Housing Prices and Macro-prudential Policies in Taiwan

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Abstract

This paper develops a small open economy dynamic stochastic general equilibrium (DSGE) model with the collateral constraints. Using Bayesian methods, the model is applied to the Taiwan’s data. We assess the welfare implications of monetary and macro-prudential policies setting counter-cyclically regulatory loan-to-value (LTV) ratio. Our analysis finds that both the optimal interest-rate rules and optimal LTV ratio rules lead to the Pareto improvements. The optimized interest-rate rule in this economy is a muted response to either credit growth or changes in housing prices, and it will increase volatility of inflation. The LTV ratio rule leaning against total credit growth could significantly increase the social welfare and the individual welfare with similar inflation variation, and it could stabilize the credit-to-GDP ratio most effectively.
1 Introduction

The subprime mortgage crisis in 2007 has served as a sharp reminder to economists and policy authorities of the importance of development in the housing market for the border economy. Moreover, the Federal Reserve Chairman Bernanke (2008) stated that "Housing and housing finance played a central role in precipitating the current crisis".\footnote{See Ben S. Bernanke Chairman, Speech At the Federal Reserve System Conference on Housing and Mortgage Markets, Washington, D.C.: Housing, Mortgage Markets, and Foreclosures (Dec. 4, 2008).} In addition, Iacoviello (2010) and Das, Gupta, and Kanda (2011) indicate that research on the housing market has become part of mainstream economics.

From the perspective of financial supervision, this crisis has highlighted that traditional micro-regulation is insufficient to ensure the soundness of the financial system. Therefore, the Basel Committee has introduced some fundamental reform named as Basel III which shifts the micro level of financial supervision towards the macro-prudential dimension. Also, Borio (2003) suggests that the objective of a macro-prudential approach is to limit financial system-wide distress and avoid output costs for the economy as a whole.

Although macro-prudential framework is an important part of the financial regulation and supervision, IMF (2011) points out that there is little consensus on what is meant by macro-prudential policy. However, the instruments targeting on housing prices and credit cycles has received primary attention from macro-prudential policymakers. In particular, the most frequently used tools are the restrictions on the LTV ratio in many countries including Taiwan (Wang, Chen, and Lin, 2017).

In terms of theoretical development, Fisher (1933) proposes the debt deflation theory to emphasize that the deterioration of credit con-
ditions is not just a passive reflection of a declining economy, but might be one of major forces of depressing economic activity. Besley (1995) indicates the credit market failures are usually the result of financial frictions like imperfect information or limited enforcement. When the short-run output fluctuations are triggered by exogenous shocks, financial frictions cause the changes of credit conditions between the market participants, and result in the persistent and amplified response referred to "financial accelerator" or "credit channel".

After Bernanke and Gertler (1989) formed the framework of DSGE model to analyze the impact of credit conditions, there are literatures incorporate this issue by the structural DSGE model with the contract of information asymmetry (Carlstrom and Fuerst, 1997; Bernanke, Gertler, and Gilchrist, 1999; Chen, 2001; Meh and Moran, 2010), or with the contract of limited enforcement (Kiyotaki and Moore, 1997; Cooley, Marimon, and Quadrini, 2004; Iacoviello, 2005; Liu, Wang, and Zha, 2013; Guerrieri and Iacoviello, 2017).

Housing demand is linked directly to agents balance-sheets and hence affects the credit conditions. Lots of literatures that study the relation between housing market and real economic activity are extended from this kind of view of credit.

In Taiwan, there are many empirical researches related to housing market. For example, Lee and Chou (2008) and Ma and Lin (2009) study the identification of housing market cycle, Chang et al. (2009), Huang, Chiang, and Chang (2017), and Peng and Tsai (2017) focus on the discussion for the housing prices. But few literatures construct the structural model to combine the housing market with business cycle. Chen and Cheng (2012) developed a DSGE model with credit frictions based on asymmetry information, to characterized the Taiwan's housing market and business fluctuation.

In contrast to the backward-looking dynamics of housing prices
proposed by Chen and Cheng (2012), we apply the collateral constraints as in Kiyotaki and Moore (1997) from the view of limited enforcement, and generate the forwarding-looking dynamics of housing prices. We bridge the housing value to the collateral constraints of entrepreneurs and borrowers, and develop a small open economy DSGE model, by using Bayesian estimation, to investigate the effect of the monetary policies and LTV ratio rules in Taiwan.

Specifically, we address the questions whether monetary policies need to respond to housing prices or credit market, and what the welfare implications of monetary policies and macro-prudential policies are in Taiwan.

Our analysis finds that both the optimized interest-rate rule and optimized LTV ratio rules lead to the Pareto improvements. The optimized interest-rate rule in Taiwan is a muted response to either credit growth or changes in housing prices, and it will increase the volatility of inflation. The LTV ratio rule leaning against total credit growth could significantly increase the social welfare and the individual welfare with lower inflation variation, and it could stabilize the credit-to-GDP ratio most effectively.

2 The Model

We extend the model featuring collateral constraints from Iacoviello (2005) and Guerrieri and Iacoviello (2017) to the small open economy framework built on Kollmann (2001), Kollmann (2002), and Dib (2011). The economy features two types of households (savers and borrowers), entrepreneurs, retailers, labor unions, a final good firm and a government.

Households consume, work, and accumulate housing (in fixed supply), while entrepreneurs produce homogenous intermediate goods
using capital, housing and hired labor. The key difference between agents is the value of their discount factors: the discount factor of savers ($\beta_s$) is higher than those of borrowers ($\beta_b$) and entrepreneurs ($\beta_e$). Monopolistically competitive retailers are only used to introduce the staggered price settings à la Calvo (1983) contracts, and wages setting from labor unions is modeled in a way analogous to price setting.

2.1 Production Sector

The structure of goods production is divided into following sectors: a final good firm, domestic intermediate goods retailers, imported intermediate goods retailers, and entrepreneurs who product intermediate goods in competitive markets. Adding retailers permits us to introduce price inertia into this economy.

2.1.1 Final good

There is a perfectly-competitive representative firm that combines homogeneous composite domestic intermediate goods $y_{di}$ and homogeneous composite imported intermediate goods $y_{mi}$ into final good, $y_t$, by using the following CES technology:

$$y_t = \left[ (1 - \omega_m) \frac{\kappa}{\omega} y_{di}^\kappa + \omega_m y_{mi}^\kappa \right]^{\frac{1}{\kappa}},$$

where, $1 > \omega_m > 0$ denotes a positive share of composite imported intermediate goods, and $\kappa > 0$ is the elasticity of substitution between composite domestic and imported intermediate goods.

Given the price of final good $P_t$, the composite domestic and imported intermediate goods prices $P_{di}$ and $P_{mi}$, the maximization prob-
lem of final goods firm is:

$$\max_{y_{dt}, y_{mt}} P_t y_t - P_{dt} y_{dt} - P_{mt} y_{mt}$$

subject to

$$y_t = \left[ (1 - \omega_m) \frac{y_{dt}}{P_t} \right]^{\frac{1}{\kappa}} + \omega_m y_{mt} \left[ \frac{y_{dt}}{P_t} \right]^{\frac{1}{\kappa}}.$$

Therefore the demand functions for composite domestic and imported intermediate goods are:

$$y_{dt} = (1 - \omega_m) \left( \frac{P_{dt}}{P_t} \right)^{-\kappa} y_t,$$

$$y_{mt} = \omega_m \left( \frac{P_{mt}}{P_t} \right)^{-\kappa} y_t.$$  \hspace{1cm} (2)

(3)

Thus, when the relative prices of composite domestic goods $P_{dt}/P_t$ rise, the demand for composite domestic intermediate goods decreases. Similar to the demand for composite imported intermediate goods. The price elasticity of these demand functions for composite domestic and imported intermediate goods is $\kappa$.

Besides, perfect competition in the final good market implies that the price level of final good is linked to domestic-output and import prices through:

$$P_t = \left[ (1 - \omega_m) P_{dt}^{1-\kappa} + \omega_m P_{mt}^{1-\kappa} \right]^{\frac{1}{\kappa}}.$$  \hspace{1cm} (4)

The Dixit-Stiglitz aggregates of composite domestic and imported intermediate goods are defined as:

$$y_{dt} = \left[ \int_{t_0}^{1} y_{dt}(s) \frac{1}{\kappa} ds \right]^{\frac{1}{\kappa}},$$

$$y_{mt} = \left[ \int_{t_0}^{1} y_{mt}(s) \frac{1}{\kappa} ds \right]^{\frac{1}{\kappa}},$$  \hspace{1cm} (5)

(6)

where $t_d, t_m > 1$ are the elasticities of substitution between the differentiated domestic and imported intermediate goods. $y_{dt}(s), y_{mt}(s)$ ($s \in [0, 1]$) are differentiated domestic and imported intermediate goods
from domestic and imported retailers. \(P_{dt}(s), P_{mt}(s)\) are the domestic prices of differentiated domestic and imported intermediate goods, respectively.

Thus, the cost minimization problems yield:

\[
y_{dt}(s) = \left( \frac{P_{dt}(s)}{P_{dt}} \right)^{-\alpha} y_{dt}, \tag{7}
\]

\[
y_{mt}(s) = \left( \frac{P_{mt}(s)}{P_{mt}} \right)^{-\alpha} y_{mt}. \tag{8}
\]

### 2.1.2 Domestic intermediate goods retailers

To motivate sticky prices we introduce the costs of adjusting nominal prices and, as in Bernanke, Gertler, and Gilchrist (1999), monopolistic competition at the retail sector. A continuum of domestic intermediate goods retailers of mass 1, indexed by \(s \in [0, 1]\), buy intermediate goods \(y_{at}\) from entrepreneurs at \(P_t^{m}\) in a competitive market, differentiate the goods at no cost into \(y_{at}(s)\). The differentiated intermediate goods is sold in the domestic market \(y_{dt}(s)\) and exported \(y_{xt}(s)\) at the same price of \(P_{dt}(s)\). So that

\[
y_{at}(s) = y_{dt}(s) + y_{xt}(s).
\]

Following McCallum and Nelson (1999), Teo (2009a), and Dib (2011), the foreign aggregate demand function for domestic exports is assumed to resemble the domestic demand function and given by:

\[
y_{xt} = \hat{\omega} \left( \frac{P_{dt}}{e_i P_{t'}} \right)^{-\kappa}, \tag{9}
\]

The corresponding price indices are given by:

\[
P_{dt} = \left( \int_0^1 P_{dt}(s)^{1-\alpha} ds \right)^{\frac{-1}{\alpha}},
\]

\[
P_{mt} = \left( \int_0^1 P_{mt}(s)^{1-\alpha} ds \right)^{\frac{-1}{\alpha}}.
\]
with $e_t$ being the nominal exchange rate; $P_t^f$ the foreign price index in foreign currency; $\tilde{\omega} > 0$ a scaling factor.\footnote{We define $\tilde{\omega} = \omega \cdot e_t^{\tilde{P}_t^f} / \Pi_a$, where $e_t$, $P_t^f$, $P_t$, $y_a$ (without subscript $t$) are the steady-state value of $e_t$, $P_t^f$, $P_t$, $y_a$, respectively. So that the ratio of export-to-GDP in steady-state is $\omega$.}

Similar to McCallum and Nelson (1999), Teo (2009a), and Dib (2011), we assume that the domestic intermediate goods retailers cannot price discriminate across markets, so that the export price in foreign currency is simply $P_{dt} / e_t$ in foreign currency. $\kappa_x > 0$ is the elasticity of substitution for export, when the relative prices $P_{dt} / (e_t P_t^f)$ rise, the exports decrease.

Domestic intermediate goods retailers adopt the Calvo (1983) staggered pricing strategy. Each period, each retailers set price optimally with the probability of $(1 - \theta_d)$. Therefore, the price remains unchanged for $1 / (1 - \theta_d)$ periods.

Given the demand curve facing each retailer by (7), the optimal price $P_{dt}^*(s)$ solves:

$$\sum_{k=0}^{\infty} \theta_d^k E_t \{ \Lambda_{t,k}(P_{dt}^*(s) - \frac{X_d}{P_{dt,k}} y_{a,t+k}^*(s)) \} = 0, \quad (10)$$

where $\Lambda_{t,k}$ is the the stochastic discount factor;\footnote{Assume the domestic intermediate goods retailers are owned by savers.} $X_{dt} = P_{dt} / P_t^w$ is the price markup which in steady state equals $X_d = t_d \omega$/(1 - $t_d$). This condition states that the retailer sets his price where expected discounted marginal revenue is equal to expected discounted marginal cost.

In a symmetric equilibrium, $P_{dt}^*(s) = P_{dt}^*$ and $y_{a,t}^*(s) = y_{a,t}^*$, following the price of composite domestic intermediate goods is:

$$P_{dt} = (\theta_d P_{dt}^{s_d} + (1 - \theta_d)(P_{dt}^*)^{-\theta_d})^{\frac{1}{1-\theta_d}}, \quad (11)$$

After linearization, we can derive a forward looking Phillips curve:

$$\ln\left(\frac{\pi_{dt}}{\pi_d}\right) = \beta_t E_t \ln\left(\frac{\pi_{d,t+1}}{\pi_d}\right) - \frac{(1 - \theta_d)(1 - \beta_t \theta_d)}{\theta_d} \ln\left(\frac{X_{dt}}{X_d}\right) + u_{dt}, \quad (12)$$
where $\pi_{dt} = \frac{P_{dt}}{T_{d,t-1}}$ is inflation rate of locally produced goods; $u_{dt}$ is the domestic cost-push shock.

### 2.1.3 Imported intermediate goods retailers

Analogous to the setting of domestic intermediate goods retailers, there are a continuum of imported intermediate goods retailers, indexed by $s \in [0, 1]$, who buy homogenous intermediate goods at the price $e_t P_t^f$ in domestic currency term (McCallum and Nelson, 1999; Teo, 2009a; Dib, 2011). Each imported goods retailers differentiates the goods into $y_{mt}(s)$, which is sold in a home monopolistically competitive market for the price $P_{mt}(s)$ to produce the imported-composite good $y_{mt}$.

Also, we assume that each retailers can change their prices with a probability with $(1 - \theta_m)$. Given the demand function by (8), the optimal $P_{mt}^*(s)$ solves:

$$
\sum_{k=0}^{\infty} \theta_m^k E_t \{ \Lambda_{t,k}(\frac{P_{mt}^*(s)}{P_{mt,t+k}} - \frac{X}{X_{m,t+k}} y_{mt,t+k}(s)) \} = 0, \quad (13)
$$

where $X_m = l_m/(l_m - 1)$ is the steady state value of the price markup $X_{mt}$ which is equal to $P_{mt}/(e_t P_t^f)$.

In a symmetric equilibrium, $P_{mt}^*(s) = P_{mt}^*$ and $y_{mt}^*(s) = y_{mt}^*$, following the price of composite domestic intermediate goods is:

$$
P_{mt} = (\theta_m P_{mt,t-1} + (1 - \theta_m)(P_{mt}^*)^{1-\theta_m})\frac{1}{1-\theta_m}, \quad (14)
$$

After log-linearizing, we can obtain the Phillips curve:

$$
\ln(\frac{\pi_{mt}}{\pi_m}) = \beta_t E_t \ln(\frac{\pi_{mt,t+1}}{\pi_m}) - (1 - \theta_m)(1 - \beta_t \theta_m)\frac{1}{\theta_m} \ln(\frac{X_{mt}}{X_m}) + u_{mt}, \quad (15)
$$

where $\pi_{mt} = \frac{P_{mt}}{T_{m,t-1}}$ is inflation rate of imported goods, $u_{mt}$ is the imported cost-push shock.
2.1.4 Entrepreneurs

The production function for the entrepreneurs is

\[ y_{at} = A_t k_{t-1}^\mu (h_t^e)^\gamma (n_t^e)^{\alpha(1-\mu^{-\nu})} (n_t^b)^{(1-\alpha)(1-\mu^{-\nu})}, \]  

(16)

where \( A_t \) is productivity shock; \( k_{t-1} \) is the capital stock; \( h_t^e \) is real estate; \( n_t^e, n_t^b \) are hours of work supplied by savers and borrowers; and \( \alpha < \mu, \nu, \alpha < 1 \) are constant parameters.

Entrepreneurs produce intermediate goods in the perfect competition environment, and sell domestic intermediate goods to domestic retailers at the price \( P_t^v \). They maximize the expected discount utility function

\[ \max E_0 \sum_{t=0}^{\infty} \beta_t z_t \Gamma_c \log (c_t^e - \epsilon_t c_{t-1}^e), \]

(17)

where \( E_0 \) denotes the expectation operator; \( c_t^e \) denotes the entrepreneurs' consumption; \( \epsilon_t \) measures habits in consumption; \( \Gamma_c \) is a scale factor to ensure the marginal utility of consumption is \( 1/c_t^e \) in steady state; \( z_t \) captures a shock to intertemporal preferences. Following Iacoviello (2005), we assume that \( \beta_c < \beta_i \) to assure the flow of funds from savers to entrepreneurs.

The budget constraint for entrepreneurs is given by:

\[ \frac{P_t}{P_{dt}} (c_t^s + i_t + \Phi_t) + q_t \Delta h_t^s + w_t^i n_t^i + w_t^h n_t^h + \frac{R_t - 1}{\pi_{dt}} b_{t-1}^e = \frac{y_{at}}{X_{dt}} + b_t^e, \]

(18)

with \( i_t \) being the investment, \( q_t \) the real housing price, \( w_t^i \) the real wage of savers, \( w_t^h \) the real wage of borrowers, \( b_t^e \) the loans in real term, \( R_t \) the nominal interest rate, \( \Phi_t = \frac{\phi}{2} (\frac{k_t}{k_{t-1}} - 1)^2 k_{t-1} \) the adjustment cost of investment (Ireland, 2003), \( \phi \geq 0 \) the parameter of adjustment cost.
After defining \( p_t \equiv \frac{D_t}{P_t} \), (18) can be written in the form:

\[
p_t (c^e_t + i_t + \Phi_t) + q_t \triangle h^e_t + w^h_t n^h_t + \frac{R_{t-1}}{\pi_{dt}} b^e_{t-1} = \frac{y_{at}}{X_{dt}} + b^e_t. \tag{19}
\]

Entrepreneurs are credit constrained, and the maximum amount \( b^e_t \) they could borrow today is

\[
b^e_t \leq m^s \exp(u_{lt}) E_t \left[ \frac{q_{t+1} \pi_{dt, t+1} h^e_t}{R_t} \right], \tag{20}
\]

where \( m^s \) denotes the LTV ratio.

As shown in Kiyotaki and Moore (1997) and Lacoviello (2005), the limit for borrowing expressed as the fraction of asset value. Suppose that, if entrepreneurs fail to repay their debt, the lenders can repossess the entrepreneurs’ assets by paying a proportional transaction cost \((1 - m^s)E_t(q_{t+1}, h^e_t)\). In addition, \( \exp(u_{lt}) \) is a shock to the LTV ratio.

The capital stock \( k_t \) evolves according to the law of motion:

\[
k_t = a^i_t i_t + (1 - \delta) k_{t-1}, \tag{21}
\]

where \( a^i_t \) is the investment-specific technology shock.

Entrepreneurs choose \( c^e_t, h^e_t, b^e_t, k_t, n^h_t \) and \( n^h_t \) to maximize the lifetime utility subject to the flow of funds constraint, technology constraint, capital law of motion and the collateral constraint. The first-
order conditions for the entrepreneur’s optimization are

\[ \lambda^e_t p_t = uc^e_t, \]  
\[ \lambda^e_t q_t = E_t \{ \beta_e \lambda^e_{t+1} \left[ \frac{v y_{a,t+1}}{X_{d,t+1}} + q(+1) \right] + m' \exp(u_{ilt}) \mu^e_{t} q_{t+1} \pi_{d,t+1} \}, \]  
\[ \lambda^e_t = E_t \left[ \beta_e \lambda^e_{t+1} \frac{R_t}{\pi_{d,t+1}} + \mu^e_t R_t \right], \]  
\[ p_t \lambda^e_t \left[ \frac{1}{a^t_i} - \Phi(1 - \frac{k}{k_{t-1}}) \right] = \beta_e E_t \left[ \lambda^e_{t+1} \left[ \frac{\mu y_{a,t+1}}{X_{d,t+1} k_t} + p_{t+1} \left( \frac{1}{a^t_i} - \frac{\Phi}{2} \left(1 - \frac{k_{t+1}}{k^t} \right) \right) \right] \right], \]  
\[ w^e_t = \frac{\alpha(1 - \mu - \nu) y_{at}}{X_{dt} n^t_i}, \]  
\[ w^h_t = \frac{(1 - \alpha)(1 - \mu - \nu) y_{ht}}{X_{dh} n^h_t}, \]

where \( \lambda^e_t, \mu^e_t \) are Lagrange multipliers, and \( uc^e_t \) denotes entrepreneur’s marginal utility of consumption defined by:

\[ uc^e_t = E_t \{ \Gamma_c \left[ \frac{z_t}{(c^e_t - \epsilon_c)} - \frac{\beta_c z_{t+1} \epsilon_c}{(c^e_{t+1} - \epsilon_c)} \right] \}. \]  

### 2.2 Households

#### 2.2.1 Savers

The economy is populated by two groups of households (savers and borrowers), each group having unit mass. Savers maximize the following expected discounted utility function:

\[ \max_{\Gamma_c} \sum_{t=0}^{\infty} \beta^t z_t (\Gamma_c \log(c_t^s - \epsilon_c c_{t-1}^s) + j_t \Gamma_h \log(h_t^s - \epsilon_h h_{t-1}^s)) - \frac{\tau_t}{1 + \eta} (n_t^s)^{1+\eta}, \]  

where \( c^s_t, h^s_t, \) and \( n^s_t \) are consumption, housing, labor hours of savers, respectively; \( \eta \) is the inverse of the Frisch labor elasticity; \( j_t \), is the housing preference shock; \( \tau_t \) is the labor supply shock; \( \epsilon_c \) and \( \epsilon_h \) measures
habits in consumption and housing, respectively; $\Gamma_c$ and $\Gamma_h$ are the scaling factors.\footnote{\(\Gamma_c = \frac{1 - \phi_c}{1 - \phi_c}, \quad \Gamma_h = \frac{1 - \phi_h}{1 - \phi_h}\)}

A saver’s revenue flow in any period comes from the wage income, lump-sum profits from retailers and labor unions, then uses the all revenue finance consumption, housing, and saving in the domestic and international credit market, as well as the adjustment costs. The budget constraint faced by savers is thus:

\[
\frac{P_t}{P_{dt}} c_t^* + q_t \Delta h_t^* + b_t^* + \frac{e_t P_t^f}{P_{dt}} (b_{t-1}^f - \frac{R_{t-1}^f}{\pi_t^f} b_{t-1}^f + \Phi_{bt}) + \text{tax}_t = \frac{w_t^* N_t^i}{X_{w_t^i}} + \frac{R_{t-1}}{\pi_{dt}} b_{t-1}^i + D iv_t^r, \tag{30}
\]

with $b_t^i$ being the domestic bonds, $b_t^f$ the foreign bonds in foreign currency term, $\pi_t^f = \frac{p_t^f}{p_{t-1}^f}$ the foreign inflation rate, $\Phi_{bt}$ the adjustment cost of foreign bonds,\footnote{To ensure that there exists a stationary solution in the small open economy, following Schmitt-Grohé and Uribe (2003), Karabarbounis (2010), and Chen and Cheng (2012), we introduce the adjustment cost of foreign bonds $\Phi_{bt} = \frac{\phi_b}{2} (b_{t}^f - b_{t}^f)^2$ parameterized by $\phi_b > 0$, where $b_{t}^f$ is the steady state value of $b_{t}^f$.} $R_t^f$ the foreign nominal interest rates, $\text{tax}_t$ the lump-sum tax, $X_{w^i,t}$ the wage markup, and $D iv_t^r$ is the lump-sum profits from retailers and labor unions.

Let $p_{mt} = \frac{p_{mt}}{P_{dt}}$, then (30) can be written in the following form:

\[
p_t c_t^* + q_t \Delta h_t^* + b_t^* + \frac{p_{mt}}{X_{mt}} (b_t^f - \frac{R_{t-1}^f}{\pi_t^f} b_{t-1}^f + \Phi_{bt}) = \frac{w_t^* N_t^i}{X_{w_t^i}} + \frac{R_{t-1}}{\pi_{dt}} b_{t-1}^i + D iv_t^r. \tag{31}
\]

Savers choose $c_t^*$, $h_t^*$, $n_t^i$, $b_t^i$, $b_t^f$ to maximize their expected utility.
The first order conditions are:

\[ \lambda^*_i p_i = uc^*_i, \]  
\[ \lambda^*_i q_i = E_i[\beta \lambda^*_{i+1} q_{i+1}] + uh^*_i, \]  
\[ z_i \tau_i (n^*_i)^\eta = \lambda^*_i \frac{w^*_i}{X_{n^*_i}}, \]  
\[ \lambda^*_i = \beta_s E_i[\lambda^*_{i+1} \frac{R_t}{\pi_{d,t+1}}], \]  
\[ [1 + \phi_b (b^f_i - b^f) \lambda^*_i \rho_{mt} X_{mt}] = \beta_s E_i[\lambda^*_{i+1} \rho_{m,t+1} R^f_i X_{m,t+1} \pi^f_i], \]

where \( \lambda^*_i \) denotes the Lagrange multiplier; \( uc^*_i \), \( uh^*_i \) denote the saver’s marginal utility of consumption and housing defined by:

\[ uc^*_i = E_i[\{ \frac{z_i}{(c^*_i - \epsilon_c) c^*_{i-1}} - \frac{\beta_s z_{t+1} \epsilon_c}{(c^*_{t+1} - \epsilon_c)} \}], \]
\[ uh^*_i = E_i[\{ \frac{j_i z_i}{(h^*_i - \epsilon_h) h^*_{i-1}} - \frac{\beta_s j_{i+1} \epsilon_h}{(h^*_{i+1} - \epsilon_h)} \}]. \]

### 2.2.2 Borrowers

Borrowers maximize the expected discount utility function:

\[ \max E_0 \sum_{t=0}^{\infty} \beta^t_b (\Gamma_c \log(c^b_t - \epsilon_c c^b_{t-1}) + j_i \Gamma_h \log(h^b_t - \epsilon_h h^b_{t-1}) - \frac{\tau_t}{1 + \eta} (n^b_t)^{1+\eta}), \]

where \( c^b_t \), \( h^b_t \), \( n^b_t \) denote the consumption, housing, and labor hours of borrowers, respectively. \( \Gamma_c \) and \( \Gamma_h \) are the scaling factors.\(^8\)

Following Funke and Paetz (2013), Mendicino and Punzi (2014), and Ng and Feng (2016), borrowers can not finance their expenditures from international credit market. And they do not own the retailers, their dividends \( Div^b_t \) only come from labor unions. The budget constraint is given by:

\[ p_t c^b_t + q_t \triangle h^b_t = \frac{w^*_i n^b_i}{X_{m^b,t}} + (b^b_t - \frac{R_{t-1}}{\pi_{d,t}} b^b_{t-1}) + Div^b_t. \]  

\(^8\)\( \Gamma_{c2} = \frac{1-\epsilon_c}{1-\beta_c \epsilon_c}, \quad \Gamma_{h2} = \frac{1-\epsilon_h}{1-\beta_h \epsilon_h} \)
Similar to entrepreneurs, borrowers are credit constrained by:

\[ b_t^b \leq m^b \exp(u_{tt}) E_t[\frac{q_{t+1}\pi_{d,t+1}h_t^b}{R_t}] \]  

(39)

Borrowers choose \( c_t^b, h_t^b, n_t^b, b_t^b \) to maximize their expected utility. The first order conditions are:

\[
\lambda_t^b p_t = uc_t^b, \tag{40}
\]

\[
\lambda_t^b q_t = E_t[\beta_t^b \lambda_{t+1}^b q_{t+1} + \mu_t^b m^b \exp(u_{tt}) q_{t+1}\pi_{d,t+1}] + uh_t^b, \tag{41}
\]

\[
\tau_t(n_t^b) \eta = \lambda_t^b \frac{w_t^b}{X_{w_t^b}}, \tag{42}
\]

\[
\lambda_t^b = E_t[\beta_t^b \lambda_{t+1}^b \frac{R_t}{\pi_{d,t+1}} + \mu_t^b R_t], \tag{43}
\]

where \( \lambda_t^b, \mu_t^b \) denote the Lagrange multiplier, \( uc_t^b, uh_t^b \) denote the borrower’s marginal utility of consumption and housing defined by:

\[
uc_t^b \equiv E_t\{\Gamma_t^b[\frac{z_t}{(c_t^b - \epsilon_c c_t^{b_{-1}})} - \frac{\beta_t^b z_{t+1}\epsilon_c}{(c_{t+1}^{b_{-1}} - \epsilon_c c_t^{b_{-1}})}]\}, \tag{44}
\]

\[
uh_t^b \equiv E_t\{\Gamma_t^b[\frac{j_t z_t}{(h_t^b - \epsilon_h h_t^{b_{-1}})} - \frac{\beta_t^b j_{t+1} z_{t+1}\epsilon_h}{(h_{t+1}^b - \epsilon_h h_t^{b_{-1}})}]\}. \tag{45}
\]

### 2.3 Labor unions

We model wage setting in a way that is analogous to price setting. We assume that there are two kind of unions, each kind labor union hires homogeneous labor services from savers and borrowers, respectively. Labor unions differentiate the labor services as in Smets and Wouters (2007), so there is some monopoly power over wages, which allows for introduce the sticky nominal wage in Calvo scheme. They offer labor services to wholesale labor packers who reassemble these services into the homogeneous labor composites \( n_t^b, n_t^b \). These packers supply labor service to entrepreneurs.

Each period, each kind of labor unions optimizes the wage with the probability \((1 - \theta_w)\). These assumptions deliver the following wage
Phillips curves:

\[
\ln\left(\frac{W^s_i}{\pi_d}\right) = \beta_s E_t \ln\left(\frac{W^s_{i+1}}{\pi_d}\right) - \epsilon_w^s \ln\left(\frac{X_{w^s_i}}{X_{w^s}}\right), \tag{46}
\]

\[
\ln\left(\frac{W^b_i}{\pi_d}\right) = \beta_b E_t \ln\left(\frac{W^b_{i+1}}{\pi_d}\right) - \epsilon_w^b \ln\left(\frac{X_{w^b_i}}{X_{w^b}}\right), \tag{47}
\]

where, for each household type, \(W^s_i = \frac{w_i^{s_t}}{w_{i-1}^{s_t}}\) denotes the wage inflation, \(X_{w^s_i}\) denotes the wage markup, and \(\epsilon_w^s = \frac{(1-\beta_w)(1-\beta_w)}{\beta_w}\).

### 2.4 Government

We assume that the central bank sets the interest rate rule according the Taylor-type rule:

\[
R_t = R\left(\frac{\pi_{dt}}{\pi_d}\right)^{\tau_n} \left(\frac{y_{at}}{y_{a,t-1}}\right)^{\tau_n} \exp(u_{Rt}), \tag{48}
\]

with \(R\) being the steady state nominal interest rate, \(r_n\) and \(\tau_n\) the policy parameters, \(u_{Rt}\) the monetary policy shock.

The government budget constraint is given by:

\[
g_t = t \alpha x_t, \tag{49}
\]

where \(g_t\) is the government expenditure which is simply assumed constant.

### 2.5 Exogenous processes

There are nine temporary and persistent shocks in this model. The temporary shocks including monetary shock \((u_{R_t})\), domestic cost-push shock \((u_{dt})\), imported cost-push shock \((u_{mt})\), and LTV ratio shock \((u_{lt})\) are i.i.d. shocks with mean zero and variance \(\sigma_{dt}^2, \sigma_{mt}^2, \sigma_{lt}^2\).

The persistent shocks are assumed to evolve according to the first-
order autoregressive processes:

\[
\begin{align*}
\ln A_t &= \rho_a \ln A_{t-1} + \epsilon_{at}, \\
\ln j_t &= (1 - \rho_j) \ln J + \rho_j \ln j_{t-1} + \epsilon_{jt}, \\
\ln z_t &= \rho_z \ln z_{t-1} + \epsilon_{zt}, \\
\ln \tau_t &= \rho_\tau \ln \tau_{t-1} + \epsilon_{\tau t}, \\
\ln a_t^i &= \rho_{a^i} \ln a_{t-1}^i + \epsilon_{al},
\end{align*}
\]

where \( \epsilon_{x}, x \in \{a, j, z, \tau, i\} \) are i.i.d. shocks with zero means and variances \( \sigma_x^2 \).

2.6 Market clearing conditions

The market clearing conditions for final good market, intermediate goods market, housing market, domestic credit market, and international credit market are:

\[
y_t - \Phi_{bt} - \Phi_1 = c_t + i_t + g_t, \quad (55)
\]

\[
y_{at} = y_{dt} + y_{xt}, \quad (56)
\]

\[
1 = h_t^a + h_t^b + h_t^c, \quad (57)
\]

\[
b_t^c = b_t^b + b_t^c, \quad (58)
\]

\[
TB_t = \frac{e_t}{P_{dt}} \left[ \frac{f_t}{\pi_{ft}} - \frac{R_t}{\pi_{ft}} b_{t-1}^d \right] = y_{xt} - \frac{e_t}{P_{dt}} y_{mt} \cdot \quad (59)
\]

where \( c_t = c_t^a + c_t^b + c_t^c \) denotes the aggregate consumption, \( TB_t \) denotes the real trade balance, \( y_{at} \) represents the real GDP.

The equilibrium is defined as the path of allocations \( \{ c_t^a, h_t^a, n_t^a, b_t^a, b_t^b, c_t^b, h_t^b, n_t^b, b_t^b, c_t^c, h_t^c, b_t^c, k_t, i_t, y_{at}, y_{dt}, y_{xt}, y_{mt}, y_{i_t}\}_{t=0}^{\infty} \), and prices \( \{ q_t, R_t, w_t^a, w_t^b, w_t^c, \pi_{dt}, \pi_{mt}, X_w^a, X_w^b, X_w^c, X_{dt}, X_{mt}, p_t, p_{mt}\}_{t=0}^{\infty} \), satisfying

\footnotesize\(^{9}\)The labor market clearing condition has been imposed at the beginning.

\footnotesize\(^{10}\)Due to the effects of foreign interest rate and foreign inflation rate shocks are found to be near zero of the output growth variations in Taiwan (Teo, 2009b), following Chen and Cheng (2012), we assume foreign interest rate and foreign inflation rate are constants.
the first order conditions of savers, borrowers, entrepreneurs, and the market clearing conditions.

3 Data and estimation

3.1 Data

The model is estimated using quarterly data of Taiwan over 1993Q1 to 2017Q2, including GDP, consumption, investment, export, import, GDP deflator, interest rate (interbank overnight call loan rate), wage, housing price.

GDP, consumption, investment, export, import, GDP deflator, and wage are obtained from Directorate-General of Budget, Accounting and Statistics (DGBAS), interest rate is obtained from AREMOS, and housing price is obtained from Taiwan Cathay Real Estate Index.\(^1\)

Except the interest rate, other data are seasonally adjusted using US Census Bureau’s X-13 ARIMA-SEATS program, transformed into real term by GDP deflator. All data other than interest rate and real wage are divided by the Taiwan’s population of age 15 years and over,\(^2\) and converted into first differences of logarithms. Interest rate is divided by four to obtain the quarterly interest rate. All data are demeaned prior to estimation. Figure 1 shows the nine data.

3.2 Calibration

Table 1 describes the calibrated parameters in this model. Some parameters directly relative to the steady state values are calibrated to match sample means in the data. For instance, the steady state value

\(^1\)Since we obtain two Taiwan Cathay Real Estate Indices with different intervals and base period, we convert the two indices to the same base period by ourselves.

\(^2\)Following Teo (2009b), Hwang (2013), we remove the influences of demographic changes using the population of age 15 years and over as a proxy for the working age population. The population data comes form AREMOS.
of domestic inflation is set to match the average GDP deflator growth rate \( \pi_d = 1.000 \) during the sample period. \( \beta_s \) is calibrated at 0.994, which corresponds to a steady state annualized nominal interest rate of 2.58% in the data. \( \omega_c \) is set to 0.5795, which is in line with the sample mean of export/GDP ratio. \( \omega_m \) is calibrated at 0.5986 to match the average import/GDP ratio of 0.5226. \( \frac{\kappa}{\mu} \) is calibrated at 0.1562 to match the average proportion of government expenditures.

Parameters not directly to the steady state values are chosen according to the literatures. \( \beta_b \) and \( \beta_c \) are commonly used at the range of 0.94 to 0.99, we set \( \beta_b = 0.98 \) and \( \beta_c = 0.97 \) to ensure that the borrowers and entrepreneurs have a large enough motive to take out a loan. \( \delta \) is calibrated at 0.025, corresponding to an annual capital depreciation rate of 10%. \( \mu \) is calibrated at 0.3, the same as the value used by Teo (2009b). \( \nu \) is calibrated at 0.03, following Iacoviello (2005). \( m^s \) and \( m^e \) are set to 0.85.\(^\text{13}\) The steady state values of markups \( X_d, X_w, \)

\(^{13}\)Chen and Wang (2007) found that the LTV ratio of the firm was about 85% in Taiwan from 1991 to 1994, while Wang, Chen, and Lin (2017) showed that the sample mean and standard deviation of
$X_{w2}$ and $X_m$ are calibrated at 1.2 according to Guerrieri and Iacoviello (2017) and Teo (2009b).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factors</td>
<td>$\beta_s = 0.994$, $\beta_b = 0.98$, $\beta_c = 0.97$</td>
</tr>
<tr>
<td>Steady state domestic inflation rate</td>
<td>$\pi_d = 1.000$</td>
</tr>
<tr>
<td>Auxiliary parameters of export and import</td>
<td>$\omega_x = 0.5795$, $\omega_m = 0.5986$</td>
</tr>
<tr>
<td>Steady state proportion of government expenditures</td>
<td>$\frac{\delta}{\delta_d} = 0.1562$</td>
</tr>
<tr>
<td>Capital depreciation rate</td>
<td>$\delta = 0.025$</td>
</tr>
<tr>
<td>Housing share and capital share</td>
<td>$\mu = 0.3$, $\nu = 0.03$</td>
</tr>
<tr>
<td>Maximum LTV ratios</td>
<td>$m_r = 0.85$, $m^e = 0.85$</td>
</tr>
<tr>
<td>Steady state markups</td>
<td>$X_d = 1.2$, $X_m = 1.2$, $X_{w1} = 1.2$, $X_{w2} = 1.2$</td>
</tr>
</tbody>
</table>

### 3.3 Estimation

We use the Bayesian methodology to estimate parameters. Table 2 and Table 3 report the prior distributions and the posterior distributions for the parameters.

The prior distributions are in accord with earlier contributions to Bayesian estimations as a whole.14

The posterior mean of housing preference parameter $j$ is 0.258, higher than the US of 0.1, and lower than the China of 0.307; the parameter $\alpha$ relates to labor income share of savers is 0.784, similar to US and China.15

---

14See Justiniano and Preston (2010), Teo (2009b), He et al. (2017), Guerrieri and Iacoviello (2017), and Hwang (2013).

15See Iacoviello (2005), Iacoviello and Neri (2010), and He et al. (2017).
The household’s degree of habit persistence in consumption $\varepsilon_c$ is 0.626, a little bit lower than Teo (2009b) of 0.744 estimated by Taiwan’s data. The habit persistence parameter in housing $\varepsilon_h$ is 0.501, which is slightly lower than $\varepsilon_c$. The price elasticities of exports $\kappa_e$ and imports $\kappa$ are 1.717 and 1.419, respectively.

Moving to the parameters $\theta_d$, $\theta_m$ and $\theta_w$ that govern the nominal rigidities. The estimate of $\theta_d$ is 0.502 implied that the domestic goods prices are re-optimized on average every 2.009 quarters in Taiwan. The imported goods prices are slightly rigid than the domestic goods prices with the $\theta_m = 0.533$. As for wages, the wage rigidity parameter $\theta_w$ is estimated at mean 0.504 suggested that the average wage-change interval on every 2.016 quarters. These numbers correspond to the estimates of Teo (2009b) for Taiwan’s economy. However, these estimated values of price and wage stickiness are much lower than those in the similar model, e.g. Iacoviello and Neri (2010) and Guerrieri and Iacoviello (2017), for U.S. economy.16

Turning to the estimates of the monetary policy rule, the mean of the reaction coefficient to inflation is estimated to be relatively high ($r_n = 2.679$), and the mean of the reaction coefficient to GDP growth $r_Y$ is 0.123. These suggest that Taiwan’s central bank pay more attention to the inflation rate (Hwang, 2013; Teo, 2009b). Cheng and Mao (2013) also showed that the Taiwan’s central bank target inflation more strictly and achieve more stable inflation after 1988.

Final about the exogenous shocks, the technology shock, housing preference shock and investment-specific technology shock are quite persistent (posterior mean of AR(1) coefficient are 0.965, 0.984 and 0.977, respectively). The imported cost-push shock, investment-specific

---

16: Comparing to the DSGE models without collateral constraint, the price and wage stickiness parameters obtained here are still much lower than Smets and Wouters (2003), Adolfsen et al. (2007), and Smets and Wouters (2007) on European and U.S. economy.
technology shock and housing preference shock are more volatile (posterior mean of standard deviation are 0.250, 0.193 and 0.160, respectively).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Density  Mean  S.D.</td>
<td>Mean  90% Confidence interval</td>
</tr>
<tr>
<td>( J )</td>
<td>beta  0.2000  0.0500</td>
<td>0.2579  0.1734  0.3429</td>
</tr>
<tr>
<td>( \eta )</td>
<td>invg  5.0000  2.0000</td>
<td>2.1970  1.7622  2.6337</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>beta  0.7500  0.0500</td>
<td>0.7837  0.6687  0.8935</td>
</tr>
<tr>
<td>( \varepsilon_c )</td>
<td>beta  0.7000  0.1000</td>
<td>0.6262  0.5646  0.6940</td>
</tr>
<tr>
<td>( \varepsilon_h )</td>
<td>beta  0.7000  0.1000</td>
<td>0.5008  0.3679  0.6311</td>
</tr>
<tr>
<td>( \phi )</td>
<td>gamm  5.0000  0.1000</td>
<td>5.0497  4.8833  5.2141</td>
</tr>
<tr>
<td>( \phi_b )</td>
<td>invg  0.0001  0.0100</td>
<td>( 3.2858 \times 10^{-5} )  ( 1.8922 \times 10^{-5} )  ( 4.6422 \times 10^{-5} )</td>
</tr>
<tr>
<td>( \theta_d )</td>
<td>beta  0.5000  0.0100</td>
<td>0.5023  0.4856  0.5184</td>
</tr>
<tr>
<td>( \theta_m )</td>
<td>beta  0.5000  0.0100</td>
<td>0.5332  0.5161  0.5509</td>
</tr>
<tr>
<td>( \theta_w )</td>
<td>beta  0.5000  0.0100</td>
<td>0.5039  0.4874  0.5203</td>
</tr>
<tr>
<td>( \kappa_x )</td>
<td>norm  1.5000  0.0500</td>
<td>1.7171  1.6446  1.7935</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>norm  1.5000  0.0500</td>
<td>1.4189  1.3462  1.4987</td>
</tr>
<tr>
<td>( r_n )</td>
<td>norm  1.5000  0.2500</td>
<td>2.6793  2.4046  2.9627</td>
</tr>
<tr>
<td>( r_{ya} )</td>
<td>beta  0.1250  0.0250</td>
<td>0.1225  0.0836  0.1606</td>
</tr>
</tbody>
</table>
4 Model properties

4.1 Housing preference shock

Figure 2 shows the impulse responses to a one standard deviation housing preference shock. The increase in the housing demand leads to a higher housing price, hence raises the real estate wealth and relaxes the collateral constraint of entrepreneurs, results in the increasing investment, and thus causes the growth in output.

Moving to the consumption, a rise in housing price enhances the collateral capacity of constrained-agents, therefore allowing them to
Figure 2: Housing preference shock
borrow more and to raise their consumption, which is called wealth effect. On the other hand, the boosting housing price increase the cost of purchasing housing and crowed out the consumption, namely substitution effect. Considering these two effects, the net effect on the aggregate consumption is positive.

Figure 2 also illustrates the responses for three alternative versions of the model in different LTV ratios. Lowering LTV ratios limits the borrowing ability of constrained-agents, following the lower increment of output, investment and consumption. Furthermore, facing the rising housing price, the model without collateral effects ($m^e, m^b = 0$) predicts the negative response of aggregate consumption which is mainly driven by the substitution effect.

For the dynamics of housing price, housing preference shock generates higher inflation increment for higher LTV ratios, following the higher interest rate under the Taylor-type rule. This suppresses the boosting of the housing price, and results in the lower increment of the housing price.

### 4.2 Technology shock

A positive technology shock of one standard deviation increases the output, induces entrepreneurs to enhance demand of investment and housing, thus the increase in housing price. The rising housing price raises the value of real estate, consequently entrepreneurs could borrow and invest more to amplify the output.

The greater supply pushes down the inflation, requires the reduction in the interest rate, increases the aggregate consumption.

In addition, Figure 3 displays the amplified effect of output, investment, consumption and loan for higher LTV ratios.
Figure 3: Technology shock
4.3 Monetary shock

Figure 4 plots the impulse responses to a monetary shock. Unlike previous two shocks, we consider the shocks which rise the interest rate by 1 percent. First, the adverse monetary shock yields a drop for the demand of consumption and housing, also causes a decline in housing price. Declining housing price results in the decreasing value of assets, tightens the borrowing constraint of entrepreneurs, hence pushes down the investment and output.

As the figure shows, with the same rises in interest rate, we could see the amplified effect of consumption, output, investment, and credit for higher LTV ratios, i.e. collateral effects magnify the aggregate variables to monetary shock.

5 Optimal policy and welfare analysis

We study the optimal monetary and macro-prudential policies by using the welfare-based evaluation rather than an ad hoc loss function. That is, policymaker maximizes the social welfare subject to the competitive equilibrium conditions and the class of interest-rate and LTV rules considered.

According to Schmitt-Grohé and Uribe (2004), Schmitt-Grohé and Uribe (2007), Kim et al. (2008), and Faia and Monacelli (2007), we measure the individual welfare conditional on the initial state, $t=0$, being the deterministic steady state, The individual welfare of each agent
Figure 4: Monetary shock
is defined by following conditional expectation of lifetime utilities:

\[
V_o^s = \max_{E_0} \sum_{t=0}^{\infty} \beta^t U_s(c_i^t, h_i^t, n_i^t),
\]

\[
V_o^b = \max_{E_0} \sum_{t=0}^{\infty} \beta^t U_b(c_i^t, h_i^t, n_i^t),
\]

\[
V_o^e = \max_{E_0} \sum_{t=0}^{\infty} \beta^t U_e(c_i^t),
\]

where \( V_o^s, V_o^b, \) and \( V_o^e \) denote the welfare of savers, borrowers and entrepreneurs, respectively.

Following Mendicino and Pescatori (2007), Rubio (2011), and Lambertini, Mendicino, and Punzi (2013), the social welfare are aggregated by the weighted sum of individual welfare:

\[
V_o^{total} = (1 - \beta_s) V_o^s + (1 - \beta_b) V_o^b + (1 - \beta_e) V_o^e.
\]

The welfare of savers, borrowers and entrepreneurs are weighted by \( (1 - \beta^s), (1 - \beta^b) \) and \( (1 - \beta^e) \), respectively. So that the social planner can equalize utility across different type of agents given a constant utility level. We evaluate the social welfare according to the different policy rules over varying parameters, and explore the optimal policy of the interest rate rules and the counter-cyclical LTV ratio rules.

### 5.1 Interest rate rules

We investigate what the optimal interest rate rule that maximize the social welfare is. Firstly, we address the welfare implication of baseline policy based on (48). And we assess the alternative interest rate rules that react to either credit growth or changes in housing price.

\[
R_t = R\left(\frac{\pi d_t}{\pi_d}\right)^{r_s}\left(\frac{y_{atl}}{y_{atl-1}}\right)^{r_s}\left(\frac{X_t}{X_{t-1}}\right)^{r_x} \exp(u_{Rt}),
\]

(60)

where \( X_t \in \{b^t, q_t\} \) and \( r_x \geq 0 \). We obtain the optimized interest rate rule by conducting the grid search over the multidimensional param-
Figure 5: Optimized parameter and conditional welfare

<table>
<thead>
<tr>
<th>Rules</th>
<th>Policy parameters</th>
<th>Welfare values</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Social</td>
<td>Saver</td>
<td>Borrower</td>
<td>Entrepreneur</td>
<td></td>
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<tr>
<td>Base line policy</td>
<td>$r_\pi = 2.68, r_{ya} = 0.12$</td>
<td>-8.3727</td>
<td>-313.5596</td>
<td>-207.8750</td>
<td>-77.7943</td>
<td></td>
</tr>
<tr>
<td>Optimized interest rate rules</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>$r_\pi = 3.3, r_{ya} = 2.5$</td>
<td>-8.3234</td>
<td>-313.5051</td>
<td>-205.5033</td>
<td>-77.7436</td>
<td></td>
</tr>
<tr>
<td>Domestic credit</td>
<td>$r_\pi = 3.3, r_{ya} = 2.5, r_b = 0$</td>
<td>-8.3234</td>
<td>-313.5051</td>
<td>-205.5033</td>
<td>-77.7436</td>
<td></td>
</tr>
<tr>
<td>Housing price</td>
<td>$r_\pi = 3.3, r_{ya} = 2.5, r_q = 0$</td>
<td>-8.3234</td>
<td>-313.5051</td>
<td>-205.5033</td>
<td>-77.7436</td>
<td></td>
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<tr>
<td>Optimized LTV ratio rules</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>$\phi_{ya} = -0.29$</td>
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<tr>
<td>Domestic credit</td>
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<td>Housing price</td>
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<td>-313.4954</td>
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<td>-77.7394</td>
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</table>
eters with constant LTV ratios.\footnote{The search ranges are set to be $r_a \in [1.1, 4]$, $r_{ya} \in [0, 3]$ and $r_X \in [0, 3]$. The grid step for each parameter is 0.1.}

Table 5 reports combination of the parameters for the optimized interest rules that maximize the social welfare function. The optimized interest rate rule features a muted response to either credit growth or changes in housing price, which means that targeting the financial variables would not improve the social welfare when the policy authority implements the optimized policy.

Besides, adopting the best Taylor-type rule not only maximizes the social welfare, but also improves the individual welfare for savers, borrowers and entrepreneurs, i.e. is Pareto optimal.

Although the best monetary policy refers to a higher response coefficient to inflation ($r_a = 3.3$), the aggressive output growth reaction coefficient ($r_{ya} = 2.5$) still induces a higher inflation volatility than the baseline policy (see Table 6 for the stabilization effect).

### 5.2 Counter-cyclic LTV ratio

Now we assess the implications of adopting the dynamic regulation on LTV ratios as macro-prudential tools. Following Mendicino and Punzi (2014), we assume that the policy authority sets the LTV ratios vary in a counter-cyclic manner:

$$m_i^t = m^t \left( \frac{X_t}{X_{t-1}} \right)^{\phi_x}, \quad i \in \{b, e\}$$

(61)

with $X_t \in \{y_{at}, b^t, q_t\}$, $m^t = 0.85$, $i \in \{b, e\}$ being the steady state LTV ratios, $\phi_x \leq 0$ being the reaction parameter. We fix the parameters of Taylor-type rule at the baseline model, and search over the range $[-20, 0]$ for the parameter $\phi_x$ to obtain the optimized LTV ratio rule.\footnote{The grid step is 0.01.} Accordingly, the LTV ratios will be tightened in response to the growth of the targeting variables, and vice versa.
### Figure 6: Stabilization effect

<table>
<thead>
<tr>
<th></th>
<th>Baseline policy</th>
<th>Optimized interest rate rule</th>
<th>Optimized LTV ratio rules</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>GDP</td>
</tr>
<tr>
<td>$c^i/y_a$</td>
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<td>0.2468</td>
<td>0.2483</td>
</tr>
<tr>
<td>$c^b/y_a$</td>
<td>0.0168</td>
<td>0.0164</td>
<td>0.0167</td>
</tr>
<tr>
<td>$c^e/y_a$</td>
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<td>0.0478</td>
<td>0.0465</td>
</tr>
<tr>
<td>$c/y_a$</td>
<td>0.2796</td>
<td>0.2820</td>
<td>0.2794</td>
</tr>
<tr>
<td>$h^i/y_a$</td>
<td>0.1580</td>
<td>0.1334</td>
<td>0.1562</td>
</tr>
<tr>
<td>$h^b/y_a$</td>
<td>0.0504</td>
<td>0.0420</td>
<td>0.0500</td>
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<tr>
<td>$b^i/y_a$</td>
<td>3.1276</td>
<td>2.6920</td>
<td>3.1019</td>
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<td>$b^b/y_a$</td>
<td>2.5397</td>
<td>2.1641</td>
<td>2.5207</td>
</tr>
<tr>
<td>$b^e/y_a$</td>
<td>1.4495</td>
<td>1.4126</td>
<td>1.4477</td>
</tr>
<tr>
<td>$y_a$</td>
<td>0.3355</td>
<td>0.3149</td>
<td>0.3347</td>
</tr>
<tr>
<td>$R$</td>
<td>0.0227</td>
<td>0.0129</td>
<td>0.0226</td>
</tr>
<tr>
<td>$\pi_d$</td>
<td>0.0114</td>
<td>0.0221</td>
<td>0.0114</td>
</tr>
<tr>
<td>$m^b, m^e$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0123</td>
</tr>
</tbody>
</table>

* Stabilization effect represents standard deviation of the second-order approximation.
### Figure 7: Level effect

<table>
<thead>
<tr>
<th></th>
<th>Baseline policy</th>
<th>Optimized interest rate rule</th>
<th>Optimized LTV ratio rules</th>
<th>GDP</th>
<th>Domestic credit</th>
<th>Housing price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c^i/y_a$</td>
<td>0.5362</td>
<td>0.5380</td>
<td>0.5360</td>
<td>0.5323</td>
<td>0.5356</td>
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<tr>
<td>$c^h/y_a$</td>
<td>0.1026</td>
<td>0.1024</td>
<td>0.1027</td>
<td>0.1041</td>
<td>0.1030</td>
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<tr>
<td>$c^e/y_a$</td>
<td>0.1747</td>
<td>0.1751</td>
<td>0.1747</td>
<td>0.1734</td>
<td>0.1749</td>
<td>0.1749</td>
</tr>
<tr>
<td>$c/y_a$</td>
<td>0.8136</td>
<td>0.8155</td>
<td>0.8135</td>
<td>0.8098</td>
<td>0.8136</td>
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<tr>
<td>$h^i/y_a$</td>
<td>0.4877</td>
<td>0.4751</td>
<td>0.4867</td>
<td>0.4664</td>
<td>0.4838</td>
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<tr>
<td>$h^h/y_a$</td>
<td>0.0483</td>
<td>0.0554</td>
<td>0.0487</td>
<td>0.0533</td>
<td>0.0500</td>
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<tr>
<td>$b^h/y_a$</td>
<td>3.5301</td>
<td>3.7061</td>
<td>3.5352</td>
<td>3.5394</td>
<td>3.5623</td>
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<td>$b^c/y_a$</td>
<td>2.9431</td>
<td>3.1046</td>
<td>2.9564</td>
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<td>$y_a$</td>
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<td>1.7375</td>
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<td>1.7508</td>
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<tr>
<td>$q$</td>
<td>67.8900</td>
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<td>67.8758</td>
<td>68.2932</td>
<td>67.8155</td>
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<tr>
<td>$R$</td>
<td>1.0042</td>
<td>1.0051</td>
<td>1.0043</td>
<td>1.0041</td>
<td>1.0043</td>
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</tr>
<tr>
<td>$\pi_d$</td>
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<td>0.9999</td>
<td>0.9993</td>
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<tr>
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<td>0.8500</td>
<td>0.8501</td>
<td>0.8615</td>
<td>0.8503</td>
<td></td>
</tr>
</tbody>
</table>

* Level effect represents theoretical stochastic mean of the second-order approximation.
The optimized LTV ratio rules targeting different variables are shown in Table 5. Comparing to the baseline model with a fixed LTV ratio, allowing for the counter-cyclical LTV policies increase the social welfare as well as the individual welfare of savers, borrowers and entrepreneurs. That is, these counter-cyclic LTV ratio rules all result in a Pareto improvement.

Across these three optimized rules, adopting the LTV ratio rule that responds to the domestic credit not only derives the best effect of social welfare, but also increases the individual welfare of all group of agents. Besides, the LTV ratio rule targeting domestic credit is the most effective way to reduce the volatility of credit-to-GDP ratio, and thereby housing price. In contrast to the other LTV ratio rules, the rule responding to domestic credit growth generates the largest variation of the LTV ratios, which effectively reduces the volatility of credit-to-GDP ratio and housing price.

This is not the cases for the rules that target GDP or housing price growth. These two rules generate the lower volatility of the counter-cyclic LTV ratios and hence mitigate the the impact on credit variation.

5.3 Overall evaluation

In terms of welfare evaluation, the optimized interest rate rule and the optimized LTV ratio rules are all Pareto optimal. Among these monetary and macro-prudential policies, allowing for the counter-cyclic LTV ratio rule reacting to domestic credit obtains the best effect of social welfare as well as individual welfare of all groups of agents.

With respect to mitigating the volatilities of key variables, the optimized interest rate rule and the three optimized LTV ratio rules could all stabilize the variations of the credit-to-GDP ratio, the housing price and the output. However, considering the impact on inflation, adopt-
ing the optimized interest rate rule would induce a higher inflation volatility. Moreover, implementing the LTV policy leaning against the domestic credit could increase the welfare most significantly under the similar inflation variation.

From the perspective of total consumption, the three LTV ratio policies could reduce the volatilities of aggregate consumption, while the optimized interest rate rule would rise the level of total consumption.

In regard to financial stability, the LTV ratio rule that targets domestic credit is the most effective way to reduce the volatility of credit-to-GDP ratio, therefore stabilize the financial system.\(^\text{10}\)

## 6 Conclusion

We extend the model featuring collateral constraints from Iacoviello (2005) and Guerrieri and Iacoviello (2017) to the small open economy framework built on Kollmann (2001), Kollmann (2002), and Dib (2011). This model is applied to the Taiwan’s data by using Bayesian technique. The estimated model allows us to assess the welfare implications of the monetary and macro-prudential policy.

Our results suggest that the optimized interest rate rule and the optimized LTV ratio rules all lead to the Pareto improvements. The optimized interest rate rule illustrates that targeting the financial variables, e.g. the credit growth or the changes in housing prices, would not improve the social welfare. Implementing the LTV policy reacting to the domestic credit could increase the social welfare as well as individual welfare most significantly under the similar inflation variation. However, adopting the optimized interest rate rule would induce the

\(^{10}\text{European Systemic Risk Board (2014) indicates that excessive credit growth has been identified as a key driver of financial crises identified four intermediate objectives.}\)
higher volatility of inflation. Furthermore, allowing for the LTV ratio rule leaning against total credit growth would obtain the best effect on stabilization of the credit-to-GDP ratio.

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