Investment Externalities in Models of Fire Sales

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Abstract

In canonical models with financial constraints, the possibility of fire sales creates a pecuniary externality that results in ex-ante overinvestment. I show that this result is sensitive to the microfoundations for fire sales. If they result from asymmetric information instead of misallocation, the overinvestment result is reversed. However, there may be a tradeoff between present and future underinvestment. Macroprudential policy may need to treat different types of investment differently.

Keywords: Fire sales, pecuniary externality, asymmetric information

JEL codes: D82, D62, G14

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Fire sales, where distressed sales depress asset prices, are a recurring feature of financial crises, and preventing or mitigating them is a central concern of macro-prudential policy. One common policy recommendation is to preventatively curb investment booms in order to reduce the scale of fire sales during crises. Lorenzoni (2008) formalizes one possible rationale for such policies: investors don't internalize the pecuniary externality that falling asset prices impose on financially constrained agents.[1] In this paper I show that this argument depends on the exact mechanism that makes asset prices fall. It is valid when fire sales involve misallocation of real assets; if instead they result from asymmetric information, a social planner would like to increase rather than reduce investment.

It seems intuitive that prices should fall when constrained agents need to sell assets, but it requires some explanation. Assets are just claims on future cash flows. If asset sales do not alter either cash flows or the discount factor of the marginal investor, there is no reason for asset prices to fall. In other words, one needs a theory of what makes asset demand downward-sloping as opposed to perfectly elastic.

A standard way to microfound downward-sloping demand curves for assets is the misallocation mechanism proposed by Shleifer and Vishny (1992) and Kiyotaki and Moore (1997), and adopted by Lorenzoni (2008). Assets are assumed to have different productivity depending on who holds them. They have a high marginal product in expert hands but a diminishing marginal product in non-expert hands. If constrained experts sell marginal units to non-experts, this lowers the marginal product in second-best use, and therefore the equilibrium price. This mechanism is an appealing microfoundation for applications where the assets in question are real assets that must be actively managed by whoever owns them. For assets that are not actively managed by the owner, such as securities backed by pools of mortgages which have a designated servicer, the fit is less clear. To a first approximation, the cash flows from these assets are the same regardless of who owns them.[2]

Empirically, fire sale effects have been documented in several different markets. These include real assets like used aircraft (Pulvino 1998) and real estate (Campbell et al. 2011) where misallocation seems like a first-order concern and financial assets like equities (Coval and Stafford 2007), corporate bonds (Ellul et al. 2011), convertible bonds (Mitchell et al. 2007) and residential mortgage-backed securities (Merrill et al. 2014) where it seems less

[1] See De Nicoló et al. (2012) and Claessens (2015) for examples of how Lorenzoni's argument has been incorporated into policy analysis.

[2] Although differences across investors in tax rates, in the correlation between a given asset and the rest of their portfolio or in beliefs about the asset's cash flows might all be sources of differences in valuation across investors even for assets that do not require management.

essential.

In Kurlat (2016), I propose a model of fire sales that does not rely on cash flows changing depending on who owns the asset. Instead, there is asymmetric information, and potential buyers differ in their expertise in evaluating assets: expert buyers can detect bad assets more accurately and refrain from buying them. In equilibrium, only investors with sufficient expertise buy assets. The cash flows obtained by the marginal buyer depend on his expertise because expertise determines which assets he purchases. The equilibrium may feature a downward-sloping relationship between asset sales and asset prices: in order to clear the market when sales are high, less-expert investors need to be induced to buy, so the price needs to fall.[3]

I embed this model of fire sales into a simplified version of Lorenzoni's three-period model and ask the same normative questions that have been asked of this canonical framework. Would a social planner who can intervene ex-ante but not ex-post want to deviate from the level of investment in the competitive equilibrium? In what way? I find that in the asymmetric information model, the normative conclusions are reversed: the social planner wants higher investment relative to the competitive equilibrium, even if this will deepen the fall in asset prices.

The reason for this reversal is that the two models imply very different types of inefficiencies. In the canonical model, the inefficiency comes from what Dávila and Korinek (2017) label a distributive externality. Incomplete markets prevent constrained investors from saving towards the state of the world where they have high marginal utility of wealth, and raising asset prices partially substitutes for this. Since the net present value of investment is zero (given how it will be misallocated in equilibrium) there is no first-order loss in reducing it relative to the competitive equilibrium in order to raise asset prices. In the model with asymmetric information, there are two important differences. First, redistributing wealth towards constrained investors to mitigate fire sales has no social benefit. Privately, they have a high marginal utility of wealth because it allows them to retain assets instead of selling them at an adverse selection discount, but since no real output is destroyed in a fire sale, this is just a transfer. Therefore the force for preventing fire sales in the canonical model is absent. Second, the social net present value of marginal investment in the competitive equilibrium is positive. What stops agents from choosing higher investment is the understanding that

[3]Of course, these are not the only two possible explanations of fire sales. Other prominent theories include models with differences of opinion (Fostel and Geanakoplos 2008, Geanakoplos 2009), cash-in-the-market pricing (Allen and Gale 1994, 1998, Acharya and Yorulmazer 2008, He and Kondor 2016) or signal extraction in the presence of noise traders (Grossman and Stiglitz 1980, Calvo 1999).

they may have to sell it at an adverse selection discount. But since this discount is just a transfer, the social planner would like investors to ignore it and choose higher investment.

I then extend the basic comparison in three dimensions. First, I allow for investment in the intermediate period in addition to the initial period. By lowering asset prices, fire sales raise the required rate of return on experts' wealth, leading to inefficiently lower intermediate-period investment. In the canonical model, this just reinforces the argument for lowering initial investment: it prevents both misallocation and future underinvestment. In the asymmetric information model, allowing for intermediate-period investment introduces a tradeoff: the social planner must balance present and future underinvestment. The direction in which the planner wants to intervene depends on the strength of fire sale effects and the sensitivity of intermediate-period investment to asset prices. Second, I examine what happens in the asymmetric information model if bad assets are deliberately created with the intent of being sold, and their quantity is endogenous. I find conditions such that this reinforces the argument for increasing ex-ante investment: if additional investment leads to lower asset prices, it also reduces incentives to waste resources in creating bad assets. Finally, I study a variant with a fixed proportion of bad assets instead of a fixed absolute quantity, and find that it does not alter the conclusions.

1 The Canonical Model

The model is a simplified version of Lorenzoni (2008), similar to the example in Moore (2013). Lorenzoni's model features aggregate uncertainty, with fire sales only in bad states of the world. This is appealing on empirical grounds but not essential for the main argument so, like Caballero and Krishnamurthy (2004), I focus on a deterministic economy.

Technology, Preferences and Markets

There are three periods: $t = 0, 1, 2$; two groups of agents: households and entrepreneurs; and two goods: consumption goods and capital. Preferences are given by $u = c_0 + c_1 + c_2$ for both groups of agents. Entrepreneurs are endowed with n consumption goods at $t = 0$; households are endowed with e_0 and e_1 consumption goods in periods $t = 0$ and $t = 1$ respectively.

Consumption goods are perishable. However, entrepreneurs have access to a technology that converts consumption goods into capital one-for-one at $t = 0$. Capital requires maintenance at $t = 1$ in order to produce output at $t = 2$. Maintenance costs are z consumption goods per unit of capital. If a unit of capital's maintenance costs are not paid, then that unit of capital becomes useless. At $t = 2$, a maintained unit of capital will produce A consumption goods if it is operated by entrepreneurs. Instead, if households operate k^H units of capital, they will produce $F\left(k^H\right)$ consumption goods, where $F(\cdot)$ is a differentiable concave function.

The economy's resource constraints are:

$$c_0^E + c_0^H + k \leq n + e_0 \tag{1}$$
$$c_1^E + c_1^H + z\alpha k \leq e_1 \tag{2}$$
$$k^E + k^H \leq \alpha k \tag{3}$$
$$c_2^E + c_2^H \leq Ak^E + F\left(k^H\right) \tag{4}$$

Equation (1) is the $t = 0$ resource constraint. The endowment $n + e_0$ is split between consumption of entrepreneurs c_0^E, consumption of households c_0^H and investment k. Equation (2) is the $t = 1$ resource constraint: the endowment is split between consumption and maintenance costs; $\alpha \in [0, 1]$ is the fraction of capital that is maintained. Equation (3) says that maintained capital is allocated to either households or entrepreneurs. Equation (4) is the $t = 2$ resource constraint.

No inter-temporal contracts are enforceable, so agents cannot borrow or lend. It is also impossible for entrepreneurs to manage capital on behalf of households: each unit of capital will be managed by its owner. There is, however, a competitive market for maintained capital at $t = 1$, with a price q.

Equilibrium Definition

Entrepreneurs solve:

$$\max_{c_0^E, c_1^E, c_2^E, k, s, k^E, \alpha} c_0^E + c_1^E + c_2^E \tag{5}$$
$$s.t.$$
$$c_0^E + k \leq n \tag{6}$$
$$c_1^E + \alpha z k \leq sq \tag{7}$$
$$k^E \leq \alpha k - s \tag{8}$$
$$c_2^E \leq Ak^E \tag{9}$$
$$c_t^E \geq 0 \quad t = 0, 1, 2; \quad k \geq 0; \quad k^E \geq 0; \quad \alpha \in [0, 1]; \quad s \in [0, \alpha k]$$

Equations (6), (7) and (9) are the $t = 0, 1, 2$ budget constraints. Entrepreneurs raise sq at $t = 1$ by selling s units of capital, and divide this between consumption and maintaining a fraction α of their capital. Equation (8) says that the capital they can carry into $t = 2$ is what they have maintained minus what they have sold. There is no intra-period financial constraint saying that entrepreneurs must meet maintenance costs before being able to sell capital. This is equivalent to assuming that capital can be sold without maintenance and maintained by the buyer.

Households solve the following problem:

$$\max_{c_0^H, c_1^H, c_2^H, k^H} c_0^H + c_1^H + c_2^H \tag{10}$$

$$s.t.$$

$$c_0^H \leq e_0 \tag{11}$$

$$c_1^H + qk^H \leq e_1 \tag{12}$$

$$c_2^H \leq F\left(k^H\right) \tag{13}$$

$$c_t^H \geq 0 \quad t = 0, 1, 2; \quad k^H \geq 0$$

Equations (11)-(13) are the budget constraints the household faces in periods 0, 1, and 2 respectively. At $t = 0$ they just consume because they have no investment technology or access to financial markets. At $t = 1$ they decide how much to consume and how much capital to buy. At $t = 2$ they just consume.

Definition 1. A competitive equilibrium is an allocation $\left\{c_0^E, c_1^E, c_2^E, c_0^H, c_1^H, c_2^H, k, s, k^E, k^H, \alpha\right\}$ and a price q such that $\left\{c_0^E, c_1^E, c_2^E, k, s, k^E, \alpha\right\}$ solves the entrepreneur's problem (5), taking q as given, $\left\{c_0^H, c_1^H, c_2^H, k^H\right\}$ solves the household's problem (10), taking q as given, and the capital market clears, so (3) holds with equality.

Equilibrium Characterization

Assume that the following conditions hold:

Assumption 1.

1. $A > 1 + z$
2. $F'(0) < A$
3. $F'(0) > \frac{Az}{A-1}$

4. $-\frac{F''(x)x}{F'(x)} < 1$ *for all* x

5. $F'\left(\frac{A-1}{A}n\right) < \frac{Az}{A-1}$

6. $e_1 > zn$

Assumption 1.1 says investing in capital (and then maintaining it) has positive net present value. Assumption 1.2 implies that capital is always more productive in the hands of entrepreneurs. Assumption 1.3 says that households are not so unproductive that the price of capital can fall to the point where no investment takes place. Assumption 1.4 ensures that there is a unique market-clearing price of capital. Assumption 1.5 ensures that entrepreneurs' endowment n is large enough that they consume at $t = 0$. Assumption 1.6 says that households' endowment at $t = 1$ is sufficiently large to meet maintenance costs for the maximum possible level of investment.

Suppose the entrepreneur enters period $t = 1$ holding k units of capital, and the market price is q. His problem reduces to

$$\begin{gathered} V(k,q) \equiv \max_{c_1^E, c_2^E, s, k^E, \alpha} c_1^E + c_2^E \\ s.t. \\ c_1^E + \alpha z k \leq sq \\ k^E \leq \alpha k - s \\ c_2^E \leq A k^E \\ \alpha \in [0,1]; \quad c_t^E \geq 0 \quad t = 1,2; \quad k^E \geq 0; \quad s \in [0, \alpha k] \end{gathered} \tag{14}$$

As shown below, Assumption 1 ensures that in equilibrium $A > q > z$. Therefore the entrepreneur will maintain all his capital, not consume at $t = 1$ and carry as much capital as possible into $t = 2$. Therefore:

$$s(q,k) = \frac{zk}{q} \tag{15}$$

so the entrepreneur sells just enough capital to meet maintenance costs, and can afford $k^E = \frac{q-z}{q}k$. This gives the entrepreneur

$$V(k,q) = \underbrace{\frac{A}{q}}_{\text{rate of return on wealth}} \quad \underbrace{(q-z)\,k}_{\text{net worth at } t=1} \tag{16}$$

At $t = 0$, the entrepreneur's problem is

$$\max_{c_0^E, k} c_0^E + V(k, q)$$
$$s.t.$$
$$c_0^E + k \leq n$$
$$k \geq 0; \quad c_0^E \geq 0$$

The solution to this problem is

$$k \begin{cases} = 0 & \text{if } q < \frac{Az}{A-1} \\ \in [0, n] & \text{if } q = \frac{Az}{A-1} \\ = n & \text{if } q > \frac{Az}{A-1} \end{cases} \tag{17}$$

Turn now to the household's problem. Assumption 1.6 ensures that e_1 is large enough so that $c_1^H \geq 0$ doesn't bind, so households' first order condition is:

$$F'\left(k^H\right) = q$$

Therefore, the market-clearing condition $s = k^H$ implies that in equilibrium the price of capital must satisfy:

$$F'\left(\frac{zk}{q}\right) = q \tag{18}$$

For a general $F(\cdot)$ function, it's possible for (18) to have more than one solution: a lower price makes entrepreneurs have to sell more units of capital to meet maintenance costs, which lowers the marginal product obtained by households, justifying the lower price. Assumption 1.4 (which is satisfied, for instance, with a Cobb-Douglas production function) ensures that this is not the case.

Lemma 1. *If equation* (18) *has a solution, it is unique. The solution* $q(k)$ *is a decreasing function.*

Lemma 1 means that this model has "fire sale" effects. Higher $t = 0$ investment means that entrepreneurs will have to liquidate more capital at $t = 1$, so households will have to absorb more units of capital, pushing down the marginal-product-in-second-best-use and therefore asset prices.

Assumptions 1.3 and 1.5 imply that for sufficiently low k^H, $F'\left(k^H\right) > \frac{Az}{A-1}$ and for $k = n$, $F'\left(k^H\right) < \frac{Az}{A-1}$. Therefore, in equilibrium the entrepreneurs must choose an interior level of investment. Using (17), this requires:

$$q = \frac{Az}{A-1} \tag{19}$$

which leaves entrepreneurs exactly indifferent between investing and consuming at $t = 0$.

Replacing (19) into (18):

$$\frac{Az}{A-1} = F'\left(\frac{A-1}{A}k\right) \tag{20}$$

Equation (20) implicitly defines the equilibrium level of investment k. The rest of the equilibrium objects follow immediately.

Welfare

I adopt the normative criterion of constrained efficiency, proposed by Hart (1975), Stiglitz (1982), Geanakoplos and Polemarchakis (1986) and Kehoe and Levine (1993). I study a constrained social planner who maximizes entrepreneurs' utility subject to delivering a minimum utility to households. The planner can dictate investment and can make transfers across agents at $t = 0$, but cannot intervene at time $t = 1$ or $t = 2$. Hence the planner is limited to choosing c_0^E, c_0^H and k.

In general it is not clear whether this type of normative exercise provides useful guidance for the analysis of optimal policy with a limited set of instruments. However, as pointed out by Dávila and Korinek (2017), this particular planning problem is equivalent to a Ramsey policy problem with a tax on investment and lump-sum redistribution, both applied at $t = 0$.

The planner solves:

$$\begin{gathered}
\max_{c_0^H, c_0^E, k} c_0^E + V\left(k, q\left(k\right)\right) \\
s.t. \\
c_0^E + c_0^H + k \leq n + e_0 \\
c_0^H + e_1 - q\left(k\right) s\left(q\left(k\right), k\right) + F\left(s\left(q\left(k\right), k\right)\right) \geq \bar{\Pi} \\
c_0^H \geq 0; \quad c_0^E \geq 0; \quad k \geq 0
\end{gathered} \tag{21}$$

Unlike individual entrepreneurs, the planner takes into account how the choice of k will determine asset prices at $t = 1$. The first constraint is just the $t = 0$ resource constraint. The second constraint imposes a minimum level of utility for households. Households consume c_0^H at $t = 0$; at $t = 1$ they consume their endowment e_1 minus what they spend buying

$s(k, q(k))$ units of capital at a unit price $q(k)$; at $t = 2$ they consume the output of the capital they purchased. Replacing the constraints into the objective, the planner's objective reduces to:

$$W(k) \equiv n + e_0 + e_1 - zk + F(s(q(k), k)) - \bar{\Pi} - k + V(k, q(k))$$

Proposition 1. *(Lorenzoni 2008) In the canonical model, the social planner can obtain a Pareto improvement by lowering investment relative to the competitive equilibrium.*

As discussed by Lorenzoni (2008) and Dávila and Korinek (2017), the equilibrium allocation is constrained inefficient: a constrained planner would be able to make everyone better off by lowering investment. In equilibrium, households consume at both $t = 0$ and $t = 1$, so their marginal rate of substitution between wealth in both periods is 1. Entrepreneurs, instead, consume at $t = 0$, so their marginal utility is 1, but are constrained at $t = 1$, when their marginal utility of wealth is $\frac{A}{q} > 1$. Hence marginal rates of intertemporal substitution are not equalized across entrepreneurs and households. If intertemporal contracts were enforceable, entrepreneurs would want to save and households would want to borrow. Lower investment, by raising the price of capital, shifts wealth from households to entrepreneurs at $t = 1$ and therefore partially substitutes for the missing credit market.[4] At the equilibrium, the direct effect of marginal investment (taking q as given) is second-order. Using (16), (18) and (19):

$$\frac{\partial W}{\partial k} = -1 - z + F'\left(\frac{zk}{q}\right)\frac{z}{q} + \frac{A(q - z)}{q} = 0 \tag{22}$$

so the only first-order effect is the redistribution of wealth at $t = 1$.

The literature has given different labels to this effect. Typically, it is described as a pecuniary externality: individual entrepreneurs do not take into account that their investment decisions affect other entrepreneurs' constraints by changing prices. In the terminology of Dávila and Korinek (2017), this is a distributive externality: prices affect the distribution of wealth across agents and dates in an environment where marginal rates of intertemporal substitution are not equalized. It is also often referred to as a "fire sale" externality: additional investment means that additional units of capital will have to be sold, which lowers its price.

Caballero and Krishnamurthy (2003, 2004), Farhi et al. (2009), Bianchi (2011), Benigno et al. (2011), Bianchi and Mendoza (2013), Jeanne and Korinek (2010, 2016), Hart and Zin-

[4] Note that entrepreneurs are savings-constrained at $t = 0$ but borrowing-constrained at $t = 1$. They want to saving towards $t = 1$ precisely because they know that they will not be able to borrow.

gales (2015), Korinek (2017) and Di Tella (forthcoming) also analyze constrained efficiency in economies with financial constraints. They show examples where ex-ante decisions affect future constraints through aggregate consumption, which affects prices of goods and/or the stochastic discount factor. The inefficiencies identified in this class of models are also sometimes described as "fire-sale externalities", but work through a different channel as the misallocation mechanism. Asriyan (2016) analyzes an example where misallocation interacts with dispersed information; Eisenbach and Phelan (2018) analyze the interaction of misallocation with oligopolistic behavior.

A central ingredient of Lorenzoni's fire-sale argument is that the demand for capital is indeed downward sloping, so that additional sales lead to lower prices. In this canonical model, this happens because the marginal product of capital in its second-best use is diminishing, as in Shleifer and Vishny (1992) and Kiyotaki and Moore (1997). In the next section, I revisit the normative analysis in a model with an alternative microfoundation for fire-sale effects.

2 An Asymmetric Information Model

The entrepreneurs' side of the economy is the same as in the canonical model. The household side follows Kurlat (2016) in assuming that there are households with different degrees of expertise for evaluating assets.

Technology, Preferences, Markets and Information

Relative to the canonical model, there are two differences. First, $F\left(k^H\right) = Ak^H$, so capital is just as productive in the hands of households as in the hands of entrepreneurs. Second, there is a specific form of asymmetric information. In addition to real entrepreneurs, there is a unit measure of fake entrepreneurs. They are endowed with n^F consumption goods at $t = 0$ plus a measure λ of "lemons", completely useless pieces of fake capital, indexed by i and distributed uniformly in the interval $[0, 1]$.

Households differ in their ability to distinguish lemons from real capital; their expertise is indexed by $\theta \in [0, 1]$ and is exogenously given. When analyzing any asset, a household will observe a binary signal $\sigma \in \{0, 1\}$. If the asset is real capital, then the household will always observe $\sigma = 1$. If the asset is a lemon, the signal will depend on the lemon's index and the household's expertise: a household of expertise θ will observe a signal $\sigma(i, \theta) = \mathbb{I}(i \geq \theta)$ when analyzing lemon i. Hence, lemons in the interval $[\theta, 1]$ will be indistinguishable from real capital for household θ. Higher θ means that a given household is more expert, since it mistakes fewer lemons for real capital.

Household θ is endowed with e_0 goods at $t=0$ and $e_1(\theta)$ goods at $t=1$, where $e(\cdot)$ is a continuous function. As in the canonical model, no intertemporal contracts are enforceable and there is a competitive market for capital at $t=1$.

Equilibrium definition

The equilibrium concept is derived from Kurlat (2016). There is a single price q at which real capital and lemons trade at $t=1$. Households can be selective in what they buy in the asset market, to the extent that their expertise allows them to tell assets apart. Therefore household θ will buy real capital plus lemons with indices in the interval $i \in [\theta, 1]$, pro-rata relative to their respective supply. All the real capital offered on sale will indeed sell, but lemons will be rationed, since some of the households will reject them. Let $\mu(i)$ be the fraction of lemons of index i put on sale that indeed sell.[5]

The problem real entrepreneurs face is exactly the same as problem (5); what market mechanism results in the price q does not affect the problem they face.

Fake entrepreneurs solve:

$$\max_{c_0^F, c_1^F, s^F(i)} c_0^F + c_1^F \tag{23}$$

$$s.t.$$

$$c_0^F \leq n^F \tag{24}$$

$$c_1^F \leq q \int_0^1 s^F(i)\,\mu(i)\,di \tag{25}$$

$$s^F(i) \leq \lambda \tag{26}$$

At $t=0$, their consumption is simply constrained by their endowment n^F. At $t=1$, their consumption is constrained by the revenue they obtain from selling lemons. If they put $s^F(i)$ lemons of type i on sale, they actually sell $\mu(i)\,s^F(i)$ of them, at a price q each. (25) results from integrating over their portfolio.

[5]Kurlat (2016) derives these equilibrium conditions from the assumption that there are markets at every possible price and with every possible rule for ordering buyers' trades, and buyers and sellers choose where to trade. See Kurlat (2016) and Kurlat (2018) for details.

Household θ solves:

$$\max_{c_0^H, c_1^H, c_2^H, \delta} c_0^H + c_1^H + c_2^H \tag{27}$$

$$s.t.$$

$$c_0^H \leq e_0 \tag{28}$$

$$c_1^H + q\delta \leq e_1(\theta) \tag{29}$$

$$c_2^H \leq \delta \frac{s}{s + \int_\theta^1 s^F(i)\, di} A \tag{30}$$

$$c_t^H \geq 0 \quad t = 0, 1, 2; \quad \delta \geq 0$$

where s is the amount of real capital sold by entrepreneurs and δ is the number of assets (capital plus lemons) that the household buys. The only difference between this problem and (10) is constraint (30). Household θ draws from a pool that includes the s units of real capital from entrepreneurs plus the $s^F(i)$ units of lemons indexed $i \in [\theta, 1]$ from fake entrepreneurs that the household is incapable of filtering out. Assets are divisible and the law of large numbers holds, so the fraction of real capital is $\frac{s}{s+\int_\theta^1 s^F(i)di}$.

Definition 2. A competitive equilibrium is an allocation $\{c_0^E, c_1^E, c_2^E, c_0^H(\theta), c_1^H(\theta), c_2^H(\theta), c_0^F, c_1^F, k, k^E, s, \delta(\theta), \alpha, s^F(i)\}$, a price q and fractions of lemons sold $\mu(i)$ such that: $\{c_0^E, c_1^E, c_2^E, k, s, k^E, \alpha\}$ solves the entrepreneur's problem (5), taking q as given; $\{c_0^H(\theta), c_1^H(\theta), c_2^H(\theta), \delta(\theta)\}$ solves household θ's problem (27), taking q, s and $s^F(i)$ as given; $\{c_0^F, c_1^F, s^F(i)\}$ solves the fake entrepreneur's problem, taking q and $\mu(i)$ as given; all the real capital put on sale indeed sells:

$$\int \frac{s}{s + \int_\theta^1 s^F(i)\, di} \delta(\theta)\, d\theta = s \tag{31}$$

and the fraction of lemon i sold is:

$$\mu(i) = \int_{\theta \leq i} \frac{1}{s + \int_\theta^1 s^F(i)\, di} \delta(\theta)\, d\theta \tag{32}$$

Condition (31) is a market clearing condition. Household θ buys $\frac{s}{s+\int_\theta^1 s^F(i)di}\delta(\theta)$ units of real capital, so adding up over households gives the total demand for real capital, which is equated to the supply s. Instead, the market for lemons does not clear, since lemon i is only accepted by households with $\theta \leq i$. Hence, the fraction that indeed sells is given by (32).

Equilibrium Characterization

Assume that the following condition holds:

Assumption 2.

$\int e_1(\theta)\, d\theta > zn + \lambda A$

Assumption 2 says that households' total $t = 1$ endowment is enough to pay the maintenance costs of the maximum level of investment, and to pay for all the lemons as though they were real capital. It ensures than some households consume at $t = 1$.

It is immediate that the solution to the fake entrepreneur's problem is $c_0^F = n^F$, $s^F(i) = \lambda$ and $c_1^F = q\lambda \int_0^1 \mu(i)\, di$. Fake entrepreneurs have no value of keeping their lemons so they always put them on sale, and at $t = 1$ they consume the proceeds of all the sales they are able to make. This implies that $\int_\theta^1 s^F(i)\, di = \lambda(1-\theta)$. Therefore the solution to the household's problem (27) is:

$$\delta(\theta) \begin{cases} = 0 & \text{if } \frac{s}{s+\lambda(1-\theta)} A < q \\ \in \left[0, \frac{e_1(\theta)}{q}\right] & \text{if } \frac{s}{s+\lambda(1-\theta)} A = q \\ = \frac{e_1(\theta)}{q} & \text{if } \frac{s}{s+\lambda(1-\theta)} A > q \end{cases} \tag{33}$$

Condition

$$\frac{s}{s + \lambda(1-\theta^*)} A = q \tag{34}$$

defines a cutoff level of expertise θ^* such that households with $\theta > \theta^*$ spend their entire endowment of $t = 1$ goods to buys assets while households with $\theta < \theta^*$ do not buy any assets at all because they understand that they would take in too many lemons and make losses.

Replacing this in the market clearing condition (31) and rearranging:

$$\int_{\theta^*}^{1} \frac{1}{s + \lambda(1-\theta)} e_1(\theta)\, d\theta = q \tag{35}$$

Also, as in Section 1, the entrepreneur's optimal investment decision is given by (17) and sales of real capital satisfy (15). Using (32), $s^F(i) = \lambda$ and (15), the fraction of asset i that can be sold in equilibrium is:

$$\mu(i) = \frac{1}{q} \int_{\theta^*}^{i} \frac{1}{s + \lambda(1-\theta)} e_1(\theta)\, d\theta \tag{36}$$

There always exists an equilibrium with $q = 0$, $k = 0$ and $\theta^* = 1$. If $q = 0$, entrepreneurs know they will be unable to afford the maintenance costs, so they do not invest. As a result, only lemons are on sale at $t = 1$, which justifies $q = 0$, and households do not buy anything. In addition, depending on parameters, there can be equilibria with $q = \frac{Az}{z-1}$ and an interior level of investment $k \in [0, n]$ and/or an equilibrium with $q > \frac{Az}{A-1}$ and investment $k = n$.

Lemma 2.

1. *Taking k as given, the system of equations* (34), (35), (15) *has a unique solution $q(k), \theta^*(k), s(k)$ for any $k \leq n$*

2. *$q'(k) < 0$ if and only if*

$$e_1(\theta^*(k)) < \frac{q(k)\lambda \int_{\theta^*}^{1} \frac{1}{[s(k)+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta}{\frac{A-q(k)}{s(k)+\lambda(1-\theta^*(k))}} \tag{37}$$

Lemma 2 is the analogue of Lemma 1. It says that, depending on the magnitude of $e_1(\theta^*)$, the model with asymmetric information may or may not have the types of fire-sale effects that are present in the canonical model.[6] Higher investment has two opposing effects on asset prices. First, it improves the overall mix of assets on sale at $t = 1$ since, by assumption, the number of lemons is fixed and more investment means more real capital needs to be sold to meet maintenance needs. Other things being equal, this pushes asset prices up rather than down, the opposite of a fire-sale effect.[7] On the other hand, absorbing the extra supply of assets on sale requires drawing less-expert households into the market, since the more-expert households exhaust their wealth. These households are rationally aware of their lower ability to filter out lemons, so other things being equal they are only willing to buy at lower prices. The net effect of higher investment on q depends on which of these two effects dominates. If $e_1(\theta^*)$ is high, this means that a small drop in the cutoff level of expertise is enough to draw a lot of wealth into the market. In this case, the first effect dominates and asset prices rise. Conversely, when $e_1(\theta^*)$ is low, the marginal level of expertise must fall more, and the fire sale effect where q falls results. This possibility is important for the comparison with the canonical model, since it shows that it's possible to obtain the same positive predictions for the relation between investment and asset prices from both microfoundations.

[6]Kurlat (2016) shows a version of this result in a model where sales of real capital are exogenous, as opposed to being dictated by the need to meet maintenance costs.

[7]Eisfeldt (2004) and Uhlig (2010) point out that simple models of trading with asymmetric information do not produce fire sales precisely for this reason.

Welfare

Consider again a constrained social planner that wants to maximize entrepreneurs' utility subject to delivering minimum levels of utility to households and fake entrepreneurs. As before, the planner can dictate investment and make transfers across agents at $t = 0$, but cannot intervene at $t = 1$ or $t = 2$. Hence the planner is limited to choosing c_0^E, $c_0^H(\theta)$, c_0^F and k. Denote by $q(k)$, $s(k)$, $\theta^*(k)$ and $\mu(i,k)$ the solution to the system of equations (34), (35), (15), (36), taking k as given. Also, let:

$$\rho(k) \equiv q(k) \int_0^1 \mu(i,k)\, di$$

be the average revenue that fake entrepreneurs obtain for each lemon they own. Replacing (36) and simplifying:

$$\rho(k) = \int_{\theta^*(k)}^1 \frac{1-\theta}{s(k) + \lambda(1-\theta)} e_1(\theta)\, d\theta \tag{38}$$

The planner solves:

$$\max_{c_0^H(\theta), c_0^E, c_0^F, k} c_0^E + V(k, q(k)) \tag{39}$$

$$s.t.$$

$$c_0^E + \int c_0^H(\theta)\, d\theta + c_0^F + k \le n + n^F + e_0 \tag{40}$$

$$c_0^H(\theta) + e_1(\theta) \max\left\{1, \frac{s(k)}{s(k) + \lambda(1-\theta)} \frac{A}{q(k)}\right\} \ge \bar{\Pi}(\theta) \tag{41}$$

$$c_0^F + \lambda \rho(k) \ge \bar{\Pi}^F \tag{42}$$

$$c_0^H(\theta) \ge 0 \quad c_0^E \ge 0 \quad c_0^F \ge 0 \quad k \ge 0$$

Constraint (40) is the $t = 0$ resource constraint. Constraint (41) imposes a minimum level of utility on household θ. At $t = 1$, the household chooses between consuming its endowment $e_1(\theta)$ and spending it to buy $\frac{e_1(\theta)}{q(k)}$ assets which yield an average of $\frac{s(k)}{s(k)+\lambda(1-\theta)} A$ consumption goods each at $t = 2$. Constraint (42) imposes a minimum level of utility for fake entrepreneurs. Replacing the constraints into the objective, the planner's objective reduces to:

$$W(k) \equiv n + n^F + e_0 - k - \bar{c}_0^F(k) - \int \bar{c}_0^H(\theta, k)\, d\theta + V(k, q(k))$$

where

$$\bar{c}_0^H(\theta, k) \equiv \max\left\{\bar{\Pi}(\theta) - e_1(\theta)\max\left\{1, \frac{s(k)}{s(k)+\lambda(1-\theta)}\frac{A}{q(k)}\right\}, 0\right\}$$
$$\bar{c}_0^F(k) \equiv \max\left\{\bar{\Pi}^F - \lambda\rho(k), 0\right\}$$

are the minimal levels of $t = 0$ consumption that the planner must assign to household θ and fake entrepreneurs respectively to ensure that they satisfy constraints (41) and (42).

Proposition 2. *In the model with asymmetric information,* $\frac{dW(k)}{dk} = A - 1 - z$ *so the social planner can obtain a Pareto improvement by raising investment relative to the competitive equilibrium.*

Proposition 2 says that this model has the opposite normative implications as the canonical model. The competitive equilibrium has underinvestment rather than overinvestment. Furthermore, this property holds regardless of whether condition (37) holds, i.e. regardless of whether higher investment leads to fire-sale effects.

In both the canonical model and the asymmetric information model, the social planner's investment decision takes into account the present value of investment and the indirect effect of investment through the effect of asset prices on $t = 1$ budget constraints. However, these effects work very differently in the two models.

In the canonical model, the net present value of investment is given by equation (22), and is equal to zero. Even though the net present value in best use is positive, only a fraction $\frac{q-z}{q}$ of the marginal unit of capital will be used by entrepreneurs; a fraction $\frac{z}{q}$ will be used by households, and the net present value of building marginal capital and assigning it to households is negative. Taking into account how capital will be allocated, the overall net present value of investment is zero. Conversely, there is social value in reallocating wealth across agents and periods. Shifting entrepreneurs wealth from $t = 0$ to $t = 1$ allows them to afford more capital and allocate it to its best use, which creates social gains.

Instead, in the asymmetric information model, the net present value of investment is always positive and equal to $A - 1 - z$, since capital will have the same marginal product no matter who holds it. The reason why the equilibrium does not produce as much investment as the social planner would like is that entrepreneurs do not capture the full value of investment at the margin. They know that they will have to sell a part of their capital to meet maintenance costs in a market where it will be partially pooled with lemons and therefore not be paid its full value. The presence of lemons in the market acts like a tax on investment, and the planner wants to counteract the effects of this tax on the level of investment.[8]

Conversely, given a level of investment, there is no social value in reallocating wealth across agents and periods in any direction. This is despite the fact that marginal rates of substitution are *not* equalized. Entrepreneurs' marginal rate of substitution is $\frac{A}{q}$, just like in the canonical model, while household θ's marginal rate of substitution is $\max\left\{\frac{s}{s+\lambda(1-\theta)}\frac{A}{q},1\right\}<\frac{A}{q}$ and fake entrepreneurs' marginal rate of substitution is 1. As in the canonical model, the reason entrepreneurs have a high marginal value of wealth at $t=1$ is that having higher wealth lets them avoid selling capital for $q<A$. If it were possible to enforce a loan of size ϵ, issued at $t=0$ and repaid at $t=1$, from an entrepreneur to a low-θ household, the bilateral gains from trade would be $\epsilon\left(\frac{A}{q}-1\right)$, just like in the canonical model. However, unlike in the canonical model, this would have a first-order negative effect on households and fake entrepreneurs, who benefit from trading in a market where real capital is on sale. Using equations (28)-(30) , (33) and (38), adding across periods and and integrating across households, total consumption by households and fake entrepreneurs is:

$$\underbrace{e_0+\int_0^{\theta^*}e_1(\theta)\,d\theta+\frac{A}{q}\int_{\theta^*}^1\left(1-\frac{\lambda(1-\theta)}{s+\lambda(1-\theta)}\right)e_1(\theta)\,d\theta}_{\text{Total household consumption}}+n^F+\underbrace{\lambda\int_{\theta^*}^1\frac{1-\theta}{s+\lambda(1-\theta)}e_1(\theta)\,d\theta}_{\text{Fake entreprneur consumption}}$$
$$=e_0+n^F+\int_0^1 e_1(\theta)\,d\theta+(A-q)\,s$$

As a result of the ϵ-sized loan, the entrepreneur would reduce his asset sales by $\frac{\epsilon}{q}$. This reduces total household and fake entrepreneur consumption by $(A-q)\frac{\epsilon}{q}$, which exactly offsets the bilateral gains from trade. Overall, since total output does not depend on the distribution of wealth at $t=1$, the social planner only cares about the positive net present value of investment and increases k until it hits nonnegativity constraints on $t=0$ consumption.

Dávila and Korinek (2017) show that in a class of models of which the canonical model is a special case, the differences between the social planner's problem and the competitive equilibrium can be summarized by three sufficient statistics: the differences in the marginal rates of substitution across agents, their trading positions, and the sensitivity of asset prices to investment. The model with asymmetric information does not fall within this class, and

[8] Kurlat (2013) shows an equivalence result between asymmetric information taxes on trading. Here the presence of maintenance costs and financial constraints implies that investment requires subsequent trading to meet maintenance costs, so asymmetric information acts like a tax on investment.

their result does not extend to it. Entrepreneurs have a higher marginal rate of substitution, are net sellers of capital, and if condition (37) holds asset prices fall when investment rises. Nevertheless, the social planner wants higher investment than in the competitive equilibrium.

Greenwald and Stiglitz (1986) show that in economies with asymmetric information there will generally be a possibility of Pareto-improving intervention. The class of policies that they consider includes intervening in the equivalent of the $t = 1$ asset market, for instance by subsidizing trades in order to reduce adverse selection. Here instead I maintain the assumption that the planner can only intervene at $t = 0$ and nevertheless find that in this particular model a social planner can bring about a Pareto improvement.

Note that one premise of the normative exercise is that the planner can redistribute across agents at $t = 0$. If there are fire sale effects, this requires taking goods from households (and possibly fake entrepreneurs) to compensate entrepreneurs for the lower prices that will result from higher investment.[9] If this redistribution is not feasible, then higher investment does not result in a Pareto improvement, and its desirability will depend on the welfare weights the planner places on each kind of agent. In the extreme case where the planner only cares about entrepreneurs (for instance, because households and fake entrepreneurs are foreigners), then avoiding fire sales justifies limiting investment irrespective of the microfoundation.

3 Extensions

New Investment Margin

One possible reason to worry about fire sales is that, whether or not they misallocate legacy assets, they may constrain the ability of important agents to undertake new investment. For instance, they may limit banks' ability to grant new loans. In this section I show that this concern is valid both models. In the canonical model, it just reinforces the argument for reducing ex-ante investment. In the asymmetric information model, it creates a tradeoff between $t = 0$ and $t = 1$ underinvestment; if $t = 1$ investment is sufficiently valuable and elastic, a case for limiting $t = 0$ investment reemerges.

Suppose entrepreneurs have an additional opportunity to invest at $t = 1$. They can convert $\psi(x)$ goods at $t = 1$ into x units of capital that will yield Ax consumption goods at $t = 2$.

[9] In the canonical model, it is the other way around: entrepreneurs need to compensate households for the profits they forgo when lower investment mitigates fire sales.

Assumption 3.

1. *$\psi(\cdot)$ is increasing and convex, with $\psi'(0) < z$*

2. *$(\psi'(x))^2 < \psi(x)\psi''(x)$ for all x*

Assumption 3.1 ensures that the equilibrium amount of new investment at $t = 1$ is positive. Assumption 3.2 means that $\psi(\cdot)$ is sufficiently convex that new investment does not respond too strongly to asset prices.

The entrepreneur's $t = 1$ problem, both in the canonical model and in the asymmetric information model, becomes:

$$
\begin{gathered}
V(k,q) \equiv \max_{c_1^E, c_2^E, s, k^E, \alpha} c_1^E + c_2^E \\
s.t. \\
c_1^E + \alpha z k + \psi(x) \leq sq \\
k^E \leq \alpha k - s + x \\
c_2^E \leq A k^E \\
\alpha \in [0,1]; \quad c_t^E \geq 0 \quad t = 1,2; \quad k^E \geq 0; \quad s \in [0, \alpha k]; \quad x \geq 0
\end{gathered}
\tag{43}
$$

The first order condition for x is:

$$\psi'(x) = q$$

The entrepreneur equates the marginal cost of delivering capital into period $t = 2$ across the two ways of obtaining it: new investment at a cost of $\psi'(x)$ or retaining maintained capital, which has an opportunity cost of q. Assumption 3.1 ensures that the entrepreneur chooses an interior solution.

Replacing this back into the entrepreneur's problem gives:

$$
V(k,q) = \underbrace{\frac{A}{q}}_{\text{rate of return on wealth}} \left[\underbrace{(q - z)k}_{\text{value of initial capital}} + \underbrace{qx(q) - \psi(x(q))}_{\text{net present value of new investement}} \right] \tag{44}
$$

where $x(q) \equiv (\psi')^{-1}(q)$. As in the baseline model, this implies that there will be an interior level of investment at $t = 0$ if and only if $q = \frac{Az}{A-1}$. Using the $t = 1$ budget constraint, sales of capital will be:

$$s(q,k)=\frac{zk+\psi(x(q))}{q} \tag{45}$$

In the canonical model, sales of capital will be purchased by households, so the price of capital will be equal to the marginal product of capital in the hands of households. The equilibrium price will therefore satisfy:

$$F'\left(\frac{zk+\psi(x(q))}{q}\right)=q \tag{46}$$

Lemma 3. *If equation* (46) *has a solution, it is unique. The solution* $q(k)$ *is a decreasing function.*

Lemma 3 is an extension of Lemma 1. It means that the fire sale effect in the canonical model extends to the variant where there is a new investment opportunity at $t=1$.

The social planner will solve problem (21), with the difference that $q(k)$ will be given by the solution to (46), the entrepreneur's value function is (44) and the amount of capital sold by entrepreneurs is given by (45).

Proposition 3. *In the canonical model with reinvestment, the social planner can obtain a Pareto improvement by lowering investment relative to the competitive equilibrium.*

Introducing a new-investment margin does not change the overinvestment result. It is still true that in equilibrium the ratio of marginal utilities across entrepreneurs and households is $\frac{A}{q}>1$ and the net present value of investment is zero, so shifting the terms of trade in favor of entrepreneurs results in a Pareto improvement.

Turn now to introducing $t=1$ investment in the asymmetric information model. Equilibrium in the asset market at $t=1$, given a level of k, is determined the same way as in the model without reinvestment. The only difference is that sales of real capital are given by equation (45) instead of equation (15). As in the baseline, there is an equilibrium with $q=0$ and no investment and possibly interior equilibria with $q=\frac{Az}{A-1}$ or corner equilibria with $q>\frac{Az}{A-1}$ and $k=n$.

Lemma 4.

1. *Taking* k *as given, the system of equations* (34), (35), (45) *has a unique solution* $q(k)$, $\theta^*(k)$, $s(k)$ *for any* $k\leq n$.

2. $q'(k)<0$ *if and only if condition* (37) *holds.*

Lemma 4 is an extension of Lemma 2. It means that the presence of fire sale effects in the variant of the asymmetric information model with reinvestment at $t = 1$ depends on the same forces as in the baseline. The uniqueness result relies on Assumption 3.2, which limits how strongly sales of real capital react to asset prices. Without it, it is possible to construct examples where higher asset prices induce higher sales of real capital to finance new investment, which improves the pool of assets on sale and justifies the higher prices, a standard source of multiple equilibria in variants of the Akerlof (1970) model.

The social planner will solve problem (39), with the only difference that $V(k, q(k))$ is defined by equation (44) and $q(k)$, $s(k)$ and $\theta^*(k)$ are defined as the solution to the system of equations (34), (35) and (45).

Proposition 4. *In the asymmetric information model with reinvestment:*

1. *If there are no fire-sale effects ($q'(k) \geq 0$), the social planner can obtain a Pareto improvement by raising investment relative to the competitive equilibrium.*

2. *If there are fire-sale effects ($q'(k) < 0$), the social planner can obtain a Pareto improvement by raising investment relative to the competitive equilibrium if the cost of $t = 1$ investment is sufficiently convex relative to the magnitude of fire-sale effects:*

$$\frac{A}{A-1} \frac{|q'(k)|}{\psi''(x(q(k)))} < 1 \tag{47}$$

Otherwise, the social planner can obtain a Pareto improvement by lowering investment relative to the competitive equilibrium.

As in the baseline asymmetric information model, the planner recognizes that entrepreneurs hold back on investment because they anticipate having to cross subsidize lemons; the planner wants to counteract this implicit tax and encourage higher investment at $t = 0$, knowing that the social net present value of $t = 0$ investment is $A - 1 - z > 0$.

On the other hand, in equilibrium there is also underinvestment at $t = 1$. Entrepreneurs choose investment at $t = 1$ according to the first order condition $\psi'(x) = q$, while the efficient level of investment (for instance, if entrepreneurs were able to borrow) would be $\psi'(x) = A$. The net present value of marginal investments at $t = 1$ is $A - \psi'(x) = A - q > 0$. If raising the amount of investment at $t = 0$ results in a change in asset prices of $q'(k)$, this results in $x'(q)\, q'(k) = \frac{q'(k)}{\psi''(x)}$ additional units of investment, with net present value $\frac{A-q}{\psi''(x)} q'(k)$.

When condition (37) holds so that the model has fire sale effects, this is a negative number: higher investment at $t = 0$ pushes down investment at $t = 1$ by lowering asset prices. Effectively, the planner faces a tradeoff between underinvestment at $t = 0$ and underinvestment at $t = 1$. Depending on how these two compare, the planner may want to raise or lower $t = 0$ investment relative to the competitive equilibrium. Condition (47) determines the direction of the net effect. The planner wants to lower $t = 0$ investment when (i) $\frac{A}{A-1}$ is high; at an interior equilibrium where $q = \frac{Az}{A-1}$, the ratio $\frac{A}{A-1} = \frac{A-q}{A-1-z}$ is the ratio of the net present value of $t = 1$ and $t = 0$ investment, so high $\frac{A}{A-1}$ means $t = 1$ investment is relatively valuable; (ii) $\psi''(x)$ is low, so the marginal cost of investment is relatively flat and $t = 1$ investment responds strongly to asset prices;[10] and (iii) $q'(k)$ is very negative, meaning strong marginal fire sale effects.

Instead, if condition (37) does not hold, then there is no tradeoff. By raising $t = 0$ investment, the planner raises asset prices and raises $t = 1$ investment. In this case, the planner unambiguously wants to raise investment relative to the competitive equilibrium.

Note that the effect on $t = 1$ investment is *not*, in the terminology of Dávila and Korinek (2017), a distributive externality. The reason why the planner wants to raise asset prices is not as a way to redistribute towards entrepreneurs at $t = 1$ and indirectly replace the missing credit market. Holding q constant, entrepreneurs use their marginal unit of wealth to retain existing capital, not to build new capital. As in the baseline asymmetric information model, whether or not entrepreneurs sell capital at $t = 1$ is neutral from the social planner's point of view, and therefore there is no social value in increasing entrepreneurs' wealth at $t = 1$. Instead, the reason why higher asset prices at $t = 1$ matter is that they determine the discount factor that entrepreneurs apply to new investment. Entrepreneurs obtain $\frac{A}{q}$ goods at $t = 2$ by dedicating one good at $t = 1$ towards retaining existing capital, and therefore require the same return from new investment. By raising asset prices the social planner lowers the required return on new investment, and therefore mitigates $t = 1$ underinvestment. One way to interpret the difference between this effect and a distributive externality is to imagine that there was a way for the social planner to (unexpectedly) intervene at $t = 1$ and transfer wealth from households to entrepreneurs, while keeping q constant. In the canonical model, this would increase efficiency because entrepreneurs could afford to dedicate more capital to its best use. Here instead keeping q constant would imply no change in $t = 1$ investment and therefore no efficiency gain.

[10] $\psi''(x)$ is the inverse of the coefficient of a regression of investment on Tobin's q, so in principle it's straightforward to measure it empirically.

Investment by Fake Entrepreneurs

One contrived feature of the asymmetric information model is that the measure of lemons is fixed and exogenous. This facilitates the comparison with the canonical model because it makes the entrepreneur's problem exactly identical. However, one may be concerned with the quality of investment in addition to the quantity. Is it the case that investment booms are associated with worse quality projects and/or lowering of credit standards? How would a social planner take this into account?

To address these questions, in this section I consider a a variant of the asymmetric information model where lemons are created deliberately with the intent of being sold, a possibility has been studied recently by Caramp (2016), Neuhann (2017) and Fukui (2018). Concretely, suppose that the amount of lemons is the result of investment by fake entrepreneurs. They can convert $\varphi(\lambda)$ goods at $t = 0$ into λ lemons (uniformly distributed in $[0, 1]$) at $t = 1$, which they can then attempt to sell in exchange for goods.

Assumption 4.

1. *$\varphi(\cdot)$ is increasing and convex, with $\varphi'(0) = 0$*
2. *$\varphi'\left(n^F\right) > A$*

Assumption 4 says that the marginal cost of the first lemon is zero, whereas if the fake entrepreneurs were to use all their wealth to produce lemons, the marginal cost would exceed the maximum price they could possibly get for them. This ensures that whenever there is positive investment in real capital, there is an interior solution for how many lemons are produced.

The fake entrepreneur's problem becomes

$$\max_{c_0^F, c_1^F, s^F(i)} c_0^F + c_1^F$$

$$s.t.$$

$$c_0^F + \varphi(\lambda) \leq n^F$$

$$c_1^F \leq q \int_0^1 s^F(i)\, \mu(i)\, di$$

$$s^F(i) \leq \lambda$$

Using the fact that the fake entrepreneur will always attempt to sell its lemons (i.e $s^F(i) = \lambda$), the first order condition for investment in lemons is:

$$\varphi'(\lambda) = q \int_0^1 \mu(i)\, di = \rho \tag{48}$$

Given a level of investment in real capital k, equilibrium in the $t = 1$ asset market requires that $\{s, q, \theta^*, \rho, \lambda\}$ satisfy: the optimal selling condition for (15); the indifference and market clearing conditions (34) and (35); the rationing formula (38) and the first order condition for fake investment (48). As in the baseline model, there always exists an equilibrium with $q = 0$ and, depending on parameters, there can be equilibria with $q = \frac{Az}{z-1}$ and $k \in [0, n]$ and/or an equilibrium with $q > \frac{Az}{A-1}$ and $k = n$.

Lemma 5.

1. *The system of equations* (15), (34), (35), (38), (48) *has a unique solution* $q(k)$, $\theta^*(k)$, $s(k)$, $\rho(k)$, $\lambda(k)$ *for any* $k \leq n$.

2. *There exist cutoffs* $\bar{e}_\lambda$ *and* $\bar{e}_q$ *such that*

   (a) $\lambda'(k) < 0$ *if and only if* $e_1(\theta^*(k)) < \bar{e}_\lambda$ *and*

   (b) $q'(k) < 0$ *if and only if* $e_1(\theta^*(k)) < \bar{e}_q$

3. *For* $\varphi''(\lambda(k))$ *sufficiently large, then* $\bar{e}_q > \bar{e}_\lambda$, *so* $q'(k) < 0$ *is a necessary condition for* $\lambda'(k) < 0$

Lemma 5 describes how asset prices and investment by fake entrepreneurs depend on real investment k. If lemons did not face rationing in equilibrium (i.e. if $\mu(i) = 1$ for all i), or if $\mu(i)$ was an exogenous constant, then the first order condition (48) would imply a direct relationship between asset prices q and revenue per unit from selling lemons ρ, and therefore investment by fake entrepreneurs λ. However, the degree of rationing faced by lemons responds endogenously to the level of investment, so q and ρ need not move in the same direction. Part 2 of Lemma 5 says that the pattern from the baseline model, established by Lemma 2, applies to both q and ρ. If there is a low level of wealth near the cutoff level of expertise, higher k will lead to falls in both asset prices and fake investment. Even though the cutoffs might be different, the basic force that leads to fire sale effects (a large fall in the marginal level of expertise among asset buyers) also leads to lower fake investment. Part

3 says that if there is enough curvature in φ so that fake investment does not respond too strongly to ρ, then there exist parameter combinations such that higher k leads to lower q but higher ρ but not the other way around, so fire sale effects are a necessary but not sufficient condition for fake investment to be decreasing in the level of real investment.

The planner solves problem (39) with the only difference that the exogenous constant λ is replaced by the function $\lambda(k)$ and constraint (40) is replaced by

$$c_0^E + \int c_0^H(\theta)\, d\theta + c_0^F + k + \varphi(\lambda(k)) \leq n + n^F + e_0$$

in order to take into account that creating $\lambda(k)$ lemons requires investing $\varphi(\lambda(k))$ resources. The implicit assumption is that the planner can control the level of investment by real entrepreneurs but cannot control fake entrepreneurs' investment, which will be given by (48). The planner just assigns a total $c_0^F + \varphi(\lambda(k))$ goods at $t = 0$ to fake entrepreneurs, understanding that how they will divide them between consumption and fake investment depends on the level of real investment and the resulting asset market equilibrium.

Proposition 5. *In the asymmetric information model with endogenous creation of lemons:*

1. *If real investment lowers the average revenue from selling lemons ($\rho'(k) \leq 0$), the social planner can obtain a Pareto improvement by raising investment relative to the competitive equilibrium.*

2. *If real investment raises the average revenue from selling lemons ($\rho'(k) > 0$), the social planner can obtain a Pareto improvement by raising investment relative to the competitive equilibrium if the cost of producing lemons is sufficiently convex:*

$$\frac{1}{A-1-z}\frac{\varphi'(\lambda(k))}{\varphi''(\lambda(k))}\rho'(k) < 1$$

Otherwise, the social planner can obtain a Pareto improvement by lowering investment relative to the competitive equilibrium.

The social planner cares about the positive net present value of investment $A - 1 - z$ and the deadweight cost of fake investment. Whenever $\rho'(k) > 0$ (which, using (48), implies $\lambda'(k) > 0$) this involves a tradeoff. Higher real investment increases the average revenue per lemon ρ and therefore induces wasteful investment by fake entrepreneurs. If this effect is large enough (which will be the case if the marginal cost of producing lemons is relatively flat, so quantities respond strongly), then it's possible that the social planner may want to reduce investment relative to the competitive equilibrium. Instead, when $\lambda'(k) < 0$, there is no tradeoff. Higher investment has a positive net present value and also discourages wasteful fake investment, so the planner unambiguously wants to raise investment relative to the competitive equilibrium.

Under the conditions of part 3 of Lemma 5, if there are no fire sale effects $(q'(k) > 0)$, this guarantees that higher real investment encourages higher fake investment, so the social planner faces a meaningful tradeoff. Instead, if there are fire sale effects $(q'(k) < 0)$ then it's possible that the planner faces no tradeoff. Since fake entrepreneurs rely on the asset market, a worsening of asset market conditions for selling lemons is actually helpful in deterring fake investment. If μ were an exogenous constant, this would immediately imply that fire sale effects are desirable: allowing for an endogenous response of fake investment strengthens the case for raising investment relative to the competitive equilibrium. Lemma 5 gives conditions under which the argument can be made precise while taking into account that q and ρ are not exactly proportional.

A constant fraction of lemons

So far I have assumed that the marginal investment the social planner controls involves creating real capital, not lemons. Another possibility is that lemons are just real investment projects gone wrong and neither entrepreneurs nor the social planner know ex-ante whether a project will turn out to be a lemon.

To explore this possibility, assume that an entrepreneur who invests k units at $t = 0$ obtains $\frac{1+\lambda}{\lambda}k$ units of real capital with probability $\frac{\lambda}{1+\lambda}$ but $\lambda(1+\lambda)k$ lemons (uniformly distributed in the unit interval) with probability $\frac{1}{1+\lambda}$, so in expectation he gets k units of real capital and λk lemons.

Entrepreneurs who end up having real capital will sell just enough to pay for maintenance, so total sales will be given by (15) as in the baseline model, while entrepreneurs who end up having lemons will always attempt to sell them. Let ρ, as before, denote the average revenue per lemon. The value at $t = 1$ for an entrepreneur who invested k is:

$$V(k; q, \rho) = \underbrace{\underbrace{\frac{\lambda}{1+\lambda}}_{\text{probability}} \underbrace{\frac{A}{q}}_{\text{return on wealth}} \underbrace{(q - z)\frac{1+\lambda}{\lambda}k}_{\text{net worth}}}_{\text{if entrepreneur obtains capital}} + \underbrace{\underbrace{\frac{1}{1+\lambda}}_{\text{probability}} \underbrace{\rho\lambda(1+\lambda)k}_{\text{net worth}}}_{\text{if entrepreneur gets lemons}}$$

$$= \left(\frac{A}{q}(q - z) + \rho\lambda\right) k$$

Note that the return on wealth for an entrepreneur who obtains capital is $\frac{A}{q}$ because one unit of wealth allows the entrepreneur to retain $\frac{1}{q}$ units of capital and therefore receive $\frac{A}{q}$ goods at $t = 2$. Instead, and entrepreneur who obtains lemons has a return on wealth of 1 because he does not retain any of his lemons and has no expertise for buying assets, so he just consumes the proceeds of selling his lemons. An interior level of investment requires:

$$\frac{A}{q}(q - z) + \rho\lambda = 1$$

Given a level of investment in real capital k, equilibrium in the $t = 1$ asset market requires that $\{s, q, \theta^*\}$ satisfy: the optimal selling condition for (15) and the indifference and market clearing formulas (34) and (35) with λk instead of λ to account for the fact that the quantity of lemons is proportional to investment:

$$\frac{s}{s + \lambda k(1 - \theta^*)} A = q \tag{49}$$

$$\int_{\theta^*}^{1} \frac{1}{s + \lambda k(1 - \theta)} e_1(\theta)\, d\theta = q \tag{50}$$

and ρ is given by equation (38), again with λk instead of λ:

$$\rho = \int_{\theta^*}^{1} \frac{1 - \theta}{s + \lambda k(1 - \theta)} e_1(\theta)\, d\theta \tag{51}$$

As in the baseline model, there always exists an equilibrium with $q = 0$ and, depending on parameters, there can be equilibria with $\frac{A}{q}(q - z) + \rho\lambda = 1$ and $k \in [0, n]$ and/or an equilibrium with $\frac{A}{q}(q - z) + \rho\lambda > 1$ and $k = n$.

Lemma 6.

1. *The system of equations* (15), (49), (50), *has a unique solution* $q(k)$, $\theta^*(k)$, $s(k)$, *for any* $k \leq n$.

2. $q'(k) < 0$

This variant of the model always features fire-sale effects. In the baseline model, higher investment improves the overall mix of assets in the economy because the number of lemons is fixed. Here, with a fixed fraction of lemons, the only effect is a shift in the marginal buyer towards a less expert household, which implies lower prices.

The planner solves problem (39) with the difference that since lemons are owned by real entrepreneurs as opposed to a separate group of fake entrepreneurs, the revenue from selling them is included in $V(k, q(k), \rho(k))$ and constraint (42) is not imposed.

Proposition 6. *In the asymmetric information model with a fixed proportion of lemons, the social planner can obtain a Pareto improvement by raising investment relative to the competitive equilibrium.*

The main insight from the baseline model is unchanged in this variant. Entrepreneurs perceive a tax on their investment due to asymmetric information in the secondary market and therefore underinvest. A social planner would choose higher investment despite realizing that this would further depress asset prices.

4 Conclusion

This paper is not the first to show that different models of fire sales can have different normative implications. For instance, Dávila and Korinek (2017) show an example of an economy with fire sales that is nevertheless constrained efficient and He and Kondor (2016) show an example where the direction of the inefficiency depends on the state of the economy. Instead, the main lesson is more specific: the microfoundation of the downward-sloping relationship that defines fires sales matters for the normative conclusions about investment.

This implies that it is worth devoting attention to try to determine what model best applies in practice, which might be context-specific. A priori, one would expect the canonical model to be a good fit for applications to investment in highly specific real assets such as airplanes, where alternative uses run into sharply diminishing marginal returns. Instead, the asymmetric information model may be a better fit for investment in complex financial instruments such as tranches of mortgage-backed CDOs, which are hard for non-experts to evaluate. If this is correct, there is a case for different macroprudential regulation depending on the setting.

Empirically, distinguishing between the models is challenging because many of the predictions similar: in both models, higher initial investment leads to falling asset prices and to increased profits for the average arbitrageur who buys at the fire sale price. An open challenge is to develop precise empirical tests that can tell the models apart.

Appendix

Proof of Lemma 1

Let $\Gamma(q,k) \equiv F'\left(\frac{zk}{q}\right) - q$. Differentiating with respect to q:

$$\frac{\partial \Gamma(q,k)}{\partial q} = -F''\left(\frac{zk}{q}\right) zkq^{-2} - 1$$

If q^* is a solution, then

$$\left.\frac{\partial \Gamma(q,k)}{\partial q}\right|_{q=q^*} = -\frac{F''\left(\frac{zk}{q^*}\right)\frac{zk}{q^*}}{F'\left(\frac{zk}{q^*}\right)} - 1 < 0$$

where the inequality follows from Assumption 1.4. Since Γ is decreasing in q at any solution, there is at most one solution.

Using the implicit function theorem:

$$\begin{aligned} q'(k) &= \frac{F''\left(\frac{zk}{q}\right)\frac{z}{q}}{F''\left(\frac{zk}{q}\right) zkq^{-2} + 1} \\ &= \frac{F''\left(\frac{zk}{q}\right)\frac{z}{q}}{F''\left(\frac{zk}{q}\right)\frac{zk}{q}F'\left(\frac{zk}{q}\right) + 1} \\ &< 0 \end{aligned}$$

where the inequality follows from the concavity of F and Assumption1.4.

Proof of Proposition 1

The planner's problem is:

$$\max_{k} W(k) \equiv n + e_0 + e_1 - zk + F(s(q(k), k)) - \bar{\Pi} - k + V(k, q(k))$$

$$s.t.$$

$$\bar{\Pi} - e_1 + q(k) s(q(k), k) - F(s(q(k), k)) \geq 0$$

$$n + e_0 + e_1 - q(k) s(q(k), k) + F(s(q(k), k)) - \bar{\Pi} - k \geq 0$$

At an interior point where the nonnegativity constraints on $t = 0$ consumption don't bind, the marginal value of investment is:

$$\frac{dW(k)}{dk} = -z - 1 + F'(s(q,k)) \left[\frac{\partial s(q(k), k)}{\partial q} q'(k) + \frac{\partial s(q(k), k)}{\partial k} \right] + \frac{dV(k, q(k))}{dk} \tag{52}$$

Using (16) and (15), this reduces to

$$\frac{dW(k)}{dk} = -z - 1 + F'\left(\frac{zk}{q(k)}\right) z \frac{q(k) - q'(k) k}{q(k)^2} + \frac{A(q(k) - z)}{q(k)} + \frac{Azk}{q(k)^2} q'(k)$$

At the equilibrium point, where $F'\left(\frac{zk}{q(k)}\right) = q(k) = \frac{Az}{A-1}$, this further reduces to:

$$\frac{dW(k)}{dk} = \frac{(A - 1 - z)(A - 1)}{Az} k q'(k) < 0$$

The inequality follows from Lemma 1 and Assumption 1.1.

Proof of Lemma 2

1. Replacing (15) into (34) and solving for q:

$$q(\theta^*) = \frac{-zk + \sqrt{(zk)^2 + 4\lambda(1-\theta^*) zkA}}{2\lambda(1-\theta^*)} \tag{53}$$

   Replacing (15) and (53) in (35) and rearranging:

$$\Phi(\theta^*) \equiv \int_{\theta^*}^{1} \frac{1}{zk + q(\theta^*)\lambda(1-\theta)} e_1(\theta) d\theta - 1 = 0 \tag{54}$$

(θ^*, q, s) is a solution to the system of equations if and only if $\Phi(\theta^*) = 0$, $q = q(\theta^*)$ and $s = \frac{zk}{q}$. To establish existence, note that since $q(\theta^*) \leq A$, and $k < n$, it follows that:

$$\Phi(\theta^*) > \int_{\theta^*}^{1} \frac{1}{zk + A\lambda} e_1(\theta)\, d\theta - 1$$

so Assumption 2 ensures that for sufficiently low θ^*, $\Phi(\theta^*) > 0$. Furthermore, for θ^* sufficiently close to 1, $\Phi(\theta^*) < 0$ and since $\Phi(\theta^*)$ is a continuous function, a solution to (54) exists. To establish uniqueness, take the derivative of $\Phi(\theta^*)$:

$$\Phi'(\theta^*) = -\frac{1}{zk + q(\theta^*)\lambda(1-\theta^*)} e_1(\theta^*) - q'(\theta^*) \int_{\theta^*}^{1} \frac{\lambda(1-\theta)}{[zk + q(\theta^*)\lambda(1-\theta)]^2} e_1(\theta)\, d\theta$$

Taking the the derivative of (53):

$$q'(\theta^*) = \frac{2\lambda zk\left(zk + 2\lambda(1-\theta^*)A - \left((zk)^2 + 4\lambda(1-\theta^*)zkA\right)^{0.5}\right)}{4\lambda^2(1-\theta^*)^2\left((zk)^2 + 4\lambda(1-\theta^*)zkA\right)^{0.5}} > 0$$

This is positive because:

$$\begin{aligned}
4\lambda^2(1-\theta^*)^2 A^2 &> 0 \\
\Rightarrow 4\lambda^2(1-\theta^*)^2 A^2 + (zk)^2 + 4\lambda(1-\theta^*)Azk &> (zk)^2 + 4\lambda(1-\theta^*)Azk \\
\Rightarrow (zk + 2\lambda(1-\theta^*)A)^2 &> (zk)^2 + 4\lambda(1-\theta^*)zkA \\
\Rightarrow zk + 2\lambda(1-\theta^*)A &> \left((zk)^2 + 4\lambda(1-\theta^*)zkA\right)^{0.5}
\end{aligned}$$

The fact that $q'(\theta^*) > 0$ implies that $\Phi'(\theta^*) < 0$. Since $\Phi(\theta^*)$ is monotonically decreasing, there is a unique solution to equation (54).

2. Rewrite (34), (35), (15) as

$$\Gamma(q, \theta^*, s; k) = 0$$

where

$$\Gamma(q, \theta^*, s; k) = \begin{pmatrix} qs + q\lambda(1-\theta^*) - sA \\ \int_{\theta^*}^{1} \frac{1}{s+\lambda(1-\theta)} e_1(\theta)\, d\theta - q \\ sq - zk \end{pmatrix}$$

and let ∇ be the gradient of Γ. Taking derivatives:

$$\nabla = \begin{pmatrix} s+\lambda(1-\theta^*) & -q\lambda & q-A \\ -1 & -\frac{1}{s+\lambda(1-\theta^*)}e_1(\theta^*) & -\int_{\theta^*}^1 \frac{1}{[s+\lambda(1-\theta)]^2}e_1(\theta)\,d\theta \\ s & 0 & q \end{pmatrix} \tag{55}$$

By the implicit function theorem

$$\begin{pmatrix} q'(k) \\ \theta^{*\prime}(k) \\ s'(k) \end{pmatrix} = -\nabla^{-1}\begin{pmatrix} \frac{\partial\Gamma_1}{\partial k} \\ \frac{\partial\Gamma_2}{\partial k} \\ \frac{\partial\Gamma_3}{\partial k} \end{pmatrix} = -\nabla^{-1}\begin{pmatrix} 0 \\ 0 \\ -z \end{pmatrix}$$

and therefore

$$q'(k) = \frac{1}{|\nabla|}\left(q\lambda\int_{\theta^*}^{1}\frac{1}{[s+\lambda(1-\theta)]^2}e_1(\theta)\,d\theta - \frac{A-q}{s+\lambda(1-\theta^*)}e_1(\theta^*)\right)z$$

where

$$\begin{aligned}
|\nabla| &= q\lambda\left(\int_{\theta^*}^{1}\frac{s}{[s+\lambda(1-\theta)]^2}e_1(\theta)\,d\theta - q\right) - e_1(\theta^*)\left[(A-q)\frac{s}{s+\lambda(1-\theta^*)}+q\right] \\
&< q\lambda\left(\int_{\theta^*}^{1}\frac{s}{[s+\lambda(1-\theta)]^2}e_1(\theta)\,d\theta - q\right) \\
&< q\lambda\left(\int_{\theta^*}^{1}\frac{1}{s+\lambda(1-\theta)}e_1(\theta)\,d\theta - q\right) \\
&= 0
\end{aligned} \tag{56}$$

where the last step uses (35). Therefore $q'(k) < 0$ if and only if

$$q\lambda\int_{\theta^*}^{1}\frac{1}{[s+\lambda(1-\theta)]^2}e_1(\theta)\,d\theta - \frac{A-q}{s+\lambda(1-\theta^*)}e_1(\theta^*) > 0$$

Rearranging gives the result.

Proof of Proposition 2

The planner's problem is

$$\max_k W(k) \equiv n + n^F + e_0 - k - \bar{c}_0^F(k) - \int \bar{c}_0^H(\theta, k)\, d\theta + V(k, q(k))$$
$$k \leq n + n^F + e_0 - \bar{c}_0^F(k) - \int \bar{c}_0^H(\theta, k)\, d\theta \tag{57}$$

At an interior point where the nonnegativity constraint on $\bar{c}_0^F(k)$ and $\bar{c}_0^H(\theta, k)$ don't bind:

$$\begin{aligned}
\frac{dW(k)}{dk} &= -1 + \frac{d}{dk}\left(\lambda\rho(k) + \int e_1(\theta) \max\left\{1, \frac{s(k)}{s(k) + \lambda(1-\theta)} \frac{A}{q(k)}\right\} d\theta\right) + \frac{dV(k, q(k))}{dk} \\
&= -1 + \frac{d}{dk}\left(\int_0^1 e_1(\theta)\, d\theta - s(k)\, q(k) + As(k)\right) + \frac{dV(k, q(k))}{dk} \\
&= A - 1 - z
\end{aligned} \tag{58}$$

The first step follows from rearranging, using (38) and using the market clearing condition (35) and the last step uses (16) and (15). As long as Assumption 1.1 (which states that investing in capital has positive net present value) holds, then $\frac{dW(k)}{dk} > 0$

Proof of Lemma 3

Let $\Gamma(q, k) \equiv F'\left(\frac{zk + \psi(x(q))}{q}\right) - q$. Differentiating with respect to q:

$$\frac{\partial \Gamma(q, k)}{\partial q} = F''\left(\frac{zk + \psi(x(q))}{q}\right)\left(\frac{\psi'(x(q))\, x'(q)\, q - zk + \psi(x(q))}{q^2}\right) - 1$$

If q^* is a solution, then

$$\left.\frac{\partial \Gamma(q, k)}{\partial q}\right|_{q=q^*} = -\frac{F''\left(\frac{zk+\psi(x(q))}{q}\right)\frac{zk+\psi(x(q))}{q}}{F'\left(\frac{zk+\psi(x(q))}{q}\right)} - 1 + F''\left(\frac{zk + \psi(x(q))}{q}\right)\frac{\psi'(x(q))\, x'(q)}{q} < 0$$

where the inequality follows from Assumption 1.4, the concavity of F and the convexity of ψ. Since Γ is decreasing in q at any solution, there is at most one solution.

Using the implicit function theorem:

$$q'(k) = \frac{F''\left(\frac{zk+\psi(x(q))}{q}\right)\frac{z}{q}}{\frac{\partial\Gamma(q,k)}{\partial q}} < 0$$

where the inequality follows from the concavity of F and the first part of the result.

Proof of Proposition 3

Using $q = F'(s(q,k)) = \psi'(x(q)) = \frac{Az}{A-1}$, expression (52) reduces to:

$$\frac{dW(k)}{dk} = \frac{1}{q^2}\left[q^3 x'(q) + \left[\psi(x(q)) + zk\right](A-q)\right] q'(k) < 0$$

Proof of Lemma 4

1. Replacing (45) into (34), $q(\theta^*)$ solves:

$$\Omega(q,\theta^*) = \frac{zk+\psi(x(q))}{q} - \frac{q}{A-q}\lambda(1-\theta^*) = 0$$

Note that

$$\begin{aligned}\frac{\partial\Omega(q,\theta^*)}{\partial q} &= \frac{\psi'(x(q))x'(q)q - zk - \psi(x(q))}{q(\theta^*)^2} - \frac{A}{(A-q)^2}\lambda(1-\theta^*)\\ &< \frac{\psi'(x(q))x'(q)q - \psi(x(q))}{q^2}\\ &= \frac{\left[\psi'(x(q))\right]^2\frac{1}{\psi''x(q)} - \psi(x(q))}{q^2} < 0\end{aligned}$$

where the last step follows from using the definition $x(q) \equiv (\psi')^{-1}(q)$ and Assumption 3.2. Also, $\lim_{q\to 0}\Omega(q,\theta^*) > +\infty$ and $\lim_{q\to A}\Omega(q,\theta^*) > -\infty$. This implies that for each $\theta^* \in [0,1]$ there is a unique $q(\theta^*)$ that satisfies (45) and (34). Also note that

$$\frac{\partial\Omega(q(\theta^*),\theta^*)}{\partial\theta^*} = \frac{q(\theta^*)}{A-q(\theta^*)}\lambda > 0$$

so, using the implicit function theorem:

$$q'(\theta^*) = \frac{-\frac{\partial\Omega(q(\theta^*),\theta^*)}{\partial\theta^*}}{\frac{\partial(q(\theta^*),\theta^*)}{\partial q}} > 0$$

Replacing $q(\theta^*)$ into (35) gives (54), so the same steps that lead to the proof of part 1 of Lemma 2 apply.

2. Rewrite the system of equations as

$$\Gamma(q,\theta^*,s;k) = 0$$

where

$$\Gamma(q,\theta^*,s;k) = \begin{pmatrix} qs + q\lambda(1-\theta^*) - sA \\ \int_{\theta^*}^{1} \frac{1}{s+\lambda(1-\theta)} e_1(\theta)\, d\theta - q \\ sq - zk - \psi(x(q)) \end{pmatrix}$$

and let ∇ be the gradient of Γ. Taking derivatives:

$$\nabla = \begin{pmatrix} s+\lambda(1-\theta^*) & -q\lambda & q-A \\ -1 & -\frac{1}{s+\lambda(1-\theta^*)} e_1(\theta^*) & -\int_{\theta^*}^{1} \frac{1}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta \\ s-\psi'(x(q))\, x'(q) & 0 & q \end{pmatrix}$$

By the implicit function theorem

$$\begin{pmatrix} q'(k) \\ \theta^{*\prime}(k) \\ s'(k) \end{pmatrix} = -\nabla^{-1} \begin{pmatrix} \frac{\partial\Gamma_1}{\partial k} \\ \frac{\partial\Gamma_2}{\partial k} \\ \frac{\partial\Gamma_3}{\partial k} \end{pmatrix} = -\nabla^{-1} \begin{pmatrix} 0 \\ 0 \\ -z \end{pmatrix} \tag{59}$$

and therefore

$$q'(k) = \frac{1}{|\nabla|} \left(q\lambda \int_{\theta^*}^{1} \frac{1}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta - \frac{A-q}{s+\lambda(1-\theta^*)} e_1(\theta^*) \right) z \tag{60}$$

where

$$|\nabla| = q\lambda \left(\int_{\theta^*}^{1} \frac{s-\psi'(x(q))\, x'(q)}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta - q \right) - e_1(\theta^*) \left[(A-q) \frac{s-\psi'(x(q))\, x'(q)}{s+\lambda(1-\theta^*)} + q \right]$$

Assumption 3.2 and (45) imply that

$$s - \psi'(x(q))\,x'(q) > 0$$

and therefore

$$\begin{aligned}
|\nabla| &< q\lambda\left(\int_{\theta^*}^{1}\frac{s - \psi'(x(q))\,x'(q)}{[s+\lambda(1-\theta)]^2}e_1(\theta)\,d\theta - q\right)\\
&< q\lambda\left(\int_{\theta^*}^{1}\frac{s}{[s+\lambda(1-\theta)]^2}e_1(\theta)\,d\theta - q\right)\\
&< q\lambda\left(\int_{\theta^*}^{1}\frac{1}{s+\lambda(1-\theta)}e_1(\theta)\,d\theta - q\right)\\
&= 0
\end{aligned}$$

so the last step of the proof of part 2 of Lemma 2 applies.

Proof of Proposition 4

At an interior point where the nonnegativity constraint on $\bar{c}_0^F(k)$ and $\bar{c}_0^H(\theta,k)$ don't bind, the same steps that lead to equation (58) result in:

$$\frac{dW(k)}{dk} = A - 1 - z + \left(\frac{A - q(k)}{\psi''(x(q(k)))}\right)q'(k) \tag{61}$$

Evaluating this expression at an interior equilibrium where $q = \frac{Az}{A-1}$ and rearranging:

$$\left.\frac{dW(k)}{dk}\right|_{k=k^*} = \frac{A-1-z}{(A-1)\,\psi''(x(q(k)))}\left[(A-1)\,\psi''(x(q(k))) + Aq'(k)\right]$$

which implies the result.

Proof of Lemma 5

1. By Lemma 2, the system (15), (34), (35) has a unique solution $q(\lambda,k), \theta^*(\lambda,k), s(\lambda,k)$ for any λ, k. Using (38), this implies that for any λ,k there is a unique

$$\rho(\lambda,k) = \int_{\theta^*(\lambda,k)}^{1} \frac{1-\theta}{s(\lambda,k)+\lambda(1-\theta)} e_1(\theta)\, d\theta \tag{62}$$

that satisfies (15), (34), (35) and (38). Taking the derivative:

$$\frac{\partial\rho}{\partial\lambda} = -\frac{1-\theta^*}{s+\lambda(1-\theta^*)} e_1(\theta^*) \frac{\partial\theta^*}{\partial\lambda} - \int_{\theta^*}^{1} \frac{(1-\theta)\left(\frac{\partial s}{\partial\lambda}+1-\theta\right)}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta$$

By the implicit function theorem:

$$\begin{pmatrix} \frac{\partial q}{\partial\lambda} \\ \frac{\partial\theta^*}{\partial\lambda} \\ \frac{\partial s}{\partial\lambda} \end{pmatrix} = -\nabla^{-1} \begin{pmatrix} q(1-\theta^*) \\ -\int_{\theta^*}^{1} \frac{1-\theta}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta \\ 0 \end{pmatrix} \tag{63}$$

where ∇ is defined by (55). Therefore:

$$\frac{\partial\theta^*}{\partial\lambda} = -\frac{1}{|\nabla|}\left[\left(-s\int_{\theta^*}^{1} \frac{1}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta + q\right) q(1-\theta^*) - (q\lambda(1-\theta^*)+As)\int_{\theta^*}^{1} \frac{1-\theta}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta\right]$$

$$\frac{\partial s}{\partial\lambda} = -\frac{1}{|\nabla|}\left[\frac{s}{s+\lambda(1-\theta^*)} e_1(\theta^*) q(1-\theta^*) + q\lambda s\int_{\theta^*}^{1} \frac{1-\theta}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta\right]$$

Equation (56) says that $|\nabla|<0$ and therefore that $\frac{\partial s}{\partial\lambda}>0$. This implies:

$$\begin{aligned}\frac{\partial\rho}{\partial\lambda} &\le -\frac{1-\theta^*}{s+\lambda(1-\theta^*)} e_1(\theta^*) \frac{\partial\theta^*}{\partial\lambda} - \int_{\theta^*}^{1} \frac{(1-\theta)^2}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta \\ &= \frac{1}{|\nabla|}\left[\begin{array}{c} \left(-s\int_{\theta^*}^{1} \frac{1}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta + q\right) q(1-\theta^*) \\ -(q\lambda(1-\theta^*)+As)\int_{\theta^*}^{1} \frac{1-\theta}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta \end{array}\right] \frac{1-\theta^*}{s+\lambda(1-\theta^*)} e_1(\theta^*) - \int_{\theta^*}^{1} \frac{(1-\theta)^2}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta\end{aligned}$$

If

$$\left(-s\int_{\theta^*}^{1} \frac{1}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta + q\right) q(1-\theta^*) - (q\lambda(1-\theta^*)+As)\int_{\theta^*}^{1} \frac{1-\theta}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta > 0$$

then $|\nabla| < 0$ implies $\frac{\partial \rho}{\partial \lambda} < 0$. Otherwise, condition (56) implies that

$$|\nabla| < e_1(\theta^*) \left[(A-q) \frac{s}{s+\lambda(1-\theta^*)} + q \right]$$

and therefore

$$\frac{\partial \rho}{\partial \lambda} \leq \frac{\left(-s \int_{\theta^*}^{1} \frac{1}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta + q\right) q(1-\theta^*) - (q\lambda(1-\theta^*) + As) \int_{\theta^*}^{1} \frac{1-\theta}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta}{e_1(\theta^*) \left[(A-q) \frac{s}{s+\lambda(1-\theta^*)} + q \right]} \frac{1-\theta^*}{s+\lambda(1-\theta^*)} e_1(\theta^*)$$

$$- \int_{\theta^*}^{1} \frac{(1-\theta)^2}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta$$

$$= -\frac{(1-\theta^*)^2 sq}{[As+q\lambda(1-\theta^*)]} \int_{\theta^*}^{1} \frac{1}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta + \frac{q^2(1-\theta^*)^2}{[As+q\lambda(1-\theta^*)]} - (1-\theta^*) \int_{\theta^*}^{1} \frac{1-\theta}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta$$

$$- \int_{\theta^*}^{1} \frac{(1-\theta)^2}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta \tag{64}$$

Using (34) and (35) and rearranging, this simplifies to:

$$\frac{\partial \rho}{\partial \lambda} \leq -\left[(1-\theta^*) \left[\int_{\theta^*}^{1} \frac{(1-\theta)}{[s+\lambda(1-\theta)]^2} \frac{s+\lambda(1-\theta^*)}{s+2\lambda(1-\theta^*)} e_1(\theta)\, d\theta \right] + \int_{\theta^*}^{1} \frac{(1-\theta)^2}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta \right] < 0 \tag{65}$$

Since $\varphi$is convex, (48) defines an increasing relationship between ρ and λ. Together with (65), this implies that there can be at most one value of λ that satisfies (15), (34), (35), (38) and (48). Assumption then implies that a solution exists.

2.

(a) Define

$$\Lambda(\rho) \equiv (\varphi')^{-1}(\rho)$$

Convexity of φ implies that Λ is an increasing function. $\rho(k)$ must solve:

$$\rho(k) = \rho(\Lambda(\rho(k)), k)$$

so

$$\rho'(k) = \frac{\frac{\partial \rho(\lambda,k)}{\partial k}}{1 - \frac{\partial \rho}{\partial \lambda} \Lambda'(\rho)}$$

By part 1, $\frac{\partial \rho}{\partial \lambda} < 0$. Therefore $\rho'(k) < 0$ if and only if $\frac{\partial \rho(\lambda,k)}{\partial k} < 0$. Taking the derivative of (62):

$$\frac{\partial \rho}{\partial k} = -\frac{1-\theta^*}{s+\lambda(1-\theta^*)} e_1(\theta^*) \frac{\partial \theta^*}{\partial k} - \int_{\theta^*}^{1} \frac{(1-\theta)}{[s+\lambda(1-\theta)]^2} \frac{\partial s}{\partial k} e_1(\theta)\, d\theta$$

Using (59):

$$\frac{\partial \theta^*}{\partial k} = \frac{1}{|\nabla|}\left[A - q + (s+\lambda(1-\theta^*))\left(\int_{\theta^*}^{1} \frac{1}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta\right)\right] z$$

$$\frac{\partial s}{\partial k} = -\frac{1}{|\nabla|}\left[e_1(\theta^*) + q\lambda\right] z$$

and therefore:

$$\frac{\partial \rho}{\partial k} = \frac{z}{|\nabla|}\left[\begin{array}{c} -\frac{1-\theta^*}{s(k)+\lambda(1-\theta^*)} e_1(\theta^*)\left[A - q + (s+\lambda(1-\theta^*))\left(\int_{\theta^*}^{1} \frac{1}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta\right)\right] \\ +\left[e_1(\theta^*) + q\lambda\right] \int_{\theta^*(k)}^{1} \frac{(1-\theta)}{[s(k)+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta \end{array}\right] \qquad (66)$$

Taking the limit:

$$\lim_{e_1(\theta^*)\to 0} \frac{\partial \rho}{\partial k} = \frac{z}{|\nabla|}\left[q\lambda \int_{\theta^*(k)}^{1} \frac{(1-\theta)}{[s(k)+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta\right] < 0$$

so by continuity there exists $\bar{e}_\lambda$ such that $\frac{\partial \rho}{\partial k} < 0$ if $e_1(\theta^*(k)) < \bar{e}_\lambda$. Since $\lambda(k) = \Lambda(\rho(k))$ and Λ is increasing, the result follows.

(b) $q(k)$ must solve

$$q(k) = q(k, \Lambda(\rho(k)))$$

so

$$q'(k) = \frac{\partial q}{\partial k} + \frac{\frac{\partial q}{\partial \lambda}\Lambda'(\rho)}{1 - \frac{\partial \rho}{\partial \lambda}\Lambda'(\rho)} \frac{\partial \rho}{\partial k} \qquad (67)$$

Lemma 2 implies that for sufficiently low $e_1(\theta^*)$, then $\frac{\partial q}{\partial k}$. Furthermore: using (63):

$$\frac{\partial q}{\partial \lambda} = \frac{1}{|\nabla|} q^2 \left[\frac{1}{s+\lambda(1-\theta^*)} e_1(\theta^*)(1-\theta^*) + \lambda \int_{\theta^*}^{1} \frac{1-\theta}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta\right] < 0 \qquad (68)$$

Therefore if $\frac{\partial \rho}{\partial k} > 0$ then $q'(k) < 0$ follows immediately. Instead if $\frac{\partial \rho}{\partial k} < 0$, the right hand side of (67) is increasing in $\Lambda'(\rho)$, so it suffices to show the result for $\Lambda'(\rho) \to \infty$. Therefore:

$$q'(k) \leq \frac{\partial q}{\partial k} - \frac{\frac{\partial q}{\partial \lambda}}{\frac{\partial \rho}{\partial \lambda}} \frac{\partial \rho}{\partial k} \tag{69}$$

Replacing (60), (68), (64) and (66) and (56) in (69) and taking the limit and rearranging, then:

$$\begin{aligned}
\lim_{e_1(\theta^*)\to 0} q'(k) &\leq \lim_{e_1(\theta^*)\to 0} \frac{\partial q}{\partial k} - \frac{\frac{\partial q}{\partial \lambda}}{\frac{\partial \rho}{\partial \lambda}} \frac{\partial \rho}{\partial k} \\
&= \left(\lim_{e_1(\theta^*)\to 0} \Delta\right) \left[\begin{array}{c} \left(\int_{\theta^*(k)}^1 \frac{(1-\theta)}{[s(k)+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta\right)^2 \\ -\int_{\theta^*}^1 \frac{1}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta \int_{\theta^*}^1 \frac{(1-\theta)^2}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta \end{array} \right] \\
&\leq 0
\end{aligned}$$

where

$$\Delta = \frac{1}{\frac{\partial \rho}{\partial \lambda}} \frac{z}{|\nabla|} \lambda \frac{sA}{s+\lambda(1-\theta^*)} > 0$$

and the inequality follows from the Cauchy-Schwarz inequality. Therefore by continuity there exists $\bar{e}_q$ such that $q'(k) < 0$ if and only if $e_1(\theta^*(k)) < \bar{e}_q$.

3. Since $\Lambda'(\rho) = \frac{1}{\varphi''(\Lambda(\rho))}$, then for sufficiently large φ'' we have $\Lambda'(\rho) \to 0$, which implies $q'(k) \to \frac{\partial q}{\partial k}$. Therefore it is sufficient to prove that $\lambda'(k) < 0$ implies $\frac{\partial q}{\partial k} < 0$. Note that:

$$(A-q)\frac{1}{s+\lambda(1-\theta^*)}\left[\int_{\theta^*}^{1} \frac{\theta^*-\theta}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta\right] < 0 < \int_{\theta^*}^{1} \frac{1}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta \int_{\theta^*}^{1} \frac{\theta-\theta^*}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta$$

Rearranging implies that:

$$\frac{q\lambda \int_{\theta^*}^1 \frac{1-\theta}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta}{\frac{1-\theta^*}{s+\lambda(1-\theta^*)}(A-q) + \int_{\theta^*}^1 \frac{\theta-\theta^*}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta} < \frac{q\lambda \int_{\theta^*}^1 \frac{1}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta}{(A-q)\frac{1}{s+\lambda(1-\theta^*)}} \tag{70}$$

Assume $\lambda'(k) < 0$. By the argument in part 2a, this requires $\frac{\partial \rho}{\partial k} < 0$, so using (66):

$$e_1(\theta^*) < \frac{q\lambda \int_{\theta^*}^1 \frac{1-\theta}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta}{\frac{1-\theta^*}{s+\lambda(1-\theta^*)}(A-q) + \int_{\theta^*}^1 \frac{\theta-\theta^*}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta}$$

Together with (70), this implies

$$e_1(\theta^*) < \frac{q\lambda \int_{\theta^*}^{1} \frac{1}{[s+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta}{(A-q)\frac{1}{s+\lambda(1-\theta^*)}}$$

and therefore, using (60), $\frac{\partial q}{\partial k} < 0$.

Proof of Proposition 5

Following the same steps that result in expression (58) and using (48), the marginal social value of investment is given by:

$$\begin{aligned}\frac{dW(k)}{dk} &= A - 1 - z - \varphi'(\lambda(k))\lambda'(k) \\ &= A - 1 - z - \frac{\varphi'(\lambda(k))}{\varphi''(\lambda(k))}\rho'(k)\end{aligned}$$

which implies the result.

Proof of Lemma 6

1. Existence and uniqueness follow by the same steps as Lemma 2

2. Let $\tilde{s} = \frac{s}{k}$ and rewrite the system of equations as:

$$\Gamma(q, \theta^*, \tilde{s}; k) = 0$$

where

$$\Gamma(q, \theta^*, \tilde{s}; k) = \begin{pmatrix} \tilde{s}A - \tilde{s}q - \lambda(1-\theta^*) \\ \tilde{s}q - z \\ \int_{\theta^*}^{1} \frac{1}{\tilde{s}+\lambda(1-\theta)} e_1(\theta)\, d\theta - qk \end{pmatrix}$$

and let ∇ be the gradient of Γ. Taking derivatives:

$$\nabla = \begin{pmatrix} -\tilde{s} & \lambda & A-q \\ \tilde{s} & 0 & q \\ -k & -\frac{1}{\tilde{s}+\lambda(1-\theta^*)} e_1(\theta^*) & -\int_{\theta^*}^{1} \frac{1}{[\tilde{s}+\lambda(1-\theta)]^2} e_1(\theta)\, d\theta \end{pmatrix}$$

By the implicit function theorem:

$$\begin{pmatrix} q'(k) \\ \theta^{*\prime}(k) \\ \tilde{s}'(k) \end{pmatrix} = -\nabla^{-1} \begin{pmatrix} \frac{\partial \Gamma_1}{\partial k} \\ \frac{\partial \Gamma_2}{\partial k} \\ \frac{\partial \Gamma_3}{\partial k} \end{pmatrix} = -\nabla^{-1} \begin{pmatrix} 0 \\ 0 \\ -q \end{pmatrix}$$

and therefore:

$$q'(k) = \frac{1}{|\nabla|}\lambda q^2$$

where

$$\begin{aligned} |\nabla| &= \lambda \left(\int_{\theta^*}^{1} \frac{1}{x+\lambda(1-\theta)} \frac{x}{x+\lambda(1-\theta)} e_1(\theta)\, d\theta - qk \right) - Ax \frac{1}{x+\lambda(1-\theta^*)} e_1(\theta^*) \\ &\leq \lambda \left(\int_{\theta^*}^{1} \frac{1}{x+\lambda(1-\theta)} e_1(\theta)\, d\theta - qk \right) - Ax \frac{1}{x+\lambda(1-\theta^*)} e_1(\theta^*) \\ &= -Ax \frac{1}{x+\lambda(1-\theta^*)} e_1(\theta^*) < 0 \end{aligned}$$

Therefore $q'(k) < 0$.

Proof of Proposition 6

Replacing the constraints in the objective function:

$$W(k) = n + e_0 - k - \int_0^1 \Pi(\theta)\, d\theta + \int_0^1 e_1(\theta)\, d\theta - \int_{\theta^*}^{1} \frac{1}{q} \left[\frac{s}{s+\lambda k(1-\theta)} A - q \right] e_1(\theta)\, d\theta + \frac{A}{q}(q-z)k + \lambda k \rho$$

Using (15), (50) and (51) and taking the derivative at a point where the nonnegativity constraints don't bind gives the result.

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