Platform Tokenization: Financing, Governance, and Moral Hazard

Jiri Chod, Nikolaos Trichakis, S. Alex Yang*

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Abstract

This paper highlights two channels through which blockchain-enabled tokenization can alleviate moral hazard frictions between founders, investors, and users of a platform: token financing and decentralized governance. We consider an entrepreneur who uses outside financing and exerts private effort to build a platform, and users who decide whether to join in response to the platform's dynamic transaction fee policy. We first show that raising capital by issuing tokens rather than equity mitigates effort under-provision because the payoff to equity investors depends on profit, whereas the payoff to token investors depends on transaction volume, which is less sensitive to effort. Second, we show that decentralized governance associated with tokenization eliminates a potential holdup of platform users, which in turn alleviates the need to provide users with incentives to join, reducing the entrepreneur's financing burden. The downside of tokenization is that it puts a cap on how much capital the entrepreneur can raise. Namely, if tokens are highly liquid, i.e., they change hands many times per unit of time, their market capitalization is small relative to the NPV of the platform profits, limiting how much money one can raise by issuing tokens rather than equity. If building the platform is expensive, this can distort the capacity investment. The resulting trade-off between the benefits and costs of tokenization leads to several predictions regarding adoption.

Keywords: blockchain, ICO, cryptocurrency, token, platform, entrepreneurial finance, agency, moral hazard

*Chod (chodj@bc.edu) is from Boston College, Trichakis (ntrichakis@mit.edu) is from Massachusetts Institute of Technology, Yang (sayang@london.edu) is from London Business School.

1 Introduction

The last decade or so has witnessed an unprecedented proliferation of the platform business model with digital matchmakers like Amazon, Alibaba, Uber, and AirBnB gaining control over a significant share of the global economic activity.[1] Each of these examples represents a platform, or two-sided marketplace, that is operated by a single entity controlled by equity holders. Recently a new way of operating digital platforms has emerged that takes advantage of the blockchain technology and differs from its traditional counterpart by (i) using its own digital currency to settle transactions and (ii) relying on decentralized, or peer-to-peer, governance. Issuing a platform digital currency, or "utility tokens," provides a way to finance the platform development without relying on equity investors. Decentralization means, loosely speaking, the absence of a central authority able to change the rules governing the platform without user consensus. An example of such blockchain-based platforms is Filecoin, a decentralized marketplace for computer storage capacity. In 2017 Filecoin raised $257 million by selling tokens that would be used for payments on its network. In December 2020, after years of development and testing, Filecoin launched its service, and the market capitalization of its tokens exceeded $1 billion.[2]

This paper studies how tokenization, that is, token financing plus decentralized governance, affects moral hazard frictions between founders, investors, and users of a platform. Although tokenization is not limited to platforms, among various types of entrepreneurial ventures platforms have received majority of token financing to date (Adhami, Giudici and Martinazzi (2018)), and have been the focus of the cryptoeconomics literature (e.g., Sockin and Xiong (2018), Li and Mann (2018), and Cong, Li and Wang (2018)).[3] The literature so far has recognized the potential of blockchain technology to address moral hazard problems inherent in corporate governance via *smart contracts* (e.g., Kaal (2019), Shermin (2017), Yermack (2017)). Against this backdrop, our paper is the first to show that platform tokenization can remedy underprovision of *non-contractible* revenue-stimulating entrepreneurial effort. In particular, we first show that tokenization can alleviate effort

[1] According to Schenker (2019), seven of the 10 most valuable companies globally are now based on the platform business model. Atluri, Dietz and Henke (2017) estimate that more than 30% of global economic activity could be mediated by digital platforms in six years' time.

[2] Between 2016 and 2019 around $35 billion have been raised through the sale of crypto tokens via Initial Coin Offerings (ICOs), Initial Exchange Offerings (IEOs), and Security Token Offerings (STOs), see e.g., Lyandres, Palazzo and Rabetti (2020).

[3] A two-sided market is a natural setting for tokenization because under a proof-of-stake protocol a large number of dispersed token holders minimizes the risk of "51% attack," a situation in which an agent or group of agents gains control of the underlying blockchain network and compromises its integrity (Yu, Lin and Tang (2018)).

underprovision due to the specific nature of token investors' claim to the venture payoff. Second, we show that this benefit can be further amplified by decentralized governance, which eliminates a potential holdup of users when joining the platform is costly. Our analysis reveals platform characteristics that are particularly conducive to adoption of the blockchain-based business model.

In our study, we model a penniless entrepreneur who intends to develop a platform matching buyers and sellers of a homogeneous good or service, and to subsequently collect a fee from each transaction. In addition to investing in physical capacity such as IT infrastructure able to accommodate a given number of users, the entrepreneur needs to exert effort to stimulate demand for the good, for example, marketing efforts that improve buyer experience. It is not just the entrepreneur but also users who need to make platform-specific investments. We assume that it is the sellers who need to make such investments to join the platform.[4]

We first examine a "traditional business model," whereby the entrepreneur raises capital to build the platform by selling equity to a venture capitalist (VC) investor. Because entrepreneurial effort cannot be contracted upon and the entrepreneur internalizes only a fraction of its payoff that corresponds to her equity share, outside equity financing leads to effort underprovision (Jensen and Meckling (1976)). Effort underprovision is the feature of our model that breaks the Modigliani-Miller paradigm and makes outside financing relevant.[5] Once the platform becomes operational, it is controlled by the firm owned by the entrepreneur and the VC. In each subsequent period, the firm sets a transaction fee, upon which potential sellers decide whether to join the platform, and trading takes place. We model the interaction between the firm and the potential and existing sellers as a dynamic game, in which all players maximize the NPV of current and future profits, while anticipating their own as well as each others' future actions.

At the core of this game lies a holdup problem, to which sellers become vulnerable once they join the platform and their cost of joining becomes sunk. Such holdup problem is well recognized in the economics literature (Grossman and Hart 1986) as well as in practice. In our model, the holdup manifests by the platform increasing the transaction fee, although in practice it could take more subtle forms.[6] Because sellers are forward-looking, they require a corresponding compensation

[4]To become an Uber driver, for example, one may need to obtain background check and a private hire vehicle licence, purchase a car meeting certain specifications, and give up an alternative employment opportunity. Amazon or Alibaba sellers may have to invest in setting up the business, purchasing inventory, etc.

[5]It is well known that in the presence of entrepreneurial moral hazard, among contracts whereby investors' payoff is non-decreasing in profit, a standard debt contract is optimal (Innes 1990). Yet, debt is usually not an option for early-stage startups that lack collateral and stable cash flows.

[6]In 2018 Uber Eats incited rider dissatisfaction by reducing the minimal per-delivery rate in London from £4.26

to join the platform in the first place. In particular, the firm's equilibrium strategy is to set a relatively low fee in the first period to attract a critical mass of sellers, and to charge a higher fee thereafter. When the cost of joining is high enough, the first-period fee may be negative, i.e., the firm may need to provide sellers with an initial subsidy.[7] Because providing such subsidy requires additional outside financing, it further dilutes the entrepreneur's stake in the venture, exacerbating effort underprovision.

To sum up, the entrepreneur's payoff in the traditional business model deviates from the first best due to the well-known agency cost of outside equity financing in the form effort underprovision, which we show to be aggravated by a potential holdup problem that increases the amount of financing required. To examine the effect of tokenization on the aforementioned moral hazard frictions and, ultimately, on the cost of outside financing, we consider a "blockchain-based business model," which differs from the traditional platform in financing and subsequent governance mechanisms. Specifically, to obtain financing, the entrepreneur issues utility tokens that will be accepted as the sole means of payment on the platform, and sells a fraction of the tokens to investors via an initial coin offering (ICO). While many different types of tokens exist in practice, we focus on the most common case, wherein token holders are not granted any control rights or claims to dividends (e.g., Bourveau et al. (2018)). The principal distinction between tokens and equity is that equity is a claim to platform profits, whereas the value of tokens is a function of platform sales. Specifically, the dollar value of tokens changing hands in any given period must equal the dollar value of platform sales in this period.[8]

This distinction has an important implication for the agency problem that stems from the entrepreneur disregarding the payoff of her effort to investors. Equity investors receive a portion of the platform profits, i.e., the difference between the platform sales and the cost of the goods sold that the sellers charge. Token investors receive a portion of the cryptocurrency market capitalization,

to £3.5 (TG (2018)). In the same year, the EU started investigating Amazon for using its merchants' transaction data to launch private label products directly competing with those merchants (FT (2018)).

[7]Subsidies to platform users are quite common in practice. Uber offers financing deals and discounts on new cars. Didi, a Chinese equivalent of Uber, spent almost one-third of its commission revenue on driver subsidies in Q4 2018 (TechInAsia (2019)). In 2019, Alibaba launched a roughly $300 million reward scheme providing subsidies to attract developers to its various platforms (KrASIA (2019)).

[8]Other differences between equity and tokens financing, which are outside the scope of our model, involve information asymmetry between entrepreneurs and investors (Chod and Lyandres (2018)), positive network externality of token adoption (Cong, Li and Wang (2018) and Li and Mann (2018)), the ability of an ICO to elicit demand information (Catalini and Gans (2018)), and the value-adding activities such as monitoring and strategic involvement usually performed by VC equity investors (Gompers and Lerner (2001)) and Hellmann and Puri (2002).

which is proportional to the platform sales. Because token investors, unlike equity investors, do not share with the entrepreneur the cost of goods sold, they require a smaller portion of the platform sales to contribute a given amount of money. By allowing the entrepreneur to retain a larger portion of the effort-sensitive platform sales, token financing increases the entrepreneur's incentives to provide effort.

Blockchain technology also provides an alternative governance mechanism. As a peer-to-peer network, a (public) blockchain ecosystem is not controlled by its founders, but instead follows a set of protocols determined at the genesis and updated over time based on user consensus. Although the technical details vary across different blockchains, a common feature of the technology is that it is extremely difficult for any single party, including the founders, to modify the rules governing the blockchain for their own benefit.[9] It is similarly difficult to modify or erase information stored in a blockchain. The immutability of blockchain-based platforms ensures that no party, including the creator, can circumvent the rules embedded in the blockchain code, creating trust between participants in the integrity of their contractual relationship. Finally, a feature of the blockchain technology known as smart contracts allows delegating contract execution to a decentralized computer network–exactly as coded, independent of human discretion, and without the possibility of opportunistic behavior of the agents (Kaal (2019)). Taken together, the above features of the blockchain technology allow the platform founders to credibly commit to a set of operating policies to the extent that would be hard to achieve with a traditional (centralized) governance due to contract incompleteness.

An implication relevant to our context is that the entrepreneur is able to credibly relinquish her power to increase the transaction fee once the platform becomes operational. By eliminating the threat of holdup, a blockchain-based platform is able to attract users without providing them with an initial subsidy even if the cost of joining is high. This lessens the entrepreneur's financing burden and the agency cost associated with it.

To summarize, our model reveals two distinct channels through which tokenization can reduce the cost of outside financing. Whereas issuing tokens alleviates effort underprovision associated with a given amount of outside financing, decentralized governance can reduce the amount of outside

[9] In the case of Bitcoin blockchain, for example, a proposal for protocol change, known as Bitcoin Improvement Proposal, can be implemented only if it receives at least 95% support among the miners of the last 2016 blocks, which take roughly 14 days to mine (Jayasuriya and Sims (2019)). Whereas the "voting rights" of Bitcoin miners are based on their computational efforts (Proof-of-Work), other consensus mechanisms are based on token ownership (Proof-of-Stake used by Ethereum II) or storage capacity (Proof-of-Replication used by Filecoin) among others.

financing required. The downside of the blockchain-based business model is that there is a limit on how much capital the entrepreneur can raise by issuing tokens. Namely, if tokens are highly liquid, i.e., they change hands many times per unit of time, their market capitalization is small relative to the NPV of the platform profits, limiting how much money one can raise by issuing tokens rather than equity. If building the platform is expensive, this can distort the capacity investment. The resulting trade-off between the benefits and costs of tokenization leads to several predictions. According to our model, tokenization is the preferable business model when (i) the success of the platform depends significantly on entrepreneurial effort, (ii) building the platform is cheap, and (iii) joining the platform is costly.

Thus far, we have been tacitly assuming that investor protection and monitoring mechanisms are in place that prevent diversion of funds by an opportunistic entrepreneur. As a robustness check, we also consider a blockchain-based model wherein the entrepreneur is able to divert part (or all) of the ICO proceeds. When this is the case, the entrepreneur's equilibrium strategy is to raise money only up to the amount that she will be willing to invest. This leads, in some cases, to underinvestment. We show that in the presence of diversion opportunities, the benefits of tokenization are greatest when most of the cost of building the platform is borne by users, which mitigates the underinvestment problem of ICO financing while aggravating the holdup problem of the traditional business model.

Our paper builds on the literature on two-sided markets, pioneered by Armstrong (2006), Caillaud and Jullien (2003), Hagiu (2006), and Rochet and Tirole (2006). These papers study platforms' pricing strategies, including the role of subsidies and price commitment to entice user adoption, but their focus is mostly on network effects. In Hagiu (2006), sellers (game developers) decide whether to join and invest in a platform (videogame console) before buyers (gamers) join it. The platform sets access prices for buyers and sellers, leading to a potential holdup of sellers by the platform: once they have invested, the platform can charge a high price to buyers, generating few user transactions. Hagiu (2006) shows that the platform can solve the holdup problem by imposing a transaction fee, which provides an incentive to charge a low price to buyers so as to generate many transactions. The implicit assumption is that the platform can commit to the fee, which is reasonable in the one-period model of Hagiu (2006). In a multi-period setting, which is of our interest, such commitment may not be feasible, at least for a traditional, centrally governed platform.

Recently, the management of two-sided platforms has started receiving attention in the operations management literature (Chen et al. 2019). The relevant papers focus on platforms' information structure (Papanastasiou, Bimpikis and Savva 2018, Allon, Drakopoulos and Manshadi 2019), campaign design (Alaei, Malekian and Mostagir 2016), optimal subsidy policy (Levi, Perakis and Romero 2017), and competition (Lai et al. 2019). We complement this literature by examining the link between a platform's operations and financing, and how it is impacted by tokenization.

The corporate governance literature has recognized that some opportunistic behavior resulting from contractual incompleteness could be alleviated via smart contracts and decentralization of governance and record keeping enabled by blockchain technology (e.g., Kaal (2019) and Yermack (2017)).[10] Holden and Malani (2019) argue specifically that the technology may allow parties to prevent holdup by credibly committing to contracts and/or making information verifiable to courts. Our paper bridges the above two literatures by demonstrating analytically how the governance capabilities of blockchains can be leveraged in the context of a two-sided market, providing one possible explanation for the recent proliferation of blockchain-based platforms.

Our paper also contributes to the emerging theoretical literature on cryptocurrency financing. A large part of this literature focuses on network effects in platform adoption. Cong, Li and Wang (2018) and Sockin and Xiong (2018) study the interaction between adoption and token prices. Bakos and Halaburda (2018) and Li and Mann (2018) demonstrate that ICO financing can be used to induce adoption under network externalities. Similar to our paper, Bakos and Halaburda (2018) show that token financing can reduce or eliminate a user subsidy and, thereby, the founders' financing burden. However, the user subsidy in Bakos and Halaburda (2018) is used to overcome coordination problem arising from network effects, whereas in our setting it is used to compensate users for future holdup. In contrast to the above papers, we show benefits of token financing that are independent of any network externalities.

Most relevant to our work is the cryptocurrency literature that focuses on moral hazard, in particular, entreprenerial effort underprovision. Chod and Lyandres (2018) and Malinova and Park (2018) show that when tokens represent claims to the venture's revenue (rather than profit), the fraction of tokens that the entrepreneur needs to sell to finance a given investment is smaller than the fraction of equity she would need to sell to finance the same investment. As a result, relative to equity financing, token financing allows the entrepreneur retain a larger stake in effort-sensitive cash-flows, which alleviates effort underprovision. Both Chod and Lyandres (2018) and Malinova and Park (2018) consider tokens that are claims to output of a monopoly in a static setting. In contrast, we consider tokens that are used as a currency on a platform whose transaction volume is determined by a dynamic game between the platform founders and users.

[10] An excellent discussion of various economic effects of blockchains and a detailed review of the related literature are provided by Halaburda and Haeringer (2019).

Closer to our setting, Canidio (2018) considers a dynamic model in which an entrepreneur develops a platform financed by issuance of platform-specific currency. Like us, Canidio (2018) shows that the entrepreneur's investment and effort deviate from the first best because she maximizes the value of her tokens, which depends on the transaction volume in a given period, rather than the NPV of the surplus generated by the platform over multiple periods. Unlike us, Canidio (2018) does not compare the agency costs to those of equity financing. Garratt and van Oordt (2019) show that relative to equity financing, token financing can instigate more entrepreneurial effort aimed at reducing production cost, which under token financing, unlike under equity financing, is fully internalized by the entrepreneur. This is different from our setting, in which entrepreneurial effort increases demand, and its benefit is directly internalized by all token holders. Gryglewicz, Mayer and Morellec (2019) study the optimal design of a token that has both utility and security features, that is, it is a platform currency and it also grants rights to dividends. Herein we take a more applied perspective by taking two financing mechanisms as given, and examining how they perform relative to one another.

The main contribution of our work to this literature is to identify a novel mechanism through which token financing alleviates effort underprovision, or, more generally, through which it reduces the amount of outside financing required to start a platform. This mechanism, which has to do with blockchain technology's ability to eliminate user holdup and, consequently, user subsidy, is absent in all of the aforementioned papers, none of which considers the cost of joining a platform, which gives rise to the holdup problem.

Another strand of the ICO literature including Catalini and Gans (2018) and Bakos and Halaburda (2019) focuses on the ability of ICO financing to elicit demand information from token valuation. This is related to Strausz (2017), who studies the trade-off between the ability to elicit demand information and moral hazard in reward-based crowdfunding, which is similar to token financing in that funds are raised in exchange for claims to a future product or service.

An operations management perspective on ICOs is taken by Gan, Tsoukalas and Netessine (2019), who study moral hazard implications of ICO financing for a newsvendor-type firm. Finally, our work belongs to the broader literature that studies implications of the blockchain technology for a firm's operations through various channels such as enhanced transparency or traceability (Chod et al. 2019, Shumkin, Hasija and Netessine 2019, Gaur and Gaiha 2020, Dong, Jiang and Xu 2020).

2 Platform economics

A penniless entrepreneur intends to build a platform matching buyers and sellers of a homogeneous good or service with the following features. Trading takes place in discrete time periods over an infinite horizon. Both buyers and sellers are allowed to join the platform in any period. To join the platform, a potential seller needs to make a platform-specific irreversible investment, which we refer to as the cost of joining and denote by c. Buyers can join the platform at no cost.

Each seller is able to supply one unit of the good in each period. Let $s(q)$ be the reservation price of the qth seller. We then sort the sellers by their reservation price in ascending order. For tractability, we assume the number of sellers are sufficiently large so that it can be credibly approximated by a continuous variable. Therefore, s can be interpreted as the one-period supply curve, which we assume to be continuously differentiable.

Demand for the good depends on the amount of effort e that the entrepreneur exerts at cost γe while building the platform. The cost of effort can be thought of in monetary terms as the opportunity cost of the entrepreneur's time. Let d be the one-period inverse demand curve, i.e., $d(e, q)$ is the price at which buyers demand q units of the good in a given period. We assume d to be differentiable with respect to both e and q, decreasing in q, and concavely increasing in e.

Building a platform that can accommodate Q sellers, and thus allows up to Q units to be traded in a given period, requires an initial monetary investment $C(Q)$. We refer to Q as platform capacity and to $C(Q)$ as the capacity investment cost. Let p_i be the number of sellers who join the platform before or during period i, and let q_i be the number of sellers who trade, or, equivalently, the quantity traded, in period i. By definition, we have $q_i \leq p_i \leq Q$, for all i. Note that once they have joined, sellers have no incentive to leave the platform; they can always remain on the platform, inactive and incurring no cost, while retaining the option to trade in future periods.

The sequence of events is the following. In period 0, which we assume to be of negligible length for the purpose of discounting, the entrepreneur secures financing, builds the platform, and exerts demand-stimulating effort.[11] In each of the subsequent periods, $1, \ldots, \infty$, new sellers are first allowed to join the platform, and then trading takes place at the price that clears the market. Demand and supply curves are deterministic and stationary, and there is no price inflation. All players have the same one-period discount factor δ, and all cash flows take place at the beginning of a period.

[11] The assumption of instantaneous period 0 is made without loss of generality since one can interpret the cost of capacity and effort as valued in period-1 dollars.

We study two business models that differ in the way the platform is financed and operated. In the traditional model, the entrepreneur raises capital by issuing equity and the platform charges a transaction fee in each period. In the blockchain-based model, the entrepreneur raises capital by issuing a cryptocurrency that is then used as the sole means of payment on the platform. She also charges a transaction fee, but relinquishes the authority to adjust it after the first period. For parsimony, we assume that in all other aspects, the two models are the same. Our results regarding the preference for one business model over the other therefore abstract from other potential differences between the models, such as development costs or regulatory framework.

3 Traditional business model

As our benchmark, we consider a model wherein the entrepreneur raises capital to build the platform by selling equity to a venture capitalist (VC). Among the various conventional ways in which entrepreneurs raise funds, we consider venture capital because early-stage startups lacking collateral and stable cash flows typically qualify for neither debt financing nor an initial public offering of equity. The platform is then built and controlled by a firm owned by the entrepreneur and the VC, and maximizing the value of equity. The sequence of events becomes as follows.

In period 0, the entrepreneur chooses platform capacity Q and the amount of outside financing y. She then approaches a VC with a take-it-or-leave-it offer of a contract that gives the VC a share α of the firm equity in exchange for his cash contribution y. The VC accepts if he at least breaks even. If so, platform capacity is built and the entrepreneur exerts private effort to boost demand.[12]

In period $i \geq 1$, the firm announces the transaction fee x_i that it will charge to sellers as a fraction of their revenue. This fee can be negative, in which case it is to be interpreted as a subsidy to the sellers. Once the fee is announced, new sellers may choose to join the platform, bringing the total number of participating sellers to p_i. Finally, q_i sellers choose to put their units on the market, and the market clears at price $d(e, q_i)$. Note that the length of periods $1, \ldots, \infty$ is implicitly defined as the amount of time for which the firm is able to commit to a certain fee.[13]

The firm's revenue in period i is fraction x_i of total sales in this period, and can be written as

$$\pi(x_i, q_i) = x_i d(e, q_i) q_i. \tag{1}$$

[12] While in practice ventures' investment strategies tend to be controlled by VCs (see e.g., Kaplan and Stromberg (2003)), in our model there is no conflict between the entrepreneur and the VC as to how much capacity to build.

[13] We note that it is common to discretize time based on the frequency that firms make operational decisions. For example, Lobel and Xiao (2017) discretize time according to the company's re-ordering frequency.

We assume that in each period, any revenue is immediately distributed to equityholders.[14] The entrepreneur's payoff is the value of her equity in the firm minus the cost of her private effort, and it can be written as

$$\Pi^{\$} = (1-\alpha)\left(\sum_{i=1}^{\infty}\delta^{i-1}\pi\left(x_i, q_i\right) + y - C(Q)\right) - \gamma e. \tag{2}$$

The superscript \$ indicates the traditional business model, in which payments on the platform are made in fiat currency; $1-\alpha$ is the entrepreneur's share of the firm equity; $\sum_{i=1}^{\infty}\delta^{i-1}\pi\left(x_i, q_i\right)$ is the NPV of the firm's revenue stream; $y - C(Q)$ is the cash contributed by the VC that is not invested in capacity, and which can be used to subsidize sellers if needed; and γe is the cost of the entrepreneur's private effort. We solve the model by backward induction.

Periods $1, \ldots, \infty$**.** Suppose that in period 0, the entrepreneur chose the optimal financing $\bar{y}$, capacity $\bar{Q}$, and effort $\bar{e}$. What follows is a repeated Stackelberg game between the firm and the sellers who have joined the platform or are considering joining. In each of the periods $1, \ldots, \infty$, the firm sets a fee (subsidy), upon which potential sellers decide whether to join, whereas existing sellers decide whether to trade. All players maximize the NPV of current and future profits while anticipating their own as well as each others' future actions. We formulate this game as a dynamic program whereby in each period the firm maximizes its NPV subject to the optimal response of both existing and potential, forward-looking sellers.

We first formulate this dynamic program in the absence of financial constraints, i.e., assuming that the firm has enough cash to pay any subsidy if needed, and then show in the proof of Lemma 1 that this is without loss, i.e., that the financially unconstrained optimal policy must be feasible. Absent financial constraints, a sufficient state representation for the firm's problem is the number of sellers who have joined the platform by incurring the cost of joining, c. Suppose that p sellers have joined by the beginning of a given period. We can formulate the firm's problem as one whereby the firm chooses both the fee, x, and the number of trading sellers, q, for that period, subject to appropriate participation constraints. Note that if the firm chooses $q \leq p$, this implies that no new sellers join the platform in this period; if $q > p$, then $q - p$ sellers join. Thus, the number of sellers who will have joined by the beginning of the next period is $p \vee q \equiv \max\{p, q\}$.

Let $V(p)$, for $p \leq \bar{Q}$, be the firm's value-to-go function that maps the number of sellers who have joined by the beginning of a period to the firm's optimal net discounted revenues from that

[14] Without this assumption, the firm could use revenue from one period as a subsidy to sellers in a subsequent period, and the dynamic, infinite-horizon game between the firm and sellers, whose state space would involve not only the number of participating sellers, but also the firm's cash balance, would become intractable.

period onward. Let further $J(z,p)$, for $z \leq p \leq \bar{Q}$, be the zth seller's surplus-to-go, i.e., the net discounted surplus that the zth seller anticipates to extract starting in a period by the beginning of which p sellers have joined. Finally, let $x^\star(p)$ and $q^\star(p)$ be the firm's optimal policies for choosing the fee and the number of traders, respectively.

According to the Bellman principle of optimality, for any $p \leq \bar{Q}$, we have

$$V(p) = \max_{x,q} \Big[xd(\bar{e},q)q + \delta V(p \vee q)\Big] \tag{3}$$

$$\text{subject to} \quad (1-x)d(\bar{e},q) - s(q) \geq 0 \tag{4}$$

$$(1-x)d(\bar{e},q) - s(q) + \delta J(q,q) \geq c, \quad \text{if } q > p \tag{5}$$

$$0 \leq q \leq \bar{Q}, \quad x \leq 1, \tag{6}$$

where $J(z,p)$ satisfies the following recursive equation:

$$J(z,p) = \Big[(1-x^\star(p))d(\bar{e},q^\star(p)) - s(z)\Big]^+ + \delta J(z, p \vee q^\star(p)), \quad 0 \leq z \leq p. \tag{7}$$

The objective in (3) comprises two parts, the revenue generated during the current period by charging fee x and having q traders, and the net discounted revenues from the future periods, which will begin with $p \vee q$ sellers having joined. Constraint (4) ensures that the qth trader extracts non-negative surplus by trading in the current period, and, implicitly, so does the zth trader, $\forall z \leq q$. Constraint (5) ensures that if $q > p$, i.e., if the qth trader joins in the current period, his anticipated net discounted surplus exceeds the cost of joining, and, implicitly, so does the surplus of all other traders who join in the current period. Note that the qth trader's anticipated net discounted surplus comprises the current-period surplus and the discounted surplus-to-go. Finally, the latter can be recursively expressed as in (7), by similarly breaking it down into the current-period surplus and the discounted surplus-to-go.

Before we characterize the optimal policy, it is useful to define

$$\mathscr{F}(e,q) \equiv q\left(d(e,q) - s(q)\right). \tag{8}$$

As quantity traded times the difference between the price at which the marginal buyer is willing to buy and the price at which the marginal sellers is willing to sell, $\mathscr{F}(e,q)$ is the maximum surplus that the firm can extract from the platform in the form of a flat transaction fee for given quantity traded q and effort level e. Let $\mathscr{F}_z$ and $\mathscr{F}_{zz}$ be the first and second partial derivatives of $\mathscr{F}$ with respect to $z \in \{e,q\}$, respectively. We assume that $\mathscr{F}_{qq}(e,q) < 0$, $\forall e \geq 0$, which is satisfied, for example, in the case of linear or iso-elastic supply and demand curves.

Lemma 1. *Suppose that platform capacity, effort, and the amount of outside financing were chosen optimally. The equilibrium fees are*

$$\bar{x}_1 = \frac{d\left(\bar{e},\bar{Q}\right) - s\left(\bar{Q}\right) - c}{d\left(\bar{e},\bar{Q}\right)}, \text{ and} \tag{9}$$

$$\bar{x}_i = \frac{d\left(\bar{e},\bar{Q}\right) - s\left(\bar{Q}\right)}{d\left(\bar{e},\bar{Q}\right)} \text{ for } i \geq 2. \tag{10}$$

The equilibrium number of sellers as well as quantity traded in each period equal platform capacity, i.e., $\bar{p}_i = \bar{q}_i = \bar{Q}$ for all i. The NPV of the firm's revenue stream is

$$\sum_{i=1}^{\infty} \delta^{i-1} \pi\left(\bar{x}_i, \bar{q}_i\right) = \frac{\mathscr{F}\left(\bar{e},\bar{Q}\right)}{1-\delta} - c\bar{Q}. \tag{11}$$

Lemma 1 stipulates that, assuming platform capacity, effort, and the amount of outside financing were chosen optimally, the firm sets the first-period fee so as to immediately bring the number of sellers up to the capacity. In period 2, the firm increases the fee and keeps it constant thereafter, while the quantity traded remains at platform capacity.

The equilibrium dynamics are driven by a holdup problem to which sellers become exposed after incurring the cost of joining the platform. Once this cost becomes sunk, the firm increases the fee so as to extract the entire surplus from the marginal seller in all future periods. Because the seller rationally anticipates this, he will join only if the first-period fee is low enough for him to fully recover the cost of joining in period 1. In other words, the fee in each period is such that the marginal seller breaks even in that period. This fee has to be lower, and possibly negative, in the first period, when sellers incur the cost of joining.

Interestingly, absent any other frictions, the holdup problem would not affect the equilibrium number of sellers, quantity traded, or the NPV of the firm's revenues. To see this, suppose that the firm were able to commit to never increasing the fee. It can be checked that in this case the optimal fee would be $\bar{x} = \frac{d\left(\bar{e},\bar{Q}\right) - s\left(\bar{Q}\right) - (1-\delta)c}{d\left(\bar{e},\bar{Q}\right)}$, the marginal seller would break even by recovering the cost of joining gradually over time, and the NPV of the firm's revenues would be again given by (11). What the holdup problem does affect, however, is the time structure of the firm's cash flows. In particular, it reduces the firm's first-period revenue while increasing its revenue in all subsequent periods. If the cost of joining is high enough, the first-period fee is negative, i.e., the firm has to provide sellers with an initial subsidy as an incentive to join. This in turn increases the amount of outside financing that the entrepreneur needs to raise, which becomes material when outside financing is costly, as we will see next.

As a technical remark, note that the policy prescribed by Lemma 1 is guaranteed to be feasible only if the entrepreneur follows the optimal strategy in period 0. This is because in deriving $\bar{\mathbf{x}}$, we assumed that the firm is not financially constrained in providing a user subsidy. Importantly, this assumption is without loss as long as the optimal strategy, $\bar{y}, \bar{Q}, \bar{e}$, is followed in period 0. The reason is the following. The policy characterized in Lemma 1 is to charge the highest fee, or offer the smallest subsidy, that induces Q sellers to join. If the firm does not have enough cash to finance this policy, it is guaranteed to end up with excess capacity and, hence, its choice of (y, Q, e) cannot be optimal.

Period 0. Before sellers join the platform, the entrepreneur exerts effort to stimulate demand for the good to be traded therein. Because effort is not contractible, the entrepreneur chooses its level after the contract with the VC and platform capacity have been determined. In doing so, she chooses the optimal effort, $\bar{e}$, as an optimal solution to the following optimization problem:[15]

$$\max_{e \geq 0} \left[(1-\alpha) \left(\frac{\mathscr{F}(e, Q)}{1-\delta} - cQ + y - C(Q) \right) - \gamma e \right]. \tag{12}$$

When choosing effort, the entrepreneur maximizes the value of her share, $1-\alpha$, of the firm equity minus her private cost of exerting effort, γe. The firm equity value equals the NPV of the fee revenue given by (11) plus the cash brought in by the VC that was not invested in capacity, $y - C(Q)$.

The entrepreneur's choice of the optimal platform capacity, $\bar{Q}$, and financing contract, $(\bar{y}, \bar{\alpha})$, corresponds to an optimal solution to the following optimization problem:

$$\begin{aligned}
\max_{Q,\, y,\, \alpha} \quad & \left[(1-\alpha) \left(\frac{\mathscr{F}(\bar{e}, Q)}{1-\delta} - cQ + y - C(Q) \right) - \gamma \bar{e} \right] && (13) \\
\text{subject to} \quad & y \geq C(Q) + \max\left\{ -\pi\left(\bar{x}_1(\bar{e}, Q), \bar{q}_1(Q)\right), 0 \right\} && (14) \\
& y \leq \alpha \left(\frac{\mathscr{F}(\bar{e}, Q)}{1-\delta} - cQ + y - C(Q) \right) && (15) \\
& Q \geq 0, \quad y \geq 0, \quad 0 \leq \alpha \leq 1. && (16)
\end{aligned}$$

The entrepreneur's objective is the same as in (12). Constraint (14) ensures that the fee policy prescribed by Lemma 1 is feasible, that is, the firm raises enough capital, y, to finance the capacity investment, $C(Q)$, as well as the potential subsidy to sellers, i.e., the first-period "revenue" if it is negative. Constraint (15) ensures that the VC breaks even, i.e., the VC's capital injection does not exceed the value of his share of the firm. The next lemma characterizes the entrepreneur's equilibrium strategy, assuming the equilibrium is interior, i.e., $\bar{Q} > 0$ and $\bar{e} > 0$.

[15] To ease notation, we write $\bar{e}(\alpha, Q, y)$ to denote the value of $\bar{e}$ when the variables it depends on take specific values. For example, if $(\bar{y}, \bar{\alpha})$ is the equilibrium VC contract, $\bar{e}(\bar{\alpha}, \bar{Q}, \bar{y})$ is the optimal effort in equilibrium. Also, we use $\bar{e}$ to denote $\bar{e}(\alpha, Q, y)$ as well as $\bar{e}(\bar{\alpha}, \bar{Q}, \bar{y})$ whenever there is no risk of confusion, and similar for other variables.

Lemma 2. *Suppose $\bar{Q} > 0$ and $\bar{e} > 0$. The entrepreneur's equilibrium payoff, $\mathbf{\Pi}^{\$}$, is given by*

$$\mathbf{\Pi}^{\$} = \max_{Q,\alpha,e} \left[\frac{\mathscr{F}(e,Q)}{1-\delta} - cQ - C(Q) - \gamma e\right] \tag{17}$$

$$\text{subject to} \quad (1-\alpha)\frac{\mathscr{F}_e(e,Q)}{1-\delta} = \gamma \tag{18}$$

$$\alpha = \frac{C(Q) - \min\{\pi(\bar{x}_1,\bar{q}_1),0\}}{\pi(\bar{x}_1,\bar{q}_1) + \frac{\delta}{1-\delta}\pi(\bar{x}_2,\bar{q}_2) - \min\{\pi(\bar{x}_1,\bar{q}_1),0\}} \tag{19}$$

$$Q > 0, \quad e > 0, \quad 0 < \alpha < 1. \tag{20}$$

The entrepreneur's payoff inside (17) is the NPV of the fee revenues minus the capacity investment cost and the cost of effort. The equilibrium effort given by (18) is below the first best level because the entrepreneur internalizes the entire cost of exerting effort, but only a fraction of the benefit that corresponds to her share of the firm equity, $1-\alpha$. Underprovision of entrepreneurial effort in the case of external financing is well established (e.g., Jensen and Meckling (1976)), and it is the feature of our model that breaks the Modigliani-Miller paradigm and makes financing relevant.

The equilibrium capacity investment too is distorted by the aforementioned agency conflict, and for the following reason. When choosing capacity, and thereby the amount of outside financing, the entrepreneur needs to take into account that the larger the VC's stake in the firm, the lower her own incentives to provide effort ex post.

Furthermore, the aforementioned inefficiencies could be exacerbated by the holdup problem. Depending on its severity, the equilibrium takes one of three possible forms.

(i) If $c < d(\bar{e},\bar{Q}) - s(\bar{Q})$, then $\pi(\bar{x}_1,\bar{q}_1) > 0$, i.e., the firm earns a positive revenue already in the first period, so only capacity investment needs to be financed by outside capital, $\bar{y} = C(\bar{Q})$. In this case the holdup problem is immaterial in the sense that the entrepreneur's equilibrium payoff is the same as if the firm could commit to charging the same fee in each period.

(ii) If $c > d(\bar{e},\bar{Q}) - s(\bar{Q})$, then $\pi(\bar{x}_1,\bar{q}_1) < 0$, i.e., the firm provides sellers with a subsidy in the first period, which increases the amount of outside financing that the entrepreneur needs to raise, $\bar{y} = C(\bar{Q}) - \pi(\bar{x}_1,\bar{q}_1)$. The additional outside financing further dilutes the entrepreneur's equity in the firm, exacerbating effort underprovision.

(iii) If $c = d(\bar{e},\bar{Q}) - s(\bar{Q})$, then $\pi(\bar{x}_1,\bar{Q}) = 0$, i.e., the firm does not charge any fee nor provides any subsidy in the first period. This is a boundary solution, in which the firm builds the largest capacity that it can populate with sellers without providing them with an initial subsidy. The effect of the holdup problem is thus a distorted capacity level, which is chosen specifically so that the entrepreneur does not have to use costly outside financing to pay sellers a subsidy.

To sum up, our benchmark model illustrates two phenomena in the context of building and operating a platform: (i) the well-known problem of effort underprovision under outside equity financing and (ii) a holdup problem that changes the time structure of a firm's cash flows, which can, in some cases, increase the amount of outside financing that the firm needs, further exacerbating (i). Next, we examine how both of these moral hazard frictions are affected by platform tokenization.

4 Blockchain-based business model

The blockchain-based business model differs from its traditional counterpart in two ways. First, the entrepreneur raises capital via an initial coin offering (ICO), i.e., by selling tokens that will be used as the sole means of payment on the platform. Second, the blockchain technology allows the entrepreneur to relinquish the control of the platform. In particular, the smart contract underlying the platform can be used to lock in the transaction fee charged to platform participants. (For completeness, we formally show that such commitment to a fixed fee is indeed the optimal policy in §6.1.) The sequence of events is the following.

In period 0, the entrepreneur issues a certain number of tokens, which we normalize to one.[16] She then raises capital by selling fraction α of the tokens to investors in an ICO. We assume that the terms of the ICO involve three legally enforceable commitments on the part of the entrepreneur: (i) not to increases the total number of tokens in circulation, (ii) not to sell the tokens retained until the platform is operational, and (iii) to invest the entire ICO proceeds in platform capacity. The first commitment, which prevents token inflation, is present in the vast majority of ICOs and can be enforced by the smart contract underlying the token (see, e.g., Catalini and Gans (2018)). The second commitment, which incentivizes the entrepreneur to exert effort, is typically ensured by gradual vesting of tokens retained by the founders post ICO.[17] The last commitment assumes that necessary regulation and monitoring mechanisms are in place that prevent diversion of the ICO proceeds by an opportunistic entrepreneur (see also §6.2).[18]

Immediately following the ICO, the entrepreneur invests the proceeds in building platform

[16] Because of this normalization, we can use the terms "number of tokens" and "fraction of tokens" interchangeably.

[17] In the case of Filecoin, for example, vesting is linear over a six-year period (Chod and Lyandres 2018).

[18] Because diversion in the emerging ICO market is still too prevalent with regulation lagging technology (see e.g., Zetsche et al. (2018)), we revisit this assumption in §6.2.

capacity Q, exerts effort e, and chooses the transaction fee x that she will collect in each period from sellers as a proportion of their revenue. The assumption that the entrepreneur alone determines the transaction fee reflects the fact that in the majority of ICOs, investors are not given any meaningful control rights (e.g., Catalini and Gans (2018) and Kaal (2018)).[19] Once the platform becomes operational, the entrepreneur sells her remaining tokens. In period 1, sellers join the platform by incurring cost c, and trading takes place. Because the fee x is guaranteed to remain the same in each subsequent period, the number of sellers and quantity traded do not change across periods, and are equal to the same quantity, which we denote with q, i.e., $p_i = q_i = q$, for all $i \geq 1$.

Let v be the token velocity defined as the number of times each token is used to purchase goods during one period. Because the value of tokens in circulation must equal $(1/v)^{\text{th}}$ of the value of goods traded during one period, $d(e, q)\, q$, and because we normalized the number of tokens to one, the price of a token must equal

$$\frac{1}{v} d(e, q)\, q. \tag{21}$$

Note that no-arbitrage principle requires the token price to be the same in each period, which implies a constant token velocity.

We take token velocity v to be a model parameter that characterizes agents' behavioural patterns and, similar to the discount factor δ, depends on the length of a period. Recall our definition of one period as the amount of time for which the firm can commit to a given fee in the traditional model. By using the same discount factor across the two models, we implicitly assumed the same duration of one period in the blockchain-based model. This means that in our model, one period is relatively long, months rather than days, and tokens are likely to change hands several times within each period. Hence, we assume that $v \geq 1$, which implies the following token dynamics within a period: (i) Buyers use the total token supply to purchase $(1/v)^{\text{th}}$ of total output. (ii) Sellers and the entrepreneur sell their tokens received as payment and fee, respectively, back to buyers. (iii) Buyers use these tokens to buy another $(1/v)^{\text{th}}$ of total output, etc., until total output changes hands. Note that each token is now used exactly v times in each period to purchase goods. Also note that the entrepreneur as well as sellers end up holding $(1/v)^{\text{th}}$ of their revenue earned in a given period in tokens, which they need to carry over to the next period. This is important because holding tokens, which are only useful to trade on the platform, across periods incurs holding cost due to the time value of money. Finally, we note that our model is not appropriate when a period is very short or tokens are very illiquid, so that $v < 1$.

[19] Adhami, Giudici and Martinazzi (2018) report that in a sample of 253 ICOs between 2014 and 2017, ICO investors could participate in governance decisions in only 25% of cases.

Let P be the ICO price of a token. Because investors can anticipate the entrepreneur's future decisions and, therefore, the future value of tokens, and because the length of period 0 is assumed to be negligible, no-arbitrage principle implies

$$P = \frac{1}{v} d(e,q)\,q. \tag{22}$$

The entrepreneur's total payoff can thus be written as

$$\Pi^{\mathrm{B}} = \alpha P + (1-\alpha)\frac{1}{v} d(e,q)\,q + \sum_{i=1}^{\infty} \delta^{i-1}\left(1 - \frac{1}{v} + \frac{1}{v}\delta\right)\pi(x,q) - C(Q) - \gamma e, \tag{23}$$

where the first term represents the proceeds from selling α tokens during the ICO; the second term represents the proceeds from selling the remaining $1-\alpha$ tokens once the platform becomes operational; the third term is the NPV of the perpetual fee revenue $\pi(x,q) = xd(e,q)\,q$, which is paid in tokens, $(1/v)^{\text{th}}$ of which the entrepreneur carries over to the next period, as remarked above; and the last two terms are the capacity investment cost and the cost of exerting effort, respectively. The entrepreneur maximizes this payoff in two stages. First, she chooses the fraction of tokens to sell in the ICO, which automatically determines the amount of money raised and, thereby, the platform capacity. After the ICO takes place, she chooses effort and the transaction fee. We solve for the equilibrium strategy by backward induction.

Post-ICO decision problem. At this stage, the ICO price of a token, P, as well as the share α of tokens sold in the ICO are already given, and the ICO proceeds uniquely determine the capacity investment

$$C(Q) = \alpha P. \tag{24}$$

The entrepreneur chooses her optimal effort, $\tilde{e}$, and transaction fee $\tilde{x}$, which then jointly determine the equilibrium number of sellers, or, equivalently, quantity traded, $\tilde{q}$, by solving

$$\max_{e,\,x,\,q} \quad \Pi^{\mathrm{B}} \tag{25}$$

$$\text{subject to} \quad c = \frac{1}{1-\delta}\left(\left(1 - \frac{1}{v} + \frac{1}{v}\delta\right)(1-x)\,d(e,q) - s(q)\right) \tag{26}$$

$$e \geq 0, \quad x \leq 1, \quad 0 \leq q \leq Q. \tag{27}$$

While we formally treat the number of sellers q as a decision variable, constraint (26) ensures that the marginal seller breaks even in the long run. This is the case if the cost of joining the platform c equals the NPV of the seller's perpetual profit. The seller's profit in a given period is the difference between the price of the good net of the transaction fee, $(1-x)\,d(e,q)$, which is paid in tokens, $(1/v)^{\text{th}}$ of which the seller holds until the next period, and the seller's reservation price $s(q)$.

Pre-ICO decision problem. At this stage, the entrepreneur chooses a fraction α of tokens to sell to investors through the ICO. In general, there could be multiple ICO token prices, P, at which investors break even.[20] To eliminate Pareto-dominated equilibria, we assume that the entrepreneur sets the token price so as to maximize payoff, subject to investors breaking even, i.e., $P = \frac{1}{v}d(q,e)q$. The platform capacity Q is then uniquely determined by the stipulation that all ICO proceeds are invested, i.e., $C(Q) = \alpha P$. Finally, the entrepreneur as well as investors rationally anticipate the entrepreneur's optimal post-ICO choice of $\tilde{e}$ and $\tilde{x}$, and the resulting equilibrium $\tilde{q}$. Formally, the entrepreneur's pre-ICO decision problem can be written as follows:

$$\max_{Q,\,\alpha,\,P} \quad \Pi^{\mathrm{B}}\left(\tilde{x}, \tilde{q}, \tilde{e}, Q, \alpha, P\right) \tag{28}$$

$$\text{subject to} \quad P = \frac{1}{v} d\left(\tilde{e}, \tilde{q}\right) \tilde{q} \tag{29}$$

$$C(Q) = \alpha P \tag{30}$$

$$Q \geq 0, \quad 0 \leq \alpha \leq 1, \quad P \geq 0. \tag{31}$$

Let $\tilde{Q}$, $\tilde{\alpha}$ be optimal for the problem above. The next lemma characterizes the entrepreneur's equilibrium strategy, assuming the equilibrium is interior, i.e., $\tilde{Q} > 0$ and $\tilde{e} > 0$.

Lemma 3. *Suppose $\tilde{Q} > 0$ and $\tilde{e} > 0$. The equilibrium transaction fee satisfies*

$$\tilde{x} = \frac{\left(1 - \frac{1}{v} + \frac{1}{v}\delta\right) d(\tilde{e}, \tilde{Q}) - s(\tilde{Q}) - (1-\delta)\, c}{\left(1 - \frac{1}{v} + \frac{1}{v}\delta\right) d(\tilde{e}, \tilde{Q})}. \tag{32}$$

The equilibrium number of sellers as well as quantity traded in each period equal platform capacity, i.e., $\tilde{p} = \tilde{q} = \tilde{Q}$. The entrepreneur's equilibrium payoff, $\mathbf{\Pi}^{\mathrm{B}}$, is given by

$$\mathbf{\Pi}^{\mathrm{B}} = \max_{Q,\,\alpha,\,e} \quad \left[\frac{\mathscr{F}\left(e, Q\right)}{1-\delta} - cQ - C(Q) - \gamma e\right], \tag{33}$$

$$\textit{subject to} \quad \frac{\mathscr{F}_e\left(e, Q\right)}{1-\delta} - \alpha \frac{1}{v} d_e(e,Q)Q = \gamma \tag{34}$$

$$\alpha d\left(e, Q\right) Q = vC(Q) \tag{35}$$

$$Q \geq 0, \quad e \geq 0, \quad 0 \leq \alpha \leq 1. \tag{36}$$

Note that the entrepreneur's equilibrium payoff $\mathbf{\Pi}^{\mathrm{B}}$ is the same function of capacity and effort as $\mathbf{\Pi}^{\$}$ in the traditional business model; cf. eq. (17). Namely, it is the NPV of the maximal surplus that can be extracted from the platform in perpetuity minus the costs of building and joining the

[20] Intuitively, a low token valuation inducing low capacity investment and low entrepreneurial effort could be equally correct as a high token valuation inducing high capacity investment and high effort.

platform and that of effort. The difference between the two business models thus boils down to the different capacity and effort choices that the models lead to.

Condition (34) reflects the fact that when choosing effort, the entrepreneur internalizes only a part of its full marginal benefit, $\frac{\mathscr{F}_e(e,Q)}{1-\delta}$. Namely, she ignores the effect that effort has on the value of tokens sold in the ICO, $\frac{\partial}{\partial e}(\alpha P) = \alpha \frac{1}{v} d_e(e,Q)Q$. Thus, similar to the traditional model, the blockchain-based model leads to effort underprovision. Its magnitude, however, is generally different, reflecting the different exposure of token investors and equity investors to entrepreneurial effort, a point we examine in detail in the next section.

Also similar to the traditional model, the effort underprovision problem distorts the equilibrium capacity. Namely, when choosing the portion of tokens to sell through the ICO, and thereby platform capacity, the entrepreneur needs to take into account that selling more tokens to finance larger capacity will dilute her stake in the venture, which will in turn weaken her incentives to provide effort post ICO. Finally, condition (35), which can be equivalently stated as $\alpha P = C(Q)$, ensures that the entrepreneur invests the entire ICO proceeds in platform capacity. The absence of a central authority capable of holding up platform participants means that the entrepreneur does not need to finance an initial subsidy that may be required in the traditional business model.[21]

5 Effect of tokenization

We now examine the entrepreneur's preference for one business model over the other. For the sake of tractability, we assume from now on that demand is iso-elastic with respect to both quantity and effort, i.e., $d(e,q) = e^a q^{-b}$, where $0 < a < b < 1$ to ensure joint concavity of the "market size," $d(e,q)q$. We also assume iso-elastic supply, $s(q) = mq^n$, where $m, n \geq 0$. Finally, we assume that $C(Q) = KQ^k$. We allow the capacity investment cost to be linear ($k = 1$), convex ($k > 1$), or concave ($k < 1$) in capacity, but we require that $k > (1-b)/(1-a)$ to ensure that "revenue is more concave than cost" and the problem is well behaved. Assuming the venture is viable, i.e., the optimal capacity and effort are not zero, we can to rewrite the entrepreneur's equilibrium payoff in each model as follows:

[21] As a technical remark, note that $\tilde{x}$ could be negative, i.e., it could be optimal for the entrepreneur to subsidize sellers in *each period*. Setting aside its implausibility in practice, such a perpetual subsidy could be always paid using the tokens retained by the entrepreneur post ICO. Thus, unlike the one-time subsidy in the traditional model, a perpetual subsidy in the blockchain-based model would not affect the amount of outside financing sought by the entrepreneur.

Corollary 1. *Suppose $\bar{Q} > 0$, $\bar{e} > 0$, $\tilde{Q} > 0$, and $\tilde{e} > 0$. The entrepreneur's equilibrium payoff in model $i \in \{\$, \text{Ƀ}\}$ can be written as*

$$\mathbf{\Pi}^i = \max_{Q>0,e>0} \quad \left[\frac{\mathscr{F}(e,Q)}{1-\delta} - cQ - C(Q) - \gamma e \right] \tag{37}$$

$$\textit{subject to} \quad \beta^i \frac{\mathscr{F}_e(e,Q)}{1-\delta} = \gamma, \tag{38}$$

$$\textit{and} \quad d(e,Q)Q \geq vC(Q) \;\textit{ if } i = \text{Ƀ}, \tag{39}$$

where

$$\beta^{\text{Ƀ}} = 1 - \frac{(1-\delta)C(Q)}{d(e,Q)Q} \quad \textit{and} \quad \beta^{\$} = \begin{cases} 1 - \frac{(1-\delta)C(Q)}{\mathscr{F}(e,Q)-(1-\delta)cQ} & \textit{if } c \leq d(e,Q) - s(Q) \\ 1 - \frac{(1-\delta)(C(Q)+cQ-\mathscr{F}(e,Q))}{\delta \mathscr{F}(e,Q)} & o/w. \end{cases} \tag{40}$$

According to the corollary, there are two differences between the payoffs attainable under the two models. The first has to do with the entrepreneur's ex-post choice of effort given by eq. (38). The second is constraint (39) that applies to the blockchain model only.

In what follows, we first examine the effect of each of these two differences separately by considering two special cases. Namely, we compare the two business models in the case entrepreneurial effort is irrelevant and in the case of sufficiently small capacity cost. We then relax both these assumptions, but focus on the case of linear capacity cost. Finally, we examine the fully general case using numerical analysis.

5.1 No sensitivity to effort

In the absence of effort considerations, i.e., when $d_e = \gamma = 0$, the only difference in payoff between the two business models is due to constraint (39). It ensures that under the blockchain model the entrepreneur does not sell more than 100% of tokens, $\alpha \leq 1$, or, equivalently, that her desired capacity investment can be financed using the ICO proceeds, $C(Q) \leq P = \frac{1}{v} d(e,Q)Q$.

The analogous constraint, $\alpha \leq 1$, does not appear in the the traditional model because it cannot be binding: selling 100% of equity to finance the capacity investment would leave the entrepreneur with zero payoff, making the venture not worth undertaking. In contrast, selling all tokens is conceivable because the entrepreneur retains the claim to the fee revenue.

Intuitively, token financing is more restrictive than equity financing because the former allows raising money only up to a fraction of one-period platform sales, whereas the latter allows raising money up to the NPV of all future profits. This gives an advantage to traditional equity financing.

Proposition 1. *In the absence of effort considerations, i.e., when $d_e = \gamma = 0$, the traditional business model weakly dominates the blockchain-based business model, i.e.,*

$$\mathbf{\Pi}^{\$} \geq \mathbf{\Pi}^{\mathrm{B}}, \tag{41}$$

and the dominance is strict if, and only if, $\frac{v}{k} > \frac{1-\delta}{1-b}$ and K is sufficiently large.

According to Proposition 1, the traditional model is superior only if token velocity v and the discount factor δ are relatively high. Because high v means that tokens change hands many times during one period, whereas high δ means that one period is short, high v as well as high δ indicate that tokens are highly liquid, i.e., they change hands many times in a unit of time. In this case, token value is small relative to the NPV of the platform profits, limiting how much money one can raise using tokens relative to equity. Moreover, this matters only if capacity is expensive, in which case the above limitation of token financing in effect constrains the desired capacity investment.

5.2 Small cost of capacity

In this subsection, we bring back effort, but we focus on the situation, in which constraint (39) is non-binding, which has to be the case when the capacity cost parameter K is sufficiently small. In this case, the only difference between the two models is the entrepreneur's choice of effort, which depends on the fraction β_i of the marginal payoff of effort, $\frac{\mathscr{F}_e(e,Q)}{1-\delta}$, that the entrepreneur internalizes. In both models, the entrepreneur ignores the payoff of her private effort that accrues to outside financiers—whether these are VC or ICO investors—resulting in *effort underprovision.* What determines its severity is the size and form of investors' claim to the venture payoff. This is what ultimately determines the preference for one business model over the other, which we characterize next.

Proposition 2. *For sufficiently small capacity cost, i.e., when $K \in [0, \varepsilon]$, the blockchain-based business model strictly dominates the traditional one, i.e.,*

$$\mathbf{\Pi}^{\mathrm{B}} > \mathbf{\Pi}^{\$}, \tag{42}$$

unless the venture is unfeasible, i.e., $\mathbf{\Pi}^{\mathrm{B}} = \mathbf{\Pi}^{\$} = 0$.

There are two distinct reasons for the superiority of the blockchain-based model, and they have to do with (i) token vs. equity financing and (ii) centralized vs. peer-to-peer governance.

(i) Financing advantage. When a venture payoff depends on entrepreneurial effort, there is an advantage of raising a given amount of outside capital by issuing tokens rather than equity.

Suppose, for simplicity, that platform participants can join at no cost and, therefore, require no subsidy in either model. Suppose further that the entrepreneur has raised $C(Q)$ to build capacity Q, either by issuing equity or by issuing cryptocurrency, and has to decide how much effort to exert. An agency problem now arises as the entrepreneur's choice of effort disregards the payoff to outside investors. In the traditional model, the investors' payoff is proportional to the value of equity, i.e., the NPV of the *profits* from operating the platform, $\frac{\mathscr{F}(e,Q)}{1-\delta}$. In the blockchain-based model, the investors' payoff is proportional to the market capitalization of the cryptocurrency, and thus, to the *transaction volume* of the platform in each period, $d(e,Q)Q$. Because the transaction volume is relatively *less sensitive to effort* than profits, disregarding the investors' share of the former does less harm. To be precise, the elasticity of $d(e,Q)Q$ with respect to e is smaller than that of $\mathscr{F}(e,Q)$, formally,

$$\frac{\partial d(e,Q)Q}{\partial e}\frac{e}{d(e,Q)Q} < \frac{\partial \mathscr{F}(e,Q)}{\partial e}\frac{e}{\mathscr{F}(e,Q)}. \tag{43}$$

To develop more intuition for this result, it is useful to abstract from the dynamic nature of the problem and focus on the fundamental distinction between equity and tokens. Equity investors receive a portion of the platform profit, which equals the platform revenue (what buyers pay) minus the platform cost (what sellers receive). Token investors receive a portion of the cryptocurrency market capitalization, i.e., a portion of the platform revenue. Because token investors, unlike equity investors, do not share with the entrepreneur the platform cost, they require a smaller portion of the platform revenue to contribute a given amount of money. By allowing the entrepreneur to retain a larger portion of the *effort-sensitive* revenue, token financing increases the entrepreneur's incentives to provide effort. This provides the blockchain-based business model with a financing advantage, which exists regardless of the potential user holdup, a point we discuss next.

(ii) Governance advantage. An additional advantage of the blockchain-based model emerges when joining a platform incurs a cost that is sufficiently large to warrant a first-period subsidy in the traditional model. By eliminating the threat of holdup and, thus, the need to offer such subsidy, the blockchain-based model reduces the amount of money that the entrepreneur needs to raise, further decreasing the agency cost associated with external financing.

To sum up, the blockchain-based model affords two distinct advantages: (i) issuing tokens rather than equity reduces the agency cost associated with raising a given amount of outside financing, and (ii) eliminating a central authority capable of holding up platform participants in some cases reduces the amount of outside financing required to launch the platform in the first place.

5.3 Linear cost of capacity

In this subsection, we consider entrepreneurial effort and, at the same time, we allow capacity to be expensive. In this case, a trade-off emerges between the potential cost and benefits associated with the blockchain model that we discussed in the previous two subsections. In this general case, it is difficult to compare the two models analytically. However, we can say something about the effect of the cost structure on the value of tokenization assuming linear capacity cost.

Suppose $C(Q) = KQ$ and consider the "full unit cost" of building and populating the platform, $K+c$, which comprises the cost of building one unit of capacity and the cost of one seller joining. Let $\tau \equiv \frac{c}{K+c}$ be the fraction of this cost that is incurred by a platform user rather than by the platform founders. In the next proposition, we characterize the effect of this parameter on the benefit of platform tokenization when the cost of joining the platform is relatively high, $c > d\left(\bar{e}, \bar{Q}\right) - s\left(\bar{Q}\right)$, requiring a traditional platform to offer sellers an initial subsidy.

Proposition 3. *Suppose that $C(Q) = KQ$ and $c > d\left(\bar{e}, \bar{Q}\right) - s\left(\bar{Q}\right)$. Keeping the total cost $c + K$ fixed, the benefit of platform tokenization increases in the proportion of this cost that is incurred by platform users, i.e.,*

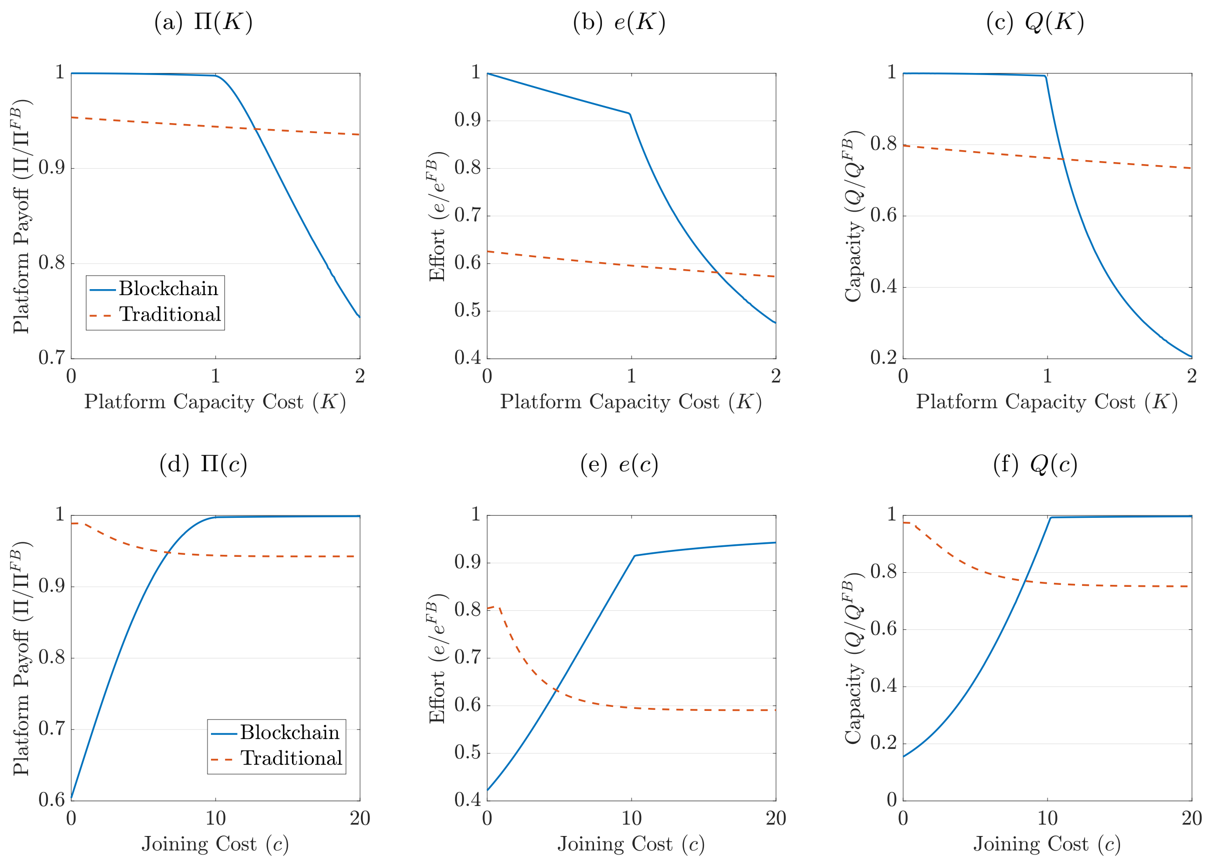
$$\frac{d}{d\tau}\left(\mathbf{\Pi}^{\text{B}} - \mathbf{\Pi}^{\$}\right) > 0. \tag{44}$$

Intuitively, the larger the portion of the investment cost that is borne by users, the more severe the holdup problem in the traditional model, and the greater the governance benefit of tokenization. A more nuanced explanation is the following. As more of the investment cost is borne by users, the entrepreneur in the blockchain-based model needs less outside financing, which alleviates the effect of the token financing constraint as well as effort underprovision. In contrast, the entrepreneur's payoff in the traditional model is unaffected by the change in the cost structure. Because the firm internalizes the users' cost of joining from the outset via the first-period subsidy, the entrepreneur needs to finance the entire cost, $K + c$, and is thus indifferent to its composition.

5.4 General case

According to Proposition 1, absent effort considerations, the traditional business model is superior only if capacity cost K is high. According to Propositions 2 and 3, in the presence of effort considerations, the blockchain model is preferable when capacity cost is low, and more so, when the cost of joining the platform is high. One would, therefore, expect that, in general, the blockchain model dominates when building the platform is cheap, joining it is costly, and demand is sensitive to entrepreneurial effort.

Figure 1: Impact of costs on payoffs and equilibrium decisions (normalized by the financially unconstrained benchmarks) when demand is sensitive to effort.



Notes. The y-axis reports the quantity normalized by the "first-best" benchmark (when the entrepreneur is financially unconstrained). Parameters: $m = 1$, $n = 1$, $a = 0.4$, $b = 0.75$, $K = 1$ (between $[0, 2]$ in Panel(a)–(c)), $c = 10$ (between $[0, 20]$ in Panel(d)-(f)), $\gamma = 100$, $\delta = 0.95$, and $v = 1$.

Figure 1 presents numerical results that confirm the above intuition. Panels (a)–(c) show the impact of platform capacity cost K on payoffs Π^{B} and $\Pi^{\$}$, and on the corresponding equilibrium capacity and effort, all normalized by their financially unconstrained counterparts. When capacity cost is low, the blockchain model, which is better at mitigating moral hazard, dominates the traditional one. However, as capacity cost increases, the token financing constraint (39) becomes binding, and limits the amount of capacity the entrepreneur can build. This tension drives down Π^{B}, and eventually leads to dominance of the traditional model. Panels (d)–(f) illustrate the effect of the joining cost c. The traditional model is superior for small c, whereas the blockchain model dominates for large c, and the reason is twofold. First, when c is small, the token financing constraint (39) is binding, thus limiting the performance of the blockchain model. For large c, the constraint becomes non-binding. Second, the holdup problem inflicting the traditional model is more pronounced for larger c. For these two reasons, the relative performance of the blockchain model increases with c.

Combining analytical and numerical results, our findings afford the following managerial implications and empirical predictions. Platform tokenization is preferable and is to be expected when (i) a large part of the cost of building the platform is borne by users and, at the same time, (ii) building the platform requires significant entrepreneurial effort or, more generally, there are significant agency or other deadweight costs associated with external financing. Finally, the blockchain model tends to dominate in the parameter space where the token financing constraint is not binding, that is, in equilibrium, the entrepreneur only sells a fraction of tokens in ICO, $\alpha < 1$. This is consistent with the practice wherein entrepreneurs often retain a considerable portion of tokens post-ICO.

6 Extensions

6.1 Optimality of commitment in the blockchain-based model

The business model described in §4 assumes that the entrepreneur uses the blockchain technology to commit to a fixed fee in all periods. In what follows, we prove that such commitment is indeed the optimal strategy by considering an alternative model that differs from our base-case blockchain-based model only in that the entrepreneur sets the fee x_i at the begining of each period.

Periods 1,...,∞. The dynamic game between the platform, which sets the fee, and users, who decide whether to join and trade, is similar to the traditional model except that payments are settled in tokens.

Note that the equilibrium price of a token has to be the same in each period; otherwise, there would be arbitrage opportunities across periods. We continue using P to denote the price of a token during the ICO, and use $P(e, Q)$ to denote the price of a token that emerges in equilibrium once the entrepreneur builds capacity Q and exerts effort e. Because x_i is allowed to vary across periods, the platform sales, $d\,(e, q_i)\, q_i$, are not guaranteed to be the same in every period, and neither is the token velocity, defined as the ratio of the platform sales and the value of tokens in circulation,

$$v_i = \frac{d\left(e, q_i\right) q_i}{P\left(e, Q\right)}, \text{ for all } i \geq 1. \tag{45}$$

Intuitively, if there is less trading in a given period, the token velocity decreases, so that eq. (45) holds true.

For now, we assume that the entrepreneur is not financially constrained in case it needs to provide a user subsidy in any given period. Similar to the traditional model, a sufficient state representation for the entrepreneur's problem is p, the number of sellers who have joined the platform by the beginning of a given period. We formulate the entrepreneur's problem as one whereby she chooses the fee, x, the number of trading sellers, q, and token velocity, v, for that period, subject to appropriate equilibrium constraints. Also similar to the traditional model, the number of sellers who will have joined by the beginning of the next period is $p \vee q$. Let $V(p)$ and $J(z, p)$, for $z \leq p \leq \tilde{Q}$, be value-to-go functions of the entrepreneur and the zth seller, respectively. Finally, let $x^\star(p)$, $q^\star(p)$, and $v^\star(p)$ be the entrepreneur's optimal policies for choosing the fee, the number of traders, and velocity, respectively.

According to the Bellman principle of optimality, for any $p \leq \tilde{Q}$, we have

$$V(p) \quad = \quad \max_{x,\, q,\, v} \quad \left[\left(1 - \frac{1}{v} + \frac{\delta}{v}\right) x d(\tilde{e}, q) q + \delta V(p \vee q)\right] \tag{46}$$

$$\text{subject to} \quad \left(1 - \frac{1}{v} + \frac{\delta}{v}\right)(1 - x) d(\tilde{e}, q) - s(q) \geq 0 \tag{47}$$

$$\left(1 - \frac{1}{v} + \frac{\delta}{v}\right)(1 - x) d(\tilde{e}, q) - s(q) + \delta J(q, q) \geq c, \quad \text{if } q > p \tag{48}$$

$$\frac{1}{v} d\left(\tilde{e}, q\right) q = P\left(\tilde{e}, \tilde{Q}\right) \tag{49}$$

$$0 \leq q \leq \tilde{Q}, \quad x \leq 1, \quad v \geq 1 \tag{50}$$

where $J(z, p)$ satisfies the following recursive equation:

$$J(z, p) = \left[\left(1 - \frac{1}{v^\star(p)} + \frac{\delta}{v^\star(p)}\right)(1 - x^\star(p)) d(\tilde{e}, q^\star(p)) - s(z)\right]^+ + \delta J(z, p \vee q^\star(p)), \quad 0 \leq z \leq p. \tag{51}$$

The dynamic program in (46)-(51) is analogous to that in (3)-(7), described in detail in §3, except that (i) the entrepreneur as well as sellers are paid in tokens, $(1/v)^{\text{th}}$ of which are always carried to the next period and need to be discounted accordingly, and (ii) the additional constraint (49) ensures that the value of a token remains unchanged across periods.

Next, we consider the following transformation of variables. Let

$$w := x + \frac{1}{v}\left(1 - \delta\right)\left(1 - x\right), \text{ so that } x = \frac{w - \frac{1}{v} + \frac{\delta}{v}}{1 - \frac{1}{v} + \frac{\delta}{v}}. \tag{52}$$

Substituting (52) into (46)-(51), and using eq. (49) to eliminate v, we obtain a dynamic program that differs from (3)-(7) only in that x is replaced by w, the per-period revenue in the objective, $xd(\bar{e}, q)q$, is replaced by

$$wd(\tilde{e}, q)\, q - (1-\delta)\, P\left(\tilde{e}, \tilde{Q}\right), \tag{53}$$

and there is an additional constraint

$$\frac{d(\tilde{e}, q)\, q}{P\left(\tilde{e}, \tilde{Q}\right)} \geq 1. \tag{54}$$

Recall that the token price $P\left(\tilde{e}, \tilde{Q}\right)$ has already formed in period 0, so the term $(1-\delta)\, P\left(\tilde{e}, \tilde{Q}\right)$ in eq. (53) is a constant that does not affect the optimal policy. Relaxing constraint (54) for now (we will show later that the optimal policy satisfies it), we can invoke Lemma 1 to obtain $\tilde{q}_i = \tilde{Q}$,

$$\tilde{w}_1 = \frac{d\left(\tilde{e}, \tilde{Q}\right) - s\left(\tilde{Q}\right) - c}{d\left(\tilde{e}, \tilde{Q}\right)}, \text{ and} \tag{55}$$

$$\tilde{w}_i = \frac{d\left(\tilde{e}, \tilde{Q}\right) - s\left(\tilde{Q}\right)}{d\left(\tilde{e}, \tilde{Q}\right)} \text{ for } i \geq 2. \tag{56}$$

From (49), we then obtain $\tilde{v}_i = \frac{d(\tilde{e}, \tilde{Q})\tilde{Q}}{P\left(\tilde{e}, \tilde{Q}\right)}$ for $i \geq 1$. Given this optimal policy, the NPV of the perpetual fee (53) is

$$\sum_{i=1}^{\infty} \delta^{i-1} \left(\tilde{w}_i d(\tilde{e}, \tilde{q}_i)\, \tilde{q}_i - (1-\delta)\, P\left(\tilde{e}, \tilde{Q}\right)\right) = \frac{\mathscr{F}\left(\tilde{e}, \tilde{Q}\right)}{1-\delta} - P\left(\tilde{e}, \tilde{Q}\right) - c\tilde{Q}. \tag{57}$$

Period 0. The token price, $P(e, Q)$, which emerges as soon as the entrepreneur builds capacity, exerts effort, and offers her remaining $1-\alpha$ tokens for sale, but before she announces the first-period fee, is given, same as in our base-case model, by

$$P(e, Q) = \frac{1}{v_0} d(e, Q)\, Q, \tag{58}$$

where $d(e, Q)\, Q$ are the anticipated platform sales and $v_0 \geq 1$ is a model parameter. Note that condition (45), which determines token velocity v_i in each period $i \geq 1$ as the ratio of the platform sales, $d(e, Q)\, Q$, and token price, $P(e, Q)$, implies that in equilibrium $v_0 = v_1 = v_2 = \ldots$. Thus, parameter v_0 can be interpreted as the anticipated token velocity, which, in equilibrium, equals the actual token velocity in each period. Finally, note that since $v_0 \geq 1$, constraint (54) is automatically satisfied in equilibrium.

In addition to the NPV of the fee revenue (57), the entrepreneur's total payoff includes the ICO proceeds, αP, the value of tokens retained post ICO, $(1-\alpha)\, P(e, Q)$, the capacity cost, $C(Q)$, and the cost of effort, γe, i.e.,

$$\Pi^{\mathrm{B}} = \frac{\mathscr{F}(e,Q)}{1-\delta} - P(e,Q) - cQ + \alpha P + (1-\alpha) P(e,Q) - C(Q) - \gamma e \tag{59}$$

$$= \frac{\mathscr{F}(e,Q)}{1-\delta} - \alpha \frac{1}{v_0} d(e,Q) Q - cQ + \alpha P - C(Q) - \gamma e. \tag{60}$$

The necessary and sufficient condition for the optimal effort, if the solution is interior, is, therefore,

$$\left(1 - \alpha(1-\delta)\frac{1}{v_0}\right) \frac{\mathscr{F}_e(e,Q)}{1-\delta} = \gamma. \tag{61}$$

Furthermore, since $P = P(e,Q)$ in equilibrium, the payoff (60) can be rewritten as

$$\Pi^{\mathrm{B}} = \frac{\mathscr{F}(e,Q)}{1-\delta} - cQ - C(Q) - \gamma e. \tag{62}$$

Comparing (61) and (62) with eq. (34) and (33) in Lemma 3, we observe that the entrepreneur's payoff as a function of Q and e, as well as the optimality condition for the latter are the same as in our base-case blockchain model.

The difference is that without the platform committing to a fixed fee, users are subject to holdup. Namely, the first-period fee, x_1, is lower than the fee in the subsequent periods, $x_2 = x_3 = \ldots$. Furthermore, the first-period fee could be negative, in which case it is a subsidy, which the entrepreneur needs to finance using the tokens retained post ICO. At optimality, the first-period fee (subsidy) given by x_1 must be feasible, otherwise not enough users would join the platform, and the entrepreneur would be better off building less capacity in the first place. However, to ensure feasibility, the entrepreneur's choice of ICO terms is subject to the following additional constraint, which ensures that the value of tokens retained post ICO plus the first-period fee is non-negative:

$$(1-\alpha) P(e,Q) + \left(1 - \frac{1}{v_1} + \delta \frac{1}{v_1}\right) x_1 d(e,q_1) q_1 \geq 0 \Longleftrightarrow \tag{63}$$

$$(\delta + v_0 - \alpha) \frac{1}{v_0} d(e,Q) - s(Q) - c \geq 0. \tag{64}$$

Because this additional constraint is the only difference between the blockchain models with and without commitment, the former is necessarily (weakly) superior to the latter.

6.2 Cash diversion

Thus far we have been assuming that the entrepreneur is able to contractually commit to investing the entire contribution of outside financiers, and subsequently carries out the commitment. While diversion by an opportunistic entrepreneur can take place under ICO as well as conventional equity-based financing, in practice as of now, it is disproportionately more common under the former

(Zetsche et al. (2018)). The sheer novelty of ICOs means that regulation and enforcement protecting investors are necessarily lagging behind (Robinson (2017)). Resulting from the low transaction cost combined with enthusiasm surrounding the new technology, token investors tend to be more dispersed and often less sophisticated than equity investors, which limits their monitoring capacity. Most important, unlike equity holders, investors in the majority of ICOs are given minimal control rights (e.g., Catalini and Gans (2018) and Kaal (2018)). In fact, many recent white papers include explicit statements such as "the founders of Telegram will be responsible for the efficient use of funds resulting from any sale of tokens" (Chod and Lyandres (2018)).

There is an apparent and somewhat ironic dichotomy between decentralized governance eliminating the possibility of opportunistic behavior and ICO financing not just allowing, but facilitating precisely such behavior. The catch-22 is that building a secure, decentralized, and fully distributed platform requires raising funds first. To capture this distinguishing, though likely transient, feature of ICOs, we next consider a blockchain-based model wherein the entrepreneur is able to divert part of the ICO proceeds. Formally, the effect of diversion opportunity boils down to the timing of the capacity investment decision, which is now made post ICO.[22] With the rest of the model remaining intact, the entrepreneur's decision problems can be written as follows:

Post-ICO decision problem. The problem is the same as (25)-(27) except that it also involves the choice of capacity, which is constrained by the ICO proceeds:

$$\max_{e,x,q,Q} \quad \Pi^{\mathrm{B}} \tag{65}$$

$$\text{subject to} \quad c = \frac{1}{1-\delta}\left(\left(1-\frac{1}{v}+\frac{1}{v}\delta\right)(1-x)\,d\,(e,q) - s\,(q)\right) \tag{66}$$

$$C(Q) \leq \alpha P \tag{67}$$

$$e \geq 0, \quad x \leq 1, \quad 0 \leq q \leq Q. \tag{68}$$

Pre-ICO decision problem. The problem is the same as (28)-(31) except that the entrepreneur does not choose capacity, but instead anticipates her optimal post-ICO capacity choice, and so do investors when pricing the tokens:

$$\max_{\alpha, P} \quad \Pi^{\mathrm{B}}\left(\tilde{x}, \tilde{q}, \tilde{e}, \tilde{Q}, \alpha, P\right) \tag{69}$$

$$\text{subject to} \quad P = \frac{1}{v} d\,(\tilde{e}, \tilde{q})\,\tilde{q} \tag{70}$$

$$0 \leq \alpha \leq 1, \quad P \geq 0. \tag{71}$$

[22] Note that this allows the entrepreneur to build no capacity and "run away" with the money if she chooses to.

To analyze this setting, we retain our assumptions from §5 regarding the functional forms of $d(e, q), s(q)$, and $C(Q)$. The next lemma characterizes the equilibrium payoff assuming the venture is viable.

Lemma 4. $\tilde{Q} > 0$, $\tilde{e} > 0$. *The entrepreneur's equilibrium payoff in the blockchain-based model with a diversion opportunity can be written as*

$$\mathbf{\Pi}^{\mathrm{B}} = \max_{Q>0, e>0} \left[\frac{\mathscr{F}(e, Q)}{1-\delta} - cQ - C(Q) - \gamma e \right] \tag{72}$$

$$\textit{subject to} \quad \beta^{\mathrm{B}} \frac{\mathscr{F}_e(e, Q)}{1-\delta} = \gamma, \tag{73}$$

$$d(e, Q)Q \geq vC(Q), \tag{74}$$

$$\frac{\mathscr{F}_Q(e, Q)}{1-\delta} - c - C_Q(Q) - \frac{C(Q)}{d(e, Q)Q} \left(d(e, Q) + d_Q(e, Q) Q \right) \geq 0. \tag{75}$$

The only difference from the base-case equilibrium characterized in Corollary 1 is the additional constraint (75), which can be written as

$$\frac{\mathscr{F}_Q(e, Q)}{1-\delta} - c - C_Q(Q) - \alpha P_Q(e, Q) \geq 0, \tag{76}$$

and has the following interpretation. First note that raising cash beyond the anticipated capacity investment cannot be optimal because any anticipated diversion would be priced in the ICO valuation, so its only effect would be to dilute the entrepreneur's holding of tokens, and to reduce her incentives to provide effort. Therefore, as the entrepreneur cannot contractually *commit* to investing the entire ICO proceeds, she can raise money only up to the amount that she will be *willing* to invest ex post. This is what constraint (76) ensures. Namely, it guarantees that the net marginal payoff of the capacity investment to the entrepreneur is non-negative when the entire ICO proceeds are invested. This payoff, i.e., the left-hand side of (76), is reduced by $\alpha P_Q(e, Q)$ because the entrepreneur does not internalize the effect of capacity investment on the value of α tokens held by outside investors. Constraint (76) thus represents an additional limit on how much money the entrepreneur can raise by issuing tokens.[23]

Despite this additional disadvantage, the blockchain-based model preserves its benefits associated with effort provision. In the limiting case of $K = 0$, i.e., when building the platform requires

[23] This is somewhat similar to underinvestment described by Chod and Lyandres (2018), which also results from the entrepreneur not internalizing the investment payoff accruing to outside token holders. The difference is that Chod and Lyandres (2018) consider neither platform nor entrepreneurial effort, and the benefit of ICO financing in their model has to do with transferring risk to diversified investors. Most important, because Chod and Lyandres (2018) consider uncertainty and different risk preferences between the entrepreneur and investors, in their model, unlike in ours, the entrepreneur can benefit from diversion even if it is rationally anticipated by investors.

only effort on the part of the entrepreneur, whereas all monetary costs are borne by platform users, the blockchain-based model does not require outside financing, and leads to first best. If, furthermore, the costs borne by users are such that $c > d\left(\bar{e}, \bar{Q}\right) - s\left(\bar{Q}\right)$, the traditional model requires outside financing of the user subsidy, and the blockchain model is strictly preferable, i.e., $\mathbf{\Pi}^{\mathrm{B}} > \mathbf{\Pi}^{\$}$. By continuity, this dominance must hold true for a sufficiently small K. In other words, tokenization is the strictly preferred strategy when building the platform requires little capital investment from the entrepreneur, but joining it is costly for users.

We make this result more precise in the next proposition. Similar to Proposition 3, it assumes linear capacity cost and focuses on the scenario in which the cost of joining is relatively high, so that the traditional business model entails a user subsidy. Recall the definition of $\tau = \frac{c}{C+c}$ as the fraction of total cost that is incurred by a platform user rather than by the platform founders.

Proposition 4. *Suppose that $C(Q) = KQ$ and $c > d\left(\bar{e}, \bar{Q}\right) - s\left(\bar{Q}\right)$. Keeping the total cost $c + C$ fixed, the benefit of platform tokenization increases in the proportion of this cost that is incurred by platform users, i.e.,*

$$\frac{d}{d\tau}\left(\mathbf{\Pi}^{\mathrm{B}} - \mathbf{\Pi}^{\$}\right) > 0. \tag{77}$$

As discussed in §5, the entrepreneur's payoff in the traditional model under the subsidy regime does not depend on the composition of the total cost, $K + c$, because both its components require outside financing. In the blockchain-based model, as more of the cost is borne by users, the entrepreneur needs to raise less outside financing, which alleviates effort underprovision as well as constraints associated with token financing.

7 Conclusions

Digital platforms matching independent buyers and sellers have become one of the most successful business models of our times, penetrating virtually every sector of the economy. Recently, a new type of digital platforms has emerged that uses blockchain technology to issue platform-specific cryptocurrencies and to decentralize governance. This paper highlights two potential benefits of tokenization in mitigating moral hazard frictions between platform founders, investors, and users. First, we show that raising outside capital by issuing tokens rather than equity reduces the well-known problem of effort underprovision. This is because the value of equity is proportional to profit, whereas the value of tokens is proportional to the transaction volume, with the latter being less sensitive to entrepreneurial effort.

Second, by providing a decentralized, fully distributed governance mechanism, blockchain technology enables to eliminate a potential holdup of users by the platform when joining it is costly. This in turn allows the platform to attract users without providing them with an initial subsidy, reducing the founders' financing burden and effort underprovision associated with it. While decentralized governance minimizes the possibility of opportunistic behavior, raising funds through issuing tokens without appropriate investor protection not only allows, but facilitates opportunism. We show that diversion opportunities of token financing provided by inadequate regulation and monitoring mechanisms can lead to underinvestment, offsetting the aforementioned benefits that tokenization has to offer.

Our model suggests that tokenization is the preferable business model when (i) the platform success depends significantly on entrepreneurial effort and/or (ii) a significant part of the cost of building the platform is borne by users, which aggravates the holdup problem of the traditional business model, while mitigating the underinvestment problem of token financing in the presence of diversion opportunities. Our paper contributes to the emerging theoretical literature on crypto-economics. While this literature has acknowledged the potential of blockchains to resolve agency problems inherent in corporate governance via smart contracts, we show that the technology can remedy, through two distinct channels, underprovision of non-contractible effort.

Our model has several limitations. First, our setting is deterministic, which means that commitment to a given policy is always optimal. With uncertainty, there is likely to be a trade-off between commitment and some flexibility. Second, our model assumes stationary supply and demand, which leads to a stationary optimal policy. In reality, a new platform is likely to follow a growth process, which could call for dynamic capacity expansion. Third, we assume that the entrepreneur exerts effort only while building the platform. In practice, entrepreneurs can continue to exert effort even after their ventures becomes up-and-running. Finally, our model abstracts from network effects, which can play an important role in platform adoption. All of these considerations are beyond the scope of this paper, and provide opportunities for future research.

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Appendix

Proof of Lemma 1: To ease notation, we let $d(\cdot) \equiv d(\bar{e}, \cdot)$ and $\mathscr{F}(\cdot) \equiv \mathscr{F}(\bar{e}, \cdot)$. Let further

$$\hat{p} \equiv \operatorname*{argmax}_{q \geq 0} \left[\frac{\mathscr{F}(q)}{1-\delta} - cq \right]$$

be the number of participating sellers that maximizes the NPV of the perpetual surplus that the firm can extract from the platform taking into account the sellers' cost of joining. Note that because building capacity is costly, the optimal capacity cannot exceed this "unconstrained" optimum, i.e., $\bar{Q} \leq \hat{p}$. This in turn implies $\mathscr{F}'(q) \geq \mathscr{F}'(\bar{Q}) \geq (1-\delta)c > 0$, for all $0 \leq q \leq \bar{Q}$. Finally, let p_0 be the smallest root of

$$\frac{\mathscr{F}(\bar{Q})}{1-\delta} - c\bar{Q} = \frac{\mathscr{F}(p_0)}{1-\delta}, \tag{78}$$

which implies $p_0 < \bar{Q}$.

Claim 1. *Suppose that optimal capacity $\bar{Q}$ and optimal effort level $\bar{e}$ had been chosen, and p sellers joined the platform before the beginning of a given period. Absent financial constraints, the equilibrium fee, $x^\star(p)$, and number of sellers, $q^\star(p)$, in that period satisfy*

$$x^\star(p) = \begin{cases} \dfrac{d(\bar{Q}) - s(\bar{Q}) - c}{d(\bar{Q})}, & 0 \leq p \leq p_0 \\ \dfrac{d(p) - s(p)}{d(p)}, & p_0 < p \leq \bar{Q}, \end{cases} \qquad q^\star(p) = \begin{cases} \bar{Q}, & 0 \leq p \leq p_0 \\ p, & p_0 < p \leq \bar{Q}. \end{cases}$$

Further, the firm's equilibrium value-to-go is given by

$$V(p) = \begin{cases} \dfrac{\mathscr{F}(\bar{Q})}{1-\delta} - c\bar{Q}, & 0 \leq p \leq p_0 \\ \dfrac{\mathscr{F}(p)}{1-\delta}, & p_0 < p \leq \bar{Q}. \end{cases}$$

To prove this claim, it suffices to show that the proposed expressions in the statement of the Lemma for the value-to-go function, V and the optimal policies, $x^\star$, $q^\star$, solve the Bellman's recursion (3)-(5), along with the following proposed expression for the surplus-to-go function

$$J(z,p) = \begin{cases} \dfrac{s(\bar{Q}) - s(z)}{1-\delta} + c, & 0 \leq p \leq p_0 \\ \dfrac{s(p) - s(z)}{1-\delta}, & p_0 < p \leq \bar{Q}, \end{cases} \qquad 0 \leq z \leq p. \tag{79}$$

It can also be readily checked using algebra that the surplus-to-go in (79) satisfies (7).

Consider now the Bellman equations for different cases depending on the value of p. Note that in the maximization problem, because the objective in (3) is increasing in x, one of the constraints will always be binding at optimality.

Case 1: $0 \leq p \leq p_0$. To find the optimal $q^\star$, we explore different regions that q could lie in.

1(a): For $q \leq p$, constraint (5) does not apply, and therefore, constraint (4) will be binding. By eliminating x, the objective can be written as a function of q as

$$f(q) = \mathscr{F}(q) + \delta V(p),$$

Given that $\mathscr{F}$ is increasing, the maximum objective value over this region is given by $\mathscr{F}(p)+\delta V(p)$. Given that also $p_0 < \bar{Q}$, we can further bound the objective value over this region from above with

$$\mathscr{F}(p_0) + \delta V(p) = \mathscr{F}(p_0) + \delta\frac{\mathscr{F}(\bar{Q})}{1-\delta} - \delta c\bar{Q} = \mathscr{F}(\bar{Q}) - (1-\delta)c\bar{Q} + \delta\frac{\mathscr{F}(\bar{Q})}{1-\delta} - \delta c\bar{Q} = \frac{\mathscr{F}(\bar{Q})}{1-\delta} - c\bar{Q}.$$

1(b): For $p < q \leq p_0$, constraint (5) also applies and we then have that $(1-x)d(q) - s(q) = (c - \delta J(q,q))^+$. By elimating x, we can obtain an upper bound on the objective value over this region:

$$\begin{aligned}
f(q) &= \mathscr{F}(q) - (c - \delta J(q,q))^+ q + \delta V(q) \\
&\leq \mathscr{F}(q) + \delta V(q) \\
&= \mathscr{F}(q) + \delta\frac{\mathscr{F}(\bar{Q})}{1-\delta} - \delta c\bar{Q} \\
&\leq \mathscr{F}(p_0) + \delta\frac{\mathscr{F}(\bar{Q})}{1-\delta} - \delta c\bar{Q} \\
&= \mathscr{F}(\bar{Q}) - (1-\delta)c\bar{Q} + \delta\frac{\mathscr{F}(\bar{Q})}{1-\delta} - \delta c\bar{Q} \\
&= \frac{\mathscr{F}(\bar{Q})}{1-\delta} - c\bar{Q},
\end{aligned}$$

where the second inequality follows from $\mathscr{F}$ being increasing in $[0,\bar{Q}]$ and $p_0 < \bar{Q}$.

1(c): For $p < q$, and $p_0 < q \leq \bar{Q}$, we have that $J(q,q) = 0$. Therefore, constraint (5) is binding and we get

$$f(q) = \mathscr{F}(q) - cq + \delta V(q) = \mathscr{F}(q) - cq + \delta\frac{\mathscr{F}(q)}{1-\delta} = \frac{\mathscr{F}(q)}{1-\delta} - cq.$$

Clearly then, $f(q)$ attains its maximum value for $q = \bar{Q}$, and evaluates to $\frac{\mathscr{F}(\bar{Q})}{1-\delta} - c\bar{Q}$. Because this is precisely the upper bound we obtained in cases 1(a) and 1(b), we conclude that for $0 \leq p \leq p_0$,

$$q^\star(p) = \bar{Q}, \quad x^\star(p) = \frac{d(\bar{Q}) - s(\bar{Q}) - c}{d(\bar{Q})}, \quad V(p) = \frac{\mathscr{F}(\bar{Q})}{1-\delta} - c\bar{Q}.$$

Case 2: $p_0 < p \le \bar{Q}$. To find the optimal $q^\star$, we explore different regions that q could lie in.

2(a): For $q \le p$, constraint (5) does not apply, as in case 1(a). By eliminating x, we get for the objective $f(q) = \mathscr{F}(q) + \delta V(p)$. This is evidently an increasing function of q, therefore it attains its maximum value for this case for $q = p$, which equals

$$f(p) = \mathscr{F}(p) + \delta V(p) = \mathscr{F}(p) + \delta \frac{\mathscr{F}(p)}{1-\delta} = \frac{\mathscr{F}(p)}{1-\delta}.$$

2(b): for $p < q \le \bar{Q}$, constraint (5) applies, and because $J(q,q) = 0$, and it is also binding. Thus,

$$f(q) = \mathscr{F}(q) - cq + \delta V(q) = \mathscr{F}(q) - cq + \delta \frac{\mathscr{F}(q)}{1-\delta} = \frac{\mathscr{F}(q)}{1-\delta} - cq \le \frac{\mathscr{F}(\bar{Q})}{1-\delta} - c\bar{Q}.$$

Further, note that for $p > p_0$, since $\mathscr{F}$ is increasing

$$\frac{\mathscr{F}(\bar{Q})}{1-\delta} - c\bar{Q} = \frac{\mathscr{F}(p_0)}{1-\delta} \le \frac{\mathscr{F}(p)}{1-\delta},$$

Comparing the two cases, we conclude that for for $p_0 < p \le \bar{Q}$,

$$q^\star(p) = p, \quad x^\star(p) = \frac{d(p) - s(p)}{d(p)}, \quad V(p) = \frac{\mathscr{F}(p)}{1-\delta},$$

which completes the proof of Claim 1.

The equilibrium policy characterized in Claim 1 has an intuitive threshold structure. If the existing number of participating sellers is low, $p \le p_0$, the firm sets a relative low fee so as to bring the total number of sellers up to the (optimally chosen) platform capacity, earning $\mathscr{F}(\bar{Q}) - c\bar{Q}$ in the current period, and $\mathscr{F}(\bar{Q})$ in each period thereafter. A downside of setting a low fee that attracts new sellers to join is that this fee needs to be charged not only to the new sellers, but also to the sellers who have joined the platform in the past. Thus, if the existing number of sellers is relatively high, $p > p_0$, attracting new sellers is not economical. Instead, the firm sets a fee that extracts the maximum surplus from the existing sellers, and does so in each period thereafter, collecting $\mathscr{F}(p)$ each time.

Because at the beginning of period 1 the platform starts with no sellers, i.e., $p = 0$, the policy characterized in Claim 1 prescribes that the firm sets a fee low enough to bring the number of sellers up to the platform capacity in period 1, and then charges a higher fee to extract the maximum surplus from the existing sellers in each period thereafter, resulting in the policy characterized in Lemma 1.

Finally, note that the first-period fee given by (9) could be negative, i.e., it could be a subsidy. In general, this subsidy is constrained by the amount of capital available, $y - C(Q)$. However, at the optimal solution, $\bar{Q}$, $\bar{e}$, and $\bar{y}$, the financially unconstrained fee given in (9) has to be feasible for the following reason. If it were not, the firm would not be able to bring participation up to the platform capacity in period 1. Furthermore, because the firm is assumed to pay out each period revenue instantly to equityholders, it would not be able to increase participation up to the platform capacity in any subsequent period. Since having permanently excess capacity cannot be optimal, the policy prescribed in Lemma 1 has to be feasible at $\bar{Q}$, $\bar{e}$, and $\bar{y}$, which completes the proof. ■

Proof of Lemma 2: Any interior solution, $\bar{e} > 0$, to (12) is given by the necessary and sufficient condition (18). Thus, problem (13)-(16) can be written as

$$\max_{Q,y,\alpha,e} \quad \left[(1-\alpha)\left(\frac{\mathscr{F}(e,Q)}{1-\delta} - cQ + y - C(Q)\right) - \gamma e\right] \tag{80}$$

$$\text{s.t.} \quad y \geq C(Q) + \max\{-\pi(\bar{x}_1, \bar{q}_1), 0\} \tag{81}$$

$$y \leq \frac{\alpha}{1-\alpha}\left(\frac{\mathscr{F}(e,Q)}{1-\delta} - cQ - C(Q)\right) \tag{82}$$

$$Q > 0, \quad y > 0, \quad 0 < \alpha < 1, \quad e > 0 \tag{83}$$

$$(1-\alpha)\frac{\mathscr{F}_e(e,Q)}{1-\delta} = \gamma. \tag{84}$$

Because the objective increases in y, constraint (82) must be binding. Using this to eliminate y, the problem can be rewritten as

$$\max_{Q,\alpha,e} \quad \left[\frac{\mathscr{F}(e,Q)}{1-\delta} - cQ - C(Q) - \gamma e\right] \tag{85}$$

$$\text{s.t.} \quad \frac{\alpha}{1-\alpha}\left(\frac{\mathscr{F}(e,Q)}{1-\delta} - cQ - C(Q)\right) \geq C(Q) + \max\{-\pi(\bar{x}_1, \bar{q}_1), 0\} \tag{86}$$

$$Q > 0, \quad 0 < \alpha < 1, \quad e > 0 \tag{87}$$

$$(1-\alpha)\frac{\mathscr{F}_e(e,Q)}{1-\delta} = \gamma. \tag{88}$$

Suppose that constraint (86) is non-binding at an optimal solution $\left(\bar{Q}, \bar{\alpha}, \bar{e}\right)$. Consider an alternative solution (Q^*, α^*, e^*), where $Q^* = \bar{Q}$, $\alpha^* = \bar{\alpha} - \varepsilon$, and e^* is a solution to (88) with $Q = Q^*$ and $\alpha = \alpha^*$. This solution is clearly feasible, and it leads to a higher value of the objective (since $\mathscr{F}(e,Q)$ is concave in e), leading to a contradiction. Hence, constraint (86) has to be binding at any optimal solution, and the problem can be written as (17)-(20). ■

Proof of Lemma 3: We first consider the post-ICO decision problem (25)-(27). In equilibrium, constraint $q \leq Q$ must be binding, otherwise the entrepreneur would be better off by building less capacity in the first place. Using constraint (26), we can obtain $\tilde{x}$ as in (32). Finally, using (23), problem (25)-(27) can be rewritten as

$$\max_{e}\left[-\alpha\frac{1}{v}d\left(e,Q\right)Q+\frac{d\left(e,Q\right)-s\left(Q\right)}{1-\delta}Q-cQ-C(Q)-\gamma e+\alpha P\right]. \tag{89}$$

Any interior solution $\tilde{e}>0$ must satisfy the necessary and sufficient first-order condition, which can be written as (34).

Next, we consider the pre-ICO decision problem (28)-(31). Using (23), (32), and the fact that $\bar{q}=Q$, and eliminating P using constraint (29), the problem can be written as (33)-(36). ■

Proof of Corollary 1: The result for $i=\text{Ƀ}$ follows by eliminating α from (33)-(36). The result for $i=\$$ follows from (17)-(20) by (i) substituting for $\mathbf{x}$ and $\mathbf{q}$ from Lemma 1, (ii) eliminating α, and (iii) realizing that any (e,Q) solving (37)-(38) automatically satisfies $\alpha\in[0,1]$. ■

Proof of Proposition 1: The first result follows from Corollary 1, namely, the fact that $\mathbf{\Pi}^{\text{Ƀ}}$ is the maximum of the same function as $\mathbf{\Pi}^{\$}$ over a smaller feasibility set.

To prove the second result, we first establish that the objective of (37), denoted within this proof as $\Pi\left(Q\right)$, is unimodal. If $k\geq 1$, $\Pi\left(Q\right)$ is clearly concave. Now suppose $k<1$. We can write $\Pi(Q)=G(Q)+H(Q)$, where

$$\begin{aligned}
G(Q):=&\frac{d\left(Q\right)Q}{1-\delta}-Qc-C\left(Q\right)=\frac{Q^{1-b}}{1-\delta}-Qc-KQ^{k}, && (90)\\
G'(Q)=&\frac{\left(1-b\right)Q^{-b}}{1-\delta}-c-KkQ^{k-1}, && (91)\\
G''(Q)=&-\frac{b\left(1-b\right)Q^{-b-1}}{1-\delta}-Kk\left(k-1\right)Q^{k-2},\text{ and} && (92)\\
H(Q):=&-\frac{s\left(Q\right)Q}{1-\delta}=-\frac{mQ^{n+1}}{1-\delta}. && (93)
\end{aligned}$$

Note that

$$G''(Q)<0\Longleftrightarrow Q<\left(\frac{b\left(1-b\right)}{\left(1-\delta\right)\left(1-k\right)kK}\right)^{\frac{1}{b+k-1}}. \tag{94}$$

Thus, $G(Q)$ is first concave and than convex. Also, $\lim_{Q\searrow 0}G'(Q)=\infty$ and $\lim_{Q\to\infty}G'(Q)=-c$, so $G(Q)$ is initially increasing and eventually decreasing. Therefore, $G(Q)$ is unimodal. Since $H(Q)$ is clearly concave and decreasing, and $\lim_{Q\searrow 0}H'(Q)=0$, $\Pi(Q)$ is unimodal.

Since constraint (39) can be written as $Q\leq(vK)^{\frac{1}{1-b-k}}$, we conclude that $\mathbf{\Pi}^{\$}>\mathbf{\Pi}^{\text{Ƀ}}\Longleftrightarrow\Pi'\left((vK)^{\frac{1}{1-k-b}}\right)>0$, i.e.,

$$K^{\frac{b}{k+b-1}}\left(\frac{1-b}{1-\delta}-\frac{k}{v}\right)>cv^{\frac{b}{1-k-b}}+\frac{m\left(n+1\right)}{1-\delta}v^{\frac{b+n}{1-k-b}}K^{\frac{n}{1-k-b}}. \tag{95}$$

If $\frac{1-\delta}{1-b}\geq\frac{v}{k}$, the LHS of (95) is non-positive, whereas its RHS is positive, so condition (95) cannot be true. Otherwise, as K increases from 0 to ∞, the LHS of (95) increases from 0 to ∞, whereas its RHS decreases from ∞ to $cv^{\frac{b}{1-k-b}}$. Thus, condition (95) is true for large enough K. ■

Proof of Proposition 2: Before proving the desired inequality, we rewrite the expressions for $\mathbf{\Pi}^{\mathrm{B}}$ and $\mathbf{\Pi}^{\$}$, assuming the optimal $Q>0$ and $e>0$ in both models.

Blockchain-based model: If $K=0$ then $d(e,Q)Q > vC(Q)$. By continuity, this is also true when $K\in(0,\varepsilon)$. Assuming this is the case, we can relax (39). Using Corollary 1, we can then write

$$\begin{aligned}
\mathbf{\Pi}^{\mathrm{B}} \quad &= \quad \max_{Q>0,e>0}\left[\frac{\mathscr{F}\left(e,Q\right)}{1-\delta}-cQ-C(Q)-\gamma e\right] && (96)\\
\text{s.t.} \quad & \quad h\left(e,Q\right)=0, && (97)\\
\text{where} \quad & \quad h\left(e,Q\right):=\left(1-\frac{\left(1-\delta\right)C(Q)}{d\left(e,Q\right)Q}\right)\frac{\mathscr{F}_e\left(e,Q\right)}{1-\delta}-\gamma. && (98)
\end{aligned}$$

Because $h\left(e,Q\right)=\left(1-\frac{(1-\delta)KQ^k}{e^aQ^{1-b}}\right)\frac{ae^{a-1}Q^{1-b}}{1-\delta}-\gamma$ and $h_e\left(e,Q\right)=\frac{a(a-1)e^{a-2}Q^{1-b}}{1-\delta}+ae^{-2}KQ^k$, we know that for a given $Q>0$, as e goes from 0 to ∞, function $h\left(e,Q\right)$ first increases from $-\infty$ and then decreases towards $-\gamma$. Consider any given $Q>0$. If $\max_e h\left(e,Q\right)<0$, then (97) has no solution, and this Q is not feasible. Otherwise, (97) has one or two solutions in terms of e. Because both these solutions are clearly below $\arg\max_e\left[\frac{\mathscr{F}(e,Q)}{1-\delta}-cQ-C(Q)-\gamma e\right]$ and this objective is concave in e, the larger solution gives a higher value of the objective. Thus, we let $e^{\mathrm{B}}(Q)$ be the single solution or the larger of the two solutions to (97). Problem (96)-(97) can be then written as

$$\begin{aligned}
\mathbf{\Pi}^{\mathrm{B}} \quad &= \quad \max_{Q>0}\left[\frac{\mathscr{F}\left(e^{\mathrm{B}}(Q),Q\right)}{1-\delta}-cQ-C(Q)-\gamma e^{\mathrm{B}}(Q)\right] && (99)\\
\text{s.t.} \quad & \quad \max_e h\left(e,Q\right)\geq 0. && (100)
\end{aligned}$$

Traditional model: Using Corollary 1, we can write

$$\begin{aligned}
\mathbf{\Pi}^{\$} \quad &= \quad \max_{Q>0,e>0}\left[\frac{\mathscr{F}\left(e,Q\right)}{1-\delta}-cQ-C(Q)-\gamma e\right] && (101)\\
\text{s.t.} \quad & \quad g\left(e,Q\right)=0, && (102)\\
\text{where} \quad & \quad g\left(e,Q\right):=\begin{cases}\left(1-\frac{(1-\delta)C(Q)}{d(e,Q)Q-s(Q)Q-(1-\delta)cQ}\right)\frac{\mathscr{F}_e(e,Q)}{1-\delta}-\gamma & \text{if } c\leq d\left(e,Q\right)-s\left(Q\right)\\ \left(1-\frac{(1-\delta)(C(Q)+cQ-d(e,Q)Q+s(Q)Q)}{\delta(d(e,Q)Q-s(Q)Q)}\right)\frac{\mathscr{F}_e(e,Q)}{1-\delta}-\gamma & \text{o/w.}\end{cases} && (103)
\end{aligned}$$

If the project is viable, $\mathbf{\Pi}^{\$}>0$, any solution to (101)-(102) must satisfy $d\left(e,Q\right)-s\left(Q\right)>0$. Consider a given $Q>0$, and let $e^{\dagger}(Q)$ be the solution to $d\left(e,Q\right)-s\left(Q\right)=0$, so that $d\left(e,Q\right)-s\left(Q\right)>0\iff e>e^{\dagger}(Q)$. Because $g\left(e,Q\right)$ is continuous in e on $(e^{\dagger}(Q),\infty)$ and $\lim_{e\searrow e^{\dagger}(Q)}g\left(e,Q\right)=-\infty$, constraint $g\left(e,Q\right)=0$ has a solution on the interval $(e^{\dagger}(Q),\infty)$ if, and only, if $\max_{e>e^{\dagger}(Q)}g\left(e,Q\right)\geq 0$. Let $e^{\$}(Q)$ be the largest of these solutions. Problem (101)-(102) can be then written as

$$\begin{aligned}
\mathbf{\Pi}^{\$} \quad &= \quad \max_{Q>0}\left[\frac{\mathscr{F}\left(e^{\$}(Q),Q\right)}{1-\delta}-cQ-C(Q)-\gamma e^{\$}(Q)\right] && (104)\\
& \quad \text{s.t.} \max_{e>e^{\dagger}(Q)} g\left(e,Q\right)\geq 0. && (105)
\end{aligned}$$

The desired inequality (42) holds because (i) the feasibility set of (104)-(105) is a subset of that of (99)-(100), and (ii) the objective in (104) is smaller than that in (99) for any Q that is feasible for both problems. Claim (i) is true because $h(e,Q) > g(e,Q)$. Claim (ii) follows from the fact that $\frac{\mathscr{F}(e,Q)}{1-\delta} - cQ - C(Q) - \gamma e$ is strictly concave in e, and the fact that

$$e^{\$}(Q) < e^{\mathrm{B}}(Q) < e^{*}(Q) := \arg\max_{e} \left[\frac{\mathscr{F}(e,Q)}{1-\delta} - cQ - C(Q) - \gamma e \right]. \tag{106}$$

To prove the two inequalities in (106), we recall that $e^{\$}(Q)$, $e^{\mathrm{B}}(Q)$, and $e^{*}(Q)$ are the unique or largests roots of $g(e,Q) = 0$, $h(e,Q) = 0$, and $\frac{\mathscr{F}_e(e,Q)}{1-\delta} - \gamma = 0$, respectively. The first inequality then follows from the fact that $h(e,Q)$ is first increasing and then decreasing in e and the fact that $g(e,Q) < h(e,Q)$. The second inequality follows from the fact that $\frac{\mathscr{F}_e(e,Q)}{1-\delta} - \gamma$ is decreasing in e and $h(e,Q) < \frac{\mathscr{F}_e(e,Q)}{1-\delta} - \gamma$.

Finally, recall our assumption that the optimal Q and e are positive in both models. If problem (99)-(100) is unfeasible, or the maximum payoff is negative, the optimal strategy is $\tilde{Q} = \tilde{e} = 0$, leading to $\mathbf{\Pi}^{\mathrm{B}} = 0$. If this is the case, then also $\mathbf{\Pi}^{\$} = 0$ because the feasibility set of (104)-(105) is a subset of that of (99)-(100), and the objective in (104) is smaller than that in (99) for any given $Q > 0$. ■

Proof of Proposition 3: It follows from Corollary 1 that $\mathbf{\Pi}^{\$}$ depends only on the total cost, $K + c$, i.e., $\frac{d}{d\tau}\mathbf{\Pi}^{\$} = 0$. It follows from the proof of Proposition 2 that

$$\mathbf{\Pi}^{\mathrm{B}} = \max_{Q>0} \left[\frac{\mathscr{F}\left(e^{\mathrm{B}}(Q), Q\right)}{1-\delta} - (K+c)\,Q - \gamma e^{\mathrm{B}}(Q) \right] \tag{107}$$

$$\text{s.t.} \quad d(e^{\mathrm{B}}(Q), Q) \geq v(1-\tau)(K+c) \tag{108}$$

$$\max_{e} h(e,Q) \geq 0, \tag{109}$$

where $e^{\mathrm{B}}(Q)$ is the largest root of $h(e,Q) = 0$; $h(e,Q) = \left(1 - \frac{(1-\delta)(c+K)(1-\tau)}{d(e,Q)}\right) \frac{\mathscr{F}_e(e,Q)}{1-\delta} - \gamma$; and $h_e\left(e^{\mathrm{B}}(Q), Q\right) \leq 0$. To prove that $\frac{d}{d\tau}\mathbf{\Pi}^{\mathrm{B}} > 0$, we first show that the objective in (107) increases in τ for any feasible Q. We have

$$\frac{d}{d\tau} \left[\frac{\mathscr{F}\left(e^{\mathrm{B}}(Q), Q\right)}{1-\delta} - (K+c)\,Q - \gamma e^{\mathrm{B}}(Q) \right] = \frac{\partial e^{\mathrm{B}}(Q)}{\partial \tau} \left(\frac{\mathscr{F}_e\left(e^{\mathrm{B}}(Q), Q\right)}{1-\delta} - \gamma \right). \tag{110}$$

It follows from the proof of Proposition 2 that $\frac{\mathscr{F}_e\left(e^{\mathrm{B}}(Q),Q\right)}{1-\delta} - \gamma > 0$. Furthermore, $\frac{\partial e^{\mathrm{B}}(Q)}{\partial \tau} > 0$ because $h_e\left(e^{\mathrm{B}}(Q), Q\right) \leq 0$ and $h_\tau(e,Q) > 0$. To complete the proof, we note that since $h_\tau(e,Q) > 0$ and $\frac{\partial e^{\mathrm{B}}(Q)}{\partial \tau} > 0$, as τ increases, so does the feasibility set of problem (107)-(109). ■

Proof of Lemma 4: Because the optimal $\tilde{x}$ and $\tilde{q}$ are the same as in the base-case model, assuming $Q > 0$ and $e > 0$, we can rewrite (65)-(68) as

$$\max_{Q>0,e>0}\left[-\alpha\frac{1}{v}d\left(e,Q\right)Q+\frac{d\left(e,Q\right)-s\left(Q\right)}{1-\delta}Q-cQ-C(Q)-\gamma e+\alpha P\right] \tag{111}$$

$$\text{s.t. } C(Q)\leq\alpha P. \tag{112}$$

Similarly, we can rewrite (69)-(71) as

$$\max_{\alpha,P\geq\mathbf{0}}\left[\frac{\mathscr{F}\left(e,Q\right)}{1-\delta}-C(Q)-cQ-\gamma e\right] \tag{113}$$

$$\text{s.t. } P=\frac{1}{v}d\left(e,Q\right)Q, \tag{114}$$

$$\alpha\leq 1, \tag{115}$$

where Q and e are given by (111)-(112).

Next, we prove, by contradiction, that constraint (112) must be binding. Suppose that $C(Q) < \alpha P$. This means that $\tilde{e}$ and $\tilde{Q}$ are the interior solution of (111), and satisfy the first-order conditions

$$\left(1-\alpha\frac{1}{v}\left(1-\delta\right)\right)\frac{\mathscr{F}_{e}\left(e,Q\right)}{1-\delta}-\gamma \;=\; 0,\ \text{and} \tag{116}$$

$$\frac{\mathscr{F}_{Q}\left(e,Q\right)}{1-\delta}-\alpha\frac{1}{v}\left(d\left(e,Q\right)+d_{Q}\left(e,Q\right)Q\right)-C_{Q}(Q)-c \;=\; 0. \tag{117}$$

Using some algebra, these can be written as

$$\left(\frac{\left(1-\frac{\alpha}{v}\left(1-\delta\right)\right)a}{\left(1-\delta\right)\gamma}\right)^{\frac{1}{1-a}}Q^{\frac{1-b}{1-a}} \;=\; e,\ \text{and} \tag{118}$$

$$\left(1-b\right)\left(\frac{a}{\gamma}\right)^{\frac{a}{1-a}}\left(\frac{\left(1-\frac{\alpha}{v}\left(1-\delta\right)\right)}{\left(1-\delta\right)}\right)^{\frac{1}{1-a}}Q^{\frac{a-b}{1-a}}-\frac{m(n+1)Q^{n}}{\left(1-\delta\right)} \;=\; c+KkQ^{k-1}. \tag{119}$$

Invoking $k > (1-b)/(1-a)$, eq. (119) implies $\frac{dQ}{d\alpha} < 0$, and eq. (118) then implies $\frac{de}{d\alpha} < 0$. Now consider the choice of α in (113)-(115). Taking the derivative of the objective w.r.t. α gives

$$\frac{d}{d\alpha}\left(\frac{\mathscr{F}\left(e,Q\right)}{1-\delta}-C(Q)-cQ-\gamma e\right)=\frac{de}{d\alpha}\left(\frac{\mathscr{F}_{e}\left(e,Q\right)}{1-\delta}-\gamma\right)+\frac{dQ}{d\alpha}\left(\frac{\mathscr{F}_{Q}\left(e,Q\right)}{1-\delta}-C_{Q}(Q)-c\right). \tag{120}$$

Note that eq. (116) implies $\frac{\mathscr{F}_e(e,Q)}{1-\delta}-\gamma > 0$, whereas eq. (117) together with $d(e,Q) = e^aQ^{-b}$ imply $\frac{\mathscr{F}_Q(e,Q)}{1-\delta}-C_Q(Q)-c > 0$. Thus, we have shown that if $C(Q) < \alpha P$, then $\frac{d}{d\alpha}\left[\frac{\mathscr{F}(e,Q)}{1-\delta}-C(Q)-cQ-\gamma e\right] < 0$, i.e., the entrepreneur is better off choosing a smaller α. Therefore, at optimal α, we must have $C(Q) = \alpha P$ and the objective in (111) must be non-decreasing in Q at Q given by $C(Q) = \alpha P$ and e given by (116), i.e.,

$$\frac{\mathscr{F}_Q(e,Q)}{1-\delta} - \alpha\frac{1}{v}\left(d(e,Q) + d_Q(e,Q)\,Q\right) - C_Q(Q) - c \geq 0. \tag{121}$$

Any given $\alpha > 0$ thus falls in one of two cases. If α is such that $\frac{\mathscr{F}_Q(e,Q)}{1-\delta} - \alpha\frac{1}{v}\left(d(e,Q) + d_Q(e,Q)\,Q\right) - C_Q(Q) - c < 0$ at Q given by $C(Q) = \alpha P$ and e given by (116), this α cannot be optimal. Otherwise, using concavity of (111) in e, $\tilde{e}$ is given by (116) and $\tilde{Q}$ is given by $C(Q) = \alpha P$. Therefore, problem (113)-(115) can be rewritten as

$$\max_{\alpha, P \geq \mathbf{0}} \quad \left[\frac{\mathscr{F}(e,Q)}{1-\delta} - C(Q) - cQ - \gamma e\right] \tag{122}$$

$$\text{s.t.} \quad P = \frac{1}{v} d(e,Q)\,Q, \tag{123}$$

$$\frac{\mathscr{F}_Q(e,Q)}{1-\delta} - \alpha\frac{1}{v}\left(d(e,Q) + d_Q(e,Q)\,Q\right) \geq C_Q(Q) + c, \tag{124}$$

$$\alpha \leq 1, \tag{125}$$

where e is given by (116) and Q is given by $C(Q) = \alpha P$. Eliminating α and P gives the desired result. ■

Proof of Proposition 4: In the proof of Proposition 3, we established that if $c > d\left(\bar{e}, \bar{Q}\right) - s\left(\bar{Q}\right)$, then $\frac{d}{d\tau}\mathbf{\Pi}^{\$} = 0$. Next, consider $\mathbf{\Pi}^{\mathrm{B}}$, which is given by

$$\mathbf{\Pi}^{\mathrm{B}} = \quad \max_{Q>0} \left[\frac{\mathscr{F}\left(e^{\mathrm{B}}(Q), Q\right)}{1-\delta} - (K+c)\,Q - \gamma e^{\mathrm{B}}(Q)\right] \tag{126}$$

$$\text{s.t.} \quad d(e^{\mathrm{B}}(Q), Q) \geq v(1-\tau)(K+c), \tag{127}$$

$$\max_e h(e,Q) \geq 0, \tag{128}$$

$$\frac{\mathscr{F}_Q\left(e^{\mathrm{B}}(Q), Q\right)}{1-\delta} - c - K - \frac{(1-\tau)(c+K)}{d\left(e^{\mathrm{B}}(Q), Q\right)}\left(d\left(e^{\mathrm{B}}(Q), Q\right) + d_Q\left(e^{\mathrm{B}}(Q), Q\right) Q\right) \geq 0 \tag{129}$$

We already know from Proposition 3 that the maximum of (126)-(128) increases with τ. Thus, to prove that $\frac{d}{d\tau}\mathbf{\Pi}^{\mathrm{B}} > 0$, it is enough to show that constraint (129) becomes looser as τ increases. That is, it is enough to show that for any $Q > 0$, we have

$$\frac{d}{d\tau}\left(\frac{\mathscr{F}_Q\left(e^{\mathrm{B}}(Q), Q\right)}{1-\delta} - c - K - \frac{(1-\tau)(c+K)}{d\left(e^{\mathrm{B}}(Q), Q\right)}\left(d\left(e^{\mathrm{B}}(Q), Q\right) + d_Q\left(e^{\mathrm{B}}(Q), Q\right) Q\right)\right) \geq 0 \tag{130}$$

Using $d(e,Q) = e^a Q^{-b}$, inequality (130) can be written as

$$(1-b)(c+K) + \frac{\partial e^{\mathrm{B}}(Q)}{\partial \tau}\frac{1-b}{1-\delta} a\left(e^{\mathrm{B}}(Q)\right)^{a-1} Q^{-b} \geq 0. \tag{131}$$

We already established $\frac{\partial e^{\mathrm{B}}(Q)}{\partial \tau} > 0$ in the proof of Proposition 3, which implies that inequality (131) holds, completing the proof. ■