

The Panel Stochastic Frontier Model with Firm Heterogeneity and Dynamic Technical Inefficiency

1. What is the question?

Among most existing models of technical efficiency measurement, the main concern usually focuses on the temporal behavior of inefficiency, not on its dynamics.

2. Why should we care about this?

Although consideration of such dynamic models is necessary, inference in such models is relatively complicated, particularly for the likelihood-based approach.

3. What is the author's answer?

This paper intends to contribute in this direction in the SF studies. We consider a panel SF model with dynamic technical inefficiency that follows a first-order autoregressive (AR(1)) process and propose to estimate the model by a likelihood-based approach.

4. How did the author get there?

A panel stochastic frontier model that allows the dynamic adjustment of the technical inefficiency as well as firms' heterogeneity and suggest using the pairwise composite-likelihood (PCL) to estimate the model. Some Monte Carlo experiments are used to compare the finite sample performance of the full maximum likelihood (FML) and PCL estimators

Notation

dynamic SF model:

$$y_{it} = x_{it}^T \beta + g_t + v_{it} - u_{it},$$

technical innovation is linear in time

$$g_t = \pi_0 + \pi_1 t$$

autoregressive (AR) process of order one

$$u_{it} = \rho u_{it-1} + u_{it}^*, \quad t = 1, \dots, T,$$

$$u_{it}^* \sim N^+(0, \sigma_{u_i}^2), \text{ for } t = 1, \dots, T,$$

$$u_{i0} \sim N^+(0, \sigma_{u_i}^2 / (1 - \rho^2)).$$

log-likelihood function of the transformed model

$$\ln L(\theta) = \sum_{i=1}^N \ln f(\varepsilon_i; \theta).$$

The full maximum likelihood estimator is defined as

$$\hat{\theta}_{ML} = \arg \max_{\theta \in \Theta} \ln L(\theta),$$

where Θ denotes the parameter space. Under the regularity conditions²,

$$\sqrt{N}(\hat{\theta}_{ML} - \theta) \sim N_d(O_d, -H(\theta)^{-1}),$$

where d is the dimension of θ and $H(\theta) = E \left[\frac{\partial^2 \ln f(\varepsilon_i; \theta)}{\partial \theta \partial \theta^T} \right]$ is the Hessian matrix.

data-generating process (DGP)

$$y_{it} = \beta_1 x_{1,it} + \beta_2 x_{2,it} + \pi_0 + \pi_1 t + v_{it} - u_{it},$$