Central Bank Digital Currency: when Price and Bank Stability Collide

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Abstract

An account-based central bank digital currency has the potential to replace demand-deposits in private banks. In that case, the central bank invests in the real economy and takes over the role of maturity transformation to allow risk-sharing among depositors. Its function as intermediary exposes the CB to demand-liquidity or 'spending' shocks by its depositors. Since demand-deposit contracts are nominal, high aggregate spending not necessarily demands excessive liquidation of real investment by the central bank. A run on a central bank can therefore manifest itself either as a standard run characterized by excessive real asset liquidation (rationing) or as a run on the price level where a small supply of real goods meets a high demand. The central bank thus trades off price stability against the excessive liquidation of real goods.

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1 Introduction

Many central banks and policy-making institutions, such as the IMF, the BIS, the Sveriges Riksbank, and the Bank of Canada, are openly debating the introduction of a central bank digital currency (CBDC). See, respectively, Lagarde (2018), Auer and Böhme (2020), Ingves (2018), and Davoodalhosseini et al. (2020).¹

The introduction and adoption of CBDCs have the potential to be a watershed for the monetary and financial systems of advanced economies. Since at least the classic formulation of Bagehot (1873), central banks have viewed their primary tasks as maintaining stable prices as well as maintaining financial stability as lender of last resort. With a CBDC, two additional and significant aspects come into play. First, a CBDC easily allows the opening of retail deposits in central banks to all private households and firms. Second, with a CBDC, central banks are in the position to lend directly to the real economy without relying on private financial intermediaries.²

In this paper, we seek to model the interplay of these roles and to evaluate the advantages and drawbacks of introducing a CBDC concerning the subsequent reorganization of the banking system and its consequences for monetary policy, allocations, and welfare.

In particular, we are keenly interested in understanding how financial intermediation will be affected by the presence of a CBDC. To do so, we will build on the tradition of the Diamond and Dybvig (1983) model, the most popular framework in the economics of banking. Such a model emphasizes the role of banks in maturity transformation: banks finance long term projects with demand deposits, which may be withdrawn at a short horizon to meet liquidity shocks. Banks, therefore, allow society to achieve allocations that are otherwise not attainable under autarky. Can this maturity transformation still occur at the socially-optimal level with a CBDC? Can a central bank do better, for instance, by avoiding runs?

¹Notice that, in this paper, we use the term CBDC to denote an account-based electronic currency in the sense of Barrdear and Kumhof (2016) and Bordo and Levin (2017). This linguistic convention has been adopted by a broad spectrum of monetary economists and policymakers. Other forms of central bank-issued electronic money, such as a token-based central bank cryptocurrency or traditional electronic reserves, beget many questions of interest, but most of them are not within the scope of our current investigation. We will analyze, nevertheless, a simple extension of our model with token-based and synthetic CBDCs and argue that most our results carry over to these two alternative cases.

²While both deposits and lending to the public at large by a central bank can be accomplished without a CBDC (as it often happened in the past; see Fernández-Villaverde et al., 2020a, for historical examples), the operational logistics without digital means become too cumbersome in a modern, large economy. Also, from the perspective of our paper, it is mainly irrelevant whether the deposits and loans in the CBDC are run directly by the central bank or by financial institutions that just implement the directives of the central bank.
We will depart from the original formulation of the Diamond and Dybvig model in a crucial aspect. While Diamond and Dybvig consider intermediation with private banks, a CBDC implies central bank intermediation. This difference is consequential because a central bank can control the price level. For example, a central bank can issue additional units of the CBDC to cover losses in its loan portfolio, implicitly diffusing the costs of the credit losses among all holders of currency.

More concretely, while classic bank runs occur due to a rationing problem (liquidation of illiquid assets) at a given price level, the central bank does not necessarily incur rationing. Instead, since contracts are nominal, the monetary authority can avoid excess liquidation of real assets by sacrificing inflation targeting. Thus, a run on the central bank can manifest itself in two ways, either as a classic run, caused by rationing of real assets, or as a run on the price level.

To allow for this feedback mechanism between the loan portfolio and the price level, we modify the basic Diamond and Dybvig (1983) model, where all contracts are real, by considering nominal contracts. To do so, we assume that real goods can only be traded against money, in particular, the CBDC, and that the agents in the economy hold accounts with CBDC balances at the central bank. This is an implicit form of a cash-in-advance constraint built on the tradition of Svensson (1983) and Lucas Jr and Stokey (1985), but suited to the digital world. In fact, a cash-in-advance constraint is more relevant in a CBDC world because other means of payment, such as the transfer of private deposits, might have disappeared.

As in Diamond and Dybvig (1983), we have three time periods (0, 1, and 2). In the economy, there exists a simple, real short-run storage technology and a real long-term investment technology that can be liquidated early, but at a penalty. Agents are symmetric in $t = 0$. In $t = 1$, the agents learn whether they prefer to consume in period one (impatient agents) or rather consume in period two (patient ones). The agent’s type is, however, private information and neither observable by other agents nor the central bank. Diamond and Dybvig show, that in such a setting, real demand-deposit contracts offered by an intermediary allow the agents to share the risk of becoming impatient.

In our model, this role as the intermediary is played the central bank that offers demand-deposit contracts to the agents. But these contracts are nominal. Unlike

\footnote{In Fernández-Villaverde et al. (2020a), we study a real version of the model. In particular, we show a simple equivalence result between financial intermediation through private banks and financial intermediation through a central bank using a CBDC and under which conditions such equivalence result collapses. Throughout the paper, we will highlight the places where dealing with a real model makes a difference with respect to our baseline results.}
private banks, the central bank can manipulate the price level, by this affecting the real allocation of the agents and thus incentives to spend CBDC or not. The manipulation of the price level occurs through the market clearing of the real goods market. At time zero, the central bank collects the agents real goods endowment and invests it in the real long-term technology, offering nominal CBDC balances (money issuance rule) in return. At the interim period, agents need to decide whether to spend their nominal balances or not. Agents decide to spend if their expected real consumption at the interim period exceeds the real consumption of the following period, in anticipation of the central bank's policy that follows. After the aggregate nominal spending decision of the agents has realized, the central bank picks a policy consisting of a liquidation policy (i.e., the percentage of real long-run projects liquidated in the second period) and a nominal interest rate policy the central bank offers on the non-spent CBDC balances. The interim supply of real goods, determined by the central bank's liquidation policy, together with the aggregate spending behavior and the money issuance rule pin down the interim price level. In particular, the central bank does not incur a real rationing problem as private banks do, since the central bank does not take as given the price level. Rather, the central bank trades off the rationing of goods with price stability. The interim liquidation policy further affects the supply of real goods in the following period. The agents spending strategy needs to be optimal given their belief about the central bank policy and the evolution of the price level.

Our main result is the existence of central bank runs (either exhaustive or partial) if the belief of the agents on the central bank policy implies that the expected real consumption in the first period exceeds consumption in the second period (where the run is exhaustive if the inequality is strict and partial if it is weak).

But what do we mean by a central bank run? Cannot the central bank issue as much CBDC as needed to service all its depositors? We will discuss how a run on the central bank has much in common with a traditional bank run in terms of its real consequences (i.e., the impact on long-term projects). Since those real consequences for allocations are the ultimate objects of interest, the label run on the central bank is surely appropriate.

Given our main result, we can show that the central bank can implement the socially-optimal amount of maturity transformation by picking an appropriate policy. In particular, to deter patient agents from spending and thus triggering a run, the central bank can threaten the agents to implement a liquidation policy that makes spending non-optimal ex-post by increasing the price level. In other words: if the central bank can credibly threaten the patient agents by setting such a liquidation
policy, the central bank deters them from spending, by this preventing a central bank run equilibrium. Therefore, the central bank can implement a unique equilibrium, where only impatient agents spend, all patient agents roll over, and the social optimum is achieved.\textsuperscript{4}

Interestingly, the implementation of a run-deterring policy is only possible because the contracts between the central bank and the agents are nominal. Since the central bank does not (have to) take the price level as given, liquidation of the real technology is at its discretion. If contracts were real, the claims of the agents in terms of the consumption are fixed already at time zero, by this implying a unique liquidation policy. Similarly, if we were to have nominal contracts, but now between a private bank and depositors, the private bank would need to take the price level as given, by this, again, pinning down the liquidation policy.

Next, we show the conditions that the central bank liquidation policy must satisfy to achieve price target and price stability (which, at this moment, we can consider as part of an exogenously given mandate). Given our intuition two paragraphs above, our next result should not be a surprise. If the central bank commits itself to a price target, the socially optimal allocation cannot be implemented. Although in this equilibrium central bank runs are also avoided, forcing the central bank to meet a price target for all realizations of beliefs of the agents exhausts the liquidation possibilities available to a central bank and precludes the right amount of investment in the long-run project. We will discuss, nevertheless, weaker versions of price stability and how those more relaxed mandates may deliver socially-optimal allocations.

Finally, we derive some results when we allow for the suspension of spending, token-based and synthetic CBDCs, and the presence of traditional cash.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces our model. Section 4 presents the main analysis of the model, defines an equilibrium, and describes some its fundamental properties. Section 5 discusses how the social optimum can be implemented. Section 6 deals with price stability and how it relates with the implementation of the social optimum. Section 7 reviews several extensions of our basic model, including alternative forms of a CBDC and cash. Section 8 concludes.

\textsuperscript{4}At first sight, this policy of the central bank might seem similar to the classic suspension-of-convertibility, which is known to exclude bank runs in the Diamond-Dybvig environment. There is a subtle, but important difference, however. Suspension of convertibility there requires the bank to stop paying customers who arrive after some fraction of withdrawers appear. Here, however, there is no suspension of accounts. Instead, the price level adjusts to reduce the amount of goods traded against the digital currency, and the central bank generates enough incentives for patient agents to wait. There is, nevertheless, a concern regarding time inconsistency to which we will return later.
2 Related literature

Our paper contributes to a growing literature on the macroeconomic implications of a CBDC. Brunnermeier and Niepelt (2019) derive an equivalence result of allocations when introducing a CBDC where the central bank redeposits CBDC funds in the private banks (pass through). Florian and Gersbach (2019) on the other hand considers competition between private deposits and CBDC and shows that the introduction of a CBDC transfers default risk to the central bank. Skeie (2019) analyzes inflation-driven digital currency runs in a nominal model where a private digital currency competes with a CBDC. Unlike Florian and Gersbach (2019) and Skeie (2019), we disregard potential competition between a CBDC and deposits at private banks, as, for instance, also analyzed in Fernández-Villaverde et al. (2020b). Instead, this paper builds on the Diamond and Dybvig (1983) model and stresses the central bank’s role of liquidity transformation when issuing a CBDC to allow agents the sharing of idiosyncratic liquidity risk.

Unlike Brunnermeier and Niepelt (2019), we are more explicit on the micro incentives of agents to run on the central bank. Unlike, Diamond and Dybvig (1983) and Fernández-Villaverde et al. (2020b) who consider real contracts, we consider nominal contracts between the agents and the central bank such that the price level becomes a crucial additional degree of freedom to the central bank. Similar to Allen and Gale (1998) and Skeie (2008), large withdrawals of nominal deposits can lead to an increase in the price level, by this reducing the real allocation and deterring runs. Unlike Skeie (2008), here, the intermediary is the central bank who can decide how much real investment to liquidate, by this controlling the goods supply and thus indirectly the price level, thus, counteracting the aggregate spending behavior of the agents when desired. Unlike Allen and Gale (1998), here, the central bank has full control over the real goods supply in $t = 1$ and $t = 2$. The central bank can potentially liquidate all or no assets early by this shifting the $t = 2$ supply of goods from zero to the maximum, thus, redistributing the goods supply across the agent groups of early and late spending depositors.

Second, we have many points of contact with Keister and Sanches (2019), who explore how the presence of a CBDC affects the liquidity premium on bank deposits and, through it, investment. Related ideas are also explored by Böser and Gersbach (2019). Our paper distinguishes itself from Brunnermeier and Niepelt (2019), Keister and Sanches (2019), and Böser and Gersbach (2019) by also discussing allocations under banking panics.
3 Our basic framework

Our framework builds on the classical Diamond-Dybvig model of banking. Time is discrete with three periods \( t = 0, 1, 2 \). There is a \([0, 1]\)-continuum of agents, each endowed with one unit of the consumption good in period \( t = 0 \). Agents are symmetric in the initial period, but can be of two types in period 1, referred to as patients and impatient. The agent’s type is randomly drawn at the beginning of period 1 and it is private information. Let \( \lambda \in (0, 1) \) denote the fraction of impatient agents, those who value consumption in period 1. In contrast, patient agents value consumption in period \( t = 2 \). Preferences are represented by a strictly increasing, strictly concave, and continuously differentiable utility function \( u(\cdot) \in \mathbb{R} \). We further assume a relative risk aversion, \(-x \cdot u''(x)/u'(x) > 1\), for all consumption levels \( x \).

**Investment.** There exists a long-term production technology in the economy. For each unit of the good invested in \( t = 0 \), the technology yields either one unit at \( t = 1 \) or \( R > 1 \) units at \( t = 2 \). Additionally, there is a storage technology between periods 1 and 2, yielding one unit of the good in \( t = 2 \) for each unit invested in \( t = 1 \). All agents can access both technologies.

**Efficient Allocation.** Let \( x_1 \in \mathbb{R}_+ \) denote the impatient agent’s consumption, and let \( x_2 \in \mathbb{R}_+ \) denote the patient agent’s consumption. The efficient allocation maximizes social welfare \( \lambda u(x_1) + (1 - \lambda) u(x_2) \) subject to \( \lambda x_1 \leq y \) and \( (1 - \lambda) x_2 \leq R (1 - y) + y - \lambda x_1 \), where \( y \in [0, 1] \) is the liquidation amount in \( t = 1 \). There exists a unique solution given by:

\[
\frac{u'(x^*_1)}{x^*_1} = R \frac{u'(x^*_2)}{x^*_2},
\]

together with \( x^*_1 = \frac{y^*}{\lambda} \) and \( x^*_2 = \frac{R(1-y^*)}{1-\lambda} \). Diamond and Dybvig (1983) have shown that \( x^*_1 < x^*_2 \) holds at the optimum and that a demand deposit contract can implement the efficient allocation. However, a demand deposit contract can also induce a a bank-run equilibrium. This outcome, by forcing the liquidation of the long-term investment, is clearly inefficient.

A key feature of analysis in Diamond and Dybvig (1983) is the use of a “real” demand deposit contract (i.e., a contract that promises to pay out goods in future periods). Our main contribution in this paper is to show that a nominal contract can lead to the unique implementation of the efficient allocation.
4 A nominal economy

We now consider an economy with a social planner that uses nominal contracts to implement the efficient allocation. The planner offers contracts in a unit of account for which it is the sole issuer. Because central banks have a monopoly on currency, the planner in our analysis can be understood as the central bank.

**Nominal Contracts.** All contracts are issued in a unit of account for which the central bank has a monopoly. Agents who sign a contract with the central bank receive a nominal payment and then trade money balances for goods. Specifically, the central bank issues a digital currency referred to as a CBDC, which takes the form of accounts at the central bank. We refer to the unit of account as digital euros. Agents can spend digital euros on their accounts by transferring them to other agents in exchange for goods.

Like physical euros, agents cannot hold negative amounts of digital euros. Indeed, in this environment, borrowing cash does not imply holding negative amounts of cash. Instead, it just means that the agent has to pay back cash at some future point, i.e, it is the debtor on a credit relation. We will discuss the distinction between this account-based system of CBDC and a token-based system as well as physical cash later in the paper.

**Timing.** The sequence of events unfolds as follows. At the beginning of the initial period, the central bank creates an account for each agent in the economy. More precisely, each agent starts at date 0 with a zero balance CBDC account. Then, the central bank agrees to deposit one unit of the good in exchange for $M > 0$ units of digital euros, to be credited to that agent’s account. Next, the central bank decides the amount of goods to be invested in the long-term technology.

In period 1, agents learn their type and decide whether to spend their CBDC balances, that is, either to withdraw or to roll them over. The central bank contract imposes that an agent either withdraws all its balances or no balance at all. This restriction simplifies the analysis by allowing us to avoid having to consider the same agents withdrawing in two periods.

Because types are unobservable, the central bank cannot deny withdrawal for a patient agent who wishes to exercise that option. Let $n \in [0, 1]$ denote the fraction of agents who decide to withdraw in $t = 1$. The central bank observes $n$ and decides the

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5For reference, we provide the classic real solution in the technical appendix ??.
fraction $y = y(n)$ of goods to be liquidated, selling that amount in the market at the unit price $P_1$. The central bank then chooses a nominal interest rate $i = i(n)$ to be paid in period 2 on the remaining CBDC balances (i.e., each digital euro held at the end of $t = 1$ turns into $1 + i(n)$ digital euros at the beginning of $t = 2$). Note that $i(n) \geq -1$, given that agents cannot hold negative amounts of digital euros.

In period 2, the remaining depositors each have $(1 + i)M$ digital euros. Here, we are implicitly assuming that some spending agents do not, in turn, sell their acquired goods to other spending agents. Agents withdraw from the central bank and use these nominal balances to buy goods in the market at a price $P_2$. The central bank then supplies $R [1 - y(n)]$ units in exchange for money balances. Figure 1 summarizes this timing.
Central Bank Policy. A central bank policy can be defined by a triple \((M, y(\cdot), i(\cdot))\), where \(y : [0, 1] \rightarrow [0, 1]\) is the central bank’s liquidation policy and \(i : [0, 1] \rightarrow [-1, \infty)\) is the interest rate policy.

Market Clearing. In periods 1 and 2, agents withdrawing from the bank exchange their money balances for goods in a Walrasian market. The market-clearing conditions are given by:

\[
\begin{align*}
 nM &= P_1 y(n) \\ (1 - n)(1 + i(n))M &= P_2 R(1 - y(n)),
\end{align*}
\]

which take the form of the quantity theory equation in each period. Given \(n\) and the central bank’s policy, these conditions determine the price level, \(P_1 = P_1(n)\) and \(P_2 = P_2(n)\), in each period:

\[
\begin{align*}
 P_1(n) &= \frac{nM}{y(n)} \\ P_2(n) &= \frac{(1 - n)(1 + i(n))M}{R(1 - y(n))}
\end{align*}
\]

The central bank chooses the initial money supply before learning the number of withdrawals in the intermediate period. However, the central bank controls the goods supply in the Walrasian market, which can be made conditional on the number of withdrawals. As a result, the central bank can control the price level in period 1. The interest payments on CBDC balances held until the final period allow the central bank to control the price level in period 2 independently of the price level set in period 1.

It is worth highlighting that the fact that the central bank has a monopoly on the unit of account in the economy allows it to control the price level. If the intermediary were a commercial bank, for instance, it would need to take the price level as given, by this having no choice on the fraction of assets to liquidate in the interim period, which could give rise to a rationing problem. Because the intermediary is the central bank with a monopoly on the unit of account used in the contracts, the liquidation policy is flexible.

Implied Real Contract. The budget constraint for an impatient agent is:

\[ x_1 = \frac{M}{P_1}, \]
and the budget constraint for a patient agent is

\[ x_2 = \frac{(1 + i(n))M}{P_2}. \]

The fraction of early withdrawals \( n \) and the liquidation policy \( y(n) \) jointly determine the allocation of goods via the market-clearing conditions:

\[
x_1(n) = \frac{y(n)}{n} \tag{5}
\]
\[
x_2(n) = \frac{1 - y(n)}{1 - n} R \tag{6}
\]

Because each agent withdrawing in the same period has the same nominal income, the liquidation amount \( y(n) \) is equally distributed across all spending agents in period 1, and the amount \( R(1 - y(n)) \) is equally distributed across all spending agents in period 2.\(^6\)

To summarize the analysis so far: in the initial period, the central bank offers a nominal contract \((M, M(1 + i(n)))\) in exchange for one unit of the good. If the consumer accepts the contract, the central bank has the option to withdraw either \( M \) digital euros in period 1 or \( M(1 + i(n)) \) digital euros in period 2. The consumer’s budget constraints then imply \((x_1, x_2) = \left( \frac{M}{P_1}, \frac{M(1 + i(n))}{P_2} \right)\). Finally, the central bank’s policy, together with the market-clearing conditions, results in the consumption amounts \((x_1(n), x_2(n)) = \left( \frac{y(n)}{n}, \frac{1 - y(n)}{1 - n} R \right)\).

**Equilibrium.** We are now ready to define a perfect Bayes Nash equilibrium, our equilibrium concept for our economy.

An equilibrium consists of an initial money supply \( M \), a liquidation policy \( y : [0, 1] \to [0, 1] \), a nominal interest rate policy \( i : [0, 1] \to [-1, \infty) \), aggregate spending behavior \( n \in [0, 1] \), and price levels \((P_1, P_2)\) such that:

1. The consumer’s deposit and withdrawal decisions are optimal, given the central bank’s policy \((M, y(\cdot), i(\cdot))\), the price level sequence \((P_1, P_2)\), and its beliefs regarding other agents’ behavior.

2. The price level clears the goods market in each period;

3. The central bank policy is optimal, given the depositors’ spending behavior \( n \).

\(^6\)These equations remain intuitive, even if \( y(n) = 0 \) or \( y(n) = 1 \). We therefore assume them to hold then as well, despite one of the price levels being ill-defined or infinite.
**Runs on the central bank.** The first important property of the equilibrium defined above is that a nominal contract, *per se*, does not rule out the possibility of a run on the central bank.

**Definition 1.** A *run on the central bank* occurs if \( n > \lambda \). The run on the central bank is called **exhaustive** if \( n = 1 \).

In a bank run, the central bank obviously is not running out of the item that it has promised to agents and that it can produce freely (i.e., it is not running out of digital money). This distinguishes it from the bank run equilibrium in Diamond and Dybvig (1983), in which a commercial bank prematurely liquidates all its assets to satisfy the demand for withdrawals in period 1, ultimately running out of resources. If \( n > \lambda \), the central bank is confronted with a run on deposits. As we will see, the real consequences of a run on the central bank with nominal contracts can be similar to its counterpart in the model with real contracts. However, we shall demonstrate that the central bank’s ability to avert a run is necessarily tied to its monopoly on currency and the implementation of a nominal contract.

Note that impatient agents will spend their entire balances in period 1, given that they have no use for the consumption good in period 2.\(^7\) Patient agents will choose to prematurely withdraw their CBDC balances only if they believe the central bank policy implies \( x_1 > x_2 \) (this is the sense in which we can call this choice a “run”). In that case, patient agents will use the storage technology to consume \( x_1 \) in period 2. Otherwise, patient agents will find it optimal to wait until the final period. These decisions depend on the central bank’s choices only through the liquidation policy \( y(\cdot) \) and not through the nominal elements \( M \) and \( i(\cdot) \).

The aggregate spending fraction \( n \) has to be consistent with these choices in equilibrium. These considerations immediately imply the following proposition.

**Proposition 2.** Given the central bank policy \((M, y(\cdot), i(\cdot))\),

1. \( n = \lambda \) is an equilibrium only if \( x_1(\lambda) \leq x_2(\lambda) \). Then, \( P_1 \) and \( P_2 \) are uniquely determined by (3) and (4).

2. A central bank run \( n = 1 \) is an equilibrium if and only if \( x_1(1) \geq x_2(1) \).

3. Only some patient agents withdraw \( \lambda < n < 1 \) in equilibrium (i.e., there is a partial run on the central bank) if and only if \( x_1(n) = x_2(n) \).

\(^7\)In case that \( y(n) = 0 \), impatient agents are indifferent between spending and not-spending. To break ties, we assume that they spend their CBDC balances in \( t = 1 \).
This proposition fully characterizes the range of equilibria, given the central bank policy. But, can this policy achieve a first-best allocation? The next section shows that, indeed, it can.

5 Implementation of the social optimum

In our model, the implementation of the social planning optimum is of particular interest to the central bank. Given the preferences and technology that we postulated above, only the real allocation of goods to the two types of agents matter, and there is no additional motive for the monetary authority to keep prices stable.

However, focusing only on real allocations is a narrow perspective. There is a vast literature arguing in favor of central banks keeping prices stable or setting a goal of low and stable inflation for reasons that are absent from our model. For instance, stable prices minimize the misallocations created nominal rigidities as in Woodford (2003). And having to hold cash to accomplish transactions, such in models of cash-in-advance or money-in-utility, create a whole range of distortions that can be minimized by a deft management of the price level (think about the logic behind the Friedman rule). Rather than extending the model to include these considerations, which will complicate the analysis for an uncertain benefit, we shall proceed by discussing, as we introduce our results, the tradeoffs between achieving the optimal real allocation of consumption and the implications of such an effort for the stability of prices.

Given that all agents behave according to their type, \( n = \lambda \), a liquidation policy \( y^* = y^*(\lambda) \) maximizes ex-ante welfare:

\[
W = \lambda u(x_1) + (1 - \lambda) u(x_2)
\]

subject to (5) and (6).

The interior first-order condition for this problem implies:

\[
u'(x^*_1) = Ru'(x^*_2),
\]

where \( x^*_1 = y^*/\lambda \) and \( x^*_2 = R(1 - y^*)/(1 - \lambda) \). Given our assumptions on the utility function, equation (8) uniquely pins down \( y^* \), which is the familiar condition arising from the optimal deposit contract in Diamond and Dybvig (1983). Together with \( R > 1 \) and the concavity of the utility function, equation (8) also implies that the
consumption of patient agents is higher than the consumption of impatient ones:

\[ x_1^* < x_2^*. \]  

(9)

Moreover, the depositors’ relative risk-aversion exceeding unity and the resource constraint yields:

\[ R(1 - \lambda x_1^*) = (1 - \lambda)x_2^*. \]  

(10)

Equations (5), (6), (8), and (10) give us \( x_1^* > 1 \) and \( x_2^* < R \).

Finally, at the socially optimal allocation, we have a liquidation policy \( y^*(\lambda) = x_1^*\lambda > \lambda \), resulting in the inequality \( P_1^* < M \) via equation (3).\(^8\) These results confirm our assertion at the start of this section that the social optimum is independent of price level stability.

Combining the previous derivation with proposition 2, we arrive at the main result of the paper.

**Proposition 3.** The central bank policy \((M, y(\cdot), i(\cdot))\) implements the social optimum \((x_1^*, x_2^*)\) in equilibrium if the central bank:

i) Sets \( y(\lambda) = y^* > \lambda \) for any \( n \leq \lambda \).

ii) Sets a liquidation policy that implies \( x_1(n) < x_2(n) \) for all \( n > \lambda \).

To understand this result, note first that the real allocation to agents and, thus, their incentives to spend or not depends on the central bank policy \((M, y(\cdot), i(\cdot))\) only through the liquidation policy \( y(\cdot) \). Given that only impatient agents are spending (i.e., \( n = \lambda \)), then a policy choice with \( y(\lambda) = y^* \) for \( \lambda \in (0, 1) \) implements the socially optimum. That is, there is a multiplicity of monetary policies that implement the first-best since the pair \((M, i(\cdot))\) is not uniquely pinned down. While the pair \((M, i(\cdot))\) does not affect the depositors’ incentives, it has an impact on prices via (3) and (4).

Second, since the central bank observes aggregate spending behavior \( n \) before it liquidates assets, it is not committed to liquidating \( y^* \) if it observes that some patient agents are also spending. To deter patient agents from spending, the central bank can threaten the agents to implement a liquidation policy \( y(\cdot) \) that makes spending

\[^8\]Following the proof in Diamond and Dybvig (1983),

\[ Ru'(R) = u'(1) + \int_1^R \frac{\partial}{\partial x} (x \cdot u'(x)) \, dx = u'(1) + \int_1^R (x \cdot u''(x) + u'(x)) \, dx < u'(1) \]  

(11)

by \(-x \cdot u''(x)/u'(x) > 1\) for all \( x \).
non-optimal ex post so that \( x_1(n) < x_2(n) \) for \( n \in (\lambda, 1] \). If the monetary authority can credibly threaten patient agents by setting such a liquidation policy, it deters them from spending, ending an equilibrium central bank run. Therefore, there is a unique equilibrium, where only impatient agents spend, all patient agents roll over, and the social optimum is always implemented.

**Definition 4.** We call a liquidation policy \( y(\cdot) \) “run-deterring” if it satisfies

\[
y^d(n) < \frac{nR}{1 + n(R - 1)}, \quad \text{for all } n \in (\lambda, 1]
\]  

(12)

Such a liquidation policy implies that “roll over” is ex post optimal \( x_1(n) < x_2(n) \) even though patient agents are withdrawing \( n \in (\lambda, 1] \).

The implementation of a run-deterring policy is only possible since the contracts between the central bank and the agents are nominal. Since the central bank does not have to take the price level as given, the liquidation of investments in the real technology is at its discretion. In the case of real contracts between a private bank and depositors, such as in Diamond and Dybvig (1983), the real claims of the agents are fixed already in \( t = 0 \), by this implying a liquidation policy for the very amount of aggregate spending \( n \). In the case of nominal contracts between a private bank and depositors, the private bank has to take the price level as given, by this, again, pinning down the liquidation policy.

**Corollary 5.** Every policy choice \((M, y(\cdot), i(\cdot)), n \in [0, 1]\) with \( y(\lambda) = y^* \) and

\[
y^d(n) < \frac{nR}{1 + n(R - 1)}, \quad \text{for all } n \in (\lambda, 1],
\]  

(13)

deters central bank runs and implements the socially optimum in the unique equilibrium. Such a deterring policy choice requires the interim price level \( P_1(n) \) to exceed the withdrawal dependent bound:

\[
P_1(n) > \frac{M}{R}(1 + n(R - 1)), \quad \text{for all } n \in (\lambda, 1].
\]  

(14)

The key to Corollary 5 is the timing of events. The central bank observes the depositors’ aggregate spending behavior \( n \) and only then decides on the overall liquidation \( y(n) \). If spending exceeded the measure of impatient agents \( n > \lambda \), the central bank disciplines spending depositors by liquidating very little, thus, reducing the real allocation \( x_1 \). The agents anticipate the punishment by the central bank ex-ante and
behave according to their type, by this deterring spending over the measure of impatient agents.

Observe that $y^d(n)$ is increasing in $n$, which implies that the constant liquidation policy

$$y(n) \equiv y^*$$

(15)

implements the socially optimal equilibrium as the unique equilibrium. However, there exist other liquidation policies that can accomplish the same result. The policy $y(n)$ delivers the same result that the classic suspension-of-convertibility option, which is known to exclude bank runs in the Diamond-Dybvig world.

There is a subtle but essential difference, however, between suspension and our liquidation policy. Suspension of convertibility there requires the bank to stop paying customers that arrive after the fraction $\lambda$ of withdrawers. One can argue that this is not in the spirit of a demand deposit contract. By contrast, in our environment, there is no restriction on agents ever to spend their digital euros in period 1, and there is no suspension of accounts. Instead, it is the amount of goods traded against those digital euros and the resulting change in the price level that generates the incentives for patient agents to rather wait.

More concretely, a low liquidation implies that the price level $P_1$ is pushed above an upper bound that is increasing in the aggregate spending. Note, however, in equilibrium, the low liquidation policy deters large spending, such that the high price level $P_1$ is a threat that realizes only off-equilibrium.

But, as every time we have an off-equilibrium threat, we should worry about the possibility of time inconsistency. In our model, we assume that the central bank can proceed with such a threat. But, what if the central bank is concerned with price stability and, therefore, refuses to induce a high price level? Notice that, in comparison with the classical treatment of time inconsistency in Kydland and Prescott (1977), the concern here is not that the central bank will be tempted to inflate too much, but that it would be tempted to inflate too little. The central bank can avoid suboptimal allocations by committing to letting inflation grows if needed.

A similar concern appears in models with a zero lower bound on nominal interest rates: a central bank wants to commit to keeping interest rates sufficiently low for sufficiently long (even after the economy is out of recession!) to get the economy out

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9Our result resembles Theorem 4 in Allen and Gale (1998) and has a similar intuition. In Allen and Gale (1998), a central bank lends to a representative bank an interest-free line of credit to dilute the claims of the early consumers so that they bear a share of the low returns to the risky asset. In their environment, private bank runs are required to achieve the first-best risk allocation.
of the zero lower bound. However, once the economy is out of the zero lower bound, there is an incentive to renegade from the commitment to lower interest rates and avoid an increase in the price level. See, for an early formulation of this argument, Krugman (1998). This concern about time consistency suggests that we must come back to explore the implications of our model for the evolution of the price level.

6 The classic policy goal: Price level targeting

As we discuss before, there are many possible reasons to explain why central banks view the stabilization of price levels (or, more generally, inflation rates) as one of their prime objectives. The model here should be viewed as part of a larger macroeconomic environment, where price stability must be taken into account. The task at hand, then, is to examine how the liquidation policy derived in the previous section and the price stability impose constraints on each other. In particular, we will document the existence of deep tensions between the objective of achieving the first best from the perspective of deterring a central bank run and the goal of price stability.

We shall distinguish two versions of the objective of price stability, as the period-1 objective might potentially be at odds with long-term price stability: full price stability and partial price stability. Let us start analyzing the former.

6.1 Full price stability

Definition 6. i) A central bank policy is P₁-stable at level \( \bar{P} \), if it achieves \( P_1(n) \equiv \bar{P} \) for the price level target \( \bar{P} \), at all spending fractions \( n \in [\lambda, 1] \).

ii) A central bank policy is price-stable at level \( \bar{P} \), if it achieves \( P_1(n) = P_2(n) \equiv \bar{P} \) for the price level target \( \bar{P} \), for all spending fractions \( n \in [\lambda, 1] \).

In our definition, price stability here is treated as a mandate even for off-equilibrium realizations of \( n \). The best way to read this is a matter of commitment to the price level \( \bar{P} \), no matter what happens, i.e., even if more than the expected equilibrium fraction of agents chooses to spend their balances in period \( t = 1 \). From (3), we can state the following proposition relating the liquidation policy of the central bank and the price-level outcome.

Proposition 7. A central bank policy is:
i) \( P_1 \)-stable at level \( \overline{P} \), if and only if its liquidation policy satisfies:

\[
y(n) = \frac{M}{\overline{P}} n, \text{ for all } n \in [0, 1]
\]  \hspace{1cm} (16)

implying a real interim allocation:

\[
x_1(n) \equiv x_1 = \frac{M}{\overline{P}} \leq 1.
\]  \hspace{1cm} (17)

ii) A central bank policy is price-stable, if and only if its liquidation policy satisfies equation (16) and its interest policy satisfies:

\[
i(n) = \frac{\overline{P} - n}{1 - n} R - 1
\]  \hspace{1cm} (18)

and

\[
\overline{P} \geq M.
\]  \hspace{1cm} (19)

Note that a price stable liquidation policy (16) requires asset liquidation in constant proportion to aggregate spending for all \( n \in [0, 1] \). Such a policy excludes rationing or all kinds of suspension policies.

To understand the previous results, equation (16) implies that \( x_1(n) \) is constant at some level \( \overline{x} \). Since the central bank cannot liquidate more than the entire investment in the real technology, \( y(n) \leq 1 \), it follows that \( \overline{x} = x_1(1) = y(1) \leq 1 \), i.e., equation (17). Equation (18) follows from (4) combined with (16). Equation (17) or, alternatively, the constraint \( i(n) \geq -1 \) for all \( n \in [\lambda, 1] \) implies (19). Recall from section 5, that the socially optimal allocation satisfies \( x^*_1 > 1 \). Therefore, we can infer state a detailed corollary showing the limitations that price stability imposes on the implementation of the social optimum.

**Corollary 8.** If the central bank commits to a \( P_1 \)-stable policy, then:

i) The socially optimal allocation is not implemented.

ii) There is a unique equilibrium where only impatient agents spend, \( n^* = \lambda \), i.e. there are no central bank run equilibria.

iii) If the central bank commits to a price-stable central bank policy, then the nominal interest rate is non-negative \( i(n) \geq 0 \) for all \( n \in [\lambda, 1] \). The interest rate \( i(n) \) is increasing in \( n \).
On item (ii) of our previous corollary, a $P_1$-stable policy deters central bank runs since equations (16) and (17) together with equations (5) and (6) imply:

$$x_2(n) = \frac{1 - n \bar{R}}{1 - n} R \geq R > 1 \geq \bar{R}$$

(20)

Therefore, patient agents will never choose to spend in period 1. To see item (iii), equation (19) implies $i(n) \geq 0$ for all $n \in [\lambda, 1]$, since $R > 1$. With equation (19), equation (18) implies that $i(n)$ is increasing in $n$.

6.2 Partial price stability

While price stability and the absence of central bank runs may be desirable, the constraint (17), i.e., the failure to implement the socially optimal real allocation is not. In particular, the implementation of the social optimum is at odds with the goal of complete price stability. Recall that optimal risk-sharing at $x_1^* > 1$ is the trigger of potential bank runs in models of the Diamond-Dybvig variety: thus part (ii) of the proposition above should not surprise.

Demanding price stability for all possible spending realizations of $n$ is thus too stringent, when $x_1(\lambda) > 1$: for sufficiently high spending levels of $n$, equation (16) exhausts the liquidation possibilities available to a central bank, as $y(n)$ can impossibly exceed unity. We therefore examine a somewhat more modest goal: a central bank may still wish to assure price stability, if it is possible at all, but may deviate from its goal in times of crises. We capture this with the following definition.

**Definition 9.** 1. A central bank policy is **partially $P_1$-stable at level** $\bar{P}$, if either it achieves $P_1(n) = \bar{P}$ for some price level target $\bar{P}$, or the central bank fully liquidates real investment $y(n) = 1$, at all spending fractions $n \in [\lambda, 1]$.

2. A central bank policy is **partially price-stable at level** $\bar{P}$, if either it achieves $P_1(n) = P_2(n) = \bar{P}$ for some price level target $\bar{P}$, or the central bank fully liquidates real investment $y(n) = 1$, for all spending fractions $n \in [\lambda, 1]$.

Obviously, $P_1$-stable central bank policies are also partially $P_1$-stable, and price-stable central bank policies are also partially price-stable.

**Proposition 10.** Suppose that $M > \bar{P} \geq \lambda M$.

1. A central bank policy is partially $P_1$-stable at level $\bar{P}$, if and only if its liquidation policy satisfies:

$$y(n) = \min \left\{ \frac{M}{\bar{P}} n, 1 \right\}$$

(21)
2. Consider a partially $P_1$-stable central bank policy at level $\overline{P}$. Define the critical aggregate spending level:

$$n_c \equiv \frac{\overline{P}}{M}$$

(22)

For all $n \leq n_c$, the price level is stable at $P_1(n) = \overline{P}$ and the real goods purchased per agent in period $t = 1$ equal:

$$x_1(n) = \overline{x}_1 = \frac{M}{\overline{P}} > 1$$

(23)

While real goods purchased per agent in period $t = 2$ equal $x_2(n) = R(1-\overline{x}_1 n)/(1-n)$. For aggregate spending in excess of the critical level, $n > n_c$, the real goods purchased per agent in period $t = 1$ equal $x_1(n) = 1/n$ at a price level $P_1(n)$ proportionally increasing with total spending $n$,

$$P_1(n) = M n$$

(24)

while $x_2(n) = 0$ for $n > n_c$. 

3. Any partially $P_1$-stable central bank policy with $M > \overline{P}$ allows an exhaustive run on the central bank to occur in equilibrium.

4. A central bank policy is partially price-stable at $\overline{P}$, if and only if its liquidation policy satisfies equation (16) and its interest policy satisfies:

$$i(n) = \frac{\overline{P} - n}{1 - n} R - 1, \quad \text{for all } n \leq n_c$$

(25)

For $n > n_c$, there is no supply of real goods in $t = 2$. Thus, $P_2$ and $i(n)$ are irrelevant then.

5. Suppose the central bank policy is partially price-stable at $\overline{P}$. The nominal interest rate turns negative for $n \in (n_0, n_c)$, where

$$n_0 = \frac{R \overline{P} - 1}{R - 1} = \frac{R_{n_c} - 1}{R - 1}.$$ 

For $R < M/\overline{P}$, the nominal interest rate is negative for all $n \in [0, n_c)$.

Proposition 10, (2) reflects the central bank’s capacity to keep the price level and the real interim allocation $x_1$ stable as long as spending remains below the critical level $n_c$. The stabilization of the price level requires liquidation of real investment proportionally to aggregate spending by factor $M/\overline{P}$. Since the central bank cannot liquidate more than its entire investment, as spending exceeds the critical level $n_c$, price level stabilization via liquidation of real assets becomes impossible. Rationing of
real goods implies that the price level has to rise and the real allocation declines in aggregate spending.

*Proof.* 1. Equation (21) follows immediately from (3) and the constraint $y(n) \leq 1$.

2. Equation (21) implies that $x_1(n) = y(n)/n$ is constant at the level $\bar{x} = M/P$, as long as $y(n) < 1$: this is the case for $n < n_c$. For $n \geq n_c$, $y(n) \equiv 1$. All goods are liquidated, so $x_1(n) = 1/n$. Equation (24) follows from equation (3).

3. This is a consequence of proposition 2 and since for $n = 1 > n_c$, $x_2(1) < x_1(1)$.

4. Equation (25) follows from (4) combined with (21).

5. This is straightforward, when plugging in (21) into $P_2(n)$ and observing that $n_0$ is positive only for $R > M/P$.

\[ \square \]

Proposition 10 is in marked contrast to Proposition 7. One could argue that when banking is interesting, i.e. $x_1 > 1$ for $n = \lambda$, then the goal of price stability induces the possibility of runs on the central bank, the necessity for negative nominal interest rates, and the abolishment of the price stability goal, if the run is too large.

### 7 Extensions

In this section we introduce several extensions of interest to our basic model. In order, we will study the case where we allow for the suspension of spending, token-based CBDCs, synthetic CBDCs and retail banking, and cash.

#### 7.1 Allowing for suspension of spending

With an account-based CBDC, there is an additional and rather drastic policy tool at the disposal of the central bank: the central bank can simply disallow agents to spend (i.e., transfer to others) more than a certain amount on their account. In other words, the bank can impose a “corralito” and suspend spending. This policy is different from the standard suspension of liquidation, as the amounts of goods to-be-made available on the goods market is a policy-induced choice that still exists separately from the suspension of spending policy. Notice also that “suspension of spending” should perhaps not be called “suspension of withdrawal.” Since there are only CBDC accounts and
they cannot be converted into something else: the amounts can only be transferred to another account.

With the suspension of spending policy, the central bank could arrange matters in such a way, that not more than the initially intended amount of money will be spent in period 1. In practice, the central bank would then either take all spending requests at once and, if the total spending requests exceeds the overall threshold, impose a pro-rata spending limit, or it could arrange and work through the spending requests in some sequence, thereby possibly imposing different limits depending on the position of a request in that queue. Needless to say, such a spending suspension might create considerable havoc and erode trust in the central bank digital currency system: these issues are outside the model considered here.

7.2 Token-based CBDC

In a token-based CBDC, a central bank issues anonymous electronic tokens to agents in period 1, rather than accounts. These electronic tokens are more akin to traditional banknotes than to deposit accounts.

Interestingly, the analysis in the previous sections still holds, since nothing of essence depended on the concrete details of operating under a deposit-based CBDC. With a token-based CBDC, agents obtain $M$ tokens in period $t = 0$, and decide how much to spend in periods $t = 1$ and $t = 2$.

With digital tokens, it is easy for a central bank to pay a nominal interest in period $t = 2$: even a negative nominal interest rate is possible. Technically, digital tokens can be designed in such a way that each unit of a token in $t = 1$ turns into a quantity $1 + i$ of tokens in $t = 2$, with $i$ to be determined by the central bank at the beginning of period $t = 2$. This is a simple task that software code can easily accomplish.

In the analysis above, the identity of the agents holding the CBDC accounts did not matter much. Thus, the same allocations can be implemented except for those that require suspension of spending, as discussed in Subsection 7.1. For the latter,

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10 This can be done with or without relying on a blockchain. In the second case, a centralized ledger to record transactions can be kept by a third-party that is separated from the central bank. That third-party could also potentially pay interest or how to impose a suspension of spending. For the purpose of this paper we do not need to worry about the operational details of such a third-party or to specify which walls should exist between it and the central bank to guarantee the anonymity of tokens.

11 Historically, we have examples of banknotes bearing positive (for instance, during the U.S. Civil War, the U.S. Treasury issued notes with coupons that could be clipped at regular intervals) and negative interests (demurrage-charged currency, such as the prosperity certificates in Alberta, Canada, during 1936). Thus, an interest-bearing electronic token is only novel in its incarnation, but not in its essence.
the degree of implementability depends on technical details outside the scope of this paper. Note that even with a token-based system, the transfer of tokens usually needs to be registered somewhere, e.g., on a blockchain. It is technically feasible to limit the total quantity of tokens that can be transferred on-chain in any given period. A pro-rata arrangement can be imposed by taking all the pending transactions waiting to be encoded in the blockchain, take the sum of all the spending requests, and accordingly divide each token into a portion that can indeed be transferred and a portion that cannot. It may be that off-chain solutions arise circumventing some of these measures, but their availability depends on the precise technical protocol of the CBDC token-based system. In the case where the token-based CBDC is operated by a centralized third-party, such an implementation is even easier.

7.3 Synthetic CBDC and retail banking

With a synthetic CBDC, agents do not hold central bank digital money directly. Rather, all agents hold accounts at their own retail bank, which in turn holds a CBDC not much different from current central bank reserves. The retail banks undertake the real investments envisioned for the central bank in our analysis above.

The key difference to the actual system of cash-and-deposit-banking system is that cash does not exist as a separate central bank currency or means of payments. That is, in a synthetic CBDC system, agents can transfer amounts from one account to another, but these transactions are always observable to the banking system and, thereby, the central bank. Likewise, agents (and banks) cannot circumvent negative nominal interest while they could do so in a classic cash-and-deposit-banking system by withdrawing cash and storing it.

For the purpose of our analysis, observability is key. Our analysis is relevant in case of a systemic bank run, i.e., if the economy-wide fraction of spending agents exceeds the equilibrium outcome. Much then depends on the interplay between the central bank and the system of retail banks. E.g., if liquidation of long-term real projects is up to the retail banks, and these retail banks decide to make the same quantity of real goods available in each period, regardless of the nominal spending requests by their depositors, then the aggregate price level will have to adjust. The central bank may seek to prevent this either by imposing a suspension of spending at retail banks or by forcing banks into higher liquidation of real projects: both would require considerable authority for the central bank.
7.4 Cash

The key difference to a fully cash-based system is that spending decisions can only be observed on the goods market, rather than by also tracing accounts or transactions on the blockchain. In principle, the payment of nominal interest rates on cash is feasible, but is demanding in practice. Excluding nominal interest rates on cash, due to these practical considerations, implies the cash-and-deposit-banking system discussed in 7.3 and the restrictions discussed there. The tools available to a central bank are now considerably more limited. These limitations may be a good thing, as they may impose a commitment technology and may thus lead to the prevention of an equilibrium systemic bank run in the first place, but the restricted tool set may be viewed as a burden ex post, should such a bank run occur.

8 Conclusion

This paper analyzes implications for price stability and financial stability when a central bank conducts maturity transformation and invests in the real economy.

In its role as intermediary, the central bank collects and invests the real goods endowments of the agents in a real production technology, offering a nominal CBDC contract in return. The contract specifies nominal payments conditional on early or late spending of CBDC balances. At an interim period, the agents learn whether they enjoy late (patient agent) or early (impatient agent) consumption and then decide whether to spend their balances. Agents who enjoy late consumption can nevertheless spend their CBDC account early by investing in a real storage technology. Agents spend early when the expected real value from early spending exceeds the expected real value from late spending. But real values depend on the central bank’s liquidation policy of real investment. A central bank run occurs if not only impatient agents but also patient agents decide to spend their CBDC balances early.

The central bank observes aggregate nominal spending and then decides how much real investment technology to liquidate, by this determining the real goods supply. The price level for real goods then adjusts such that the nominal CBDC spending clears the real goods market. In contrast, a private intermediary would need to take the price level as given such that the price level jointly with aggregate nominal spending pins down the necessary liquidation of the technology.

As the main result, we show that the central bank can always implement the socially optimal allocation in the unique equilibrium and deter the central bank run equilibrium.
To do so, the central bank needs to deter agents who enjoy late consumption from spending their CBDC balances early. The monetary authority does so by threatening to run high price levels given spending is too high, such that spending early was ex post sub optimal. Ex-ante, depositors anticipate the central bank’s behavior, and do not spend when learning that they are patient, such that in equilibrium the central bank’s threat is never implemented.
References


