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Motivation

• Sociology & political science literatures regarding segmentation and alienation of households in American society → emergence of disenchanted underclass & vanishing middle class with groups emerging, disappearing or reshaping over time (Temin, 2017).

• Parallel expression in economic segmentation & alienation of household groupings in terms of income size distribution.

• Household income inequality has grown steadily in the US. Following Gibrat (1931), classic models of income processes (Freidman, 1957; Hall, 1978; Blundell & Preston, 1998; Batistin, Blundell & Lewbel, 2006) → current income/consumption does not affect expected rate of growth nor the variance of its growth rate.

• But subgroup segmentation/alienation in this context is an open question.
Motivation (Cont’d)

• One problem with studying these phenomena is that a stringent specification of the nature of the subgroups → undue influence on conclusions of the analysis.

• Is this really innocuous?

• Choosing quantiles or cohorts fixes class sizes, while tying class locations to the overall distributional location locks the progress of subgroups into the progress of the overall distribution.

• **Solution:** Semiparametric (Empirical Bayes’) analyses of a mixture of subgroup distributions developed, which facilitates the determination of class sizes, their locations and spreads, with the number of classes being independently determined. In addition, correlates of household class membership probabilities can be explored.
Outline

1 Motivation
2 Alienation, Polarization & Segmentation
3 Gibrat’s Law, Mixture Distributions & Determination of Income Classes
   - Gibrat’s Law Theory
   - Relation to Gibrat’s Law
4 Empirical Approach
   - Latent Class Model
   - Measuring Segmentation & Polarization
5 Results
   - Data
   - Preliminary Results, 4 Component Concerns
6 Conclusion
Intricacies in Analysis

- ↗ overall inequality ⇒ nothing regarding subgroup inequalities.
- Overall inequality can ↗ with ↗ within subgroup inequalities, while at the same time between subgroup inequality is ↓ diminishing.
- Similarly overall inequality can ↗, with ↓ within subgroup inequalities, while at the same time between subgroup inequality ↗.
- Essence of distinction between polarization & inequality (Duclos, Esteban & Ray, 2004) which allows those measures to go in different directions.
Intuition: Location Shift
Intuition: Identification
Theoretical Basis: Gibrat’s Law

- Right tail of distribution of wealth, income, firm size, or city size, conforms well to a Pareto distribution.

- Essentially models income as accumulation of random multiplicative shocks.

- Based on astronomer Jacobus Kapteyn → assume that underlying such distributions was a simple Gaussian process: a large number of small additive influences, operating independently of each other, would generate a normally distributed variate (“Law of Laplace”).

- Gibrat postulated the simplest such process be the logarithm of the variate.

- Champernowne (1953) & Simon (1955) showed that Pareto distributions arise naturally if the time series behavior of variable in question satisfies Gibrat’s law: current position of variable does not influence its expected rate of growth, nor variance of growth rate.
Theoretical Basis: Gibrat’s Law, Cont’d

- Battistin, Blundell & Lewbel (2006) noted significant departures from ln-normality in income data → violation of Gibrat’s law. Showed empirically that the within cohort distribution of consumption expenditures across households is closer to ln normality than income.

- Perhaps there are no violations, or that there is more to it than meets the eye?
Theoretical Basis: Income Size Distribution

- Denote household income variate as $X$ with an overall distribution $f(X)$.
- Class is considered latent with undetermined number of classes indexed $k = 1, \ldots, K \rightarrow$ common income process within household class.
- Following Modigliani & Brumberg (1954), Friedman (1957), Blundell & Preston (1998), & Batistin, Blundell & Lewbel (2006), agents from each class with “no bequest” preferences are assumed to maximize the present value of lifetime happiness,
  \[
  \int_0^T U_k(C_t) e^{-r^*_kt} dt
  \]
  subject to the present value of lifetime wealth
  \[
  \int_0^T X_t e^{-rt} dt
  \]
Theoretical Basis: Income Size Distribution, Cont’d

- where $U_k(.)$ is an instantaneous consumption utility function, $r_k^*$ is the class rate of time preference, $r$ is the market lending rate.

- This all leads to a consumption smoothing model of the form:
  $$C_t = e^{g_k t} C_0$$ with $g_k = \frac{r - r_k}{\gamma_k}$ (where $\gamma_k$ is a class CRRA parameter), & a one period process $C_t = e^{g_k} C_{t-1}$ or $\ln(C_t) = g_k + \ln(C_{t-1})$ reflective of permanent income process for each class of a form based upon the law of proportionate effects (Gibrat’s Law) where its income in period $t$ has form:
  $$X_{k,t} = (1 + \delta_{k,t}) X_{k,t-1},$$ where $\delta_{k,t}$ is a random variable with mean $\delta_k$, & variance $\sigma_k^2$. 

Following Gibrat (1931), if the income process has a starting value $x_{k,0}$ → letting $X = \ln(x)$, after $T$ periods the variate $x_{k,T}$ will be such that:

$$X_{k,T} = \ln(x_{k,T})$$

$$\sim N \left( \ln(x_{k,0}) + T \left( \delta_k + 0.5\sigma^2_k \right), T\sigma^2_k \right) = N \left( \mu_k, T, \sigma^2_k, T \right)$$

where $\delta_{k,t}$ corresponds to period $t$ income growth rate of the $k^{th}$ class, with $\sigma^2_{k,T}$ the $T^{th}$ period variance.

In-normality holds an added convenience here since, following Aitchison and Brown (1957), when $X$ is lnnormally distributed with $E(X) = e^{\theta + 0.5\sigma^2}$, where $\theta$ & $\sigma^2$ are respectively the mean & variance of $\ln(X)$, & Gini Coefficient is given by $\left[2\Phi \left( \frac{\sigma_{k,T}}{2} \right) - 1 \right] \rightarrow$ subgroup Gini’s grow over time since $\sigma^2_{k,T}$ grows over time.
Mixture Distribution & Income Classes

- Given $K$ classes where the proportion of the population in class $k$ at time $T$ is $w_{k,T}$, $f(X)$ the distribution of income at time $T$ is:

$$f(X_T) = \sum_{k=1}^{K} w_{k,T} f(X_T; \mu_{k,T}, \sigma_{k,T}^2)$$

(1)

where $\sum_{k=1}^{K} w_{k,T} = 1$.

- Assume $k > j \iff \mu_{k,T} > \mu_{j,T}$. For a given $K$, & independent random sample $X_i, i = 1, \ldots, N$, the $3K - 1$ parameters of the semi-parametric mixture distribution can be estimated by pseudo maximum likelihood methods (Anderson et. al., 2016 & 2018) $\rightarrow$ EM Algorithm.
This approach does not depend upon arbitrarily determined boundaries, since class boundaries are fuzzy → all that can be determined is $\Pr(X, k)$, the probability that an agent with $x$ is in class $k$, which is given by:

$$\Pr(X_T, K) = \frac{w_{kT} f(X_T; \mu_{kT}, \sigma_{kT}^2)}{\sum_{j=1}^{K} w_{jT} f(X_T; \mu_{jT}, \sigma_{jT}^2)}$$

With this information, subgroup decomposition of inequality measures can be explored, & polarization & segmentation measures developed.
Mixture Distribution & Income Classes

• Class membership correlates are examined by modifying (1) as

\[
f(X_i, T) = \sum_{k=1}^{K} w_{i,k,T} f(X_i, T; \mu_k, \sigma_k^2)
\]

where \( w_{i,k,T} \) is prior probability of agent \( i \) with income \( x_i \) belonging to class \( k \), given correlates vector \( z_{i,T} \), it may be modelled as:

\[
w_{i,k,T} = \Pr(1(i \in C_k) = 1) = g(\beta_k' z_{i,T}) \tag{2}
\]

for \( k = 2, \ldots, K \), and

\[
w_{i,1,T} = 1 - \sum_{k=2}^{K} g(\beta_k' z_{i,T})
\]

where \( g(.) \) is the link function, eg. a logistic function.
Decomposing GINI

•

\[
\text{GINI} = \frac{1}{2E(x)} \int_0^\infty \int_0^\infty f(y) [f(x)|x - y|dx]dy
\]

where \( E(x) = \mu = \sum_{k=1}^K w_k \mu_k \), unlike some other inequality measures, is generally not subgroup decomposable (Bourguignon 1971).

• This is advantageous when \( f(.) \) construed as mixture distribution.
• When class income distributions have compact, mutually exclusive support (i.e. segmented) → GINI can be decomposed into weighted sum of within subgroup GINI’s ($G_k$) plus a between group GINI ($B$) (Mookerjee & Shorrocks, 1982) in the form:

$$GINI = \sum_{k=1}^{K} w_k^2 \frac{\mu_k}{\mu} G_k + \sum_{k=1}^{K} \sum_{h=1}^{K} \frac{w_k w_h}{2\mu} |\mu_k - \mu_h|$$

$$= \sum_{k=1}^{K} w_k^2 \frac{\mu_k}{\mu} G_k + B$$
Decomposing GINI, Cont’d

- When constituent classes are not segmented → GINI may be more generally written as:

\[
GINI = \sum_{k=1}^{K} w_k^2 \frac{\mu_k}{\mu} G_k + B \\
+ \frac{2}{\mu} \sum_{k=2}^{K} \sum_{j=1}^{k-1} w_k w_j \int_{0}^{\infty} f_k(y) \left[ \int_{y}^{\infty} f_j(x)(x - y) dx \right] dy \\
= \sum_{k=1}^{K} w_k^2 \frac{\mu_k}{\mu} G_k + B + F
\]

where \( F \) is the Non-Segmentation Factor.

- \( \therefore \) \( F \) is a function of extent of distributional overlap → measure of degree to which class distributions are not segmented.
Derived Measures

- A segmentation index \( S \) is given by:

\[
S = 1 - \frac{F}{GINI}
\]

\( S \in [0, 1] \).

- As noted, Polarization has as much to do with Segmentation as the distance between groups (Alienation) \( \rightarrow \) net effect of the two forces. The index in (3) only reflects the former.

- Joint effect of Segmentation & Alienation \( \rightarrow \) \( GINI \) based Polarization Index \( P \) can be constructed in the form:

\[
P = S^\alpha B^{1-\alpha}
\]

(4)
Data

- Used odd years, 1977-2015.
- Income is adult equivalized using the square root rule.
- Household incomes below $1,200 were dropped → 498,333 observations for odd years.
- Covariates include: Age of Head & Spouse, Educational Attainment, Family Size, Degree of Assortative Matching, Occupation of Head & Spouse, Metro, Region, & Trend.
Summary Statistics
## Preliminary Results, 4 Component

### Class Covariates (4 Component Model)

<table>
<thead>
<tr>
<th>Component</th>
<th>$2^{nd}$</th>
<th>$3^{rd}$</th>
<th>$4^{th}$</th>
<th>$2^{nd}$</th>
<th>$3^{rd}$</th>
<th>$4^{th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>0.1176</td>
<td>0.2979</td>
<td>0.5505</td>
<td>&lt; Grade 9,</td>
<td>-1.3810</td>
<td>-2.7235</td>
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<tr>
<td></td>
<td>(0.0038)</td>
<td>(0.0042)</td>
<td>(0.0077)</td>
<td>Husband</td>
<td>(0.0168)</td>
<td>(0.0237)</td>
</tr>
<tr>
<td>Age$^2$</td>
<td>-0.0013</td>
<td>-0.0031</td>
<td>-0.0052</td>
<td>$\geq$ Grade 9, w/o</td>
<td>-0.6796</td>
<td>-1.1756</td>
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<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0001)</td>
<td>High Sch, Husband</td>
<td>(0.0101)</td>
<td>(0.0112)</td>
</tr>
<tr>
<td>Age of Spouse</td>
<td>0.1532</td>
<td>0.3284</td>
<td>0.4501</td>
<td>University Grad, Husband</td>
<td>0.4306</td>
<td>1.0735</td>
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<tr>
<td></td>
<td>(0.0035)</td>
<td>(0.0039)</td>
<td>(0.0071)</td>
<td></td>
<td>(0.0116)</td>
<td>(0.0095)</td>
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<tr>
<td>Age$^2$ of Spouse</td>
<td>-0.0013</td>
<td>-0.0029</td>
<td>-0.0042</td>
<td>&lt; Grade 9, Wife</td>
<td>-1.2322</td>
<td>-2.4136</td>
</tr>
<tr>
<td></td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0001)</td>
<td></td>
<td>(0.0189)</td>
<td>(0.0290)</td>
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<tr>
<td>Family Size</td>
<td>-0.0640</td>
<td>-1.3331</td>
<td>-2.5279</td>
<td>$\geq$ Grade 9, w/o</td>
<td>-0.5069</td>
<td>-0.8396</td>
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<tr>
<td></td>
<td>(0.0204)</td>
<td>(0.0237)</td>
<td>(0.0457)</td>
<td>High Sch, Wife</td>
<td>(0.0103)</td>
<td>(0.0110)</td>
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<tr>
<td>Family Size$^2$</td>
<td>-0.0286</td>
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<td>University Grad, Wife</td>
<td>0.3789</td>
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<td></td>
<td>(0.0023)</td>
<td>(0.0029)</td>
<td>(0.0062)</td>
<td></td>
<td>(0.0120)</td>
<td>(0.0099)</td>
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<tr>
<td>Family Size × Trend</td>
<td>-0.0137</td>
<td>0.0174</td>
<td>0.0075</td>
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<td></td>
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<td>(0.0018)</td>
<td>(0.0021)</td>
<td>(0.0038)</td>
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</tr>
<tr>
<td>Family Size$^2$ × Trend</td>
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<td>-0.0024</td>
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<tr>
<td></td>
<td>(0.0002)</td>
<td>(0.0003)</td>
<td>(0.0005)</td>
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</tr>
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<td>Black</td>
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<tr>
<td></td>
<td>(0.0133)</td>
<td>(0.0148)</td>
<td>(0.0280)</td>
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<tr>
<td>Not in Metro</td>
<td>-0.1873</td>
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<tr>
<td></td>
<td>(0.0078)</td>
<td>(0.0082)</td>
<td>(0.0154)</td>
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<tr>
<td>Trend</td>
<td>-0.0957</td>
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<td>-0.3462</td>
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<tr>
<td></td>
<td>(0.0037)</td>
<td>(0.0040)</td>
<td>(0.0066)</td>
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</tbody>
</table>

Standard errors are in parenthesis. Educational Attainment is compared against high school graduate for both head & spouse.
Class Covariates (4 Component Model), Cont’d

<table>
<thead>
<tr>
<th>Component</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Husband</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technical/</td>
<td>-0.2349</td>
<td>-0.6779</td>
<td>-1.3166</td>
<td>-1.6002</td>
<td>-2.7451</td>
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<td>Sales/Admin</td>
<td>(0.0112)</td>
<td>(0.0096)</td>
<td>(0.0134)</td>
<td>(0.0092)</td>
<td>(0.0099)</td>
<td>(0.0162)</td>
</tr>
<tr>
<td>Service</td>
<td>-1.0459</td>
<td>-2.0564</td>
<td>-3.7812</td>
<td>0.1825</td>
<td>0.7303</td>
<td>1.1988</td>
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<td>Service</td>
<td>(0.0142)</td>
<td>(0.0151)</td>
<td>(0.0335)</td>
<td>(0.0109)</td>
<td>(0.0091)</td>
<td>(0.0135)</td>
</tr>
<tr>
<td>Farming/Forestry/Fishing</td>
<td>-1.9552</td>
<td>-3.5204</td>
<td>-4.3223</td>
<td>-0.8114</td>
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<tr>
<td>Farming/Forestry/Fishing</td>
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<td>(0.0445)</td>
<td>(0.0110)</td>
<td>(0.0131)</td>
<td>(0.0315)</td>
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<td>Production/</td>
<td>-0.2987</td>
<td>-0.9483</td>
<td>-2.7201</td>
<td>0.3297</td>
<td>0.9857</td>
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<td>Craft/Repair</td>
<td>(0.0110)</td>
<td>(0.0106)</td>
<td>(0.0224)</td>
<td>(0.0333)</td>
<td>(0.0434)</td>
<td>(0.0751)</td>
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<tr>
<td>Operator/Fabricator/Laborer</td>
<td>-0.5611</td>
<td>-1.5929</td>
<td>-3.6602</td>
<td>-0.3421</td>
<td>-0.9357</td>
<td>-1.6814</td>
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<tr>
<td>Operator/Fabricator/Laborer</td>
<td>(0.0114)</td>
<td>(0.0117)</td>
<td>(0.0192)</td>
<td>(0.0147)</td>
<td>(0.0170)</td>
<td>(0.0502)</td>
</tr>
<tr>
<td>New England Div</td>
<td>0.4280</td>
<td>0.7382</td>
<td>0.7436</td>
<td>-0.0981</td>
<td>-0.2955</td>
<td>-0.7006</td>
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<tr>
<td>East-South</td>
<td>(0.0147)</td>
<td>(0.0137)</td>
<td>(0.0205)</td>
<td>(0.0165)</td>
<td>(0.0177)</td>
<td>(0.0307)</td>
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<tr>
<td>Mid Atlantic Div</td>
<td>0.2736</td>
<td>0.4991</td>
<td>0.6731</td>
<td>-0.3136</td>
<td>-0.5197</td>
<td>-0.6958</td>
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<tr>
<td>West-South</td>
<td>(0.0134)</td>
<td>(0.0128)</td>
<td>(0.0193)</td>
<td>(0.0139)</td>
<td>(0.0146)</td>
<td>(0.0232)</td>
</tr>
<tr>
<td>East-North Central Div</td>
<td>0.4396</td>
<td>0.7237</td>
<td>0.5688</td>
<td>0.0412</td>
<td>-0.0562</td>
<td>-0.3672</td>
</tr>
<tr>
<td>Mountain Div</td>
<td>(0.0126)</td>
<td>(0.0122)</td>
<td>(0.0192)</td>
<td>(0.0133)</td>
<td>(0.0137)</td>
<td>(0.0221)</td>
</tr>
<tr>
<td>Central Div</td>
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<td>0.3196</td>
<td>-0.1142</td>
<td>-0.0848</td>
<td>0.2287</td>
<td>0.5089</td>
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<td>Pacific Div</td>
<td>(0.0134)</td>
<td>(0.0134)</td>
<td>(0.0219)</td>
<td>(0.0135)</td>
<td>(0.0131)</td>
<td>(0.0192)</td>
</tr>
</tbody>
</table>

Standard errors are in parenthesis. Occupation for husband is compared against managerial/professional occupations, while that for the wife is compared against Technical/Sales/Administrative Support. Regional effects are compared against South Atlantic Div.
Results
Preliminary Results, 4 Component

On Class Covariates

• Education, relative to completed high school, & age (both parents) \( \uparrow \) probability of higher class membership (the latter with diminishing effect).

• Family size \( \downarrow \) probability of higher class membership.

• Black, relative to white & non-metro, relative to metro both \( \downarrow \) probability of higher class membership.

• For husbands, relative to managerial & professional, all occupational categories \( \downarrow \) the probability of higher class membership. For wives relative to technical/admin/sales support, all occupational categories \( \downarrow \) probability of higher class membership, except for managerial & professional occupations which \( \uparrow \) probability of higher class membership.

• Regionally, East-South Central & West-South Central have negative effect on higher income class membership. New England, Mid-Atlantic, & East-North Central have positive effect on higher income class membership.
Figure 1: Class Membership Shares
Results

Preliminary Results, 4 Component

Figure 1a: Class Membership Shares, Combined Middle
Results

Preliminary Results, 4 Component

On Class Shares

• Steadily ↘ middle class.
• Clear structural break in 1991.
• ↗ poor & high income groups over the period.
Figure 2: Class Mean Income Processes
Preliminary Results, 4 Component

Class Average Mean Income Processes, Cont’d

**Table:** Growth Regressions Class Membership Probabilities

<table>
<thead>
<tr>
<th>Class</th>
<th>2 Year Coefficient</th>
<th>Unrestricted</th>
<th>Restricted ($\chi^2_6 = 1.4658$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>$\Delta$</td>
<td>0.0023 (0.5418)</td>
<td>0.0032 (1.8197)</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ dum(77-91)</td>
<td>-0.0004 (0.0584)</td>
<td>-0.0006 (0.1981)</td>
</tr>
<tr>
<td>Lower-Middle</td>
<td>$\Delta$</td>
<td>-0.0029 (0.9191)</td>
<td>-0.0032 (1.8197)</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ dum(77-91)</td>
<td>-0.0010 (0.2017)</td>
<td>0.0006 (0.1981)</td>
</tr>
<tr>
<td>Upper-Middle</td>
<td>$\Delta$</td>
<td>-0.0036 (1.2970)</td>
<td>-0.0032 (1.8197)</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ dum(77-91)</td>
<td>0.0022 (0.4828)</td>
<td>0.0006 (0.1981)</td>
</tr>
<tr>
<td>High</td>
<td>$\Delta$</td>
<td>0.0042 (0.9448)</td>
<td>0.0032 (1.8197)</td>
</tr>
<tr>
<td></td>
<td>$\Delta$ dum(77-91)</td>
<td>-0.0008 (0.1034)</td>
<td>-0.0006 (0.1981)</td>
</tr>
</tbody>
</table>
Figure 3: Class Standard Deviation of Income
On Class Shares

- Steady growth rate in subgroup income means (3.1% implied annual growth rate).
- ▲ variation over the period in all classes, consistent with Gibrat’s law.
- Jump in high income group variation post structural break.
Results

Preliminary Results, 4 Component

Figure 4. GINI, ALIENATION, IDENTIFICATION and POLARIZATION INDICES
Results

Preliminary Results, 4 Component

Change in Alienation/Segmentation & Polarization

- \( \uparrow \) Gini.
- \( \uparrow \) Alienation (relative group differences).
- Constant Identification (degree of segmentation)
- \( \uparrow \) Polarization (Monotonic Increasing function of Identification & Segmentation)
Results

Concerns

Number of Classes & Structural Break

<table>
<thead>
<tr>
<th>Component</th>
<th>AIC</th>
<th>BIC</th>
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- Model is suggesting more than 4 components → Overfitting? ⇒ Currently estimating > 6 components.
- At least one structural break in 1991. Is it significant? → Currently estimating ≥ 4 components splitting data into two sets to verify significance of structural break, & whether there is Δ in number of classes.
- Additional classes principally drawn from middle components ⇒ more complicated narrative.
Other Issues

- Has the transition matrix changed over the years? Data not panel. → Examine Latent-Markov transition on state/metro level data over same years. → But examines Alienation/Segmentation/Polarization between states.

- How much of these cross time changes result from ruling government at state & federal levels? → including voting pattern at both levels.

- Ideally, need large panel to discern both Latent Class & Latent Markov covariates.
Conclusion

• Need a more robust approach to examining Segmentation/Alienation/Polarization → Use Latent Class model to discern number of components.

• Violation of Gibrat’s Law not seen when Latent Class model used.

• When approach applied to US:
  1. Segmentation seems to have been constant over the past 4 decades.
  2. Observed divisiveness may possibly be due to falling middle income shares, increasing poverty share, & ballooning income of highest status class → Polarization

• However, caveat to this agreement with common narrative:
  1. There are more than 4 components.
  2. Preliminary results suggest this clean narrative is more complicated.