Macroprudential Policy Coordination with International Capital Flows

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Abstract

We theoretically illustrate how macroprudential policy spillovers through international capital flows can lead to uncoordinated policy choices that are tighter than would occur with coordination. We consider a symmetric two-country macro model in which countries have limited ability to issue state-contingent contracts in international markets. Accordingly, output endogenously depends on the relative share of wealth held by each country. Because markets are incomplete, welfare can be improved by regulating countries' borrowing positions. Tighter macroprudential policy in country A (limiting leverage or capital inflows) stabilizes country A and endogenously increases the frequency with which A is relatively more wealthy than country B. Thus, tight policy in A provides incentives for B to choose tight policy as well so that B is not poor on average relative to A. We numerically solve for the coordinated and uncoordinated equilibria when countries choose among countercyclical macroprudential policies.

Keywords: International Capital Flows, Capital Controls, Macroeconomic Instability, Macroprudential Regulation, Policy Coordination, Spillovers, Financial Crises.

JEL classification: E44, F36, F38, F42, G15.

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1 Introduction

Following the Great Recession, economists and policy makers have debated the appropriate uses of macroprudential regulation to promote financial and macroeconomic stability.[1] Additionally, many economists have shown greater interest in using capital controls to mitigate the potential adverse consequences of volatile global flows.[2] The effectiveness and implementation of macroprudential policies are complicated by potential spillovers through international capital markets. At the micro-level, Buch and Goldberg (2016) have documented cross-border spillover effects in international lending due to domestic macroprudential policies, such as capital requirements and loan-to-value ratio limits.[3] However, there is likely to be macro-level spillovers arising from changes in aggregate dynamics and global interactions. While there is a rich literature studying the the effects of macroprudential regulation and capital controls, less is understood about how countries should coordinate macroprudential policies given these cross-border interactions.[4]

In light of these considerations, our paper theoretically considers how global spillovers through international capital markets can affect countries' policy choices when policies are not coordinated. We use a two-country, two-good, stochastic macroeconomic model in which countries have limited ability to issue equity in international markets, based off Brunnermeier and Sannikov (2015). International financial markets are imperfect, but there are no trade frictions other than, potentially, regulations imposed by each country limiting domestic leverage or capital inflows. As a result of financial market imperfections, global output depends endogenously on the relative share of wealth in each country, and debt imbalances (leverage, capital inflows) create volatility that increases the fraction of the time the global economy spends with misallocated capital (one or the other country has very low levels of relative wealth). Greater leverage and capital inflows lead to a better static allocation of capital, with the tradeoff that increased volatility can hurt dynamic global stability.

[1] Macroprudential policies include bank capital requirements, counterparty concentration limits, interbank exposure limits, loan-to-value ratios, and reserve requirements.

[2] e.g., Costinot et al. (2014); Farhi and Werning (2014); Caballero and Simsek (2016).

[3] The authors find that "banks with higher initial capital were poised to increase lending internationally...when foreign countries tightened their capital requirements." See also Obstfeld (2012, 2015), Shin (2012), and Rey (2015) for concerns about global financial linkages.

[4] The literature on macroprudential policy and pecuniary externalities includes Caballero and Krishnamurthy (2001, 2004), Aoki et al. (2009, 2010), Bianchi (2010, 2011), Mendoza (2010), and Bianchi and Mendoza (2010).

In this setting, Brunnermeier and Sannikov (2015) show that closed capital accounts can lead to higher welfare as a result of this tradeoff.

We consider when each country separately chooses policies limiting domestic leverage or capital inflows. Since countries trade through imperfect international financial markets, their outcomes depend on the actions of other countries, and foreign spillovers have the potential to positively or adversely the effectiveness of each country's policies. Our main result is that tighter macroprudential regulations in country A will (i) decrease global volatility *when country A is relatively poor*, (ii) improve the "terms of trade hedge" for country A *when country A is relatively poor*, and (iii) as a result, the frequency with which A is relatively poor will decrease.[5] Hence, macroprudential regulation by A will increase the frequency with which country B is relatively poor, which creates strategic incentives for B to respond with tighter macroprudential measures. In other words, even if the regulation by country A can improve welfare for both countries, regulation in country A provides incentives for B to tighten its policy to increase the frequency of being relatively rich.

In light of these results, we then see how policy coordination affects the tightness of regulations chosen by each country. We find that when countries can coordinate macroprudential policies, they choose countercyclical regulation that completely limits leverage when capital is efficiently allocated (closed capital inflows), but allows limited leverage (limited capital inflows) when either country is in crisis (low relative wealth) and capital is misallocated. We then numerically solve for the Nash Equilibrium countercyclical policies when countries cannot coordinate. For our baseline parameters, we find that the Nash policies call for tighter regulation.

While macroprudential regulation in our model is highly stylized, the key insight should apply across a wide range of potential environments. While our model lacks any distinction between the effects of limits on leverage or capital inflows, the key mechanism of our model is that, when international credit markets are imperfect, a policy that stabilizes country A will lead country A to be relatively richer more frequently.[6] This is a negative spillover to country B, which increases

[5] Phelan (2016) considers a closed economy model with a banking sector to show that, when equity markets are imperfect, a more stable economy is rich more frequently.

[6] In our model regulation is a simple leverage constraint, which in the model is equivalent in the aggregate to limiting capital inflows. While in our model leverage limits and controls on capital inflows are identical, in reality these instruments can have very different roles. Korinek and Sandri (2016) quantitatively find the optimal capital control and macroprudential regulation for emerging economies to mitigate contractionary exchange rate depreciations and reduce

the incentive for B to enact stabilizing policies. While a richer setup is likely to generate positive spillovers through other sets of pecuniary externalities, we expect that the negative externality present in our model would continue to be present.[7] Thus, while the quantitative importance of our mechanism will depend on the full set of global interactions, on the margin the mechanism we highlight will lead to tighter uncoordinated policies relative to global coordination.

Related Literature Our paper follows the stochastic continuous-time macro literature, pioneered by Brunnermeier and Sannikov (2014, 2015, 2016) and He and Krishnamurthy (2012, 2013, 2014), who apply continuous-time methods to analyze the non-linear global dynamics of economies with financial frictions, building on seminal results from Bernanke and Gertler (1989); Kiyotaki and Moore (1997); Bernanke et al. (1999). Most closely related, Brunnermeier and Sannikov (2015) demonstrate that closed capital accounts can improve welfare relative to open accounts because closing capital accounts improve global stability arising from incomplete markets. While the global economy is stable near the steady state with high output and growth, away from the steady state, the economy features high asset price volatility and nonlinear amplifications that can be dampened by capital controls. Similarly, Phelan (2016) illustrates that leverage limits improve macroeconomic stability by endogenously increasing the frequency with which the banking sector is well-capitalized, and this can increase welfare

We extend the analysis of Brunnermeier and Sannikov (2015), who only consider completely closed capital accounts as a policy instrument, by allowing countries to choose regulations that occasionally bind, either limiting capital inflows or limiting leverage, and by allowing countries to choose potentially different policies. By giving countries more flexibility in their policy choices, our model admits interesting policy spillovers and strategic considerations. Our focus on countercyclical leverage policy is closely related to Geanakoplos (2003, 2010), who advocates limiting leverage during good times and increasing leverage during crises to address pecuniary externalities arising from collateral constraints.

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the amount and riskiness of financial liabilities. Nonetheless, the authors find it is optimal for emerging economies to employ both instruments in order to improve stability.

[7] For example, Buch and Goldberg (2016) find that international spillovers vary across prudential instruments and are heterogeneous across banks.

Furthermore, our model contributes to the literature on coordinated policy in international settings due to spillovers. Banerjee et al. (2016) explore the impact of spillovers from the macroeconomic policies of advanced economies to emerging market economies. Ghosh and Masson (1991) find that with learning, coordinated policies outperform activist uncoordinated policies or exogenous money targets. In addition to the literature on macroprudential policy and pecuniary externalities cited earlier, Hahm et al. (2011) evaluate the impact of macroprudential policies when applied to open emerging economies as opposed to advanced economies. Korinek and Simsek (2016) apply macroprudential policy in the case of liquidity traps with overborrowing, finding that interest rate policy is inferior in dealing with excessive leverage. Farhi and Werning (2016) incorporate nominal rigidities and financial market frictions into a general theory and provides a simple formula that characterizes optimal financial market intervention.

There is a large literature studying the effectiveness of macroprudential regulation and capital controls, as well as the potential spillover effects. Analysis of early experiences with macroprudential instruments shows that some tools can reduce banks' asset growth within countries (Claessens et al., 2013). However, effectiveness may be weakened when risky or excessive lending moves outside of the regulatory perimeter to non-covered entities or activities (Bengui and Bianchi, 2014; Aiyar et al., 2014; Reinhardt and Sowerbutts, 2015). Kuttner and Shim (2016) find that debt-to-income ratios significantly affect housing credit growth. Bruno et al. (2016) document the role of macroprudential policies and capital controls in mitigating credit growth in 12 Asian economies. Berrospide et al. (2016) find that some regulatory changes spill over.[8] For evidence on the limited effectiveness of partial or limited capital controls see Klein (2012) and Klein and Shambaugh (2015). Empirical evidence about the effect of capital account liberalizations are mixed, e.g., Obstfeld and Taylor (2004) and Magud et al. (2011).

Outline The remainder of the paper is organized as follows. Section 2 presents the model and the equilibrium conditions. Section 3 solves for equilibrium in a loosely calibrated numerical case

[8]Specially, a foreign country's tightening of limits on loan-to-value ratios and local currency reserve requirements increase lending growth in the United States; a foreign tightening of capital requirements shifts lending by U.S. global banks away from the country where the tightening occurs to the United States; tighter U.S. capital regulation reduces lending by large U.S. global banks to foreign residents.

and illustrates the effects of symmetric and asymmetric regulations. Section 4 solves for optimal coordinated and uncoordinated policies in two cases: constant (fixed) leverage limits and piecewise limits that allow counter-cyclical or pro-cyclical policy. Section 5 concludes.

2 The Model

This section presents a slightly modified version of the two-country model in Brunnermeier and Sannikov (2015). The global economy is populated by agents who live in two different countries, A and B. Agents use capital to produce the two intermediate consumption goods a and b, and agents in country A have productive advantages at producing good a (vice versa B and b). Capital trades in a competitive market, but agents in each country may be subject to capital constraints owing to macroprudential policies in each country. Financial frictions limit international credit flows to risk-free debt (non-state contingent contracts).

2.1 Setup

Technology. Time is infinite and continuous. Capital can be used to produce either good a or good b, which are then combined into an aggregate (final) consumption good. Final consumption is given by

$$y_t = (y_t^a)^{1/2}(y_t^b)^{1/2}, \tag{1}$$

where y_t^a is the supply of good and y_t^b is the supply of good b.

Agents in either country can produce goods a and b using linear production technologies, but agents in country A have a superior technology for good a while agents in country b have a superior technology for good b. In particular, given k_t units of a capital, agents in country A produce good a at rate $\bar{a}k_t$ and good b at rate $\underline{a}k_t$, where $\bar{a} > \underline{a} > 0$. Country B agents face the reverse situation, with good a produced at rate $\underline{a}k_t$ and good b at rate $\bar{a}k_t$.

Denote the aggregate amount of world capital available at time t by K_t, and denote the share of world capital held by agents in country A and in country B by ψ_t^A and ψ_t^B, respectively. Furthermore, denote the fraction of world capital devoted to production of goods a and b by ψ_t^{Aa}, ψ_t^{Ab},

ψ_t^{Ba}, and ψ_t^{Bb}, where the first superscript denotes the country. By definition,

$$\psi_t^{Aa} + \psi_t^{Ab} + \psi_t^{Ba} + \psi_t^{Bb} = 1. \tag{2}$$

The aggregate supply of good a and b then are given by

$$Y_t^a = (\bar{a}\psi_t^{Aa} + \underline{a}\psi_t^{Ba})K_t, \quad Y_t^b = (\underline{a}\psi_t^{Ab} + \bar{a}\psi_t^{Bb})K_t, \tag{3}$$

yielding the total supply of the aggregate good

$$Y_t = (Y_t^a)^{1/2}(Y_t^b)^{1/2}. \tag{4}$$

Let the final good be the numeraire. Then the prices of a and b can be written as

$$P_t^a = \frac{1}{2}\left(\frac{Y_t}{Y_t^a}\right)^{1/s}, \quad P_t^b = \frac{1}{2}\left(\frac{Y_t}{Y_t^b}\right)^{1/s}. \tag{5}$$

There is a single type of physical capital. We model productivity shocks as shocks directly to capital, which can be interpreted as shocks to "effective capital." Capital in country I evolves according to

$$\frac{dk_t}{k_t} = gdt + \sigma^I dZ_t^I, \tag{6}$$

where dZ_t^I is a Brownian motion. The two Brownian motions, dZ_t^A and dZ_t^B, are independent and exogenous. This specification is tractable and admits the interpretation of global aggregate shocks with idiosyncratic (negatively correlated) country specific shocks. Thus, the shocks can capture country-specific productivity gains as well as redistributive shocks (e.g. international law suits). It follows that aggregate capital follows the law of motion

$$\frac{dK_t}{K_t} = gdt + \psi_t^A\sigma^A dZ_t^A + \psi_t^B\sigma^B dZ_t^B. \tag{7}$$

For modeling tractability we suppose that capital grows at a constant rate g. This simplifies the analysis and the computations but does not significantly effect our results.[9]

Preferences. All agents have log utility with intertemporal preferences described by the expected utility function

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\log c_t dt\right], \tag{8}$$

where c_t is the consumption of the final good at time t and $\rho \in (0,1)$ is the discount factor.

Markets for Physical Capital and Risk-free Bonds. Agents can trade physical capital in a competitive international market. We denote the equilibrium market price of capital per unit by q_t. Hence, capital k_t has market value $q_t k_t$. We postulate that q_t evolves endogenously according to

$$\frac{dq_t}{q_t} = \mu_t^q dt + \sigma_t^{qA} dZ_t^A + \sigma_t^{qB} dZ_t^B, \tag{9}$$

where μ_t^q, σ_t^{qA}, and σ_t^{qB} will be determined in equilibrium.

Agents can also trade a risk-free bond that is in zero net supply. We denote the endogenously determined risk-free return by dr_t^F. Agents can borrow or save in the risk-free asset, but they may face borrowing limits imposed by regulation in their country.

Returns from Capital. The return from holding capital can be written as diffusion processes by summing capital gains $d(q_t k_t)/(q_t k_t)$, which has two volatility terms, and the dividend yield from using capital to produce good a or b, which has no volatility terms. We denote the return from an agent in country I buying physical capital and using it to produce good j by dr_t^{Ij}. Given equations (6) and (9) for the laws of motion for capital and the capital price and using Ito's Lemma, returns

[9] Our results regarding how regulation affects equilibrium is the same if we follow Brunnermeier and Sannikov (2015) and suppose that capital grows at a rate $\Phi(\iota_t)$, where ι_t is the investment rate. However, solving for the Nash Equilibrium with uncoordinated policy is computationally intensive, which is why we use the simpler setup. We obtain similar results if we suppose that the capital growth rate is a function of the capital price without directly modeling investment.

are given by

$$dr_t^{Aa} = \left(\frac{\bar{a}P_t^a}{q_t} + \mu_t^q + g + \sigma^A \sigma_t^{qA}\right) dt + (\sigma^A + \sigma_t^{qA}) dZ_t^A + \sigma_t^{qB} dZ_t^B, \tag{10}$$

$$dr_t^{Ab} = \left(\frac{\underline{a}P_t^b}{q_t} + \mu_t^q + g + \sigma^A \sigma_t^{qA}\right) dt + (\sigma^A + \sigma_t^{qA}) dZ_t^A + \sigma_t^{qB} dZ_t^B, \tag{11}$$

$$dr_t^{Ba} = \left(\frac{\underline{a}P_t^a}{q_t} + \mu_t^q + g + \sigma^B \sigma_t^{qB}\right) dt + \sigma_t^{qA} dZ_t^A + (\sigma^B + \sigma_t^{qB}) dZ_t^B, \tag{12}$$

$$dr_t^{Bb} = \left(\frac{\bar{a}P_t^b}{q_t} + \mu_t^q + g + \sigma^B \sigma_t^{qB}\right) dt + \sigma_t^{qA} dZ_t^A + (\sigma^B + \sigma_t^{qB}) dZ_t^B. \tag{13}$$

Notably, country A (B) shocks affect the returns to capital used in country A (B) directly (k_t changes) and indirectly through the effect on the capital price q_t; country B (A) shocks affect returns only indirectly through the effect on the capital price.

Incomplete Markets, Financial Frictions, and Macroprudential Regulation. The key financial friction in this model is the inability of countries to issue equity to each other. Agents can only trade risk-free bonds to purchase capital, and as a result, markets are incomplete because they cannot trade the equivalents of Arrow securities but are limited to non-contingent financial contracts. In addition, agents cannot sell short investment in the production of good a and b. Incomplete international markets can be motivated by home bias in equity holdings and micro-founded by agency problems and asymmetric information.[10]

The heart of our analysis involves macroprudential regulations in each country. We model macroprudential regulation as borrowing limits requiring that leverage not exceed a country-specific threshold. As will become clear, in the aggregate, imposing leverage limits is equivalent to imposing limits on capital inflows.

Consumption and Portfolio Choice Each agent decides her consumption rate c_t as well as how to allocate remaining wealth. Agents face a portfolio choice problem of how much capital to invest in the production of goods a and b, and how much to invest in the risk-free bond. We denote the

[10]Heathcote and Perri (2013) provide an explanation for the empirically observed bias toward domestic asset holdings. See Jensen and Meckling (1976), Bolton and Scharfstein (1990), and Holmström and Tirole (1997) for theories of agency problems and asymmetric information limiting state-contingent contracts.

portfolio weights by $(x_t^a, x_t^b, 1-x_t^a-x_t^b)$, where x_t^a is the fraction of wealth invested in capital used to produce good a, x_t^b is the fraction of wealth invested in capital used to produce good b, and $1-x_t^a-x_t^b$ is the fraction of wealth invested in the risk-free asset. Portfolio weights x_t^a and x_t^b must be non-negative.

Denote L_t^A and L_t^B as the leverage constraints imposed on country A and B, respectively. These constraints impose that the market value of an agent's borrowing (debt) cannot exceed L_t^I times their wealth (leverage of 0 implies no borrowing). In other words, for an agent in country I

$$x_t^a + x_t^b \leq L_t^I + 1. \tag{14}$$

Given a consumption rate c_t^I and portfolio weights $(x_t^a, x_t^b, 1-x_t^a-x_t^b)$, the the net worth n_t^I of an agent evolves according to

$$\frac{dn_t^I}{n_t^I} = x_t^a dr_t^{Ia} + x_t^b dr_t^{Ib} + (1-x_t^a-x_t^b)dr_t^F - \frac{c_t^I}{n_t^I}dt. \tag{15}$$

Thus, investors' problems can be summarized as maximizing utility (8) subject to the budget constraint (15) together with the solvency constraint $n_t \geq 0$ and the borrowing constraint (14). Since agents have log utility, optimal consumption is to consume a fraction ρ of net wealth, implying $c_t = \rho n_t$ for all agents.

Definition 1 (Equilibrium)**.** *For any initial allocation of wealth, an equilibrium is a map from histories of shocks $\{Z_s^A, Z_s^B, s \in [0,t]\}$ to the allocation of capital $(\psi_t^{Aa}, \psi_t^{Ab}, \psi_t^{Ba}, \psi_t^{Bb})$ and the aggregate consumption good (C_t^A, C_t^B) as well as price q_t and risk-free rate dr_t^F such that*

1. *All agents solve their optimal consumption and portfolio choice problems, subject to the solvency constraint on their net worth and leverage constraints.*

2. *Capital, consumption, and debt markets clear.*

2.2 Solving for Equilibrium

Since countries cannot issue equity internationally but can only trade in capital and risk-free assets, agents' portfolio decisions depend on their level of wealth, and so equilibrium depends on the aggregate level of wealth in each country. For example, as country A's wealth increases, its aggregate capital holdings will increase, which will increase the fraction of global capital used to produce good a. Thus, capital allocations and final good production will depend on country's relative wealth, which will vary in response to global shocks. We use stochastic continuous-time methods to solve for global equilibrium dynamics. We solve for a recursive (or Markov), rational-expectations equilibrium in which the single state variable is the relative wealth of the two countries.

Denote the aggregate net worth of agents in country A at time t by N_t. Then the relative share of net wealth held by country A is defined to be $\eta_t \equiv \frac{N_t}{q_t K_t}$. Thus, η_t represents the share of global wealth held by country A. The aggregate portfolio choice of country A can be written as

$$\left(\frac{\psi_t^{Aa}}{\eta_t}, \frac{\psi_t^{Ab}}{\eta_t}, 1 - \frac{\psi_t^{A}}{\eta_t}\right), \tag{16}$$

and the aggregate portfolio choice of country B as

$$\left(\frac{\psi_t^{Ba}}{1-\eta_t}, \frac{\psi_t^{Bb}}{1-\eta_t}, 1 - \frac{\psi_t^{B}}{1-\eta_t}\right), \tag{17}$$

where $\psi_t^A = \psi_t^{Aa} + \psi_t^{Ab}$ and $\psi_t^B = \psi_t^{Ba} + \psi_t^{Bb}$. Equilibrium, therefore, consists of an endogenous law of motion for η_t and capital allocations and prices which are functions of the state variable η_t. Since all agents consume a fraction ρ of their wealth, market clearing for the final consumption good implies that the equilibrium capital price satisfies

$$q_t = \frac{(\bar{a}\psi_t^{Aa} + \underline{a}\psi_t^{Ba})^{1/2}(\underline{a}\psi_t^{Ab} + \bar{a}\psi_t^{Bb})^{1/2}}{\rho}. \tag{18}$$

Asset-Pricing Equations. Since all agents have log utility, when leverage constraints do not bind investors choose portfolios so that the Sharpe ratio of investments equal the volatility of net worth.

Iin equilibrium, when leverage constraints do not bind, returns on production must satisfy

$$\mathbb{E}[dr_t^{Aa}] - dr_t^F = \frac{\psi_t^A}{\eta_t}(\sigma^A + \sigma_t^{qA})^2 + \frac{\psi_t^A}{\eta_t}(\sigma_t^{qB})^2, \tag{19}$$

$$\mathbb{E}[dr_t^{Ab}] - dr_t^F \le \frac{\psi_t^A}{\eta_t}(\sigma^A + \sigma_t^{qA})^2 + \frac{\psi_t^A}{\eta_t}(\sigma_t^{qB})^2, \tag{20}$$

$$\mathbb{E}[dr_t^{Ba}] - dr_t^F \le \frac{\psi_t^B}{1-\eta_t}(\sigma_t^{qA})^2 + \frac{\psi_t^B}{1-\eta_t}(\sigma^B + \sigma_t^{qB})^2, \tag{21}$$

$$\mathbb{E}[dr_t^{Bb}] - dr_t^F = \frac{\psi_t^B}{1-\eta_t}(\sigma_t^{qA})^2 + \frac{\psi_t^B}{1-\eta_t}(\sigma^B + \sigma_t^{qB})^2, \tag{22}$$

where the term on the right hand side can be interpreted as the risk premium that agents must earn in order to invest in the production of good a or b.[11] Notice that the excess return on the production of the disadvantaged good is less than or equal to the risk premium, which reflects that in equilibrium there exist ranges of η_t where country A produces only good a and country B produces only good b. However, when leverage constraints bind, the excess returns can exceed the risk premium (the right-hand side of the equations above). The leverage constraint will bind for agents in country A when, subject to no constraints, $\psi_t^A/\eta_t \ge L^A + 1$ and for country B when, subject to no constraints, $\psi_t^B/(1-\eta_t) \ge L^B + 1$. Thus $\psi_t^A/\eta_t \le L^A + 1$ and $\psi_t^B/(1-\eta_t) \le L^B + 1$ over the state space. A country with positive leverage invests using capital inflows. Thus, limiting leverage has the effect, in the aggregate, of restricting capital inflows.

Characterizing Equilibrium Using the returns equations, together with market clearing for capital and consumption, we can characterize equilibrium as a system of differential equations in the capital price q_t. We first characterize equilibrium when leverage constraints do not bind, which follows immediately from Brunnermeier and Sannikov (2015).

Proposition 1. *When leverage constraints do not bind, the equilibrium law of motion of η will be endogenously given as*

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^{\eta A} dZ_t^A + \sigma_t^{\eta B} dZ_t^B, \tag{23}$$

[11] See Brunnermeier and Sannikov (2015) for technical details.

where

$$
\begin{aligned}
\mu_t^{\eta} = &\left(\frac{\psi_t^A}{\eta_t}\right)^2 \left[(1-\eta_t)^2(\sigma^A+\sigma_t^{qA})^2+(1+\eta_t^2)(\sigma_t^{qB})^2\right] \\
&-(\psi_t^B)^2\left(\frac{\eta_t}{1-\eta_t}\right)\left[(\sigma^B+\sigma_t^{qB})^2+(\sigma_t^{qA})^2\right] \\
&+\left(\frac{\psi_t^A}{\eta_t}\right)\left(\psi_t^B\right)\left[(2\eta_t-1)(\sigma^A+\sigma_t^{qA})\sigma_t^{qA}+(2\eta_t+1)(\sigma^B+\sigma_t^{qB})\sigma_t^{qB}\right] \\
\sigma_t^{\eta A} = &\frac{1-\eta_t}{\eta_t}\psi_t^A(\sigma^A+\sigma_t^{qA})-\psi_t^B\sigma_t^{qA}, \\
\sigma_t^{\eta B} = &\frac{1-\eta_t}{\eta_t}\psi_t^A\sigma_t^{qB}-\psi_t^B(\sigma^B+\sigma_t^{qB}).
\end{aligned}
$$

Asset prices satisfy

$$
\begin{aligned}
&\frac{\bar{a}\left(P_t^a-P_t^b\right)}{q_t}+\sigma^A\sigma_t^{qA}-\sigma^B\sigma_t^{qB}= \\
&\qquad \frac{\psi_t^A}{\eta_t}\left((\sigma^A+\sigma_t^{qA})^2+\left(\sigma_t^{qB}\right)^2\right)-\frac{1-\psi_t^A}{1-\eta_t}\left((\sigma^B+\sigma_t^{qB})^2+\left(\sigma_t^{qA}\right)^2\right),
\end{aligned}
\tag{24}
$$

The state space is divided into 3 regions. For $\eta < \eta^a$, both countries produce good a and country B produces good b. For $\eta > \eta^b$ both countries produce good b and country A produces good a. For $\eta \in [\eta^a, \eta^b]$ countries specialize, using only their most productive technology. Goods prices satisfy

$$
\frac{\underline{a}}{\bar{a}} \leq \frac{P_t^b}{P_t^a} \leq \frac{\bar{a}}{\underline{a}}, \tag{25}
$$

where the first (second) inequality becomes equality in the left (right) region of the state space.

However, equilibrium is slightly modified when leverage constraints bind. Crucially, leverage constraints affect equilibrium prices and allocations only when they bind, but when constraints do not bind equilibrium prices and allocations are the same as in an economy in which leverage constraints never bind. Additionally, with *constant* leverage constraints over the binding region, we can analytically solve for the capital price $q(\eta)$ as well as its derivative $q'(\eta)$. When leverage constraints cease to bind, the capital price (and as a result capital allocations) are the same as in an economy in which leverage constraints never bind. Second, over the range of η where the leverage constraint binds, η_t follows a different law of motion until the leverage constraint no longer binds, at which point the law of motion reverts to (23).

Proposition 2. *When country A's leverage constraint binds, $\psi_t^A/\eta_t = 1 + L^A$, and η_t follows*

$$\begin{aligned}
\frac{d\eta_t}{\eta_t} = {} & \frac{\psi_t^A}{\eta_t}\left(\frac{\bar{a}(P_t^a - P_t^b)}{q_t}\right) + \left(\frac{\bar{a}P_t^b}{q_t}\right) - \rho + \left(1 + \frac{L^A}{1-\eta_t}\right)\left((\sigma_t^{qA})^2 + (\sigma^B + \sigma_t^{qB})^2\right) + (\psi_t^A)^2((\sigma^A)^2 + (\sigma^B)^2) \\
& - \left(\frac{\psi_t^A}{\eta_t}\right)\left[(\sigma^A + \sigma_t^{qA})\sigma_t^{qA} + (\sigma^B + \sigma_t^{qB})\sigma_t^{qB}\right] + \left(\frac{\psi_t^A}{\eta_t} - \psi_t^A\right)(\sigma^A\sigma_t^{qA} - \sigma^B\sigma_t^{qB}) \\
& + \psi_t^A\left[(1+L^A)(\sigma^B\sigma_t^{qB} - (\sigma^A + \sigma_t^{qA})) + 2(\sigma^A\sigma_t^{qA} - \sigma^B(\sigma^B + \sigma_t^{qB})) - \frac{L^A}{1-\eta_t}((\sigma_t^{qA})^2 + (\sigma^B + \sigma_t^{qB})^2)\right] \\
& + \left[(1-\eta_t)(1+L^A)\sigma^A + L^A\sigma_t^{qA}\right]dZ_t^A + \left[L^A\sigma_t^{qB} - (1 - \eta_t(1+L^A))\sigma^B\right]dZ_t^B,
\end{aligned} \tag{26}$$

and q_t evolves according to

$$\begin{aligned}
\frac{dq_t}{q_t} = {} & \frac{q'(\eta_t)}{q(\eta_t)}\eta_t\mu_t^\eta dt + \frac{q'(\eta_t)}{q(\eta_t)}\eta_t((1-\eta_t)(1+L^A)\sigma^A + L^A\sigma_t^{qA})dZ_t^A \\
& + \frac{q'(\eta_t)}{q(\eta_t)}\eta_t(L^A\sigma_t^{qB} - (1-\eta_t(1+L^A))\sigma^B)dZ_t^B \\
& + \frac{1}{2}\frac{q''(\eta_t)}{q(\eta_t)}\eta_t^2[((1-\eta_t)(1+L^A)\sigma^A + L^A\sigma_t^{qA})^2 + (L^A\sigma_t^{qB} - (1-\eta_t(1+L^A))\sigma^B)^2]dt
\end{aligned} \tag{27}$$

where the volatilities σ_t^{qA} and σ_t^{qB} can be explicitly written as

$$\sigma_t^{qA} = \frac{\frac{q'(\eta_t)}{q(\eta_t)}\eta_t(1-\eta_t)(1+L^A)\sigma^A}{1 - \frac{q'(\eta_t)}{q(\eta_t)}\eta_t L^A}, \quad \sigma_t^{qB} = -\frac{\frac{q'(\eta_t)}{q(\eta_t)}\eta_t(1-\eta_t(1+L^A))\sigma^B}{1 - \frac{q'(\eta_t)}{q(\eta_t)}\eta_t L^A}. \tag{28}$$

Furthermore, when countries specialize in production

$$q(\eta) = \frac{\bar{a}}{\rho}\sqrt{\eta(1+L_A)(1-\eta(1+L_A))}, \tag{29}$$

$$q'(\eta) = \frac{q}{2}\left[\frac{1-2\eta(1+L_A)}{\eta(1-\eta(1+L_A))}\right], \tag{30}$$

and when countries do not specialize,

$$q(\eta) = \frac{a\sqrt{\tau}}{2\rho}(1+\eta(1+L_A)(\tau-1)), \tag{31}$$

$$q'(\eta) = q\left(\frac{(1+L_A)(\tau-1)}{1+\eta(1+L_A)(\tau-1)}\right), \tag{32}$$

where $\tau = \frac{\bar{a}}{\underline{a}}$*. When leverage constraints bind, the range of* η *for which countries specialize or not can be solved in closed form using the above expressions.*

We can similarly solve for capital prices, allocations, and equilibrium evolutions when B's constraints bind.

Welfare The goal of our analysis is to understand how countries choose regulation in order to maximize the welfare of its agents. We can evaluate the effects of leverage constraints on the welfare of agents in a country using Propositions 1 and 2. From Brunnermeier and Sannikov (2015), the value function for the representative agent for country A is of the form

$$V^A(N_t,\eta_t) + \frac{\log N_t}{\rho} + h^A(\eta_t),$$

where $h^A(\eta_t)$ depends on the market frictions in the model. We can write $N_t = \eta_t q_t K_t$, so

$$V^A(N_t,\eta_t) = V^A(\eta_t) = \frac{\log \eta_t}{\rho} + \frac{\log K_t}{\rho} + \frac{\log q(\eta_t)}{\rho} + h^A(\eta_t).$$

Solving the Hamilton-Jacobi-Bellman equation is equivalent to solving the following second-order differential equation for $H^A(\eta_t) \equiv \log(q(\eta_t))/\rho + h^A(\eta_t)$.

$$\begin{aligned}\rho H^A = \log(\rho q(\eta_t)) + \frac{\mu_t^\eta}{\rho} - \frac{(\sigma_t^{\eta A})^2 + (\sigma_t^{\eta B})^2}{2\rho} + \frac{g}{\rho} \\ - \frac{(\psi_t^A\sigma^A)^2 + (\psi_t^B\sigma^B)^2}{2\rho} + \mu_t^\eta \eta_t (H^A)' + \frac{(\sigma_t^{\eta A})^2 + (\sigma_t^{\eta B})^2}{2}\eta_t^2 (H^A)''.\end{aligned} \tag{33}$$

$V^B(N_t,\eta_t) = V^B(\eta_t)$ follows a symmetric equation.

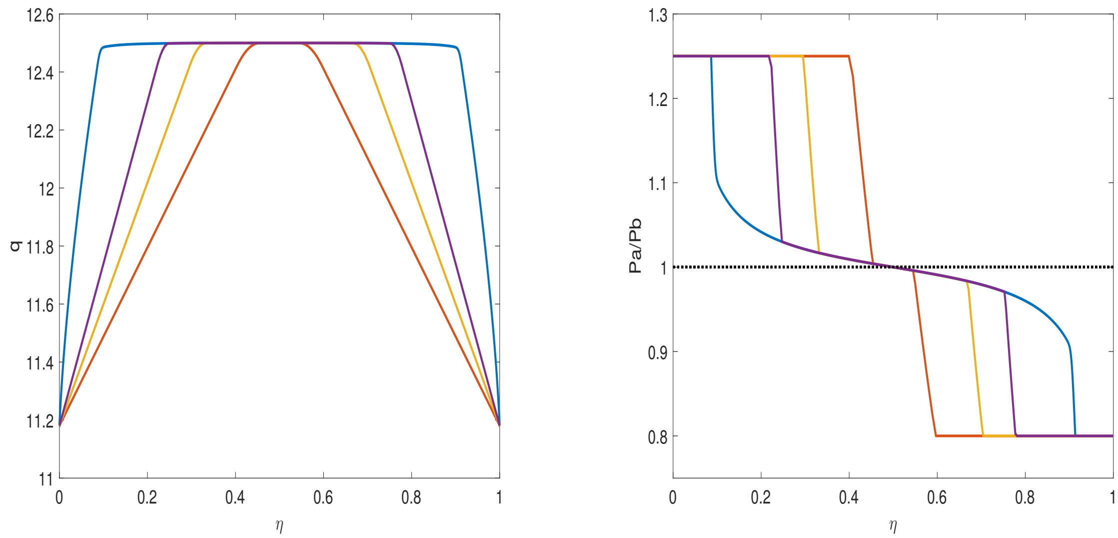
3 Numerical Example

To illustrate the effects of leverage constraints on equilibrium and welfare, we solve the model numerically using parameters roughly calibrated to match developed countries (our qualitative results are robust across a range of parameters). The most important parameters are the volatilities, which we set to $\sigma^A = \sigma^B = 3\%$, which is roughly the volatility of TFP shocks. We normalize productivity to $\bar{a} = 1$ and set $\underline{a} = 0.8$, implying gains from specialization in trade of 25 percent. The discount factor has negligible effects on the results (we set $\rho = 4\%$) and the growth rate only affects the level of welfare (we set $g = 2\%$). We first look at how constraints affect prices and the stationary distribution (stability) of the global economy. We first consider symmetric constraints, and for clarity, let $L^A = L^B = L$, and then we consider asymmetric constraints. Then we consider the effects of constraints on welfare.

The economy is typically stable, with a stochastic steady state at $\eta = .5$ where capital is best allocated (countries are symmetric), and the economy tends to drift toward $\eta = .5$ after shocks move the system away. Equilibrium allocations and evolutions of η_t are presented in the appendix.

3.1 Prices and stability

Figure 1 plots equilibrium capital prices $q(\eta)$ and the ratio of goods prices. Ordered from loosest to tightest, Blue refers to the unconstrained case, purple to $L = 1$, yellow to $L = 0.5$, and red to $L = 0.1$. Panel (a) illustrates that tighter leverage constraints result in lower capital prices with slope. Figure 2 plots the stationary distribution of η. As discussed, the economy is typically near the stochastic steady state, and the frequency of time spent away from the steady state (near 0 or 1) is greater when leverage constraints are looser. As leverage constraints tighten, the density becomes more concentrated in the middle, indicating that the economy is more likely to stay near $\eta = 0.5$, where capital is best allocated. Thus, large fluctuations in η associated with a capital price crash become increasingly rare, reflecting lower systemic volatility, so agents will actually attain a superior consumption flow over an infinite time horizon, despite their borrowing restrictions. Part of the reason is that the drift of $d\eta_t$ drives η back to 0.5 much faster with leverage constraints, except in the case of exceptionally adverse shocks. (See the appendix for plots for



(a) Capital price, q_t (b) Ratio of goods prices P_t^a/P_t^b

Figure 1: Equilibrium prices. Blue: unconstrained. Purple: $L = 1$. Yellow: $L = .5$. Red: $L = .1$.

drift and volatility.)

The model's symmetry makes it easy to see how asymmetric constraints affect equilibrium. Consider when $L^A = 0.1$ and $L^B = 0.5$ (country A has tighter constraints). For $\eta \in (0, .5)$ the capital price would follow the red line in Figure 1(a), which was the capital price when both countries symmetrically imposed $L = .1$. For $\eta \in (.5, 1)$ the capital price would follow the yellow line, which is the capital price when both countries symmetrically chose $L = .5$. Most interesting, though, is the effect on the stationary distribution, which is now asymmetric, reflecting that countries are no longer symmetric (due to policies). In particular, when one country imposes a tighter leverage constraint, the economy will spend more time in regions where the less-constrained country takes on leverage. Tighter leverage constraints improve stability for that country, increasing the relative frequency of having a high level of wealth.

We illustrate this result in two ways. Figure 3 plots the stationary density when $L^A = 0.1$ and $L^B = 0.5$, and when countries symmetrically choose $L = .1$ and $L = .5$. When A imposes tighter constraints, the distribution shifts right: the mode continues to be at $\eta = 0.5$, but the mean is higher. The economy is more concentrated around $\eta = 0.5$ than when $L^A = L^B = 0.5$ but less

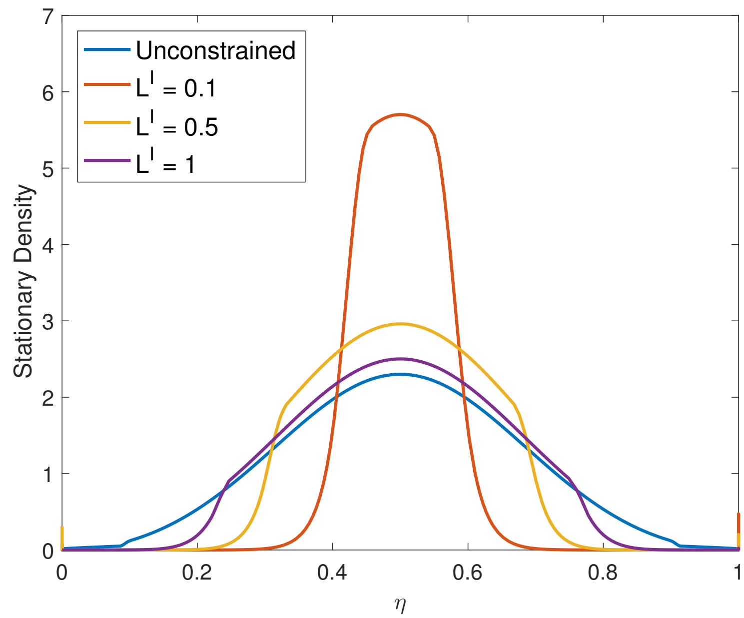


Figure 2: Stationary density of η under symmetric leverage constraints. Blue: unconstrained. Purple: $L = 1$. Yellow: $L = .5$. Red: $L = .1$.

concentrated than had $L^A = L^B = 0.1$. In addition, the economy spends more time in the region $(0.5, 0.7)$ than had both countries imposed looser leverage constraints and comparatively less time in $(0.3, 0.4)$. In this case, A's constraints make country A stable relative to B and thus A is more likely to grow and acquire wealth compared to B. As a result the global economy endogenously spends more time with B relatively poorer.

Figure 4 plots the stationary distribution when A has no leverage constraint but B does, and when countries symmetrically impose either no or tight constraints. Similarly to the previous case, when country B imposes tighter constraints the distribution shifts toward $\eta = 0$, with the mode continuing to be at $\eta = 0.5$ but the mean significantly lower. In this case, B's constraints make country B stable relative to A and thus B is more likely to grow and acquire wealth compared to A. As a result the global economy endogenously spends more time with A relatively poorer.

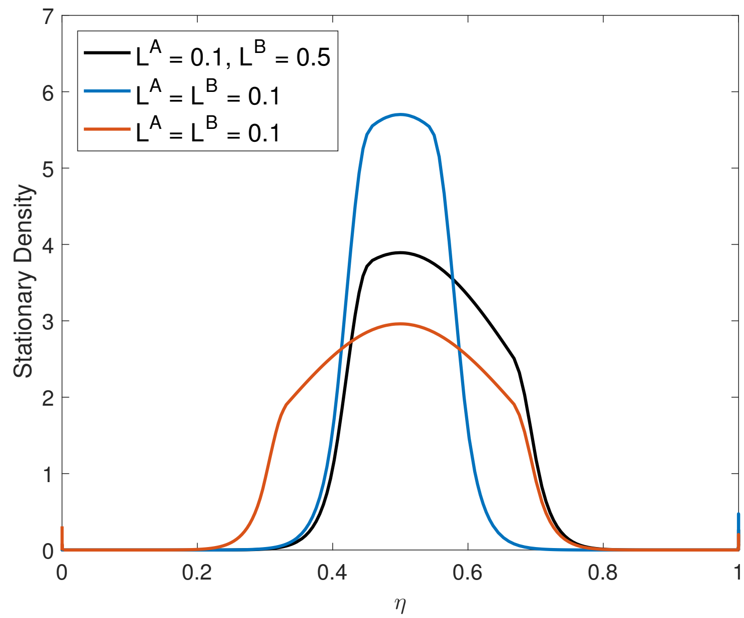


Figure 3: Stationary distributions with asymmetric leverage constraints. Black (asymmetric): $L^A = 0.1$ and $L^B = 0.5$. Blue (symmetric): $L^A = L^B = 0.1$. Red (symmetric): $L^A = L^B = 0.5$.

3.2 Welfare Analysis

Figure 5 displays country A's welfare as a function of η.[12] Similar to the welfare plot in Brunnermeier and Sannikov (2015), for low η, leverage constraints reduce welfare, but at higher η, regulation yields higher welfare. Furthermore, as η continues to increase, welfare when $L^A = 0.1$ becomes higher than welfare when $L^A = 0.5$, suggesting that as leverage constraints tighten, the welfare benefits only accrue at higher and higher η.

We examine welfare from another perspective in Panel (b) where we plot the value function against a monotone transformation of η using the CDF of the stationary distribution. After this transformation, it appears that leverage constraints almost always improve welfare in the region $(0, 0.5)$, and we argue that this is the correct way to evaluate the impact of macroprudential regulation on welfare, although the reason for this conclusion may not be immediately clear. While the HJB accounts for the future behavior of η, computing the value function requires an initial value for η, which is what the horizontal axis represents in Figure 5. Thus, plotting welfare against η is

[12]The value functions approach $-\infty$ as η approaches zero, so we cut the plot off at -100 to make them clearer. Additionally we restrict the domain to $(0, 0.5)$.

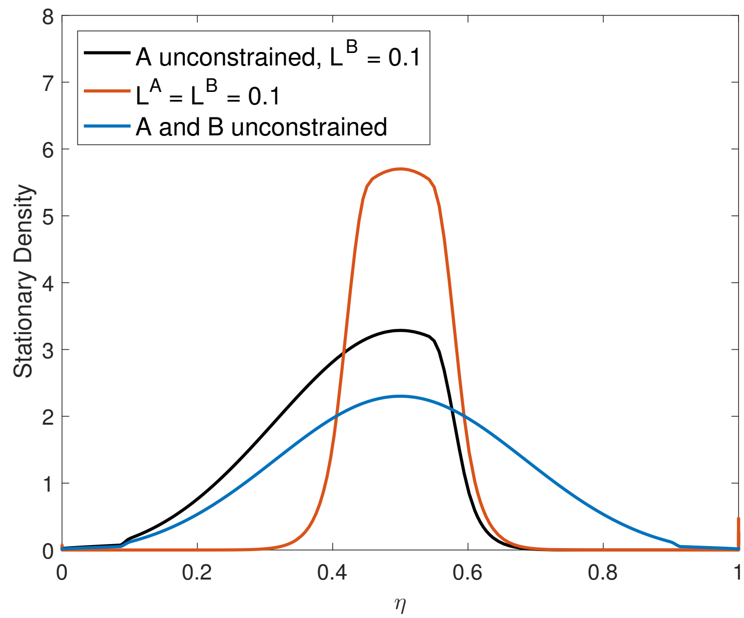
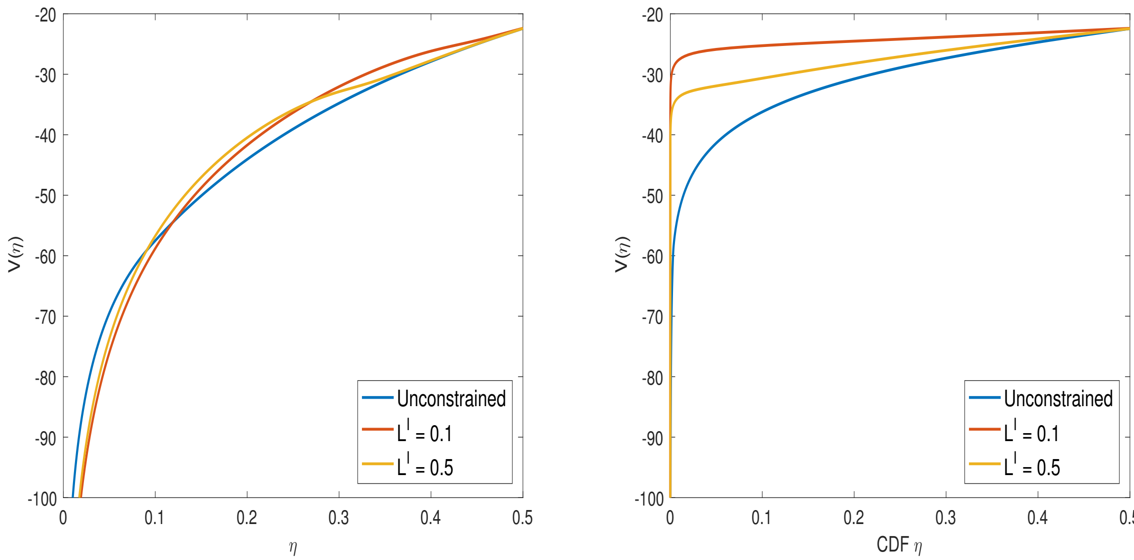


Figure 4: Stationary distributions with asymmetric leverage constraints. Black (asymmetric): A has no leverage constraint, $L^B = 0.1$. Blue (symmetric): no constraints. Red (symmetric): $L^A = L^B = 0.1$.

misleading because it presumes that η is drawn from a uniform distribution. Since there does not exist a clear prior on what the initial wealth distribution ought to be, a sensible initial distribution is not the uniform distribution but the stationary distribution of η because it describes the probability density of η without conditioning on time. In other words, drawing from the stationary distribution reflects the fact that the economy tends to stay around $\eta = 0.5$. Since the economy does not have a clear starting point, one might as well assume that the economy has existed before time $t = 0$, and the best guess of η_0 would then be a random draw from the stationary distribution.

The fact that the initial η ought to be drawn from the stationary distribution underpins why it matters what leverage constraint the other country chooses. For example, in Figure 4 when country A applies no constraint while $L^B = 0.1$, the density of η below 0.5 is larger than it used to be. Thus, we could expect a majority of initial values for η to be drawn from this portion of the state space, so a low η_0 becomes more likely, and Figure 5 Panel (a) demonstrates that the lower η_0 is, the worse country A's utility becomes. We illustrate this insight in Figure 6 with a comparison of welfare with symmetric and asymmetric constraints, and it is immediately clear that welfare



(a) Welfare of country A (b) Welfare against the CDF transformation of η

Figure 5: Welfare with symmetric leverage constraints.

has been affected. In Figure 5(b), as the CDF approaches 0.5, the value functions under different but symmetric capital controls become indistinguishable. With asymmetric constraints, however, even when the CDF equals 0.5, welfare differs significantly depending on the policy that country A adopts. In particular, even though $L^B = 0.5$, country A's best response is not $L^A = 0.5$ because country A's welfare appears almost always greater when $L^A = 0.1$ than when $L^A = 0.5$ over $(0,1)$.

Thus, strategic considerations do indeed arise from cross-border spillovers. Depending on the policies adopted, the stationary distribution of wealth will vary, and this affects welfare across time. When a country imposes comparatively looser constraints, its agents take on too much debt after adverse shocks, and although individually rational, it slows down the economy's return to the optimal allocation of capital.

4 Coordinated and Uncoordinated Policies

In light of the previous results, in this section we let countries choose policies to maximize welfare in each country. We consider two classes of policy tools: fixed leverage limits, and piece-wise

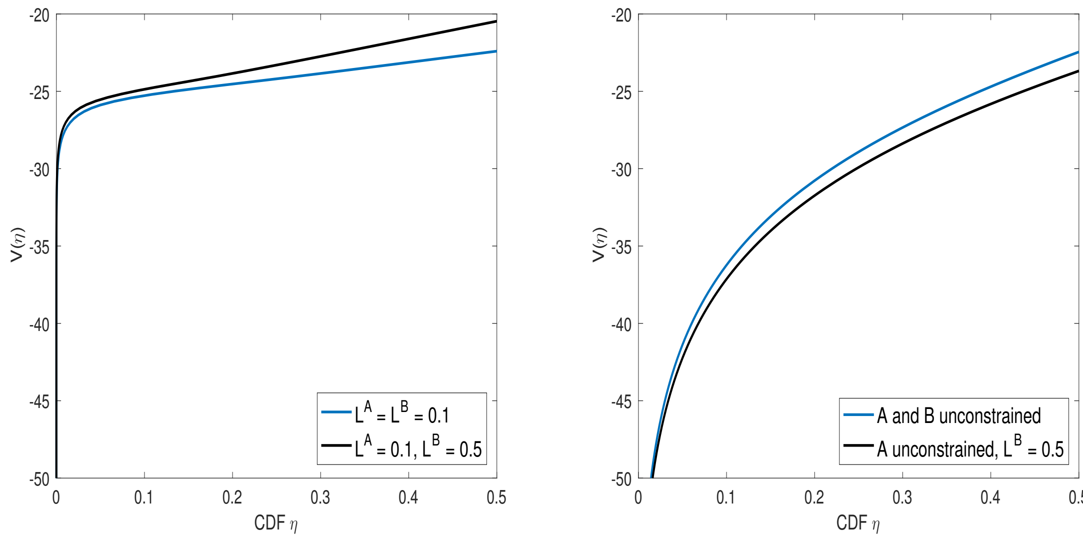


Figure 6: Welfare with asymmetric leverage constraints, plotted against the CDF transformation of η.

countercyclical limits.

4.1 Fixed Leverage Limits

We solve for the optimal set of policies in two ways. First, since countries are symmetric, we suppose that the initial condition is $\eta = 0.5$ (equal wealth shares) and we compute the social optimum (coordinated policies) to maximize $V^{A+B}(0.5)$. We then compute the uncoordinated policy choices (L_N^A, L_N^B) that satisfy a Nash equilibrium policy choice for each country. Second, we take the ex-ante perspective that policy makers do not know the initial condition for η but use the stationary distribution of η to calculate the probability distribution for the initial condition. Since countries are symmetric, we maximize $\mathbb{E}[V^{A+B}(\eta)]$ where the expectation is calculated using the equilibrium stationary distribution given policy constraints. We compute the optimal coordinated and uncoordinated policy choices given this ex-ante objective function.

Table 1 presents the results using the initial condition and compares to the welfare in competitive equilibrium (no constraints). Table 2 presents the results using expected welfare.

In both cases, the Nash equilibrium is to completely close capital accounts, while the social optimum is to allow a small amount of leverage/capital flows. To calculate the Nash equilibrium,

Table 1: Optimal coordinated and uncoordinated fixed leverage limits given symmetric initial wealth shares.

	L^A	L^B	$V^{A+B}(0.5)$
Social optimum	.0728	.0728	-44.8038
Nash	0	0	-44.851
Competitive equilibrium	N/A	N/A	-44.8796

Table 2: Optimal coordinated and uncoordinated fixed leverage limits to maximize ex-ante welfare.

	L^A	L^B	$\mathbb{E}[V^{A+B}(\eta)]$
Social optimum	.0387	.0387	-44.9600
Nash	0	0	-44.9748
Competitive equilibrium	N/A	N/A	-47.9165

we iterate best responses for A, starting at multiple initial levels for B. Since the best responses iterate to the same level regardless of the starting value for B, we are confident that the equilibrium is unique for these parameters.

Our main result—that uncoordinated constraints are tighter than coordinated constraints—holds broadly across parameters. For all parameters we've considered the Nash equilibrium is zero. Additionally, however, for low values of $\underline{a}$ the optimal policy is to completely close capital accounts, consistent with the result in Brunnermeier and Sannikov (2015), and so the Nash and coordinated equilibria correspond. This is consistent with the interpretation that, the more significant is the productivity gain from specialization, the more important is the terms of trade hedge. Accordingly, for $\underline{a} = 0.9$, the socially optimal leverage limit is slightly higher. Furthermore, the social optimum appears to be monotonic in risk (σ), with higher σ leading to looser constraints. In this case, higher risk means that crises are more likely (larger shocks) and so looser constraints alleviate the costs of crises.[13]

[13]For very high levels of risk ($\sigma = 10\%$), it appears that multiple Nash equilibria are possible: one with closed capital accounts (leverage is zero) and one with high leverage almost at laissez-faire.

4.2 Countercyclical Limits

We now allow countries more flexibility when choosing macroprudential policies. To that end, we first considered symmetric, coordinated piecewise policies described by (L_1, L_2, η_*). For $\eta < \eta_*$ country A limits leverage to L_1, and for $\eta \in (\eta_*, .5)$ country A limits leverage to L_2, and symmetrically for country B. A completely state-contingent policy is unrealistic to consider in practice, given issues of time-consistency and implementability (Klein, 2012). (Additionally, solving a completely flexible state-contingent policy is too computationally difficult.) The simple, piecewise rule we consider is flexible enough to allow countries to choose pro-cyclical or countercyclical policies (or completely closed capital accounts), and could realistically be approximated in practice.

Maximizing welfare in this way, countries optimally chose countercyclical policy. In particular, countries choose $L_2 = 0$, completely limiting leverage (or capital inflows) when the economy is near the stochastic steady state, which is when capital is allocated more efficiently, but choose $L_1 > 0$ when the economy is away from steady state and capital is misallocated. Furthermore, the leverage limit binds (leverage equals L_1) because there continue to be welfare gains from limiting the pecuniary externality.

Given these results, we solve for the optimal coordinated and uncoordinated policies restricted to piecewise countercyclical constraints. We suppose countries can adopt a piecewise leverage constraint described by the pair (L_*^I, η_*^I), where $I = A, B$. For the purpose of exposition, first consider $I = A$. For $\eta \leq \eta_*^A$, country A imposes L_*^A as its leverage constraint, and for $\eta > \eta_*^A$, country A closes its capital inflows. Country B's policy behaves similarly, except that when $\eta < \eta_*^B$, country B closes its capital inflows, and when $\eta \geq \eta_*^B$, B adopts leverage constraint L_*^B. In an economy with symmetric capital volatilities, countries adopt $L_*^I > 0$ and $\eta_*^I < 0.5$, which improves welfare in both countries relative to the unconstrained competitive equilibrium, indicating that countercyclical macroprudential policy can be effective. As before, we solve for the optimal policies (i) with the initial condition $\eta = 0.5$ and (ii) with the ex-ante perspective using the equilibrium stationary distribution to calculate expected welfare.[14] Table 3 presents the results using the initial condition and compares to the welfare in competitive equilibrium (no constraints). Table 4 presents the

[14]We have not been able to rule out the possibility of multiple equilibria given the computational difficulty of choosing best responses in two variables.

results using expected welfare.

Table 3: Optimal coordinated and uncoordinated policies given symmetric initial wealth shares.

	L_*^A	L_*^B	η_*^A	η_*^B	$V^{A+B}(0.5)$
Social Optimum	.0983	.0983	.4714	.5286	-44.7138
Nash	.1197	.1197	.3999	.6001	-44.7992
Competitive	N/A	N/A	0.5	0.5	-44.8796

Table 4: Optimal coordinated and uncoordinated policies to maximize ex-ante welfare.

	L_*^A	L_*^B	η_*^A	η_*^B	$\mathbb{E}[V^{A+B}(\eta)]$
Social Optimum	.2393	.2393	.4439	.5561	-44.8050
Nash	.1003	.1003	.4075	.5925	-44.9538
Competitive	N/A	N/A	0.5	0.5	-47.9165

The uncoordinated policies are tighter in two ways in at least one of two ways. First, the Nash leverage limit L_1^I is tighter than in the coordinated case. Second, capital inflows are closed off for a larger range of η. From the ex-ante view, Nash policy tighter in both senses while with the initial condition $\eta = 0.5$, only the second statement is true. However, there is a drastic difference in η_*^I while there is only a small difference in L_*^I, so we argue that Nash policy is still on balance tighter. Notably, the welfare gains from policy—coordinated or otherwise—are larger when calculated in an ex-ante way. This is because leverage constraints improve welfare by increasing global economic stability so that the stationary distribution is more concentrated around $\eta = 0.5$. As a result, using the stationary distribution to calculate welfare adds an additional benefit to the welfare calculation since the distribution used for the expectation is less disperse (the welfare function $V(\eta)$ is concave).

The result that Nash policies are tighter than coordinated policies holds across a range of parameters. The key parameters determining the welfare costs from instability arising from incomplete markets are the productivity loss $\underline{a}$ and volatility σ. (We provide robustness results in the appendix.) We find that across a variety of parameters, coordinated policies are looser than Nash policies in some sense. In particular, for several parameters, such as $\underline{a} = 0.6, 0.7$, coordinated policies are looser in both leverage constraint and η^I_*. For others, such as $\underline{a} = 0.5$, while the leverage constraint is somewhat looser in Nash, the magnitude of the Nash η^I_* is comparatively much tighter than the coordinated choice. When volatility is high, the economy is less stable (i.e., the stationary distribution is more spread out) for any level of leverage limits, but based on our results with $\sigma = 5\%$, coordinated policy is still looser.

Several dimensions of this model can be further extended to provide greater insight as to how global macroprudential regulation should be conducted. First, our analysis ignores any important heterogeneity within countries. Indeed, Buch and Goldberg (2016) find that heterogeneity among banks lead to different responses to cross-border spillovers. Thus, a homogenous constraint may not be the best choice, so further research should be conducted to determine how agent heterogeneity influences the optimal constraint.[15] Second, in our model capital flows are driven by changes in relative wealth—there is no other heterogeneity across countries. However, a robust literature on global imbalances addresses how differences in financial sectors across countries affect capital flows.[16] Including heterogeneity in this dimension would likely provide additional forces for strategic interactions between countries.

5 Conclusion

We have theoretically illustrated how macroprudential policy spillovers through international capital flows can lead to uncoordinated policy choices that are tighter than would occur with coordination. Macroprudential regulation, when effective, increases economic stability. When international credit markets are imperfect, more stable countries are more likely to be relatively wealthy compared to less stable countries. As a result, tight macroprudential policy in one country provides strategic incentives for tight policy in the other. As a result, policy coordination allows countries

[15] To capture some of the additional mechanisms that could be present in a richer model, we extended our analysis to suppose that the capital growth rate is an increasing function of the capital price, $g(q)$. Specifically, we suppose that if q falls below a threshold, adverse selection in capital production leads the growth rate to drop. Our results are qualitatively identical in this environment and quantitatively more significant.

[16] See for example Willen (2004); Caballero et al. (2008); Mendoza et al. (2009); Angeletos and Panousi (2011); Maggiori (2013); Phelan and Toda (2016); Fostel et al. (2017).

to maintain looser constraints than would arise with uncoordinated policy.

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Appendices

A Proofs

A.1 Proof of Proposition 2: Law of Motion for η.

To write out the law of motion for η_t, we first find how N_t and $1/(q_t K_t)$ evolve, and then we use Ito's Product Rule to find $d\eta_t/\eta_t$. The net worth of country A evolves according to

$$\frac{dN_t}{N_t} = x_a^A dr_t^{Aa} + x_b^A dr_t^{Ab} + (1 - x_a^A - x_b^A) dr_t^F - \rho dt.$$

Since we know the laws of motion for q_t and K_t, Ito's Product Rule implies that

$$\frac{d(q_t K_t)}{q_t K_t} = (\mu_t^q + \mu_t^K + \psi_t^A \sigma^A \sigma_t^{qA} + \psi_t^B \sigma^B \sigma_t^{qB})dt + (\psi_t^A \sigma^A + \sigma_t^{qA})dZ_t^A + (\psi_t^B \sigma^B + \sigma_t^{qB})dZ_t^B.$$

Ito's Quotient Rule then implies that

$$\begin{aligned}
\frac{d(1/q_t K_t)}{1/q_t K_t} &= \left(\frac{d(q_t K_t)}{q_t K_t}\right)^2 - \frac{d(q_t K_t)}{q_t K_t} \\
&= [(\psi_t^A \sigma^A + \sigma_t^{qA})^2 + (\psi_t^B \sigma^B + \sigma_t^{qB})^2 - \mu_t^q - \mu_t^K - \psi_t^A \sigma^A \sigma_t^{qA} \\
&\quad - \psi_t^B \sigma^B \sigma_t^{qB}]dt - (\psi_t^A \sigma^A + \sigma_t^{qA})dZ_t^A - (\psi_t^B \sigma^B + \sigma_t^{qB})dZ_t^B.
\end{aligned}$$

Applying Ito's product rule then yields the general form for the evolution of η_t:

$$\frac{d\eta_t}{\eta_t} = \frac{dN_t}{N_t} + \frac{d(1/q_t K_t)}{1/q_t K_t} + \mathrm{Cov}\left[\frac{d(1/q_t K_t)}{1/q_t K_t}, \frac{dN_t}{N_t}\right].$$

We now consider when $\psi_t^A/\eta_t \geq L^A + 1$. First, the leverage constraint can only bind when $\eta_t \in [0, 0.5]$, $\psi_t^{Ab} = 0$, which allows us to write country A's portfolio choice as $(1 + L^A, 0, -L^A)$, where $\psi_t^A = \psi_t^{Aa} = \eta_t(1 + L^A)$. Substitution of rates of return yields after cancelling and rearranging terms

$$\frac{d\eta_t}{\eta_t} = [(1+L^A)\left(\frac{\bar{a}P_t^a}{q_t}\right) + L^A(\mu_t^q + \mu_t^K) + (1+L^A)(1-\eta_t)(\sigma^A\sigma_t^{qA}) - \rho - L^A dr_t^F$$
$$+ (\eta_t(1+L^A)\sigma^A + \sigma_t^{qA})^2 + ((1-\eta_t(1+L^A))\sigma^B + \sigma_t^{qB})^2 - (1-\eta_t(1+L^A))\sigma^B\sigma_t^{qB}$$
$$- (1+L^A)(\sigma^A + \sigma_t^{qA})(\eta_t(1+L^A)\sigma^A + \sigma_t^{qA}) - (1+L^A)\sigma_t^{qB}((1-\eta_t(1+L^A))\sigma^B + \sigma_t^{qB})]dt$$
$$+ ((1-\eta_t)(1+L^A)\sigma^A + L^A\sigma_t^{qA})dZ_t^A + (L^A\sigma_t^{qB} - (1-\eta_t(1+L^A))\sigma^B)dZ_t^B$$

To calculate dr_t^F, we note that country B's portfolio choice must pin down the risk-free rate. Because the leverage constraint binds,

$$\mathbb{E}[dr_t^{Aa}] - dr_t^F > \frac{\psi_t^A}{\eta_t}(\sigma^A + \sigma_t^{qA})^2 + \frac{\psi_t^A}{\eta_t}(\sigma_t^{qB})^2,$$

so we cannot use the same procedure outlined in the appendix of Brunnemeier and Sannikov (2015). Instead, we note that country B must produce good b, implying that

$$dr_t^F \leq \mathbb{E}[dr_t^{Bb}] - \frac{\psi_t^B}{1-\eta_t}(\sigma_t^{qA})^2 - \frac{\psi_t^B}{1-\eta_t}(\sigma^B + \sigma_t^{qB})^2.$$

However, the inequality cannot be strict because that would imply that country B would prefer to produce more of good b, so B would not be optimizing its portfolio choice. In addition, given an η_t where country A wants to take on more leverage but cannot due to the constraint, country B must reduce its optimal investment in risk-free bonds without leverage constraints and either invest that extra capital in producing good a or lower the risk-free rate so that excess returns from producing good b rise. Therefore, the above equation holds with equality and substituting it in yields

$$\begin{aligned}
\frac{d\eta_t}{\eta_t} = & \left[(1+L^A)\left(\frac{\bar{a}P_t^a}{q_t}\right) - L^A\left(\frac{\bar{a}P_t^b}{q_t}\right) + (1+L^A)(1-\eta_t)(\sigma^A\sigma_t^{qA}) - \rho\right]dt \\
& - \left[L^A(\sigma^B\sigma_t^{qB}) - L^A\left(\frac{1-\eta_t(1+L^A)}{1-\eta_t}\right)((\sigma^B+\sigma_t^{qB})^2 + (\sigma_t^{qA})^2)\right]dt \\
& + [(\eta_t(1+L^A)\sigma^A + \sigma_t^{qA})^2 + ((1-\eta_t(1+L^A))\sigma^B + \sigma_t^{qB})^2 - (1-\eta_t(1+L^A))\sigma^B\sigma_t^{qB}]dt \\
& - (1+L^A)(\sigma^A+\sigma_t^{qA})(\eta_t(1+L^A)\sigma^A + \sigma_t^{qA})dt \\
& - (1+L^A)\sigma_t^{qB}((1-\eta_t(1+L^A))\sigma^B + \sigma_t^{qB})dt \\
& + ((1-\eta_t)(1+L^A)\sigma^A + L^A\sigma_t^{qA})dZ_t^A + (L^A\sigma_t^{qB} - (1-\eta_t(1+L^A))\sigma^B)dZ_t^B,
\end{aligned}$$

Similarly, when $\psi_t^B/(1-\eta_t) \geq L^B+1$, the leverage constraint will only bind for $\eta_t \in [0.5, 1]$, which allows us to write country B's portfolio choice as $(0, 1+L^B, -L^B)$, where $\psi_t^B = \psi_t^{Bb} = (1-\eta_t)(1+L^B)$. This partly pins down country A's portfolio choice, requiring that $\psi_t^A = \eta_t(1+L^B) - L^B$ and thus $\psi_t^A/\eta_t = 1+L^B - L^B/\eta_t$. Since the portfolio shares must sum to one, we have that coutnry A loans out $1-\psi_t^A/\eta_t = L^B(1/\eta_t - 1)$ Therefore, after substituting in returns and cancelling terms,

$$\begin{aligned}
\frac{d\eta_t}{\eta_t} = & [\frac{\psi_t^{Aa}}{\eta_t}\left(\frac{\bar{a}P_t^a}{q_t} + \mu_t^q + \mu_t^K + \sigma^A\sigma_t^{qA}\right) + \frac{\psi_t^{Ab}}{\eta_t}\left(\frac{\underline{a}P_t^b}{q_t} + \mu_t^q + \mu_t^K + \sigma^A\sigma_t^{qA}\right) - \rho \\
& + L^B\left(\frac{1}{\eta_t} - 1\right)dr_t^F + (\psi_t^A\sigma^A + \sigma_t^{qA})^2 + (\psi_t^B\sigma^B + \sigma_t^{qB})^2 - \mu_t^q - \mu_t^K - \psi_t^A\sigma^A\sigma_t^{qA} - \psi_t^B\sigma^B\sigma_t^{qB} \\
& - \left[(1+L^B) - \frac{L^B}{\eta_t}\right](\sigma^A + \sigma_t^{qA})(\psi_t^A\sigma^A + \sigma_t^{qA}) - \left[(1+L^B) - \frac{L^B}{\eta_t}\right]\sigma_t^{qB}(\psi_t^B\sigma^B + \sigma_t^{qB})]dt \\
& + \left[(1+L^B) - \frac{L^B}{\eta_t}\right](\sigma^A + \sigma_t^{qA}) - (\psi_t^A\sigma^A + \sigma_t^{qA})dZ_t^A + \left[(1+L^B) - \frac{L^B}{\eta_t}\right]\sigma_t^{qB} - (\psi_t^B\sigma^B + \sigma_t^{qB})dZ_t^B
\end{aligned}$$

For this range of η_t, it is the case that country A's returns pin down the risk-free rate, so after substituting in

$$dr_t^F = \mathbb{E}[dr_t^{Aa}] - \frac{\psi_t^A}{\eta_t}(\sigma^A + \sigma_t^{qA})^2 - \frac{\psi_t^A}{\eta_t}(\sigma_t^{qB})^2,$$

the law of motion for η_t becomes

$$\begin{aligned}\frac{d\eta_t}{\eta_t} = [&\left(1-\frac{\psi_t^{Ab}}{\eta_t}\right)\left(\frac{\bar{a}P_t^a}{q_t}\right)+\frac{\psi_t^{Ab}}{\eta_t}\left(\frac{\underline{a}P_t^b}{q_t}\right)-\rho+(1-\psi_t^A)\sigma^A\sigma_t^{qA}\\ &-\left(1-\frac{\psi_t^A}{\eta_t}\right)\frac{\psi_t^A}{\eta_t}[(\sigma^A+\sigma_t^{qA})^2+(\sigma_t^{qB})^2]+(\psi_t^A\sigma^A+\sigma_t^{qA})^2+(\psi_t^B\sigma^B+\sigma_t^{qB})^2-\psi_t^B\sigma^B\sigma_t^{qB}\\ &-\left[(1+L^B)-\frac{L^B}{\eta_t}\right](\sigma^A+\sigma_t^{qA})(\psi_t^A\sigma^A+\sigma_t^{qA})-\left[(1+L^B)-\frac{L^B}{\eta_t}\right]\sigma_t^{qB}(\psi_t^B\sigma^B+\sigma_t^{qB})]dt\\ &+\left[(1+L^B)-\frac{L^B}{\eta_t}\right](\sigma^A+\sigma_t^{qA})-(\psi_t^A\sigma^A+\sigma_t^{qA})dZ_t^A+\left[(1+L^B)-\frac{L^B}{\eta_t}\right]\sigma_t^{qB}-(\psi_t^B\sigma^B+\sigma_t^{qB})dZ_t^B\end{aligned}$$

A.2 Proof of Proposition 2: Law of Motion for q_t.

Assume q_t is a twice-continuously differentiable function of η_t. Then Ito's Lemma implies that

$$dq_t = q'(\eta_t)d\eta_t + \frac{1}{2}q''(\eta_t)(d\eta_t)^2$$

Thus, if we divide both sides by q_t and expand terms, q_t evolves according to

$$\frac{dq_t}{q_t} = \left[\frac{q'(\eta_t)}{q(\eta_t)}\eta_t\mu_t^\eta + \frac{1}{2}\frac{q''(\eta_t)}{q(\eta_t)}\eta_t((\sigma_t^{\eta A})^2+(\sigma_t^{\eta B})^2)\right]dt + \frac{q'(\eta_t)}{q(\eta_t)}\eta_t\sigma_t^{\eta A}dZ_t^A + \frac{q''(\eta_t)}{q(\eta_t)}\eta_t\sigma_t^{\eta B}dZ_t^B,$$

where $\sigma_t^{\eta A}$ is the volatility from dZ_t^A and $\sigma_t^{\eta B}$ is the volatility from dZ_t^B. Substituting in the drift and volatilities from Proposition 1 yields the laws of motions for q_t in Proposition 2. Additionally, when the leverage constraint for country A binds, we have that

$$\begin{aligned}\sigma_t^{qA} &= \frac{q'(\eta_t)}{q(\eta_t)}\eta_t\sigma_t^{\eta A} \Rightarrow \sigma_t^{qA} = \frac{q'(\eta_t)}{q(\eta_t)}\eta_t((1-\eta_t)(1+L^A)\sigma^A + L^A\sigma_t^{qA})\\ \sigma_t^{qB} &= \frac{q'(\eta_t)}{q(\eta_t)}\eta_t\sigma_t^{\eta B} \Rightarrow \sigma_t^{qB} = \frac{q'(\eta_t)}{q(\eta_t)}\eta_t(L^B\sigma_t^{qB} - (1-\eta_t(1+L^B))\sigma^B)\end{aligned}$$

Rearranging yields the result. Similarly, when the leverage constraint for country B binds,

$$\begin{aligned}
\sigma^{qA} &= \frac{q'(\eta)}{q(\eta)}\eta\left[\left(\frac{\psi^A}{\eta}\right)(\sigma^A+\sigma^{qA})-(\psi^A\sigma^A+\sigma^{qA})\right] \\
&= \frac{q'(\eta)}{q(\eta)}\eta\left[\left(\frac{\psi^A}{\eta}-\psi^A\right)\sigma^A\right]+\frac{q'(\eta)}{q(\eta)}\eta\left(\frac{\psi^A}{\eta}-1\right)\sigma^{qA} \\
\sigma^{qB} &= \frac{q'(\eta)}{q(\eta)}\eta\left[\frac{\psi^A}{\eta}\sigma^{qB}-(\psi^B\sigma^B+\sigma^{qB})\right] \\
&= -\frac{q'(\eta)}{q(\eta)}\eta\psi^B\sigma^B+\frac{q'(\eta)}{q(\eta)}\eta\left(\frac{\psi^A}{\eta}-1\right)\sigma^{qB}
\end{aligned}$$

Rearranging and substitution yields a similar closed-form expression for σ^{qA} and σ^{qB} when constraints bind for country B.

A.3 Proof of Proposition 2: Expressions for q and q'

Lemma 1. *Suppose reinvestment is unnecessary for capital formation, and let $\tau = \bar{a}/\underline{a}$. Furthermore, suppose that σ^A and σ^B are sufficiently close such that $\psi^{Ab} = 0$ whenever country A is levered. Under our other assumptions on the model, if leverage constraints for country A bind, then equilibrium capital shares are characterized by*

$$\begin{aligned}
\psi^{Aa}(\eta) &= \eta(1+L_A) \\
\psi^{Ab}(\eta) &= 0 \\
\psi^{Ba}(\eta) &= \begin{cases} \dfrac{1-(1+\tau)(1+L_A)\eta}{2} & \textit{if } \eta \le \dfrac{1}{(1+L_A)(1+\tau)}, \\ 0 & \textit{otherwise.} \end{cases} \\
\psi^{Bb}(\eta) &= 1-\psi^{Aa}(\eta)-\psi^{Ba}(\eta).
\end{aligned}$$

Proof. Suppose leverage constraints bind for country A. Then by definition of leverage constraints and our hypothesis on ψ^{Ab},

$$\frac{\psi^{Aa}}{\eta} = 1+L_A \Rightarrow \psi^{Aa}(\eta) = \eta(1+L_A).$$

The expression for $\psi^{Bb}(\eta)$ folows from market-clearing. To finish this section of the proof, the excess returns condition for country B requires that $\psi^{Ba} > 0$ if and only if

$$\mathbb{E}[dr^{Bb}] = \mathbb{E}[dr^{Ba}],$$

which holds only when

$$\frac{P^a}{P^b} = \frac{\bar{a}}{\underline{a}} = \tau.$$

Since the consumption good is the numeraire, we have that

$$\frac{Y^b}{Y^a} = \tau \Rightarrow \bar{a}\tau\psi^{Aa} + \bar{a}\psi^{Ba} = \bar{a}\psi^{Bb}.$$

Divide both sides by $\bar{a}$ and apply market-clearing for capital and the condition of a binding leverage constraint to attain

$$\tau\psi^{Aa} + \psi^{Ba} = 1 - \psi^{Ba} - \psi^{Aa} \Rightarrow \psi^{Ba}(\eta) = \frac{1-(\tau+1)\psi^{Aa}}{2} = \frac{1-(\tau+1)(1+L_A)\eta}{2}.$$

This is positive when $\eta \leq (1+L_A)^{-1}(1+\tau)^{-1}$, and since we require capital shares to be nonnegative, $\psi^{Ba} = 0$ when this condition is not satisfied $\square$

We now proceed to prove the final statement of the proposition. Since agent preferences are identical and log utility, aggregate flow consumption will be proportional to aggregate wealth. By market-clearing for flow consumption,

$$\rho qK = Y = (Y^a)^{1/2}(Y^b)^{1/2} = \left(\bar{a}\psi^{Aa}K + \underline{a}\psi^{Ba}K\right)^{1/2}\left(\bar{a}\psi^{Bb}K + \underline{a}\psi^{Ab}K\right)^{1/2}.$$

Let $\Upsilon^a = \bar{a}\psi^{Aa} + \underline{a}\psi^{Ba}$, $\Upsilon^b = \bar{a}\psi^{Bb} + \underline{a}\psi^{Ab}$, and $\Upsilon = (\Upsilon^a)^{1/2}(\Upsilon^b)^{1/2}$. Divide both sides by K, and we attain

$$\rho q = \Upsilon.$$

To acquire q', note that Υ^a and Υ^b are functions of the capital shares ψ^{Aa}, ψ^{Ab}, ψ^{Bb}, and ψ^{Ab}. Since we have closed-form expressions for capital shares as functions of η, we can directly differentiate

Υ^a and Υ^b with respect to η. Thus, differentiating both sides with respect to η yields

$$
\begin{aligned}
q'(\eta) &= \frac{1}{\rho}\left[\frac{1}{2}(\Upsilon^a)^{-1/2}(\Upsilon^a)'\left(\Upsilon^b\right)^{1/2} + \frac{1}{2}\left(\Upsilon^b\right)^{-1/2}\left(\Upsilon^b\right)'(\Upsilon^a)^{1/2}\right] \\
&= \frac{(\Upsilon^a)^{1/2}\left(\Upsilon^b\right)^{1/2}}{2\rho}\left[\frac{(\Upsilon^a)'}{\Upsilon^a} + \frac{\left(\Upsilon^b\right)'}{\Upsilon^b}\right] \\
&= \frac{\Upsilon}{2\rho}\left[\frac{(\Upsilon^a)'}{\Upsilon^a} + \frac{\left(\Upsilon^b\right)'}{\Upsilon^b}\right]
\end{aligned}
$$

Using Lemma 1, we can write Υ^a, Υ^b, and their derivatives in terms of η, L^A, and fundamental parameters. Simplification will yield the final statement in Proposition 2.

B Computational Algorithm

Here, we provide a description of the algorithm used to compute dynamics and welfare.

Unconstrained Case. Using market-clearing for consumption to express the capital shares as functions of the capital price q, the asset-pricing relationship in Proposition 2 from Brunnermeier-Sannikov (2015) form is an implicit differential equation with initial conditions $(q(0), q'(0))$.

1. We apply a small perturbation to ψ^{Aa} and η and apply market-clearing to retrieve $q(0)$. We then estimate $q'(0)$ using the equilibrium asset-pricing relationship.

2. Use Matlab's *decic* function to compute consistent initial conditions while fixing $\psi^{Aa}(0) = 0$.

3. Using Matlab's *ode15i*, calculate (q, qp) from $\eta \approx 0$ to the level of η at which $\psi^{Aa} = 0$ and the capital price q is maximized. With these values, we can also compute μ^{η}, $\sigma^{\eta A}$, and $\sigma^{\eta B}$.

4. Repeat steps 1-3 but solving from $\eta \approx 1$ to the η at which q is maximized.

5. Estimate $H(0)$ and $H(1)$ numerically are approximately zero.

6. Interpolate the capital price, capital shares, and drift and volatilities of $d\eta_t/\eta_t$ with *interp1* to

construct the second-order differential equation for $H(\eta)$. Solve the ODE using the boundary value problem solver *bvp4c*.

7. Use the Kolmogorov Forward Equations to compute the stationary density. Numerically integrate to retrieve the CDF and $\mathbb{E}[V(\eta)]$.

Binding Leverage Constraint. Since we can explicitly write capital shares as functions of η using the fact that the leverage constraint binds, we can acquire q and q' in closed form by market-clearing.

1. Determine for what range of η the leverage constraint binds for countries A and then B using our computations from the unconstrained case. Using our closed-form expressions for q and q', explicitly compute capital shares and dynamics over those η insert these values into the relevant matrix from the unconstrained case.

2. Use steps 5, 6, and 7 from the unconstrained case to compute welfare and the stationary density.

Piecewise Leverage Constraint. For clarity, assume $L_1^A = L_1^B = L_1$ and $L_2^A = L_2^B = L_2$.

1. Compute the constrained equilibrium twice, once using L_1 and again using L_2.

2. Consider the matrices from using L_1. Cut the matrices for dynamics, η, q, and q' into two groups: values whose corresponding $\eta \leq \eta_*^A$ or $\eta \geq \eta_*^B$.

3. Consider the matrices from using L_2. Slice the matrices for those values whose $\eta \in (\eta_*^A, \eta_*^B)$.

4. Join these matrices produced from these three sections. Proceed using steps 5, 6, and 7 from the unconstrained case.

Solving for the Coordinated and Uncoordinated Equilibrium. We assume here that $L_2 = 0$. We first consider the coordinated algorithm and then the uncoordinated one.

1. Create a function that takes as arguments L_1^A, η_*^A, L_1^B, and η_*^B and gives as output the expectation of the value function. Multiply by -1 to turn the problem into a minimization problem.

2. Apply *fminsearch* to the function, minimizing over the vector $[L_1^I, \eta_*^I]$, as given symmetric σ^A and σ^B, the coordinated equilibrium will have symmetric policies.

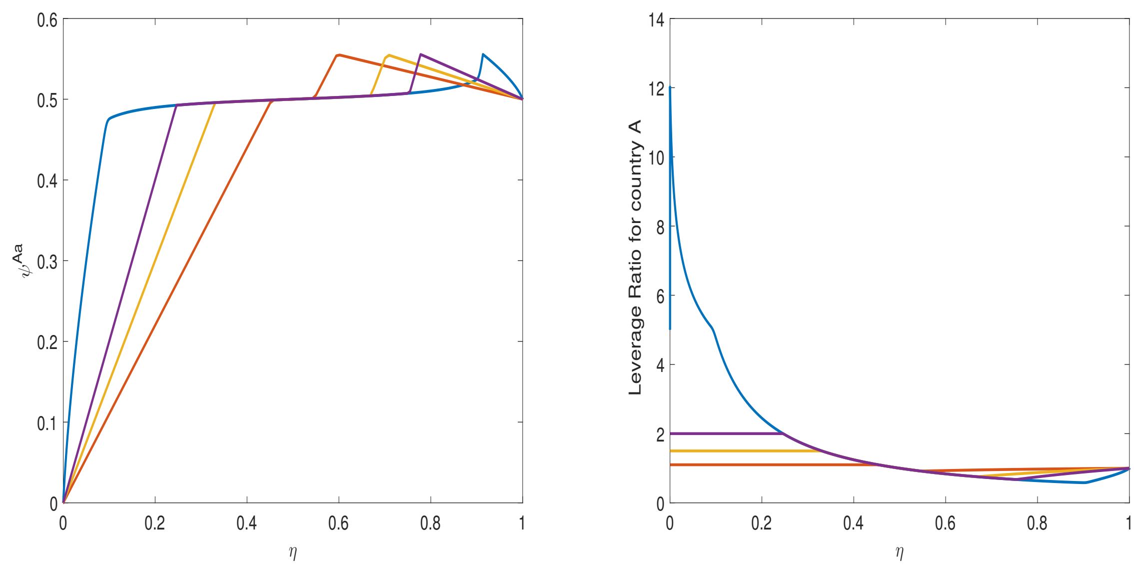
3. Set L_1^B and η_*^B to the coordinated values. Holding them fixed, now minimize the expectation of the value function with respect to L_1^A and η_*^A to determine country A's best response. Change L_1^B and η_*^B to country A's best response. Repeat until Nash equilibrium is found (i.e., policy responses converge to a fixed point).

4. If the coordinated equilibrium yields lower utility, repeat step 2 using a different initial value, as there may exist several local minima.

C Additional Figures

Figure 7 displays the allocations of world capital between country A and B. Panel (a) plots capital held by country A and used for good a (ψ^{Aa}). When leverage constraints bind ψ^{Aa} varies linearly by $L+1$, but ψ^{Aa} equals the unconstrained curve when constraints do not bind(i.e., country A's optimal choice of ψ^{Aa} can now be achieved even with the leverage constraint). The jumps on the right side of the state space correspond to when B's leverage constraints bind. Panel (b) plots country A's leverage over the state space. When the constraint binds, the leverage ratio is constant, as evidenced by the flat dashed lines for $\eta < 0.5$. When country A holds a majority of world wealth, its leverage ratio dips below 1, indicating that some share of its portfolio is now being lent out (capital outflows), and eventually the ratio returns to 1, reflecting that at $\eta = 1$ or $\eta = 0$, one country holds all capital. Results for B are symmetric.

Figure 7 illustrates more clearly how leverage constraints affect prices. When leverage constraints are tighter, A cannot hold as much capital and will consequently produce less of good a. Since country A has a comparative advantage in producing a, the supply of a decreases much faster, resulting in a sharper increase in the price ratio. Furthermore, the "terms of trade hedge" discussed in Brunnermeier and Sannikov (2015) is nullified much quicker, as country B will shift into the production of good a much faster due to country A's reduced production, and this requires that $\underline{a}P_t^a = \bar{a}P_t^b$.

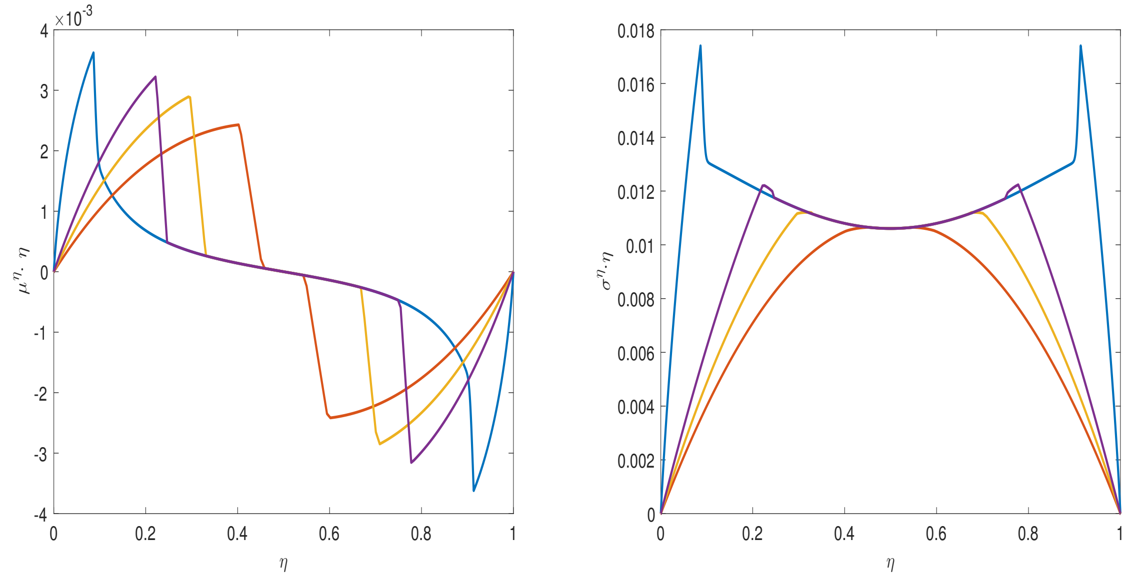


(a) Capital Share ψ_t^{Aa}

(b) A's leverage ratio ψ_t^{Aa}/η_t

Figure 7: Equilibrium capital allocations and leverage. Blue: unconstrained. Purple: $L = 1$. Yellow: $L = .5$. Red: $L = .1$.

Figure 8 plots the equilibrium drift and volatility terms for the state variable η. For $\eta \in (0, 0.5)$, $\mu^\eta > 0$ while for $\eta \in (0.5, 1)$, $\mu^\eta < 0.5$ with $\mu^\eta = 0$ at $\eta = 0.5$. As a result, $\eta = .5$ is an attracting basin. Symmetric leverage constraints do not change this property, but $\mu^\eta \cdot \eta$ peaks much sooner because the price of good a relative to good b increases up much faster as the economy moves away from the stochastic steady state. Consequently, dividend yields increase, causing investment in good a to deliver a higher return for smaller deviations from $\eta = 0.5$ when leverage constraints are imposed, so η returns to 0.5 at a much faster rate. On the other hand, leverage extends the range of η for which the terms of trade hedge is effective. Comparing Panel (b) with the ratio of goods prices makes it clear that $\mu^\eta \cdot \eta$ peaks when country B starts production of good a. The drift keeps going up precisely because the price ratio increased, but once $P^a/P^b = \bar{a}/\underline{a}$, the terms of trade hedge is nullified. Once this occurs, country A's decreasing share of world capital has a



(a) Drift of $d\eta_t$, $\mu^\eta \cdot \eta$.

(b) Volatility of $d\eta_t$, $\sigma^\eta \cdot \eta$.

Figure 8: Equilibrium evolution of relative wealth share (state variable) η_t. Blue: unconstrained. Purple: $L = 1$. Yellow: $L = .5$. Red: $L = .1$.

dominant influence on $\mu^\eta \cdot \eta$ and pushes it down. This follows from the fact that country A's net worth evolves according to

$$\frac{dN_t}{N_t} = \psi_t^{Aa} dr_t^{Aa} + \psi_t^{Ab} dr_t^{Ab} + (1 - \psi_t^A) dr_t^F - \rho dt. \tag{34}$$

Since $\psi_t^{Ab} = 0$ on $\eta \in [0, 0.5]$ and $\mathbb{E}[dr_t^{Aa}] > dr_t^F$, an increasingly smaller ψ^{Aa}, all else equal, leads to a slower drift of dN_t and thus of $d\eta_t$ as well. As a result, $\mu^\eta \cdot \eta$ eventually becomes larger in the unconstrained case than in the constrained cases when $\eta \in [0, 0.15]$.

Panel (b) plots the volatility of $d\eta_t$, which is obtained by computing the standard deviation of $d\eta_t/\eta_t$ and multiplying through by η_t. Explicitly, $\sigma^\eta = \sqrt{(\sigma^{\eta A})^2 + (\sigma^{\eta B})^2}$. Consistent with the results in Phelan (2016), the volatility of $d\eta_t$ is lower when leverage constraints are imposed. One can interpret this as evidence of lower systemic volatility because it suggests smaller changes in η when the economy is hit with exogenous shocks. Equation (26) explains why this occurs. All else equal, as $L^A \to 0$, the impact of σ^{qA} decreases, so $\sigma^{\eta A} \to (1 - \eta_t)\sigma^A$. Similarly, when L^A approaches zero, it removes the effect of σ^{qB} on $\sigma^{\eta B}$, and this downward pressure outweighs the increase in the size of σ^B's coefficient.

D Policy Coordination Robustness

This section presents the results for calculating coordinated and uncoordinated policies, varying $\underline{a}$ and σ, consider countercyclical macroprudential policies.

Table 5: Optimal coordinated and uncoordinated policies: $\underline{a} = 0.5$.

	L_*^A	L_*^B	η_*^A	η_*^B	$V^{A+B}(0.5)$
Social Optimum	.1527	.1527	.4354	.5646	-44.7440
Nash	.2629	.2629	.3353	.6647	-44.7828
Competitive	N/A	N/A	0.5	0.5	-44.8796

	L_*^A	L_*^B	η_*^A	η_*^B	$\mathbb{E}[V^{A+B}(\eta)]$
Social Optimum	.3592	.3592	.3997	.6003	-44.9198
Nash	.8426	.8426	.2737	.7263	-44.9518
Competitive	N/A	N/A	0.5	0.5	-47.6552

Table 6: Optimal coordinated and uncoordinated policies: $\underline{a} = 0.6$.

	L_*^A	L_*^B	η_*^A	η_*^B	$V^{A+B}(0.5)$
Social Optimum	.3200	.3200	.4122	.5878	-44.7638
Nash	.2325	.2325	.3500	.6500	-44.7878
Competitive	N/A	N/A	0.5	0.5	-44.8796

	L_*^A	L_*^B	η_*^A	η_*^B	$\mathbb{E}[V^{A+B}(\eta)]$
Social Optimum	.9976	.9976	.4006	.5994	-44.7910
Nash	.5173	.5173	.3057	.6943	-44.9624
Competitive	N/A	N/A	0.5	0.5	-47.6963

Table 7: Optimal coordinated and uncoordinated policies: $\underline{a} = 0.7$.

	L_*^A	L_*^B	η_*^A	η_*^B	$V^{A+B}(0.5)$
Social Optimum	.1891	.1891	.4223	.5777	-44.7744
Nash	.1248	.1248	.3804	.6196	-44.7886
Competitive	N/A	N/A	0.5	0.5	-44.8796

	L_*^A	L_*^B	η_*^A	η_*^B	$\mathbb{E}[V^{A+B}(\eta)]$
Social Optimum	.5140	.5140	.3968	.6032	-44.8710
Nash	.1248	.1248	.3726	.6274	-44.9190
Competitive	N/A	N/A	0.5	0.5	-47.7561

Table 8: Optimal coordinated and uncoordinated policies: $\underline{a} = 0.9$.

	L_*^A	L_*^B	η_*^A	η_*^B	$V^{A+B}(0.5)$
Social Optimum	.7801	.7801	.4679	.5321	-44.7928
Nash	.1811	.1811	.4083	.5917	-44.8624
Competitive	N/A	N/A	0.5	0.5	-44.8797

	L_*^A	L_*^B	η_*^A	η_*^B	$\mathbb{E}[V^{A+B}(\eta)]$
Social Optimum	.0726	.0726	.4611	.5389	-45.0488
Nash	.1024	.1024	.4357	.5643	-45.2742
Competitive	N/A	N/A	0.5	0.5	-48.3699

Table 9: Optimal coordinated and uncoordinated policies: $\sigma = 2.5\%$.

	L_*^A	L_*^B	η_*^A	η_*^B	$V^{A+B}(0.5)$
Social Optimum	.1990	.1990	.4378	.5622	-44.5990
Nash	.0546	.0546	.4263	.5737	-44.6472
Competitive	N/A	N/A	0.5	0.5	-44.7118

	L_*^A	L_*^B	η_*^A	η_*^B	$\mathbb{E}[V^{A+B}(\eta)]$
Social Optimum	.4724	.4724	.4389	.5611	-44.2422
Nash	.1492	.1492	.3975	.6025	-44.7690
Competitive	N/A	N/A	0.5	0.5	-47.6353

Table 10: Optimal coordinated and uncoordinated policies: $\sigma = 5\%$.

	L_*^A	L_*^B	η_*^A	η_*^B	$V^{A+B}(0.5)$
Social Optimum	N/A	N/A	.4200	.5800	-45.7310
Nash	.2667	.2667	.3750	.6250	-45.8312
Competitive	N/A	N/A	0.5	0.5	-45.8218

	L_*^A	L_*^B	η_*^A	η_*^B	$\mathbb{E}[V^{A+B}(\eta)]$
Social Optimum	.2922	.2922	.3696	.6304	-46.3458
Nash	.2180	.2180	.3721	.6279	-46.3706
Competitive	N/A	N/A	0.5	0.5	-49.5006