Productivity Investment, Power Law, and Welfare Gains from Trade

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Why Do We Care About Productivity Distributions?


Quantifying gains from trade.

But:


2. Truncation to productivity distribution due to firm selection. Because weak firms die out...
Motivation

Productivity is a result of R&D and investment activities!
Why does the empirical distribution exhibits power law / Pareto tail?
How does productivity distribution respond to trade liberalization?
What is the implication on welfare gains from trade?
Our Model

1. Incorporates firm-level productivity investment decision. Sutton (1991)
2. Heterogeneous investment efficiency (talent / entrepreneurship).
Our Results

The productivity distribution always has a Pareto tail.

Requires almost no assumptions on the distribution of talent.

Robust against:

1. Investment cost function (a subclass of smoothly varying function).
2. Demand system (asymptotic CES).
Our Results

Intensive margin matters so it’s more than a specification issue.

Trade liberalization results in:

1. Extensive margin: more firm selection.
2. Intensive margin:
   Exporters invest more.
   Non-Exporters invest less (e.g. Pavcnik (2002), Fernandes (2007), and Baldwin and Gu (2009)).
3. New gains from trade through variable trade cost.
4. Less welfare elasticity.
Exogenous Distribution

Source: Figure 4 of Nigai (2017)
Exogenous Distribution


Pareto: Chaney (2008), Melitz and Redding (2015), and hundreds of studies.

Lognormal: Eeckhout (2004), Head et al. (2014)

Bee et al. (2017): Neither Pareto nor lognormal!

The distribution is exogenously assumed!
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The distribution is exogenously assumed!
(This is important so must be repeated by three times)
Endogenous Productivity

Matching between firms and workers: Monte (2011) and Sampson (2014)
Endogenous Productivity

Power law is not addressed.

Gains from trade is not examined.

Bas and Ledezma (2015): the effect of trade liberalization on exporter is ambiguous.
Welfare Gains from Trade

Arkolakis et al. (2012):

(R1) Trade balances.
(R2) Constant ratio between aggregate profit and revenue.
(R3) Constant bilateral trade elasticity $d \ln (\lambda_x/\lambda_0) / d \ln \tau$ for all $x$ countries.

$\lambda_i$ denotes the expenditure on products of country $i$ in the domestic country $0$.

If (R1)-(R3) holds, then welfare depends on $\lambda_0$ only.

Krugman model and Melitz model alike!
Symmetric preference and income.

Utility for consuming each variety $v$: $U = \int_{v \in \Upsilon} u(q(v))\,dv$.

Implied demand per variety: $p(v) = D(q(v); A)$.

$A$ is endogenously determined.
Monopolistic competitive firms.
Labor is the only input, and is considered as numeraire.
Entry cost: $\kappa_e$.
Production cost: $q/\varphi + \kappa_D$.
Productivity $\varphi$ is endogenously determined through investment.
Investment function:

$$\varphi = B(t \cdot k).$$

Labor input $k$.

Talent / entrepreneurship $t \in (t_L, \infty)$ with $t_L \geq 0$.

$B'(t \cdot k) > 0$, $B''(t \cdot k) < 0$.

For convenience, the cost of investment is:

$$k = \frac{B^{-1}(\varphi)}{t} \equiv \frac{V(\varphi)}{t} \equiv \gamma V(\varphi).$$

The talent index $\gamma \in (0, \gamma_H)$ follows a distribution with p.d.f. $f(\gamma)$. 
Basic Setting

Total Profit:

\[ \Pi(\varphi) = \pi(\varphi) - \gamma V(\varphi), \]
\[ \pi(\varphi) = pq - \varphi^{-1} q - \kappa_D. \]

Timing:

1. **Entry Stage:** Each firm pays \( \kappa_e \) to enter, and then observes \( \gamma \) respectively.
2. **Investment Stage:** Each firm decides whether to invest, and if yes, the level of \( \varphi \).
3. **Production Stage:** Each firm decides whether to produce, and if yes, the price of its variety.
Preliminary Example

CES demand: \( q = A^{\frac{1}{\sigma}} p^{-\frac{1}{\sigma}}. \)

Power function: \( k ( \varphi ) = \gamma \varphi^\beta \)

Optimal output: \( q ( \varphi ) = A \rho^\sigma \varphi^\sigma, \) where \( \rho \equiv (\sigma - 1)/\varphi \) and \( A \equiv L/P^{1-\sigma}. \)

Investment Stage: each firm solves

\[
\max_{\varphi} \Pi (\varphi) = \frac{A \rho^\sigma}{\sigma - 1} \varphi^{\sigma-1} - \gamma \varphi^\beta.
\]

Optimal productivity:

\[
\tilde{\varphi} (\gamma) = \frac{A \rho^\sigma}{\beta} \gamma^{-\frac{1}{\theta}},
\]

where \( \theta \equiv \beta - \sigma + 1 > 0 \) must hold to ensure the existence of optimality.
Zero cutoff profit condition (ZCP):

\[ \Pi (\tilde{\varphi} (\gamma); \gamma) \geq 0 \text{ if and only if } \gamma \leq \gamma_D. \]

Entry Stage: the free entry condition

\[ \int_0^{\gamma_D} \Pi (\tilde{\varphi} (\gamma); \gamma) \, dF (\gamma) = \kappa_e \]

picks down \( A \) along with ZCP and \( \tilde{\varphi} (\gamma) \).
Preliminary Example

Productivity distribution:

\[ g(\varphi) = \frac{f(\gamma(\varphi))}{F(\gamma_D)} A\left(\frac{\rho^\sigma}{\beta}\right) \theta \varphi^{-\theta-1}. \]

Note that \( \frac{\partial \gamma(\varphi)}{\partial \varphi} < 0 \) and \( \lim_{\gamma \to 0} \varphi(\gamma) = \infty. \)

\( A, \sigma, \beta, \rho, \theta, \gamma_D \) are all independent of \( \varphi. \)

If \( \lim_{\gamma \to 0} f(\gamma) = K > 0, \) then

\[ g(\varphi) \approx \frac{K}{F(\gamma_D)} A\left(\frac{\rho^\sigma}{\beta}\right) \theta \varphi^{-\theta-1}. \]
Preliminary Example

1. Since \( \lim_{\gamma \to 0} f(\gamma) = K \), we can express \( f(\gamma) \) as
   \[ f(\gamma) = K + h(\gamma), \text{ where } \lim_{\gamma \to 0} h(\gamma) = 0. \]

2. Therefore, \( g(\varphi) = A \left( \frac{\rho^\sigma}{\beta} \right) \theta K \varphi^{-\theta-1} + A \left( \frac{\rho^\sigma}{\beta} \right) \theta h(\gamma) \varphi^{-\theta-1} \).

3. Clearly, \( \lim_{\varphi \to \infty} \varphi^{-\theta-1} = 0 \), \( \lim_{\varphi \to \infty} h(\gamma(\varphi)) = 0 \), and
   \[ \lim_{\varphi \to \infty} h(\gamma(\varphi)) \varphi^{-\theta-1} = 0. \]

4. Since
   \[ \lim_{\varphi \to \infty} \frac{h(\gamma(\varphi)) \varphi^{-\theta-1}}{\varphi^{-\theta-1}} = \lim_{\varphi \to \infty} h(\gamma(\varphi)) = 0, \]
   it implies that the rate of convergence of \( h(\gamma(\varphi)) \varphi^{-\theta-1} \) dominates that of \( \varphi^{-\theta-1} \). Thus, there is a \( \varphi_0 \) where for all \( \varphi \geq \varphi_0 \)
   \[ g(\varphi) \approx \frac{K}{F(\gamma_D)} A \left( \frac{\rho^\sigma}{\beta} \right) \theta \varphi^{-\theta-1}. \]
If $\gamma \sim U[0, \gamma_H]$ and $\kappa_D = 0$
Smooth Variation

Definition

Definition 1. A function \( v(x) \) is a *regularly varying function* if and only if \( v(x) \) can be expressed as

\[
v(x) = x^\alpha l(x),
\]

where \( l(x) \) is a slowly varying function, i.e., for any \( \lambda > 1 \),

\[
\lim_{x \to \infty} \frac{l(\lambda x)}{l(x)} = 1.
\]

Definition

Definition 2. A *Smoothly Varying Function* is an infinitely differentiable regularly varying function \( v(x) \), such that for all \( n \geq 1 \)

\[
\lim_{x \to \infty} \frac{x^n v^{(n)}(x)}{v(x)} = \beta (\beta - 1) \ldots (\beta - n + 1),
\]

where \( v^{(n)}(x) \) denotes for the \( n \)-th derivative of \( v(x) \).
Power Law of Productivity

Assumption

**Assumption 1.** The inverse demand for each variety is a smoothly varying function $p = D(q; A) \equiv q^{-\frac{1}{\sigma}} Q(q; A)$, where $\sigma > 1$ and $\lim_{q \to \infty} Q(q; A) = C_Q > 0$. The investment cost is a smoothly varying function $k(\phi) = \gamma V(\phi) \equiv \gamma \phi^\beta L(\phi)$, where $\beta > 1$ and $\lim_{\phi \to \infty} L(\phi) = C_L > 0$.

Proposition

**Proposition 1.** Under Assumption 1, suppose that $\lim_{\gamma \to 0} f(\gamma) = K > 0$, and $\theta \equiv \beta + 1 - \sigma > 0$, the productivity distribution is approximately

$$g(\phi) \approx \frac{K}{F(\gamma D)} \frac{C_Q^\sigma}{C_L} \rho^\sigma \frac{\theta}{\beta} \phi^{-\theta-1}.$$
Why Smooth Variation?

Because smoothly varying functions are general!

For example, polynomial functions are smoothly varying.

Demand systems that are asymptotically CES are widely applied.

<table>
<thead>
<tr>
<th>Demand Class</th>
<th>Inverse Demand Function</th>
<th>$C_Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CES</td>
<td>$p(q) = q^{-\frac{1}{\sigma}}A^{\frac{1}{\sigma}}$</td>
<td>$A^{\frac{1}{\sigma}}$</td>
</tr>
<tr>
<td>CREMR</td>
<td>$p(q) = q^{-\frac{1}{\sigma}}A^{\frac{1}{\sigma}}(1 - \omega q^{-1})^{\frac{\sigma-1}{\sigma}}$ where $q &gt; \sigma \omega$</td>
<td>$A^{\frac{1}{\sigma}}$</td>
</tr>
<tr>
<td>CEMR</td>
<td>$p(q) = q^{-\frac{1}{\sigma}}(A^{\frac{1}{\sigma}} + \alpha q^{-\frac{\sigma-1}{\sigma}})$</td>
<td>$A^{\frac{1}{\sigma}}$</td>
</tr>
<tr>
<td>Bipower inverse</td>
<td>$p(q) = q^{-\frac{1}{\sigma}}A^{\frac{1}{\sigma}}(1 + \hat{A}^{\frac{1}{\sigma}}q^{-\frac{1}{\sigma}})^{\frac{\sigma-1}{\sigma}}$ where $\sigma &gt; \zeta$</td>
<td>$A^{\frac{1}{\sigma}}$</td>
</tr>
<tr>
<td>Pollak</td>
<td>$p(q) = q^{-\frac{1}{\sigma}}A^{\frac{1}{\sigma}}(1 - \hat{A}q^{-1})^{\frac{1}{\sigma}}$</td>
<td>$A^{\frac{1}{\sigma}}$</td>
</tr>
</tbody>
</table>

**Table**: Smoothly Varying Inverse Demand
Why Smooth Variation?

Because smoothly varying function can approximate the tail behaviors of regularly varying functions.

Theorem

(Theorem 1.8.2, Bingham et al. (1989)) For a regularly varying function $f$, there exists smoothly varying functions $f_1$ and $f_2$, with $f_1 \sim f_2$ and $f_1 \leq f \leq f_2$ on some neighbourhood of infinity. In particular, for a regularly varying function $f$ there exists a smoothly varying function $g$ such that $g \sim f$. 

Yi-Fan Chen, Wen-Tai Hsu, and Shin-Kun Peng
Power Law of Firm Size

Let $s = pq$

**Corollary**

**Corollary 1.** Under Assumption 1, suppose that

$$\lim_{\gamma \to 0} f(\gamma) = K > 0,$$

and $\theta \equiv \beta + 1 - \sigma > 0$, the distribution of firm size $s$ follows the power law with a tail index $\frac{\theta}{\sigma - 1}$, i.e.,

$$\lim_{s \to \infty} g(s) \approx \frac{K}{F(\gamma_D)} \frac{C_Q^{\beta\sigma}}{C_L^{\sigma - 1}} \left( \frac{\sigma - 1}{\sigma} \right)^\beta \frac{\theta}{\beta\sigma} s^{\frac{-\theta}{\sigma - 1} - 1}.$$
Setting

CES utility.

Power function investment cost.

Total cost of exporting: $\frac{\tau q}{\varphi} + \kappa x$.

Timing:

1. Entry Stage.
2. Investment Stage.
3. Production Stage: each firm can further decides whether to export and the price to charge.
Optimality

Production optimality implies that

$$\pi_D(\varphi) = \frac{A \rho^\sigma}{\sigma - 1} \varphi^{\sigma - 1} - \kappa_D,$$

$$\pi_X(\varphi) = \tau^{1-\sigma} \frac{A \rho^\sigma}{\sigma - 1} \varphi^{\sigma - 1} - \kappa_X,$$

where

$$A \equiv \frac{L}{P^{1-\sigma}}.$$
Optimal Investment

Profit for non-exporters and exporters:

\[ \Pi_D (\varphi) = \pi_D (\varphi) - \gamma \varphi^\beta, \]
\[ \Pi_X (\varphi) = \pi_D (\varphi) + \pi_X (\varphi) - \gamma \varphi^\beta. \]

Optimal Productivity

\[ \varphi = \begin{cases} 
A^{\frac{1}{\theta}} \left( \frac{\rho^\sigma}{\beta} \right)^{\frac{1}{\theta}} \gamma^{-\frac{1}{\theta}} & \text{for non-exporting firms,} \\
(1 + \tau^{1-\sigma})^{\frac{1}{\theta}} A^{\frac{1}{\theta}} \left( \frac{\rho^\sigma}{\beta} \right)^{\frac{1}{\theta}} \gamma^{-\frac{1}{\theta}} & \text{for exporting firms.} 
\end{cases} \]
Zero Cutoff Profit Conditions

Firms must not make negative profits: \( \Pi_D(\gamma) \geq 0 \) and \( \Pi_X(\gamma) \geq \Pi_D(\gamma) \).

Therefore:

\[
\gamma_D \equiv \left[ \kappa_D^{-1} A_\theta^\beta \left( \frac{\rho^\sigma}{\beta} \right)^\frac{\beta}{\theta} \left( \frac{\beta}{\sigma - 1} - 1 \right) \right]^{\frac{\theta}{\sigma - 1}},
\]

\[
\gamma_X \equiv \left[ \kappa_X^{-1} \left[ (1 + \tau^{1-\sigma})^\frac{\beta}{\theta} - 1 \right] A_\theta^\beta \left( \frac{\rho^\sigma}{\beta} \right)^\frac{\beta}{\theta} \left( \frac{\beta}{\sigma - 1} - 1 \right) \right]^{\frac{\theta}{\sigma - 1}}.
\]
Assumption 2. Assume that

\[
\frac{\gamma_X}{\gamma_D} \equiv \delta \equiv \left( \frac{\kappa_D}{\kappa_X} \right) \frac{\theta}{\sigma-1} \left[ \left( 1 + \tau^{1-\sigma} \right)^{\frac{\beta}{\theta}} - 1 \right]^{\frac{\theta}{\sigma-1}} < 1,
\]

i.e., the fixed exporting cost \( \kappa_X \) must be large enough.

This means that \( \kappa_X > \kappa_D \).

Otherwise, all firms are exporters.
Firms are subjected to free entry

\[ \overline{\pi} = k_e, \]

where

\[ \overline{\pi} = \int_0^{\gamma_X} \Pi_X(\gamma) \, dF(\gamma) + \int_{\gamma_X}^{\gamma_D} \Pi_D(\gamma) \, dF(\gamma). \]

The aggregate price relates mass of entrants with \( A \equiv L/P^{1-\sigma} \).

\[ P^{1-\sigma} = M_e \left[ \int_{\gamma_X}^{\gamma_D} \rho^{-1} \varphi(\gamma)^{\sigma-1} \, dF(\gamma) + \int_0^{\gamma_X} \rho^{-1} \varphi(\gamma)^{\sigma-1} \, dF(\gamma) \right] \]

\[ + M_e \int_0^{\gamma_X} \tau^{1-\sigma} \rho^{-1} \varphi(\gamma)^{\sigma-1} \, dF(\gamma). \]
Equilibrium Productivity

Equilibrium productivity:

\[
\varphi(\gamma) = \begin{cases} 
\kappa_D^\frac{1}{\beta} \frac{\beta}{\beta \sigma} \left( 1 - \frac{\beta}{\sigma-1} \right)^{-\frac{1}{\beta}} \gamma_D^{-\frac{1}{\theta}} & \text{if } \gamma \in (\gamma_X, \gamma_D] \\
(1 + \tau^{1-\sigma}) \frac{1}{\theta} \kappa_D^{\frac{1}{\beta}} \frac{\gamma_D}{\beta \theta} \left( 1 - \frac{\beta}{\sigma-1} \right)^{-\frac{1}{\beta}} \gamma_D^{-\frac{1}{\theta}} & \text{if } \gamma \in [0, \gamma_X] 
\end{cases}
\]

Let \( \varphi_D \equiv \kappa_D^\frac{1}{\beta} \frac{\beta}{\beta \sigma} \left( 1 - \frac{\beta}{\sigma-1} \right)^{-\frac{1}{\beta}} \gamma_D^{-\frac{1}{\theta}} \), \( \varphi_{DX} \equiv (1 + \tau^{1-\sigma}) \frac{1}{\theta} \kappa_D^{\frac{1}{\beta}} \frac{\gamma_D}{\beta \theta} \left( 1 - \frac{\beta}{\sigma-1} \right)^{-\frac{1}{\beta}} \gamma_D^{-\frac{1}{\theta}} \), and \( \varphi_X \equiv (1 + \tau^{1-\sigma}) \frac{1}{\theta} \kappa_D^{\frac{1}{\beta}} \frac{\gamma_D}{\beta \theta} \left( 1 - \frac{\beta}{\sigma-1} \right)^{-\frac{1}{\beta}} \gamma_X^{-\frac{1}{\theta}} \).
Equilibrium Productivity

\[ g(\varphi) = \begin{cases} 
\frac{\left(1 + \tau^{1-\sigma} \kappa_{\frac{\theta}{\beta}} \gamma_{\frac{\sigma-1}{\beta}} \left(\frac{\beta}{\sigma-1} - 1\right)^{-\frac{\theta}{\beta}} \varphi^{-\theta}\right)}{F(\gamma_{D})} \cdot \left[ \kappa_{D} \gamma_{D} \left(\frac{\beta}{\sigma-1} - 1\right)^{-\frac{\theta}{\beta}} \right] \theta \varphi^{-\theta-1} & \text{if } \varphi \in [\varphi_D, \varphi_{DX}) \\
0 & \text{if } \varphi \in [\varphi_{DX}, \varphi_X) \\
\frac{\left((1 + \tau^{1-\sigma}) \kappa_{\frac{\theta}{\beta}} \gamma_{\frac{\sigma-1}{\beta}} \left(\frac{\beta}{\sigma-1} - 1\right)^{-\frac{\theta}{\beta}} \varphi^{-\theta}\right)}{F(\gamma_{D})} \cdot \left[ (1 + \tau^{1-\sigma}) \kappa_{D} \gamma_{D} \left(\frac{\beta}{\sigma-1} - 1\right)^{-\frac{\theta}{\beta}} \right] \theta \varphi^{-\theta-1} & \text{if } \varphi \in [\varphi_X, \infty) 
\end{cases} \]
Proposition 1 perfectly holds here.

\( \tau \) affects the productivity of large firms directly.

The country size \( L \) does not affect productivity.
**General Power Function (GPF) Class**

**Definition**

**Definition 3.** (Mrazova, Neary and Parenti [2017]) The distribution of $\varphi$ is of GPF class if its c.d.f. can be expressed as $H \left( \theta_0 + \theta_1 \varphi^{\theta_2} \right)$, where $\theta_0$, $\theta_1$ and $\theta_2$ are parameters, and $H (\cdot)$ is a monotonic function.

**Corollary**

**Corollary 2.** Let $G (\cdot)$ denotes the c.d.f. of productivity. The productivity distribution belongs to the General Power Function (GPF) with a Pareto tail, where

$$G (\varphi) = \begin{cases} 
1 - F \left( \frac{\theta}{ \kappa_D \gamma_D^\beta} \left( \frac{\beta}{\sigma-1} - 1 \right)^{-\frac{\theta}{\beta}} \varphi^{-\theta} \right) \frac{1}{F(\gamma_D)} & \text{if } \varphi < \varphi_X \\
1 - F \left( (1 + \tau^{1-\sigma}) \frac{\theta}{ \kappa_D \gamma_D^\beta} \left( \frac{\beta}{\sigma-1} - 1 \right)^{-\frac{\theta}{\beta}} \varphi^{-\theta} \right) \frac{1}{F(\gamma_D)} & \text{if } \varphi \geq \varphi_X 
\end{cases}$$
Proposition 2

Figure: The Effect of an Increment of $\kappa_D$
Proposition 2

Figure: The Effect of an Increment of $\kappa_X$
Proposition 2

Figure: The Effect of an Increment of $\tau$

$\varphi(\gamma)$

$\gamma^X \quad \gamma^I \quad \gamma_D \quad \gamma_D'$
Welfare Gains from Trade

Welfare equation of Melitz (2003):

\[
d \ln W_{0ACR} = \frac{d \ln \lambda_0}{1 - \sigma - \frac{\eta_{ACR}^D}{\phi_{D0}}} - \frac{d \ln M_{e0}}{1 - \sigma - \frac{\eta_{ACR}^D}{\phi_{D0}}},
\]

where \( \frac{\eta_{ACR}^D}{\phi_{D0}} > 0 \).

Welfare equation of productivity model:

\[
d \ln W_0 = \frac{d \ln \lambda_0 - d \ln M_{e0} - \frac{\tilde{\lambda}_{X0}}{\lambda_0} \left( T \xi - \frac{\eta_{X0}}{\Gamma_{X0}} \Xi \right) d \ln \tau}{1 - \sigma - (\sigma - 1) \frac{\sigma - 1}{\theta} - \beta \left( \frac{\tilde{\lambda}_{D0}}{\lambda_0} \frac{\eta_{D0}}{\Gamma_{D0}} + \frac{\tilde{\lambda}_{X0}}{\lambda_0} \frac{\eta_{X0}}{\Gamma_{X0}} \right)},
\]

where \( T > 0, \xi < 0, \Xi > 0, \tilde{\lambda}_{X0} > 0, \tilde{\lambda}_{D0} > 0, \tilde{\lambda}_{X0} + \tilde{\lambda}_{D0} = \lambda_0, \frac{\eta_{D0}}{\Gamma_{D0}} > 0, \frac{\eta_{X0}}{\Gamma_{X0}} > 0 \).
Welfare Gains from Trade

Melitz model has the extensive margin $\eta^{ACR}_D / \Phi_D$ only.

We have:

1. **Extensive margin:** $\frac{\tilde{\lambda}_D}{\lambda_0} \frac{\eta_D}{\Gamma_D}$, $\frac{\tilde{\lambda}_X}{\lambda_0} \frac{\eta_X}{\Gamma_X}$, and $\frac{\tilde{\lambda}_X}{\lambda_D} \frac{\eta_X}{\Gamma_X} \Xi d \ln \tau$.
   Key: $d \ln \gamma_X = \beta d \ln P_0 - \Xi d \ln \tau$.

2. **Intensive margin:** $\frac{\tilde{\lambda}_X}{\lambda_D} T \xi d \ln \tau$ and $(\sigma - 1) \frac{\sigma - 1}{\theta}$
   The direct effect of $\tau$, and the substitution effect.
Welfare Gains from Trade

Benchmark: $g(\varphi) = \theta \varphi^{-\theta - 1}$ v.s. $f(\gamma) = \gamma_{H}^{-1}$.

Melitz:

$$d \ln W_{0}^{ACR} = \frac{d \ln \lambda_{0}}{-\theta},$$

$$\varepsilon_{0x}^{ACR} = \varepsilon^{ACR} = -\theta \ \forall x.$$

Our model:

$$d \ln W_{0} = -\frac{d \ln \lambda_{0}}{\beta} + \frac{\tilde{\lambda}_{X_{0}}}{\lambda_{0}} T \xi - \frac{\theta - \sigma + 1}{\theta} \Xi d \ln \tau,$$

$$\varepsilon_{0x} = \varepsilon = 1 - \sigma + \xi - \frac{\tilde{\lambda}_{X_{0}}}{\lambda_{0}} T \xi - \frac{\theta - \sigma + 1}{\theta} \left(1 - \frac{\tilde{\lambda}_{X_{0}}}{\lambda_{0}} \right) \Xi \ \forall x.$$
Welfare Gains from Trade

Within model comparison: firm selection v.s. no firm selection.

(R3) is not important if no firm selection

\[
d \ln W_0^{NoSelection} = \frac{d \ln \lambda_0}{1 - \sigma - (\sigma - 1) \frac{\sigma-1}{\theta}} - \frac{\xi d \ln \tau}{1 - \sigma - (\sigma - 1) \frac{\sigma-1}{\theta}},
\]

\[
\varepsilon^{NoSelection} = 1 - \sigma.
\]

Similar channel but different magnitude.
Welfare Gains from Trade

Our model v.s. Melitz:

1. The variable trade cost affects the welfare directly: more sensitive to trade.
2. Less welfare elasticity: less sensitive to trade.

Our model with and without selection:

2. The effect from $d \ln \tau$ differs.
We obtain the following results under a general setting.

1. Microfundation for power law in productivity and firm size.
2. Intensive margin of productivity matters a lot!
3. Provides empirical insights on the new channel of gains from trade.