = 0 = c_{01}$, and accordingly

$c_{10} = c_{11}$ by (TRA). Adjusting the corresponding w_{ts}'s, the optimal contract becomes

$$
\begin{array}{cccccccc}
 & & s & & & & s & \\
 & c_{00} & = & c_{01} & & w_{00} & < & w_{01} \\
t & \wedge & & \wedge & t & \wedge & & \vee \\
 & c_{10} & = & c_{11} & & w_{10} & = & w_{11}
\end{array},
$$

which is the contract of Regime 1.

When the marginal cost of u_{00} is low such that $MC\left(\bar{u}\right) \leq 0$, it is worthwhile to put money on the worst signals even to the extent that $c_{00} = c_{10}$. Since $c_{00} = c_{10}$ implies $w_{00} = w_{10}$, the contract specifies that $w_{01} = w_{11}$ by (TRP). The optimal contract thus takes the form:

$$
\begin{array}{cccccccc}
 & & s & & & & s & \\
 & c_{00} & > & c_{01} & & w_{00} & < & w_{01} \\
t & \shortparallel & & \wedge & t & \shortparallel & & \shortparallel \\
 & c_{10} & < & c_{11} & & w_{10} & < & w_{11}
\end{array},
$$

which is the contract of Regime 2.

There are two characteristics across the three regimes deserving closer attention. First, the agent's compensation is (weakly) increasing in the principal's report, while the principal's wage payment is (weakly) increasing in the agent's report. Second, Regime 1 and Regime 2 are symmetric in the following sense: in Regime 1 the agent's compensation solely depends on the principal's signal ($c^*_{11} = c^*_{10}$ and $c^*_{01} = c^*_{00}$), while in Regime 2 the principal's wage payment only relies on the agent's signal ($w^*_{11} = w^*_{01}$ and $w^*_{10} = w^*_{00}$). Regime 3 is a mixture of these two regimes.

3.1 The Influence of the Information Structure

In the following, we investigate how the prevalence of various regimes is influenced by the information structure. We divide this investigation into two parts: the effect of the disincentive power of the worst signal and then the effect of the relative informativeness of the parties' signals.

As discussed earlier, exploiting the agent's information calls for a positive compensation when both parties receive bad signals, which in turn reduces the agent's incentives to exert effort. The level of $|\alpha'_{00}|$ measures the disincentive power of putting money on the worst signal: the larger the $|\alpha'_{00}|$, the more reluctant the agent to work in anticipation of a reward in the worst state, and therefore the more costly to exploit the agent's information. Clearly, when $|\alpha'_{00}|$ increases from a low initial value, the optimal contract will transit from Regime 2 to 3, and then eventually to 1. The following corollary summarizes this result.

Corollary 2. *If $|\alpha'_{00}|$ and α'_{01} increase with the same amount so that α'_{10} and α'_{11} remain unchanged, then the optimal contract will transit from Regime 2 to 3, and then to 1 provided that the initial contract falls into Regime 2.*

Proof. Notice that $\sum_{t,s\in\{0,1\}} d\alpha'_{ts} = 0$, and we have restricted our attention to the case that $d\alpha'_{00} + d\alpha'_{01} = 0$ and $d\alpha'_{10} = d\alpha'_{11} = 0$. Thus we have

$$\begin{aligned} dMC\left(u_{00}\right) &= \sum_{t,s\in\{0,1\}} \frac{\partial MC\left(u_{00}\right)}{\partial \alpha'_{ts}} d\alpha'_{ts} \\ &= \frac{-1}{\alpha'_{11}+\alpha'_{10}} \left[\left(\alpha_{11}+\alpha_{01}\right) \tilde{u}'(u_{11}) + \left(\alpha_{10} + \frac{\alpha_{10}}{\alpha_{11}}\alpha_{01}\right) \tilde{u}'(u_{10}) \right] d\alpha'_{00} \\ &< 0, \end{aligned}$$

and the result follows by Proposition 2. □

We proceed to analyze the effect of the relative informativeness of the parties' signals. We begin with two extreme cases:

Definition 1. *t is a sufficient statistic for s with respect to e if $\alpha'_{t1}/\alpha_{t1} = \alpha'_{t0}/\alpha_{t0}$ for $t \in \{0,1\}$; s is a sufficient statistic for t with respect to e if $\alpha'_{1s}/\alpha_{1s} = \alpha'_{0s}/\alpha_{0s}$ for $s \in \{0,1\}$.*

t (s) being a sufficient statistic for s (t) means that the principal's (agent's) signal carries all the relevant information about the effort level, and the agent's (principal's) signal is non-informative in the sense that it adds nothing to the inference power (Hölmström, 1979). When the agent's signal is non-informative for the effort level, exploiting the agent's information does not worth its cost, and it should be expected that Regime 1 prevails. In contrast, when the agent's signal conveys all the relevant information about the effort level, the agent's information should be exploited to the greatest extent possible, and Regime 2 should prevail.

To measure the relative informativeness of the parties' signals, we parameterize their beliefs about the value of the agent's information. In particular, suppose that both parties commonly believe that t is a sufficient statistic for s with probability p, and s is a sufficient statistic for t with probability $1-p$. The parameter p thus measures the relative informativeness of the agent's (or the principal's) signal: the larger the p, the less (more) informative the agent's (the principal's) signal. When p approaches 1 from 0, we would expect that the optimal contract will transit from Regime 2 to 3, and then to 1. The following proposition verifies this intuition.

Proposition 3. *Assume that, with probability p, t is a sufficient statistic for s, while with probability $1-p$, s is a sufficient statistic for t , then there exists a lower cutoff $\underline{p} \in (0,1)$ and an upper cutoff $\bar{p} \in (\underline{p}, 1)$, such that for $0 \le p \le \underline{p}$, Regime 2 prevails; for $\underline{p} < p < \bar{p}$, Regime 3 prevails; and for $\bar{p} \le p \le 1$, Regime 1 prevails.*

Proof. See Appendix A. □

Proposition 3 shows that the main result (Proposition 3) in MacLeod (2003)—the agent's compensation does not depend on his own report when the principal's signal is a sufficient statistic of the agent's—is correct for binary-signal case. However, this result is incorrect for 3-signal case. For example, let $u(c) = c$, $V' = 1$, $\alpha = (0.0088, 0.0327, 0.0399, 0.0327, 0.1277, 0.1596, 0.0399, 0.1596, 0.399)$, and $\alpha' = (-0.11, 0.0308, 0.048, -0.41, 0.12, 0.1918, -0.5, 0.15, 0.4795)$, where $\alpha \equiv (\alpha_{00}, \alpha_{01}, \alpha_{02}, \alpha_{10}, \alpha_{11}, \alpha_{12}, \alpha_{20}, \alpha_{21}, \alpha_{22})$ and $\alpha' \equiv (\alpha'_{00}, \alpha'_{01}, \alpha'_{02}, \alpha'_{10}, \alpha'_{11}, \alpha'_{12}, \alpha'_{20}, \alpha'_{21}, \alpha'_{22})$. In this case, the unique optimal solution is $\mathbf{w} = (11.27, 0, 9.85, 6.79, 3.36, 8.08, 0, 5.3, 7.98)$ and $\mathbf{c} = (11.27, 0, 0, 6.79, 3.36, 0, 0, 5.3, 7.98)$, where $\mathbf{w} \equiv (w_{00}, w_{01}, w_{02}, w_{10}, w_{11}, w_{12}, w_{20}, w_{21}, w_{22})$ and $\mathbf{c} \equiv (c_{00}, c_{01}, c_{02}, c_{10}, c_{11}, c_{12}, c_{20}, c_{21}, c_{22})$. That is, the agent's compensation depends on both parties' signals even if the principal's signal is a sufficient statistic of the agent's.

3.2 The Influence of Effort Level in the Binary Output Case

In this section, we investigate how the prevalence of various regimes is influenced by the effort level. We concern the binary output case, that is, the output is either success or failure. Following MacLeod (2003), let joint probabilities α_{ts} to be *linear* in effort level:

Assumption 4. *$\alpha_{ts}(e) = \beta_{ts}e + \gamma_{ts}$, where β_{ts}, $\gamma_{ts} > 0$, $\forall t, s \in \{0, 1\}$.*

Under Assumption 4, we may regard the effort level, $e \in [0, 1]$, as the probability of success. The success output induces the probability distribution $\beta_{ts} + \gamma_{ts}$ on the signal space $\{(t, s)|t, s = 0, 1\}$, while the failure output induces the probability distribution γ_{ts} on the signal space $\{(t, s)|t, s = 0, 1\}$. Hence, $\alpha_{ts}(e) = \beta_{ts}e + \gamma_{ts}$ denotes the probability of signal (t, s) being realized given effort level e.

It is intuitive, that the higher the effort level being implemented, the more power the principal has. Specifically, we may assume that the expected net benefit to the principal is

$\Pi(\tilde{e}; B) = \tilde{e}B - C^*(\tilde{e})$, where B denotes the benefit when the project is successful, and $C^*(\tilde{e})$ denote the cost minimization function in the first step. It is easy to show that the optimal effort level $\tilde{e} = e$ is increasing in B.[8] Hence, if a principal proposes a project with higher B, she may be regarded as one with more power. Such power comes from the principal's vision (to delegate a good project), but not from the cost function, and hence we call it *power of vision*.

From the discussion in the previous section, we know that, if the principal's signal has more power, that is, the principal's information is more precise than the agent's, Regime 1 prevails. Such power, which we call *power of information*, directly affects the cost function. We have no idea that if a principal with greater power of vision is also equipped with greater power of information. However, we may expect that, the principal with greater power of vision, who tends to implements higher effort level, is more likely to adopt Regime 1. It is verified in the following proposition.

Proposition 4. *Assume that the agent is risk-neutral. If there exists an effort level, e_1, such that the optimal contract implementing e_1 is in Regime 1, then the optimal contract implementing any higher effort level than e_1 is also in Regime 1.*

Proof. See Appendix A. □

We offer an explanation for Proposition 3 from the viewpoint of information structure. Since Corollary 1 rules out the possibility of Regime 3, we just need to show that Regime 2 is less preferred as the implemented effort level gets higher. If the agent sees the signal $s = 0$, he would think that $(t, s) = (1, 0)$ is more likely to revealed if e increases, since MLRC says that $\alpha_{10}(e)/\alpha_{00}(e)$ increases with e. He then would like to misreport $s = 1$ since $c_{11} > c_{10}$ in Regime 2. In order to restore such condition, by considering (TRA) (or Lemma 5), the principal shall

[8] Since $\frac{\partial^2 \Pi}{\partial \tilde{e} \partial B} = 1 > 0$, $\Pi(\tilde{e}; B)$ satisfies strictly increasing differences in (e, B). Then, the optimal effort level $\tilde{e} = e$ is increasing in B. See Sundaram (1996): Theorem 10.4, Theorem 10.7, and Theorem 10.12.

either reduce the gap between c_{11} and c_{10}, or keep the gap by increasing c_{00} at the same time. In the latter case, note that $w_{l0} = c_{10} = w_{00} = c_{00}$ in Regime 2, (TRA) (or Lemma 5) implies $w_{11} = c_{11}$ increases, which in turn results in a higher w_{01} through (TRP) (or Lemma 3). In sum, the principal pays more in all the contingencies, which is unattractive and less likely to be optimal. In the former case, Regime 1 ($c_{11} = c_{10}$) is more likely to emerge.

In view of Proposition 4, we know that, if the agent is risk-neutral, three patterns may emerge, and each pattern reflects how the prevalence of various regimes is influenced by the effort level. We summarize in the following corollary.

Corollary 3. *If the agent is risk-neutral, and given Assumption 4, there are three patterns reflecting the relationship of regimes and effort level: (i) Regime 1 prevails for any implemented effort level $e \in (0, 1)$; (ii) Regime 2 prevails for any implemented effort level $e \in (0, 1)$; (iii) there exists a cutoff point $e_0 \in (0, 1)$, such that Regime 1 prevails for high effort level $e \in (e_0, 1)$, and Regime 2 prevails for low effort level $e \in (0, e_0)$. In sum, the implemented effort level ranks higher in Regime 1 than in Regime 2.* [9]

If the agent is risk-averse, Regime 3 may prevail. Since Regime 3 could be thought as a mixture of Regime 1, in which principal has more authority, and Regime 2, in which agent has more authority, we may conjecture that, as the implemented effort level increases, Regime 2 comes first, Regime 3 follows, and Regime 1 finally emerges. However, the explicit result is not so definite as the risk-neutral case. In general, the second derivatives of utility function $u(\cdot)$ and cost function $C(\cdot)$ affect the result. Roughly speaking, if $\tilde{u}''(\cdot)C''(\cdot)$ is sufficiently small, i.e., $u(\cdot)$ or $C(\cdot)$ tends to be linear, then the aforementioned conjecture holds. The detailed

[9] A numerical example to verify this statement is given as follows: $u(c) = c, v(e) = e^2, (\beta_{00}, \beta_{01}, \beta_{10}, \beta_{11}) = (-0.08, -0.02, -0.04, 0.14), (\gamma_{00}, \gamma_{01}, \gamma_{10}, \gamma_{11}) = (0.34, 0.16, 0.22, 0.28)$. Then, the cutoff point is $e_0 \simeq 0.8100$, that is: Regime 1 prevails for any effort level $e > e_0$ to be implemented, while Regime 2 prevails for any effort level $e < e_0$ to be implemented.

discussion is left in Appendix B.

4 Conclusion

This paper revisits the properties of the static optimal contract with subjective performance measures when the principal's and the agent's signals are correlated. We show that, because of the truthful revelation requirement, the optimal contract always entails loss of information: it is always true that one party's pay (or cost) depends on the other party's information only, despite that its private signal also has informational value. Whether the agent's compensation is a function of his own information depends on the relative precision of the principal's information. Moreover, even if a party's information is not fully incorporated into the contract, it nonetheless is used as an instrument to prevent the other party from strategically manipulating its signal.

Our model is of binary outcomes. An extension to the general n-outcome case may be difficult, but it is interesting and worthy of further study.

Appendix A. Omitted Proofs

Proof of Proposition 2

We first prove a series of lemmas which characterize the optimal contract. The strategy of the proof is to reduce the original optimization problem into one with a single variable with a single constraint.

Lemma 2. $c_{11} > 0$, $c_{01} = 0$.

Proof. Lemma 1 implies that either $u(c_{11})-u(c_{10}) = u(c_{00})-u(c_{01}) = 0$ or both $u(c_{11})-u(c_{10}) > 0$ and $u(c_{00}) - u(c_{01}) > 0$. In the former case, $V'(e) = [u(c_{10}) - u(c_{01})][\alpha'_{11}(e) + \alpha'_{10}(e)] > 0$. Since $\alpha'_{11}(e)+\alpha'_{10}(e) > 0$ by Assumption 2, we have $c_{10} > 0$. Hence, $c_{11} = c_{10} > 0$. In the latter case, $c_{11} > c_{10} \geq 0$. To show that $c_{01} = 0$, suppose instead that $c_{01} > 0$. Note that $c_{00} \geq c_{01}$ by Lemma 1, we first reduce c_{00} and c_{01} slightly such that $u(c_{00}) - u(c_{01})$ remains unchanged and (IR) still holds. Then, (TRA), or equivalently, (2), is not affected. Now, consider (ICF) with the new values of c_{00} and c_{01}. The right-hand side of (ICF) increases because $\alpha'_{11}(e) + \alpha'_{10}(e) > 0$ by Assumption 2. It follows that $V'(e) = \sum_{t,s\in\{0,1\}} ku(c_{ts})\alpha'_{ts}(e)$ for some $k \in (0,1)$. Let $\hat{c}_{ts} \in [0, c_{ts}]$ such that $u(\hat{c}_{ts}) = ku(c_{ts})$. Then, $\{\hat{c}_{ts}\}_{t,s\in\{0,1\}}$ satisfies (TRA) and (ICF). Moreover, the reconfiguration can be made so slight that k is so close to 1, and that (IR) still holds.

By Assumption 1, $c_{11} > 0$, and thus $\hat{c}_{11} < c_{11}$. Note also that $\hat{c}_{01} < c_{01}$. Then, we reduce w_{11} and w_{01} with the same amount such that (LL) holds and (TRP), or equivalently, (1), is not affected. Let $\hat{w}_{ts}$ denote the new values of wage payment. The new contract $\{\hat{w}_{ts}, \hat{c}_{ts}\}_{t,s\in\{0,1\}}$ is a feasible solution to (P) and yields a lower expected payment. Hence, $c_{01} > 0$ cannot be optimal. □

Lemma 2 shows that, when the agent reports the good signal, he receives positive wage only if the principal concurs: The principal's signal is essentially used to restrain the agent's incentive to strategically misreport.

Lemma 3. $w_{01} - w_{11} = (w_{10} - w_{00})\frac{\alpha_{10}(e)}{\alpha_{11}(e)}$. *That is, the principal is indifferent between reporting $t = 0$ and $t = 1$ when she observes $t = 1$.*

Proof. The lemma just says that the first inequality of (1) is binding. If $w_{01} = 0$, Lemma 1 implies that $w_{11} = 0$, then all the inequalities of (1) are binding. If $w_{01} > 0$ and the inequality is slack, we can reduce the value of w_{01} so that (1) and (LL) still hold (because $c_{01} = 0$ by Lemma 2), while the expected wage payment is reduced. Therefore, at the optimum the inequality must be binding. □

Let $x^\dagger \equiv \max\{x, 0\}$, we can express w_{ts} in terms of c_{ts} in the next lemma.

Lemma 4. $w_{11} = c_{11}$, $w_{00} = c_{00}$, $w_{10} = \max\{c_{10}, c_{00}\}$, *and* $w_{01} = c_{11} + (c_{10} - c_{00})^\dagger \cdot \frac{\alpha_{10}(e)}{\alpha_{11}(e)}$.

Proof. By Lemma 3, the principal's objective function can be rewritten as

$$\begin{aligned} C =& w_{11}\alpha_{11}(e) + w_{10}\alpha_{10}(e) + w_{00}\alpha_{00}(e) + \left[w_{11} + (w_{10} - w_{00})\frac{\alpha_{10}(e)}{\alpha_{11}(e)} \right] \alpha_{01}(e) \\ =& w_{11}\left[\alpha_{11}(e) + \alpha_{01}(e)\right] + w_{10}\left[\alpha_{10}(e) + \frac{\alpha_{10}(e)\alpha_{01}(e)}{\alpha_{11}(e)}\right] + w_{00}\left[\alpha_{00}(e) - \frac{\alpha_{10}(e)\alpha_{01}(e)}{\alpha_{11}(e)}\right]. \end{aligned} \tag{3}$$

By Assumption 2, the coefficient of w_{00} is positive, so are that of w_{11} and w_{10}. The constraints concerning w_{ts} are (1) and (LL). Since Lemma 3 is implemented, it suffices to check (LL) and $w_{10} \geq w_{00}$ with $w_{01} \geq w_{11}$. Accordingly, for the objective function being minimum, $w_{11} = c_{11}$, and $w_{00} = c_{00}$. Then, $w_{10} = \max\{w_{00}, c_{10}\} = \max\{c_{00}, c_{10}\}$. Moreover, again by Lemma 3, we have $w_{01} = c_{11} + (\max\{c_{00}, c_{10}\} - c_{00})\frac{\alpha_{10}(e)}{\alpha_{11}(e)} = c_{11} + (c_{10} - c_{00})^\dagger \cdot \frac{\alpha_{10}(e)}{\alpha_{11}(e)}$. □

Lemma 5. $u(c_{11}) - u(c_{10}) = u(c_{00})\frac{\alpha_{00}(e)}{\alpha_{10}(e)}$. *That is, the agent is indifferent between reporting $s = 0$ and $s = 1$ when he observes $s = 0$.*

Proof. By Lemma 2, $c_{01} = 0$, this lemma just says that the second inequality of (2) is binding. If $c_{00} = 0$, all the inequalities of (2) are binding. If $c_{00} > 0$ and the inequality is slack, we can decrease the value of c_{00} slightly so that (2) and (IR) still hold. Now, consider (ICF) with the new value of c_{00}. The right-hand side of (ICF) increases because $\alpha'_{00}(e) < 0$ by Assumption 2. Follow the same arguments as that of Lemma 2, there exist $\hat{c}_{ts} \in [0, c_{ts}]$ such that $\{\hat{c}_{ts}\}_{t,s \in \{0,1\}}$ satisfies (TRA), (ICF), and (IR). The change of the principal's payment, which follows from Lemma 4 and (3), is

$$\Delta_{11}\left[\alpha_{11}(e) + \alpha_{01}(e)\right] + \Delta_{10}\left[\alpha_{10}(e) + \frac{\alpha_{10}(e)\alpha_{01}(e)}{\alpha_{11}(e)}\right] + \Delta_{00}\left[\alpha_{00}(e) - \frac{\alpha_{10}(e)\alpha_{01}(e)}{\alpha_{11}(e)}\right],$$

where $\Delta_{11} = \hat{c}_{11} - c_{11} < 0$, $\Delta_{00} = \hat{c}_{00} - c_{00} < 0$, and $\Delta_{10} = \max\{\hat{c}_{00}, \hat{c}_{10}\} - \max\{c_{00}, c_{10}\} < 0$. Hence, we find a new contract which reduces the principal's wage payment. Then, at the optimum the inequality must be binding. □

Lemma 6. $c_{10} \geq c_{00}$.

Proof. Suppose that $c_{00} > c_{10} (\geq 0)$. By Lemma 2, $c_{00} > 0 = c_{01}$. Then, by Lemma 1, $c_{11} > c_{10}$. We first raise c_{10} slightly such that both c_{00} and c_{11} are still strictly greater than c_{10}, and (2) still holds, which can be done since the second inequality of (2) is binding by Lemma 5, and so the first inequality of (2) is slack. Note that the the principal's wage payment is unaffected, because, by Lemma 4, no expression of w_{ts} is regarding c_{10} while $c_{00} > c_{10}$.

Now, consider (ICF) with the new value of c_{10}. The right-hand side of (ICF) increases because $\alpha'_{11}(e) + \alpha'_{10}(e) > 0$ by Assumption 2. Follow the same arguments as that of Lemma 2, we can find another feasible contract entailing a lower expected payment. Hence, $c_{00} > c_{10}$ cannot be optimal. □

Combining Lemmas 2, 4, and 6, we know that $w_{01} \geq c_{11} > 0 = c_{01}$, and $w_{10} = c_{10}$. That is, the only case in which money burning occurs is when the principal reports $t = 0$ and the agent reports $s = 1$.

Let $u_{ts} = u(c_{ts}), \forall t, s \in \{0, 1\}$. Lemma 1 through 6 allow us to rewrite the principal's expected payment as a function of a single variable, u_{00}. To simplify notations, we now write $\alpha_{ij}(e)$ as α_{ij}, and $c^*(e)$ as c^*. Rearranging Lemma 5 and (IC$_\text{F}$), we have

$$\begin{pmatrix} \alpha'_{10} & \alpha'_{11} \\ -1 & 1 \end{pmatrix} \begin{pmatrix} u_{10} \\ u_{11} \end{pmatrix} = \begin{pmatrix} V' - u_{00}\alpha'_{00} \\ u_{00}\frac{\alpha_{00}}{\alpha_{10}} \end{pmatrix}. \tag{4}$$

Solving (4), we obtain

$$u_{10} = \frac{V' - \left(\alpha'_{00} + \frac{\alpha_{00}}{\alpha_{10}}\alpha'_{11}\right) u_{00}}{\alpha'_{11} + \alpha'_{10}}; \tag{5}$$

$$u_{11} = \frac{V' + \left(\frac{\alpha_{00}}{\alpha_{10}}\alpha'_{10} - \alpha'_{00}\right) u_{00}}{\alpha'_{11} + \alpha'_{10}}. \tag{6}$$

By Lemmas 4 and 6, the principal's objective function can be rewritten in terms of c_{ts}'s:

$$\begin{aligned} C^* &= c_{11}\alpha_{11} + c_{10}\alpha_{10} + \left[c_{11} + (c_{10} - c_{00})\frac{\alpha_{10}}{\alpha_{11}}\right]\alpha_{01} + c_{00}\alpha_{00} \qquad (7) \\ &= (\alpha_{11} + \alpha_{01})c_{11} + (\alpha_{10} + \frac{\alpha_{10}}{\alpha_{11}}\alpha_{01})c_{10} + (\alpha_{00} - \frac{\alpha_{10}}{\alpha_{11}}\alpha_{01})c_{00} \\ &= (\alpha_{11} + \alpha_{01})\tilde{u}(u_{11}) + (\alpha_{10} + \frac{\alpha_{10}}{\alpha_{11}}\alpha_{01})\tilde{u}(u_{10}) + (\alpha_{00} - \frac{\alpha_{10}}{\alpha_{11}}\alpha_{01})\tilde{u}(u_{00}), \end{aligned}$$

where $\tilde{u}(\cdot)$ is the inverse function of $u(\cdot)$, i.e., $\tilde{u}(u_{ts}) = c_{ts}$, $\forall t, s \in \{0, 1\}$. Since u_{10} and u_{11} are functions of u_{00} only by (5) and (6), C is then a function of a single variable u_{00}. Note that the constraints regarding the relationship between c_{ts}'s, which can be summarized as Lemma 5 and $0 \le c_{00} \le c_{10}$, are now transformed in u_{ts}'s. Since Lemma 5 has been replaced by (4), it remains to consider $0 \le c_{00} \le c_{10}$, which now reads as $0 \le u_{00} \le u_{10}$, or equivalently, $0 \le u_{00} \le \frac{V'}{\frac{\alpha_{00}}{\alpha_{10}}\alpha'_{11} - \alpha'_{01}} \equiv \bar{u}$ (by (5) and the fact that $\sum_{ij}\alpha'_{ij} = 0$). The principal's problem therefore reduces to choosing u_{00} to minimize C, subject to the constraint that $0 \le u_{00} \le \bar{u}$.

The marginal cost of u_{00} for the principal is

$$MC(u_{00}) \equiv \frac{dC}{du_{00}} = \frac{1}{\alpha'_{11}+\alpha'_{10}} \begin{bmatrix} (\alpha_{11}+\alpha_{01})\left(\frac{\alpha_{00}}{\alpha_{10}}\alpha'_{10}-\alpha'_{00}\right)\tilde{u}'(u_{11}) \\ -\left(\alpha_{10}+\frac{\alpha_{10}}{\alpha_{11}}\alpha_{01}\right)\left(\alpha'_{00}+\frac{\alpha_{00}}{\alpha_{10}}\alpha'_{11}\right)\tilde{u}'(u_{10}) \end{bmatrix} + (\alpha_{00}-\frac{\alpha_{10}}{\alpha_{11}}\alpha_{01})\tilde{u}'(u_{00}). \tag{8}$$

Note that C is convex in u_{00} because

$$\begin{aligned} \frac{d^2C}{du_{00}^2} &= \left(\frac{1}{\alpha'_{11}+\alpha'_{10}}\right)^2 \begin{bmatrix} (\alpha_{11}+\alpha_{01})\left(\frac{\alpha_{00}}{\alpha_{10}}\alpha'_{10}-\alpha'_{00}\right)^2\tilde{u}''(u_{11}) \\ +\left(\alpha_{10}+\frac{\alpha_{10}}{\alpha_{11}}\alpha_{01}\right)\left(\alpha'_{00}+\frac{\alpha_{00}}{\alpha_{10}}\alpha'_{11}\right)^2\tilde{u}''(u_{10}) \end{bmatrix} + (\alpha_{00}-\frac{\alpha_{10}}{\alpha_{11}}\alpha_{01})\tilde{u}''(u_{00}) \\ &\geq 0, \end{aligned}$$

where the inequality follows from Assumption 1. Recall that the domain of u_{00} is $[0,\bar{u}]$. Therefore, if $MC(0) \geq 0$, the minimizer is $u_{00}=0$; if $MC(\bar{u}) \leq 0$, the minimizer is $u_{00}=\bar{u}$; if $MC(0) < 0 < MC(\bar{u})$, the minimizer is some $u_{00} \in (0,\bar{u})$. For each case, we can summarize the ranking of w_{ts}'s and c_{ts}'s, as shown in Proposition 2, by Lemma 1, Lemma 2, and Lemma 4.

Proof of Proposition 3:

The marginal cost function of u_{00} (see (8)) can be rewritten as

$$\begin{aligned} MC(u_{00}) &= \frac{\alpha_{11}+\alpha_{01}}{\alpha_{11}(\alpha'_{11}+\alpha'_{10})}\left[\alpha_{11}\left(\frac{\alpha_{00}}{\alpha_{10}}\alpha'_{10}-\alpha'_{00}\right)\tilde{u}'(u_{11})-\alpha_{10}\left(\alpha'_{00}+\frac{\alpha_{00}}{\alpha_{10}}\alpha'_{11}\right)\tilde{u}'(u_{10})\right] \\ &\quad +(\alpha_{00}-\frac{\alpha_{10}}{\alpha_{11}}\alpha_{01})\tilde{u}'(u_{00}). \end{aligned}$$

Let $MC(u_{00}) = MC^P(u_{00})$ when t is a sufficient statistic for s. We have

$$\begin{aligned} MC^P(u_{00}) &\geq \frac{\alpha_{11}+\alpha_{01}}{\alpha_{11}(\alpha'_{11}+\alpha'_{10})}\left[\alpha_{11}\left(\frac{\alpha_{00}}{\alpha_{10}}\alpha'_{10}-\alpha'_{00}\right)-\alpha_{10}\left(\alpha'_{00}+\frac{\alpha_{00}}{\alpha_{10}}\alpha'_{11}\right)\right]\tilde{u}'(u_{10}) \\ &= -\frac{\alpha_{11}+\alpha_{01}}{\alpha_{11}(\alpha'_{11}+\alpha'_{10})}(\alpha_{11}+\alpha_{10})\alpha'_{00}\tilde{u}'(u_{10}) \\ &> 0, \end{aligned}$$

where the first inequality holds because $u_{11} \geq u_{10}$, $\frac{\alpha_{00}}{\alpha_{10}}\alpha'_{10} - \alpha'_{00} > 0$ by Assumption 3, $\alpha'_{11} + \alpha'_{10} > 0$ by Assumption 2, and $\alpha_{00} - \frac{\alpha_{10}}{\alpha_{11}}\alpha_{01} > 0$ by Assumption 1; the second equality follows from $\frac{\alpha'_{11}}{\alpha_{11}} = \frac{\alpha'_{10}}{\alpha_{10}}$; and the third inequality follows from $\alpha'_{00} < 0$. Therefore, when t is a sufficient statistic for s, regime (i) prevails by Proposition 2.

Let $MC(u_{00}) = MC^A(u_{00})$ when s is a sufficient statistic for t. We have

$$
\begin{aligned}
MC^A(u_{00}) &= -\frac{\alpha_{11} + \alpha_{01}}{\alpha_{11}}\alpha_{00}\tilde{u}'(u_{10}) + (\alpha_{00} - \frac{\alpha_{10}}{\alpha_{11}}\alpha_{01})\tilde{u}'(u_{00}) \\
&\leq -\frac{\alpha_{01}(\alpha_{10} + \alpha_{00})}{\alpha_{11}}\tilde{u}'(u_{00}) \\
&< 0,
\end{aligned}
$$

where the first equality follows from $\frac{\alpha'_{10}}{\alpha_{10}} = \frac{\alpha'_{00}}{\alpha_{00}}$, and the second inequality follows from $u_{00} \leq u_{10}$. Consequently, when s is a sufficient statistic for t, regime (ii) prevails by Proposition 2.

The marginal cost function of u_{00} with parameterized belief p is

$$
MC(u_{00}; p) = pMC^P(u_{00}) + (1 - p)MC^A(u_{00}).
$$

Note that $MC(u_{00}; p)$ is strictly increasing in p because $\frac{\partial MC(u_{00};p)}{\partial p} = MC^P(u_{00}) - MC^A(u_{00}) > 0$. Since $MC^A(\bar{u}) < 0 < MC^P(\bar{u})$, there exists a $0 < \underline{p} < 1$ such that $MC(\bar{u}; \underline{p}) = 0$. For the similar reason, there exists a $0 < \bar{p} < 1$ such that $MC(0; \bar{p}) = 0$. In addition, because $MC(u_{00}; p)$ is strictly increasing in both u_{00} and p, we know that $\underline{p} < \bar{p}$. Accordingly, for $0 \leq p \leq \underline{p}$, $MC(\bar{u}; p) \leq 0$; for $\underline{p} < p < \bar{p}$, $MC(0; p) < 0 < MC(\bar{u}; p)$; for $\bar{p} \leq p \leq 1$, $MC(0; p) \geq 0$, and the result follows by Proposition 2.

Proof of Proposition 4:

By Corollary 1, we only need to compare the cost in Regime 1 (which is denoted by C_1) and that in Regime 2 (which is denoted by C_2). In Regime 1, we have

$$\begin{aligned} w_{01}^* &= \frac{V'(e)\left(1+\frac{\alpha_{10}(e)}{\alpha_{11}(e)}\right)}{\beta_{11}+\beta_{10}}; \\ c_{11}^* &= c_{10}^* = w_{11}^* = w_{10}^* = \frac{V'(e)}{\beta_{11}+\beta_{10}}; \\ c_{00}^* &= c_{01}^* = w_{00}^* = 0. \end{aligned}$$

Hence,

$$C_1 = \left(\alpha_{11}+\alpha_{10}+\alpha_{01}+\frac{\alpha_{10}\alpha_{01}}{\alpha_{11}}\right)\frac{V'}{\beta_{11}+\beta_{10}}.$$

In Regime 2, we have

$$\begin{aligned} c_{11}^* &= w_{11}^* = w_{01}^* = \frac{V'(e)\left(1+\frac{\alpha_{00}(e)}{\alpha_{10}(e)}\right)}{\beta_{11}\frac{\alpha_{00}(e)}{\alpha_{10}(e)}-\beta_{01}}; \\ c_{00}^* &= c_{10}^* = w_{00}^* = w_{10}^* = \frac{V'(e)}{\beta_{11}\frac{\alpha_{00}(e)}{\alpha_{10}(e)}-\beta_{01}}; \\ c_{01}^* &= 0. \end{aligned}$$

Hence,

$$C_2 = (\alpha_{11}+\alpha_{10}+\alpha_{01})\frac{V'}{\beta_{11}+\frac{\alpha_{10}}{\alpha_{10}+\alpha_{00}}(\beta_{10}+\beta_{00})}.$$

If regime 1 is preferred, $C_1 \leq C_2$, and it is equivalent to

$$\left[1+\frac{\alpha_{10}\alpha_{01}}{\alpha_{11}(\alpha_{11}+\alpha_{10}+\alpha_{01})}\right]\left[\beta_{11}+\frac{\alpha_{10}}{\alpha_{10}+\alpha_{00}}(\beta_{10}+\beta_{00})\right] \leq \beta_{11}+\beta_{10}. \tag{9}$$

Note that the *RHS* of (9) is positive constant. Hence the proposition is equivalent to: if $LHS \leq \beta_{11}+\beta_{10}$ for some $e_1 \in (0,1)$, then, $\forall e > e_1$, $LHS \leq \beta_{11}+\beta_{10}$.

We just need to consider the case that the term in the second brackets is positive, for the other case the inequality obviously holds. It suffices to show that, under this premise, *LHS* is decreasing in e. The term in the first brackets can be rewritten as

$$1+\frac{1}{\frac{\alpha_{11}}{\alpha_{10}}\cdot\frac{\alpha_{11}}{\alpha_{01}}+\frac{\alpha_{11}}{\alpha_{10}}+\frac{\alpha_{11}}{\alpha_{01}}},$$

where Assumption 3 implies that the denominator is increasing in e . Hence, this term decreases with e. The term in the second brackets also decreases with e since Assumption 3 implies that $\frac{\alpha_{10}}{\alpha_{10}+\alpha_{00}}$ is increasing in e and Assumption 1 says that $\beta_{10}+\beta_{00}$ is negative. Since the term in both brackets are positive, we conclude that LHS is decreasing in e, attaining our goal.

Appendix B. The Influence of Effort Level: General Discussion

Note that Proposition 2 provides criteria to determine which regime prevails by $MC(0)$ and $MC(\bar{u})$. We shall check how these two values change with e. First, we have

$$\begin{aligned} MC(0) =& \frac{\alpha_{11}+\alpha_{01}}{\alpha'_{11}+\alpha'_{10}}\left(-(\alpha_{11}+\alpha_{10})\alpha'_{00}+\frac{\alpha_{11}\alpha_{00}}{\alpha_{10}}\alpha'_{10}-\alpha_{00}\alpha'_{11}\right)\tilde{u}'\left(\frac{V'}{\alpha'_{11}+\alpha'_{10}}\right) \\ &+(\alpha_{11}\alpha_{00}-\alpha_{10}\alpha_{01})\tilde{u}'(0). \end{aligned}$$

Let $M_1 \equiv M_1(e)$, respectively $N_1 \equiv N_1(e)$, denote the coefficient of $\tilde{u}'(\frac{V'}{\alpha'_{11}+\alpha'_{10}})$, respectively the coefficient of $\tilde{u}'(0)$. Let $\kappa_1 \equiv \kappa_1(e) \equiv \frac{\tilde{u}'(\frac{V'}{\alpha'_{11}+\alpha'_{10}})}{\tilde{u}'(0)}$. Then, $MC(0) \geq 0$ is equivalent to

$$f_1(e) \equiv M_1\kappa_1 + N_1 \geq 0. \tag{10}$$

Our first goal is to offer a sufficient condition to make the following statement hold:

P1. If Regime 1 prevails for effort level $e_1 \in (0,1)$ to be implemented, then Regime 1 prevails for any effort level $e \in (e_1, 1)$ to be implemented.

Note that **P1** holds if and only if the following statement holds:

Q1. If there exists $e_1 \in (0,1)$ such that $f_1(e_1) = 0$, then $f'_1(e_1) \geq 0$.

Hence, we may transform our goal to make **Q1** hold.

At $e = e_1$, (10) takes equality, we have $\kappa_1(e_1) = -\frac{N_1(e_1)}{M_1(e_1)} > 0$. Then, it follows that

$$\begin{aligned} f'(e_1) &= M'_1(e_1)\kappa_1(e_1) + M_1(e_1)\kappa'_1(e_1) + N'_1(e_1) \\ &= -\frac{M'_1(e_1)}{M_1(e_1)}N_1(e_1) + M_1(e_1)\kappa'_1(e_1) + N'_1(e_1). \end{aligned}$$

Denote $\xi_1(e) = -\frac{N_1(e)}{M_1(e)}$. Then, $f'(e_1) = -M_1(e_1)(\xi_1'(e_1) - \kappa_1'(e_1))$. Note that $-M_1(e_1) > 0$, then $f'(e_1) \geq 0$ if and only if

$$\xi_1'(e_1) \geq \kappa_1'(e_1), \tag{11}$$

which is the exact criterion to make **Q1**, or **P1** hold. We shall note that $\xi_1(e) \equiv -\frac{N_1(e)}{M_1(e)}$ is fully determined by α_{ts}'s, while $\kappa_1(e)$ involves the utility and cost function.

When $\alpha_{ts}(e)$ is linear, we know

$$\kappa_1'(e_1) = \frac{1}{(\alpha_{11}' + \alpha_{10}')\tilde{u}(0)}\tilde{u}''\left(\frac{V'(e_1)}{\alpha_{11}' + \alpha_{10}'}\right)V''(e_1).$$

Hence, $\kappa_1'(e_1)$ is zero if the agent is risk-neutral, then (11) reads as $\xi_1'(e_1) \geq 0$. However, we have confirmed this (strict) inequality by Proposition 4 (and its proof), i.e., $\xi_1'(e_1)$ is positive. Then, (11) holds if $\tilde{u}''\left(\frac{V'(e_1)}{\alpha_{11}'+\alpha_{10}'}\right)V''(e_1)$ is sufficiently small, which justified we mentioned in the main text. We Summarize the first part, as follows:

S1. If $\tilde{u}''\left(\frac{V'(e_1)}{\alpha_{11}'+\alpha_{10}'}\right)V''(e_1)$ is sufficiently small, then **P1** holds.

In the second part, we shall calculate $MC(\bar{u})$ first:

$$\begin{aligned} MC(\bar{u}) =& \frac{\alpha_{11} + \alpha_{01}}{\alpha_{11}' + \alpha_{10}'}\alpha_{11}\left(\frac{\alpha_{00}}{\alpha_{10}}\alpha_{10}' - \alpha_{00}'\right)\tilde{u}'\left(\frac{\alpha_{10} + \alpha_{00}}{\alpha_{00}\alpha_{11}' - \alpha_{10}\alpha_{01}'}V'\right) \\ &+ \left(\alpha_{11}\alpha_{00} - \alpha_{10}\alpha_{01} - \frac{\alpha_{11} + \alpha_{01}}{\alpha_{11}' + \alpha_{10}'}(\alpha_{10}\alpha_{00}' + \alpha_{00}\alpha_{11}')\right)\tilde{u}'\left(\frac{\alpha_{10}}{\alpha_{00}\alpha_{11}' - \alpha_{10}\alpha_{01}'}V'\right). \end{aligned}$$

Let $M_2 \equiv M_2(e)$, respectively $N_2 \equiv N_2(e)$, denote the coefficient of $\tilde{u}'(\frac{\alpha_{10}+\alpha_{00}}{\alpha_{00}\alpha_{11}'-\alpha_{10}\alpha_{01}'}V')$, respectively the coefficient of $\tilde{u}'(\frac{\alpha_{10}}{\alpha_{00}\alpha_{11}'-\alpha_{10}\alpha_{01}'}V')$. Let $\kappa_2 \equiv \kappa_2(e) \equiv \frac{\tilde{u}'(\frac{\alpha_{10}+\alpha_{00}}{\alpha_{00}\alpha_{11}'-\alpha_{10}\alpha_{01}'}V')}{\tilde{u}'(\frac{\alpha_{10}}{\alpha_{00}\alpha_{11}'-\alpha_{10}\alpha_{01}'}V')}$. Then, $MC(\bar{u}) \leq 0$ is equivalent to

$$f_2(e) \equiv M_2\kappa_2 + N_2 \geq 0. \tag{12}$$

Our second goal is to offer a sufficient condition to make the following statement hold:

P2. If Regime 2 prevails for effort level $e_2 \in (0, 1)$ to be implemented, then Regime 2 prevails for any effort level $e \in (0, e_2)$ to be implemented.

Note that **P2** holds if and only if the following statement holds:

Q2. If there exists $e_2 \in (0, 1)$ such that $f_2(e_2) = 0$, then $f_2'(e_2) \geq 0$.

Hence, we may transform our goal to make **Q2** hold.

Making use similar argument and similar notations in the first part, we know that, $f_2'(e_2) \geq 0$, if and only if,

$$\xi_2'(e_2) \geq \kappa_2'(e_2), \tag{13}$$

where $\xi_2(e_2) = -\frac{N_2(e)}{M_2(e)}$. This is the exact criterion to make **Q2**, or **P2** hold.

Note that $\kappa_2'(e_2)$ is zero if the agent is risk-neutral, then (13) reads as $\xi_2'(e_2) \geq 0$. However, we have confirmed this (strict) inequality by Proposition 4 (and its proof). [10] That is, $\xi_2'(e_2)$ is positive. Then, (13) holds if $\kappa_2'(e_2)$ is sufficiently small. We Summarize the second part as follows:

S2. If $\kappa_2'(e_2)$ is sufficiently small, then **P2** holds.

Given **P1** an **P2**, we conclude, that Regime 2 comes first, Regime 3 follows, and Regime 1 finally emerges, as the implemented effort level increases.

[10] Proposition 4 is equivalent to the following: Assume that the agent is risk-neutral. If there exists an effort level, e_2, such that the optimal contract implementing e_2 is in Regime 2, then the optimal contract implementing any lower effort level than e_2 is also in Regime 2.

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