PRIVATE RENEGOTIATIONS AND GOVERNMENT INTERVENTIONS IN DEBT CHAINS*

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Abstract

We propose a model of strategic debt renegotiation in which businesses are sequentially interconnected through their liabilities. This financing structure, which we refer to as a debt chain, gives rise to externalities, as a lender's willingness to provide concessions to its privately-informed borrower depends on how the lender's own liabilities are expected to be renegotiated. Our analysis reveals how government interventions that aim to prevent default waves should account for these private renegotiation incentives and their interlinkages. Our results shed light on the effectiveness of subsidies and debt reduction programs following economic shocks such as pandemics or financial crises.

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1 Introduction

The COVID-19 pandemic has imposed unprecedented hardships on businesses worldwide. During the second quarter of 2020 more than 20% of U.S. small businesses either permanently or temporarily shut down and the default rate on U.S. private loans reached 8.1%.[1] In response to these events, private parties and governments have been implementing measures aimed at preventing large-scale default waves. Whereas businesses have renegotiated debt contracts, a pervasive response to changes in economic conditions (see Roberts and Sufi 2009), governments have enacted policies providing subsidies and have intervened in private contracts.[2]

In this paper, we argue that the effectiveness of such private and public efforts is crucially influenced by the fact that businesses tend to be sequentially interconnected through their liabilities, a financing structure we refer to as a *debt chain*. For example, a retail store might have a large account payable owed to its inventory supplier. The inventory supplier, in turn, might have a loan from a local credit union, which might also have financial obligations to a large national bank. Perhaps this large national bank is partly financed with bonds held by a pension fund that owes retirement benefits to workers, etc. The prevalence of these debt chains implies that private debt renegotiations have important externalities that governments should recognize when designing and targeting their interventions.

To analyze this issue, we develop a tractable model of strategic renegotiations in debt chains. Our model accounts for the fact that agents are generally heterogeneously exposed to economic shocks and have private information about their individual financial conditions (see Chava and Roberts 2008, Adelino, Gerardi, and Willen 2013, Roberts 2015, for related empirical evidence). Moreover, bargaining between a borrower and its lender is bilateral, giving rise to the possibility that an agent's bargaining power impedes the efficiency of not only one credit relationship but that of the whole chain (see Chava and Roberts 2008, Roberts and Sufi 2009, Denis and Wang 2014, for related empirical evidence). Each lender decides whether to reduce the present value of payments promised

[1] See "Small-Business Failures Loom as Federal Aid Dries Up" in the *New York Times*, September 1, 2020, and "Pandemic Puts Pressure on Private Debt as Default Rates Rise" in the *Wall Street Journal*, July 28, 2020.

[2] Prominent examples include the CARES act passed by congress (see U.S. Treasury Department 2020) and the CDC's eviction ban (see CDC 2020).

by its borrower, accounting for the fact that a decrease in the probability of default lowers expected default costs. Our analysis reveals how private renegotiation decisions are generically interrelated in a debt chain: a lender's willingness to provide concessions to its borrower — sometimes referred to as "taking a haircut" — depends on its own liabilities and how they are expected to be renegotiated (see Murfin 2012, Chodorow-Reich and Falato 2020, for related empirical evidence). The more a lender's own liabilities are expected to be reduced, the likelier that agent is to internalize default costs that are incurred when its own borrower defaults, thereby increasing the lender's incentives to be lenient. On the other hand, a lender who is deeply indebted typically finds it suboptimal to reduce its borrower's liabilities — while the probability of being paid is higher after making concessions, the payment collected in case of no default is lower and possibly insufficient to cover the lender's own liabilities. Whereas a tough renegotiation strategy may then be privately optimal, it not only increases the potential for costly default in the specific bilateral credit relationship but also creates negative externalities to renegotiation efforts elsewhere in the chain. If a lender such as the local credit union in the example above takes a tough stance with its borrower (i.e., the inventory supplier), so will the inventory supplier with its own borrower (i.e., the retail store). Furthermore, if the large national bank expects the credit union to take this tough stance with the inventory supplier, it might reduce the bank's incentives to renegotiate the credit union's liabilities to a default-free level. As a result, an unaccommodating renegotiation strategy in a particular credit relationship can trigger tough renegotiations and increased default probabilities everywhere else in the debt chain.

Accounting for the endogenous responses of all debt-chain members, we analyze how targeted government policies affect renegotiation outcomes throughout a chain. First, we show that providing subsidies to "downstream" borrowers like the retail store (whose debt payments are expected to flow up the chain) can be particularly effective in eliminating default waves. Such subsidies generally have to cover only a fraction of the potential shortfall the targeted borrower is facing, since the lender also has private incentives to reduce the debt to a default-free level. That is, private renegotiation is an important factor determining the magnitudes of government subsidies needed to avoid default, and our analysis reveals under which economic conditions these subsidies can be particularly small while still being fully effective. Importantly, providing subsidies to a borrower like the retail store also strengthens "upstream" lenders' incentives to renegotiate their borrowers' debt to default-free levels. By boosting the maximum debt payment the retail store can pay without defaulting on its inventory supplier, a subsidy to the retail store can first lead the inventory supplier, then the local credit union, and then the large national bank to more efficiently renegotiate with their respective borrowers. As a result, awarding a subsidy to a downstream borrower can be highly effective in preventing default waves, compared to giving the same subsidy to an upstream borrower, due to the recursivity of debt-chain members' optimal renegotiation decisions.

Second, we show how government interventions affecting the allocation of bargaining power in private renegotiations can help prevent default waves. In particular, preventing an upstream lender from being able to choose its renegotiation strategy and instead mandating a reduction of its borrower's liabilities can incentivize downstream agents to voluntarily renegotiate the debt owed to them to levels that also avoid default of their respective borrowers. For example, reducing how much the local credit union owes to the large national bank may first lead the credit union, and then the inventory supplier to more efficiently renegotiate with their respective borrowers. If poorly designed, this type of intervention can, however, backfire as it may significantly reduce how much the bank's bondholders would collect from efficiently renegotiating the bank's debt. Such an intervention may thereby result in the pension fund that holds the bank's bonds toughening its renegotiation strategy and increasing default risk in the chain.

A key friction impeding efficient renegotiation in our environment is lenders' imperfect information. Absent this friction, each lender would know its borrower's financial condition and would have no incentive to ask for more than the borrower can actually pay. As a result, inefficiencies associated with default would be avoided throughout the chain. In contrast, with asymmetric information, each lender faces a generic tradeoff when renegotiating with its borrower. On the one hand, significantly lowering how much a borrower owes increases the probability of repayment and reduces the probability of having to trigger inefficiencies associated with bankruptcy. On the other hand, not providing concessions implies a higher amount being collected if the borrower happens to be able to make its payment. The uncertainty the lender faces about the borrower's financial condition as well as the expected renegotiation outcomes elsewhere in the chain significantly affect the tradeoff associated with a lender's renegotiation decision. How much the inventory supplier knows about the retail store's ability to pay its debt and whether it expects to have its own loan renegotiated by the local credit union determine the optimal renegotiation strategy regarding the retail store's liabilities.

Finally, as a third policy implication, our analysis reveals how the timing of the renegotiation process, relative to the arrival of private information available on payment dates, is an important determinant of inefficiencies. In particular, we show that a default-free equilibrium for the whole chain only exists if renegotiation occurs before agents have access to all the information based on which they will ultimately make their default decisions. If, at the time of renegotiation, each agent in the chain has only imprecise information about the conditions it will face at a future payment date, each agent still has more optimistic beliefs about its asset values than under the worst possible ex-post scenario. As a result, relative to this agent-specific worst-case scenario, each agent still assigns positive probabilities to higher asset values and thus, has stronger incentives to follow a lenient renegotiation strategy with its own borrower. In other words, since incentives to be lenient are nonlinear in an agent's asset value (agents get increasingly aggressive as their asset values decline), renegotiation outcomes in the whole chain are more efficient when they occur before each agent has had the chance to obtain sufficiently precise negative information about its idiosyncratic condition. An implication of this mechanism is that government policies facilitating *early* renegotiation following a large shock tend to be desirable as they tend to lead to more efficient outcomes.[3]

Literature review. Our paper sheds light on debt renegotiation decisions in debt chains and how they are impacted by government interventions. We contribute to the existing literature on renegotiation, which to date abstracts from debt chains and the associated externalities of each renegotiation decision. Riddiough and Wyatt (1994a) study the dynamic decision whether to reorganize a single distressed firm, and Riddiough and Wyatt (1994b) analyze reputational effects of renegotiation when a lender has several loans that mature sequentially. Bolton and Scharfstein (1996) show how creditor dispersion can impede the efficient renegotiation of debt (see also He and Xiong (2012),

[3]More generally, the benefits of early renegotiation uncovered by our analysis shed light on the fact that renegotiation indeed tends to occur early in the life of a loan (see Roberts and Sufi 2009).

Brunnermeier and Oehmke (2013), Donaldson et al. (2020), and Zhong (2020) for related analyses of the effects of creditor dispersion). Gârleanu and Zwiebel (2009) analyze the design and renegotiation of debt covenants, showing that adverse selection problems lead to the allocation of greater ex-ante decision rights to the creditor.

Our paper is related to models of sequential strategic interactions in financial markets. Di Maggio and Tahbaz-Salehi (2015) study sequential lending relationships, but unlike us, they are interested in the use of collateral in origination decisions, rather than in debt renegotiation. They show how the allocation of collateral affects an intermediation chain's ability to shepherd liquidity towards a good investment opportunity. Relatedly, shedding light on the benefits of intermediation chains, Glode and Opp (2016) show that trading through moderately informed intermediaries can improve the efficiency of asset allocations in over-the-counter markets.

Our paper also relates to the theoretical literature studying the impact of debt and limited liability on firm decisions. This literature shows how outstanding debt can affect firms' incentives to invest (see Myers 1977), take risks (see Jensen and Meckling 1976), charge high prices for their products (see Brander and Lewis 1986), and negotiate with stakeholders like unionized workers (see Perotti and Spier 1993, Matsa 2010). In our model of debt chains, most agents are both lenders and borrowers, and we show how an agent's outstanding debt as a borrower, which depends on renegotiations with its lender, weakens that agent's willingness as a lender to renegotiate its borrower's liabilities. By providing concessions to a struggling borrower, a lender makes the distribution of payment outcomes more concentrated and reduces the probability that its borrower will default. Due to limited liability, the benefits of these concessions are, however, not fully internalized by a lender who is at risk of defaulting on its own liabilities. Moreover, optimal renegotiation decisions and equilibrium default risk are generically interrelated across debt-chain members, due to the presence of externalities. Our policy analysis identifies which debt-chain member(s) should be targeted by government interventions to maximize the total benefits throughout a chain.

Finally, our analysis complements insights from the existing literature on cascades and contagion in financial networks, which abstracts from the strategic renegotiation of debt contracts under asymmetric information. Allen and Gale (2000), Elliott, Golub, and Jackson (2014), and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) study different channels through which small economic shocks can spread and expand through networks of firms connected by financial obligations. Allen, Babus, and Carletti (2012) study the interaction between asset commonality and funding maturity in generating this type of contagion. Babus and Hu (2017) study how agents' incentives to default on their financial obligations can be weakened by a star network, in which a central intermediary monitors everyone's trading history. Taschereau-Dumouchel (2020) studies how firms' failure to produce inputs can lead to cascades of firm shutdowns. Our paper contributes to this literature by analyzing the optimal renegotiation strategies of lenders/borrowers who are part of a debt chain. In an environment that features the asymmetric information and bargaining power problems inherent in most bilateral renegotiations, our analysis accounts for agents' strategic responses to each other and to government policies. In particular, it shows how upstream renegotiation outcomes affect the optimal renegotiation decisions of downstream agents, contrasting with a typical default cascade which would propagate from the final borrower's balance sheet to that of the initial lender (i.e., from downstream to upstream agents).

Roadmap. In the next section, we introduce the model. In Section 3, we analyze the equilibrium renegotiation strategies of debt-chain members and the interconnectedness of these strategies. In Section 4, we analyze how various government policies affect renegotiation outcomes throughout a debt chain. In Section 5, we discuss the robustness of our model's key insights, and the last section concludes.

2 The Environment

In this section, we introduce our model of renegotiation in debt chains.

Agents and asset endowments. We consider an environment with $N \geq 3$ agents. At date $t = 1$, each agent j owns an endowment asset that takes a random value v_j at date $t = 2$. All agents discount future values at a rate of zero. The cumulative distribution function (CDF) of v_j, based on public information available at date $t = 1$, is denoted by $F_j(v_j)$. We denote by $\underline{v}_j$ and $\overline{v}_j$ the lower and upper bounds of the support of v_j, respectively. As of date $t = 1$, the asset values v_j are independently distributed across agents, reflecting the notion that agents face heterogeneous financial conditions, while still allowing for an aggregate shock that hit before date $t = 1$ and shaped the distributions $F_j(v_j)$ (see Section 5 for a related discussion).

Existing debt obligations. Capturing the central notion of a debt chain, the N agents are linked through existing debt obligations. In particular, at date $t = 1$, each agent $j \geq 2$ owes agent $(j-1)$ a payment equal to $\overline{d}_j$ that is due at date $t = 2$. We consider a setting where the initial face values $\overline{d}_j$ were chosen at a prior date (e.g., at an unmodeled date $t = 0$) based on the information available at that time. Our paper's focus on the renegotiation of existing debt contracts (rather than the initial security design problem) is motivated by the relevance of such phenomena after an economy is hit by large negative shocks such as the current worldwide pandemic, which was essentially unanticipated prior to the end of 2019.[4]

Debt contract settlement and default costs. If at date $t = 2$, an agent j defaults on the payment of its (potentially renegotiated) face value d_j, the lending agent $(j-1)$ aims to seize the remaining assets that agent j owns, which generally consist of the endowment asset worth v_j at $t = 2$ and the funds agent j collects from agent $(j+1)$. However, agent $(j-1)$ can collect only a fraction $(1-\rho) < 1$ of agent j's assets, where the parameter ρ captures the deadweight losses associated with liquidation, the bankruptcy process, and the potential losses of customers, employees, and suppliers.[5] These deadweight losses are the key source of surplus losses in the model (see Section 5 for an alternative specification for default costs).

Debt contracts are settled at date $t = 2$ starting with agent N's liability, then agent $(N-1)$'s liability, up until agent 2's liability that is owed to agent 1. This specification increases the tractability

[4]For example, the commissioner of the National Basketball Association (which generates over \$8B of worldwide revenues per year) explained the need to renegotiate the league's collective bargaining agreement (CBA) with the players as: *"This CBA was not built for an extended pandemic (...) There's not a mechanism in it that works to properly set the cap when you've got so much uncertainty, when our revenue could be \$10 billion or it could be \$6 billion. Or less."* https://www.espn.com/nba/story/_/id/30123004/nba-nbpa-extend-negotiating-window-cba-modifications-oct-30.

[5]Andrade and Kaplan (1998), Almeida and Philippon (2007), Korteweg (2010), Davydenko, Strebulaev, and Zhao (2012), Glover (2016), Dou et al. (2020), and Greenwood, Iverson, and Thesmar (2020) provide empirical evidence of the magnitudes of these costs.

of our model by ensuring that each agent j observes the realized value of its debt claim to agent $(j+1)$ (i.e., the face value payment or the recovery value) before deciding on whether to default itself on what it owes to agent $(j-1)$.[6]

Private information. At date $t=1$, each agent j obtains a private signal $s_j \in \Omega_s$ that is informative about the future realization of the endowment asset value v_j. We denote by $F_j(v_j|s_j)$ the conditional CDF of v_j as perceived by agent j. Introducing this private signal allows us to identify relevant differences in the implications that imperfect vs. private information have for the efficiency of the strategic renegotiation process. When dates $t=1$ (renegotiation) and $t=2$ (payment due date) are very close to each other, agent j is likely to have almost perfect information about its endowment asset value at date $t=2$. In contrast, when renegotiation ($t=1$) occurs a long time before the actual payment is due ($t=2$), then, at date $t=1$, agent j is likely to face significant uncertainty (imperfect information) about this value.

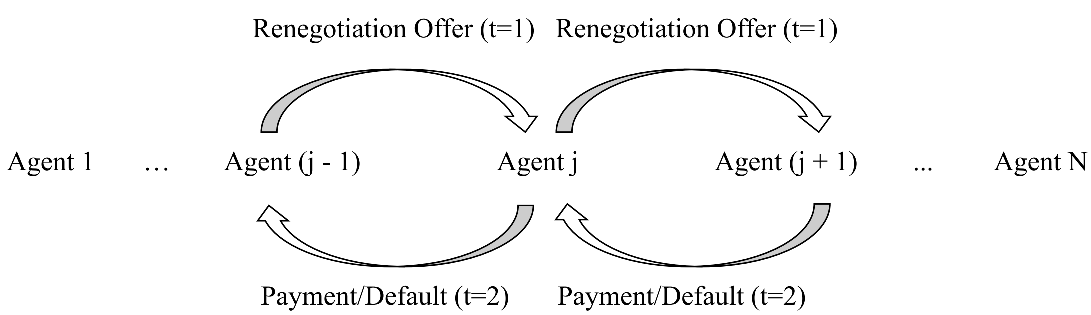

Figure 1: The figure illustrates the chain of renegotiation offers, payments, and default decisions.

Renegotiation. At date $t=1$, agents can renegotiate their debt contracts. Specifically, agent $(j-1)$ chooses whether to make a concession to agent j by lowering the face value of the debt contract to $d_j \leq \overline{d}_j$. Formally, agent $(j-1)$ proposes the new face value with a take-it-or-leave-it

[6]If contracts were settled in the reverse order, agents could not rely on payments from the debt claims they own to fulfill their liabilities. In this case, a firm could consider issuing additional securities to bridge a temporary shortfall caused by the delayed settlement of the debt claim it owns. However, such issuance would generally involve a security design decision and associated signaling concerns, thereby complicating the model and obfuscating the main insights.

offer to agent j. It is a dominant strategy for agent j to accept a new face value as long as it is weakly lower than the initial face value $\overline{d}_j$. However, at date $t = 2$, agent j will optimally use its limited liability and potentially default on this renegotiated face value. Renegotiation offers and outcomes are not publicly observable at date $t = 1$. Figure 1 gives an overview of the setup by illustrating the chain of renegotiation offers, payments, and default decisions (see Section 5 for a discussion of renegotiation in alternative network structures).

While renegotiation in our model pertains to adjustments in the face value of debt, renegotiation in practice more broadly affects the *present value* of payments promised by a borrower, which can also occur via payment delays or adjustments to coupon payments. Considering other contractual features and renegotiation margins that also match this description would yield similar economic insights.

Timeline. To summarize, the timeline of the model is as follows.

- Date $t = 1$: Renegotiation
  - (i) Each agent j obtains a signal s_j that is informative about its future endowment asset value v_j.
  - (ii) Each agent $j = 1,...,(N-1)$ simultaneously makes a take-it-or-leave-it offer to its debtor $(j+1)$, specifying a new face value d_{j+1}.
  - (iii) Each agent $j = 2,...,N$ decides whether to accept the newly proposed face value.
- Date $t = 2$: Payment
  - (i) Each agent j observes its endowment asset value v_j.
  - (ii) Debt contracts are settled sequentially, starting with the contract owed by agent N, then the contract owed by agent $(N-1)$, and so on.

3 Equilibrium Renegotiation and Default

In this section, we first characterize agents' optimization problems as borrowers and lenders. We then derive explicit conditions for default-free equilibrium outcomes in debt chains, considering both discrete and continuous distributions.

3.1 Renegotiation and Equity Values

An agent's equity value depends on where the agent is located in the debt chain and on other agents' renegotiation strategies.

Agent N. Agent N is special in that it does not hold a claim against any other agent in the chain. At date $t = 1$, it is a dominant strategy for agent N to accept any renegotiation offer below the pre-existing face value, $d_N < \overline{d}_N$. At date $t = 2$, agent N knows the endowment asset value v_N and, using its limited liability, optimally pays the face value d_N as long as:

$$d_N \leq v_N. \tag{1}$$

Otherwise, agent N defaults.

Agent (j – 1). Our analysis will primarily focus on the conditions under which efficient renegotiation can occur, that is, the conditions under which default-free, subgame-perfect Nash equilibria exist.[7]

Importantly, these conditions emphasize each agent's incentives to deviate to strategies that trigger defaults among debt-chain members and how to dampen these incentives. Apart from our interest in efficient renegotiation, this focus will greatly increase the tractability of our analysis,

[7] Given this focus, our analysis does not investigate equilibrium multiplicity, but rather establishes necessary and sufficient conditions for the existence of equilibria where default does not occur on the equilibrium path. Moreover, for the distributional assumptions and parameterizations we consider as examples in this paper, such multiplicity does not exist. More generally, renegotiation complementarities could become particularly relevant in the presence of circular debt linkages (i.e., if agent 1 owed a payment to agent N and agent N would strategically renegotiate agent 1's liabilities). Yet we intentionally consider a chain structure in our baseline model to account for typical asymmetries across agents in their upstream vs. downstream positions in credit relationships. In Section 5 we also discuss the robustness of our main insights to alternative network structures.

which features $(N-1)$ rounds of strategic debt renegotiation with asymmetric information. In Section 5, we discuss how the main insights derived from our baseline analysis also apply when default does occur on the equilibrium path.

In a default-free equilibrium, agent $(j-1) < (N-1)$ can rationally anticipate that agent j will collect the anticipated equilibrium face value d_{j+1} from agent $(j+1)$, which is helpful for predicting agent j's wealth.[8] Given this, agent $(j-1)$ anticipates that agent j will not default on an offer d_j as long as:

$$d_j \leq v_j + d_{j+1}. \tag{2}$$

For agent $(j-1)$, proposing a new face value d_j is equivalent to choosing a *marginal debtor type* $v_j^* = d_j - d_{j+1}$ that would be just indifferent between defaulting and not defaulting at date $t = 2$. All date-2 debtor types greater or equal to v_j^* will be *included* by this offer, in the sense that they will not default on the new face value. All debtor types below v_j^* will be *excluded* in the sense that they will default at date $t = 2$. Correspondingly, we can write agent $(j-1)$'s optimization problem as choosing a marginal debtor type v_j^* to maximize its expected equity value given its signal s_{j-1}:

$$\begin{aligned}\Pi_{j-1}(v_j^*) = \; &\Pr[v_j < v_j^*] \cdot \mathbb{E}\left[\max\left(v_{j-1} + (1-\rho)(v_j + d_{j+1}) - d_{j-1}\;,\;0\right) \mid s_{j-1}, v_j < v_j^*\right] \\ &+ \Pr[v_j \geq v_j^*] \cdot \mathbb{E}\left[\max\left(v_{j-1} + v_j^* + d_{j+1} - d_{j-1}\;,\;0\right) \mid s_{j-1}\right]. \end{aligned} \tag{3}$$

The equity value (3) shows that agent $(j-1)$'s renegotiation offer will generally depend on the agent's private information about v_{j-1}, as represented by the signal s_{j-1}. The max operators in equation (3) reflect agent $(j-1)$'s own limited liability: whenever the total payoff would be negative after paying off the debt, agent $(j-1)$ prefers to default and to get a payoff of zero. The extent to which the agent anticipates using limited liability depends on its information about the future value of its endowment asset and on the expected renegotiation offer from agent $(j-2)$.

[8] Agent $(N-1)$ differs from agents 1 to $(N-2)$ in that its debtor, agent N, does not have a debt claim to another agent's assets (or equivalently, we could assume that there was an agent $(N+1)$ but with $\overline{d}_{N+1} = 0$).

Agent 1. The first agent in the debt chain is special in that it does not owe anything to another agent. We can again write this agent's expected equity value for a given signal s_1 as a function of the marginal debtor type:

$$\Pi_1(v_2^*) = \mathbb{E}[v_1 \mid s_1] + \Pr[v_2 < v_2^*] \cdot (1-\rho)\left(\mathbb{E}[v_2 \mid v_2 < v_2^*] + d_3\right) + \Pr[v_2 \geq v_2^*] \cdot (v_2^* + d_3)\ . \tag{4}$$

3.2 Default-Free Renegotiation

To capture the prevalent view that default waves are undesirable outcomes, we have introduced deadweight losses that are realized in the event of bankruptcy.[9] For a default-free equilibrium to occur, each agent must find it optimal to renegotiate the debt level of its borrower to the lowest possible *total* asset value that this agent might have at date $t = 2$. We denote this value by $\underline{d}_j$. Conditional on its information at $t = 1$, an agent j's endowment asset delivers at least a value equal to $\underline{v}_j$ at date $t = 2$. Moreover, in a *default-free* equilibrium, agent j also collects the renegotiated face value d_{j+1} from agent $(j+1)$ with probability 1. As a result, the *total* value of agent j's assets at date $t = 2$ is bounded from below at $(\underline{v}_j + d_{j+1})$. Note that $\underline{d}_j$ is not per se the lowest possible value of an agent's total assets. Rather it is the lowest possible value conditional on its information at $t = 1$ *and* on being in an equilibrium in which agents $(j+1)$ through N do not default, that is, the newly proposed face values are indeed collected with probability 1. In contrast, if a default did occur on the equilibrium path among agents $(j+1)\ldots N$, then agent j would possibly end up having less financial wealth than $(\underline{v}_j + d_{j+1})$.

The new face values proposed by the lenders in a default-free equilibrium correspondingly satisfy the recursive relation:

$$\underline{d}_j \equiv \underline{v}_j + d_{j+1}, \tag{5}$$

[9] Even when borrowers use their debt issuance proceeds to fund negative-NPV projects, renegotiating their liabilities at $t = 1$ can be efficient, provided that their investment decisions were already made prior to $t = 1$ and liquidation is not optimal at that point. Moreover, a default that appears to be efficient from a partial equilibrium perspective (for a given credit relationship) might, in bad economic times, trigger default waves elsewhere in the debt chain that are harmful to the whole economy. Overall, we thus view the deadweight losses from excessive defaults as a first-order concern after large economic shocks such as the COVID-19 pandemic (see, e.g., Becker and Oehmke 2021).

provided that the initial face value $\overline{d}_j$ exceeds this value, that is, $\overline{d}_j \geq \underline{d}_j$. Otherwise, the face value remains at its initial level. Moreover, if $\overline{d}_j \geq \underline{d}_j$ for all j, the recursive relation (5) yields the following explicit formulae:

$$\underline{d}_j = \sum_{i=j}^{N} \underline{v}_i. \tag{6}$$

Whereas equation (6) indicates that the default-free renegotiated face values represent the accumulated lower bounds of the endowment asset values, higher renegotiated face values would apply if we introduced additional default costs that are internalized by the debtors, such as for example, a reputation cost from defaulting (see Section 5 for details).

3.3 The Case with Binomially Distributed Asset Values

To illustrate our main insights, we first consider the case in which each agent's endowment asset value is binomially distributed. Specifically, each agent j owns an asset that might either be worth $\underline{v}_j$ or $\overline{v}_j$ at date $t = 2$, where $\underline{v}_j < \overline{v}_j$. Furthermore, to make every negotiation decision non-trivial, we consider the case where $\overline{d}_j > \sum_{i=j}^{N} \underline{v}_i$ for each agent j, that is, all initial face values are greater than their default-free counterparts (relaxing this assumption would simply add credit relationships that do not need to be renegotiated). Finally, we assume that $\overline{d}_j < \overline{v}_j + \underline{d}_{j+1}$ so that no agent j defaults with probability 1 if its debt is not renegotiated down. We now state our first main result.

Proposition 1. *In the case of binomially distributed asset values, private renegotiation leads to a default-free debt chain on the equilibrium path whenever the following conditions hold:*

$$\frac{1 - F_2(\underline{v}_2)}{F_2(\underline{v}_2)} \left(\overline{d}_2 - \underline{d}_2\right) \leq \rho \cdot \underline{d}_2, \tag{7}$$

and for $j = 3, \ldots, N$:

$$\frac{1 - F_j(\underline{v}_j)}{F_j(\underline{v}_j)} \left(\overline{d}_j - \underline{d}_j\right) \leq \mathbb{E}[\min(\rho \cdot \underline{d}_j \ , \ v_{j-1} - \underline{v}_{j-1}) \mid s_{j-1}] \ \ \forall s_{j-1} \in \Omega_s. \tag{8}$$

In what follows, we prove the results of Proposition 1 and discuss them in detail.

Agent (j – 1). In a default-free equilibrium, agent $(j-1)$ expects that the debt of any agent $k \neq j$ will be renegotiated to $\underline{d}_k = \sum_{i=k}^{N} \underline{v}_k$. Thus, agent $(j-1)$ faces the following renegotiation choices. First, agent $(j-1)$ can keep agent j's debt at the initial level $\overline{d}_j$. Agent j then makes the promised debt payment when its type is $v_j = \overline{v}_j$ at date $t = 2$ (recall that $\overline{d}_j < \overline{v}_j + \underline{d}_{j+1}$), but defaults when it is $v_j = \underline{v}_j$ (recall that $\overline{d}_j > \underline{v}_j + \underline{d}_{j+1} = \underline{d}_j$). That is, the marginal included type is $\overline{v}_j$; the low type $\underline{v}_j$ is excluded. Given this strategy, agent $(j-1)$'s equity value at $t = 1$ is:

$$\begin{aligned}\Pi_{j-1}(\overline{v}_j) = F_j(\underline{v}_j) \cdot \mathbb{E}\left[\max\left(v_{j-1} + (1-\rho)(\underline{v}_j + \underline{d}_{j+1}) - \underline{d}_{j-1}\ ,\ 0\right) \mid s_{j-1}\right] \\ + (1 - F_j(\underline{v}_j)) \cdot (\mathbb{E}[v_{j-1} \mid s_{j-1}] + \overline{d}_j - \underline{d}_{j-1}).\end{aligned} \tag{9}$$

Alternatively, agent $(j-1)$ can renegotiate agent j's liabilities to a default-free level $\underline{d}_j$, in which case the marginal included type is $\underline{v}_j$. Agent $(j-1)$ then collects:

$$\begin{aligned}\Pi_{j-1}(\underline{v}_j) &= \mathbb{E}\left[\max\left(v_{j-1} + \underline{d}_j - \underline{d}_{j-1}\ ,\ 0\right) \mid s_{j-1}\right] \\ &= \mathbb{E}[v_{j-1} \mid s_{j-1}] - \underline{v}_{j-1}.\end{aligned} \tag{10}$$

Correspondingly, efficient renegotiation to the default-free level is privately optimal for agent $(j-1)$ whenever $\Pi_{j-1}(\underline{v}_j) \geq \Pi_{j-1}(\overline{v}_j)$ for all possible signal realizations s_{j-1}. This condition for the private optimality of efficient renegotiation simplifies to inequality (8) in Proposition 1.

One can think of the left-hand side of condition (8) as the benefit of following a tough renegotiation strategy and the right-hand side as its cost. When the benefit is lower than the cost (i.e., when condition (8) holds), agent $(j-1)$ is lenient, ensuring that agent j does not default. The left-hand side of condition (8) shows that the benefit of following a tough renegotiation strategy is higher when the initial face value $\overline{d}_j$ is large relative to the level that would be required to avoid default, $\underline{d}_j$. Moreover, the benefit of following this strategy is affected by the relative odds of facing a high borrower type vs. a low type, as represented by the odds ratio $\frac{1-F_j(\underline{v}_j)}{F_j(\underline{v}_j)}$.

On the other hand, the cost of following a tough renegotiation strategy, as represented by the right-hand side of condition (8), is affected by two channels. First, default costs are incurred through a tough renegotiation strategy. If agent j is the low type, it has assets worth $\underline{d}_j$. Under this renegotiation strategy, this low type defaults, leading to default costs $\rho \cdot \underline{d}_j$. If agent $(j-1)$ did not have any liabilities, it would fully internalize those losses, but otherwise, it internalizes losses only to the extent that it has sufficient equity value to absorb them. In a default-free equilibrium, agent $(j-1)$'s equity value at $t=2$ is given by $(v_{j-1} - \underline{v}_{j-1})$. As such, in each state of the world at $t=2$, agent $(j-1)$ at the margin internalizes at most the minimum of the default costs and its equity value (absent default), as indicated by the min operator in condition (8). The possibility of a positive equity value for agent $(j-1)$ stems from an information rent. In a default-free equilibrium, agent $(j-1)$'s lender, agent $(j-2)$, chooses a new face value that allows agent $(j-1)$ to avoid default even if it is the lowest type $\underline{v}_{j-1}$. Agent $(j-2)$ optimally makes that choice given the limited information it has about agent $(j-1)$'s asset values and given that agent $(j-1)$ also makes a renegotiation offer to agent j at a time when it has imperfect information about its own future asset value v_{j-1}. As a result, when agent $(j-1)$'s actual type exceeds this lowest type, the equity value at date $t=2$ is strictly positive. A higher equity value and the associated skin-in-the-game, in turn, discourage agent $(j-1)$ from choosing a tough renegotiation strategy for its own borrower, agent j.

Discussion: The role of private information and the timing of renegotiations. The magnitude of this skin-in-the-game effect depends on the private signal s_{j-1} that agent $(j-1)$ obtains at date $t=1$. The worse the signal, the less likely it is that agent $(j-1)$ will have an asset worth more than the lower bound $\underline{v}_{j-1}$ at date $t=2$. Thus, agent $(j-1)$ is less willing to renegotiate down its borrower's liabilities after receiving a bad interim signal about the value of its own endowment asset. In fact, if agent $(j-1)$ is perfectly informed about v_{j-1} and observes a bad asset value realization $v_{j-1} = \underline{v}_{j-1}$, the right-hand-side of condition (8) takes the value zero. Given our initial assumption that $\overline{d}_j \geq \underline{d}_j = \sum_{i=j}^{N} \underline{v}_i$, condition (8) then cannot be satisfied. If, at the renegotiation stage, agent $(j-1)$ already observes that its asset value is $\underline{v}_{j-1}$, the agent knows that a default-free renegotiation strategy will generate zero equity value. Thus, the agent is better off taking a harder stance by keeping agent j's debt level at the initial level $\overline{d}_j$. This optimal response to a bad signal implies that an equilibrium without default risk does not exist if the signal agents obtain is perfectly informative (that is, if an agent j observes v_j already at $t = 1$).

An immediate implication of this channel is that the timing of the arrival of private information, relative to the renegotiation process, is an important determinant of default risk in a debt chain. If the timing of renegotiation ($t = 1$) is such that agents still learn a substantial amount of information about their asset values after the renegotiation, the information rent entering condition (8) is larger, facilitating efficient renegotiation. In practice, agents are likely to learn more after the renegotiation process if the renegotiation takes place early, relative to the actual payment date. As a result, *early* renegotiation after a large shock tends to be desirable, as it tends to lead to more efficient outcomes. We will investigate this issue in more detail in Section 4.

Agent 1. As stated above, agent 1's decision to renegotiate with agent 2 is different due to the fact that agent 1 does not owe liabilities to another agent. In a default-free equilibrium, if agent 1 keeps agent 2's face value at the initial debt level $\overline{d}_2$, agent 1 can expect to collect:

$$\Pi_1(\overline{v}_2) = \mathbb{E}[v_1 \mid s_1] + F_2(\underline{v}_2)(1-\rho)(\underline{v}_2 + \underline{d}_3) + (1 - F_2(\underline{v}_2))\overline{d}_2. \tag{11}$$

If, on the other hand, agent 1 renegotiates agent 2's debt to its default-free level $\underline{d}_2$, agent 1 can expect to collect:

$$\Pi_1(\underline{v}_2) = \mathbb{E}[v_1 \mid s_1] + \underline{d}_2. \tag{12}$$

The condition for the optimality of agent 1's efficient renegotiation simplifies to inequality (7) in Proposition 1. Agent 1 fully internalizes the default costs that are triggered when the marginal agent-2 type $\underline{v}_2$ defaults, because agent 1 does not have any lenders that would potentially participate in absorbing these losses.

Discussion: The role of lender indebtedness. An important force in our model is the fact that lenders are generally indebted themselves. We now illustrate how this channel reduces downstream agents' incentives to renegotiate with their own borrowers.

In our baseline setting, agent 1 is by definition the first chain member and thus, solely a lender. Without any liabilities, agent 1 is willing to renegotiate with agent 2 if and only if condition (7) is satisfied. Suppose instead that agent 1 owed $\overline{d}_1$ to a fictional agent 0. Using the derivations above, we know that in a default-free equilibrium agent 1 would then be willing to renegotiate with agent 2 if and only if:

$$\frac{1-F_2(\underline{v}_2)}{F_2(\underline{v}_2)}(\overline{d}_2-\underline{d}_2) \leq \mathbb{E}\left[\min\left(\rho\cdot\underline{d}_2\ ,\ v_1-\underline{v}_1\right) \mid s_1\right]. \tag{13}$$

This condition is more restrictive than the one derived in our original environment for a given signal s_1 whenever:

$$\mathbb{E}\left[\min\left(\rho\cdot\underline{d}_2\ ,\ v_1-\underline{v}_1\right) \mid s_1\right] \leq \rho\cdot\underline{d}_2. \tag{14}$$

This inequality is guaranteed to be weakly satisfied and is strictly satisfied if default costs are large and the signal agent 1 receives is bad (which means $v_1-\underline{v}_1<\rho\cdot\underline{d}_2$ with positive probability for at least some realizations of s_1).

When owing debt to another agent, a lender internalizes only a fraction of the deadweight costs of default. Thus, agent j has lower incentives to renegotiate down the debt of agent $(j+1)$ if agent j is also indebted. Importantly, agent $(j-1)$'s bargaining power generally leads to higher liabilities for agent j, rendering this channel more severe. Agent $(j-1)$ is expected to use its bargaining power to extract any safe gains that agent j might secure when renegotiating with agent $(j+1)$. Thus, agent j has stronger incentives to take risks by opting for a tougher renegotiation stance with agent $(j+1)$.

Numerical example. We now further illustrate the intuition behind our first results through a numerical example. We assume that $N=3$, where agent 3 owes $\overline{d}_3=\$125K$ to agent 2 who owes $\overline{d}_2=\$325K$ to agent 1. Each agent j has an endowment asset that is equally likely to take the values $\underline{v}_j=\$100K$ or $\overline{v}_j=\$250K$. We set $\rho=0.6$, that is, only 40% of the borrower's asset value can be recovered in case of default.

In this scenario, agent 2 can choose between keeping agent 3's debt at its existing level $\overline{d}_3 =$ \$125K or renegotiating it to its default-free level $\underline{d}_3 = \underline{v}_3 =$ \$100K. Now, suppose that before renegotiating with agent 3, agent 2 receives a signal s_2 that either updates the probability of its own endowment asset value being high to 0.75 (i.e., the good signal) or to 0.25 (i.e., the bad signal). If agent 2 expects agent 1 to renegotiate its debt to $\underline{d}_2 = \underline{v}_2 + \underline{v}_3 =$ \$200K, agent 2 is unwilling to renegotiate agent 3's debt to $\underline{d}_3 =$ \$100K after the bad signal realization. Agent 2 prefers to keep asking agent 3 for \$125K, despite the 50% probability of default, than renegotiating agent 3's debt to its default-free level of \$100K. Formally, condition (8) is violated when agent 2 receives the bad signal:

$$\begin{aligned} &\frac{1-F_3(\underline{v}_3)}{F_3(\underline{v}_3)}\left(\overline{d}_3 - \underline{d}_3\right) = \left(\frac{1-0.5}{0.5}\right)(\$125K - \$100K) = \$25K \\ &> \mathbb{E}\left[\min\left(\rho \cdot \underline{d}_3\ ,\ v_2 - \underline{v}_2\right) \mid s_2\right] = 0.25 \cdot 0.6 \cdot \$100K + 0.75 \cdot \$0 = \$15K. \end{aligned} \tag{15}$$

Moreover, even if agent 1 expects agent 2 to renegotiate down agent 3's debt (which will not happen in equilibrium), agent 1 is unwilling to renegotiate agent 2's debt from its existing level $\overline{d}_2 =$ \$325K to its default-free level $\underline{d}_2 = \underline{v}_2 + \underline{v}_3 =$ \$200K. Formally, condition (7) is violated:

$$\begin{aligned} &\frac{1-F_2(\underline{v}_2)}{F_2(\underline{v}_2)}\left(\overline{d}_2 - \underline{d}_2\right) = \left(\frac{1-0.5}{0.5}\right)(\$325K - \$200K) = \$125K \\ &> \rho \cdot \underline{d}_2 = 0.6 \cdot \$200K = \$120K. \end{aligned} \tag{16}$$

Thus, absent outside interventions, no lender is willing to renegotiate its borrower's debt to an efficient, default-free level in this example (even if it expects that the other lender would do so). We will revisit this example again below when we analyze the impact of government interventions.

3.4 The Case with Continuously Distributed Asset Values

While the binomial case above provided a simple and intuitive illustration of some of our main insights, our results are by no means specific to this distributional assumption. In this section, we investigate the case with continuously distributed asset values. Specifically, the distribution of the asset values v_j has a density function $f_j(v_j)$, which takes strictly positive and finite values everywhere on the support $v_j \in [\underline{v}_j, \overline{v}_j]$. Unlike in the binomial setting where a lender's renegotiation decision effectively involves choosing either $\overline{d}_j$ or $\underline{d}_j = \underline{v}_j + \underline{d}_{j+1}$ as the face value, the continuous setting enriches the renegotiation stage in that the lender optimally chooses from a larger relevant set of marginal borrower types $v_j^* \in [\underline{v}_j, \overline{v}_j]$ and corresponding face values $d_j = v_j^* + \underline{d}_{j+1}$.

For tractability, we impose a standard regularity condition that the hazard rate $\frac{f_j(v_j)}{1-F_j(v_j)}$ is increasing on the support $[\underline{v}_j, \overline{v}_j]$. This condition ensures that (local) first-order conditions are sufficient for global optimality. Moreover, agent j obtains a signal $s_j \in \Omega_s = [\underline{s}_j, \overline{s}_j]$ at date $t = 1$ that implies that the conditional density of the value of its endowment asset at date $t = 2$ is given by $f_j(v_j|s_j)$. We assume that this conditional density takes finite values everywhere on the support $[\underline{v}_j, \overline{v}_j]$ for all possible signal realizations $s_j \in \Omega_s$. Further, let $f_j(v_j, s_j)$ denote the joint density of v_j and s_j.

Proposition 2. *In the case of continuously distributed asset values, private renegotiation leads to a default-free debt chain on the equilibrium path whenever the following conditions hold:*

$$\frac{1-F_2(\underline{v}_2)}{f_2(\underline{v}_2)} \leq \rho \cdot \underline{d}_2, \tag{17}$$

and for $j = 3, ..., N$:

$$\frac{1-F_j(\underline{v}_j)}{f_j(\underline{v}_j)} \leq \mathbb{E}[\min(\rho \cdot \underline{d}_j \ , \ v_{j-1} - \underline{v}_{j-1}) \mid s_{j-1}] \quad \forall s_{j-1} \in \Omega_s. \tag{18}$$

Again, we prove and discuss the results of the proposition in the main text.

Agent (j–1). If agent $(j-1)$ expects the debt of any agent $k \neq j$ to be renegotiated to $\underline{d}_k$, its marginal benefit of increasing the marginal debtor type v_j^* is given by (see the Appendix for the derivation of this expression):

$$\begin{aligned}\Pi'_{j-1}(v_j^*) = {} & f_j(v_j^*) \cdot \mathbb{E}\left[\max\left(v_{j-1} + (1-\rho)(v_j^* + \underline{d}_{j+1}) - \underline{d}_{j-1}\ ,\ 0\right)\ \mid s_{j-1}\right] \\ & - f_j(v_j^*) \cdot \mathbb{E}\left[\max\left(v_{j-1} + v_j^* + \underline{d}_{j+1} - \underline{d}_{j-1}\ ,\ 0\right)\ \mid s_{j-1}\right] \\ & + (1 - F_j(v_j^*))\Pr[v_{j-1} \geq \underline{d}_{j-1} - \underline{d}_{j+1} - v_j^* \mid s_{j-1}]. \end{aligned} \tag{19}$$

Given the regularity condition we imposed on the hazard rate, the sufficient and necessary condition for an equilibrium in which agent $(j-1)$ chooses a face value that ensures that agent j does not default is:

$$\Pi'_{j-1}(\underline{v}_j) \leq 0, \tag{20}$$

that is, $v_j^* = \underline{v}_j$ is the optimal choice for agent $(j-1)$. A default-free equilibrium requires this condition to hold for all possible signal realizations that agent $(j-1)$ might observe, $s_{j-1} \in \Omega_s$. Plugging in the relation for default-free debt levels derived in Section 3, we obtain condition (18) in Proposition 2 (see the Appendix for details).

This condition is similar to the one we obtained in the case of binomially distributed asset values (see condition (8)). In particular, the right-hand sides of the two conditions (8) and (18) are identical. Moreover, the left-hand side of condition (18) is the continuous analogue of the corresponding terms in condition (8). While the lender considers a discrete deviation from $\underline{d}_j$ to $\overline{d}_j$ in the binomial setting, condition (18) now focuses on a marginal increase from $\underline{d}_j$. As a result, the initial face value $\overline{d}_j$ does not explicitly enter the condition for a default-free equilibrium in the case of continuously distributed asset values.

Agent 1. As explained previously, agent 1's decision to renegotiate with agent 2 is special since agent 1 does not owe debt to another agent. With continuously distributed asset values, the condition for the optimality of agent 1's efficient renegotiation simplifies to inequality (17) in Proposition 2.

4 Policies Supporting Efficient Renegotiation

In this section, we analyze and compare how different types of government interventions affect renegotiation outcomes in debt chains. Specifically, we first consider two types of interventions that aim to reduce the shortfall between a targeted borrower's assets and liabilities: government subsidies and mandated debt reductions. Then, we show that policies that incentivize early renegotiation may reduce the negative impact of private information on efficient renegotiation (i.e., private renegotiation that avoids costly default on the equilibrium path).

In practice, both subsidies and mandated debt reductions are policy tools that are associated with significant costs and constraints (e.g., budget constraints, taxation costs, moral hazard, and reputational concerns). Absent such costs and constraints, preventing default would be straightforward to achieve: the government could either mandate that all debt contracts are nullified, or it could provide abundant subsidies that would ensure that all borrowers can fulfill their financial obligations. Given that deadweight losses of default are the key source of inefficiency in our environment, these policies would yield the first-best level of output. However, recognizing the presence of significant costs and constraints related to a large-scale implementation of these policies in practice, we analyze the effectiveness of *minimal* targeted interventions in eliminating inefficient default waves. This analysis highlights the endogenous responses by all debt-chain members to policies that only target a subset of agents.

4.1 Subsidies

In this section, we characterize how providing a subsidy to a struggling borrower does not only improve the recipient's ability to make its payments, but also incentivizes upstream lenders to renegotiate the debt that is owed to them to default-free levels.[10] Thus, a subsidy provided to downstream borrowers can have a large effect due to the interconnectedness of financial obligations and renegotiations.

[10] Several policy proposals that circulated at the onset of the COVID-19 crisis suggested awarding grants and subsidies to struggling businesses. See, for example, Hanson et al. (2020), Hubbard and Strain (2020), and Saez and Zucman (2020).

For ease of exposition we focus in the following corollary on the case of continuously distributed asset values. Thereafter, we also provide a numerical example with binomial distributions to further illustrate the effects of government subsidies.

Corollary 1. *Let Ψ denote the set of joint distribution functions $f_j(v_j, s_j)$ for $j = 1, ..., N$ associated with efficient renegotiation in a debt chain of N agents for a given value of ρ and absent government interventions. Further, let Ψ^{g_k} denote the corresponding set if the government provides a subsidy $g_k = g > 0$ to agent k. Suppose that $g < \overline{d}_j - \underline{d}_j$ for all j. Providing the subsidy to agent $k = N$ is most effective in expanding the set of default-free debt chains, that is, $\Psi^{g_k=g} \subset \Psi^{g_N=g}$ for any $k < N$.*

Proof. First, note that providing a subsidy to agent 1 has no effect on renegotiation outcomes. Next, suppose the government provides a subsidy $g_k = g$ to an agent $k \geq 2$. Agent k then effectively obtains an endowment cash flow of $(v_k + g)$ instead of v_k absent subsidies and we can adjust the efficient-renegotiation conditions provided in Proposition 2 as follows:

$$\frac{1-F_2(\underline{v}_j)}{f_2(\underline{v}_2)} \leq \rho \cdot (g + \underline{d}_2), \tag{21}$$

$$\frac{1-F_j(\underline{v}_j)}{f_j(\underline{v}_j)} \leq \mathbb{E}[\min(\rho \cdot (g + \underline{d}_j)\ ,\ v_{j-1} - \underline{v}_{j-1}) \mid s_{j-1}] \ \text{ for } j = 3, ..., k \text{ and } \forall s_{j-1} \in \Omega_s, \tag{22}$$

$$\frac{1-F_j(\underline{v}_j)}{f_j(\underline{v}_j)} \leq \mathbb{E}[\min(\rho \cdot \underline{d}_j\ ,\ v_{j-1} - \underline{v}_{j-1}) \mid s_{j-1}] \ \text{ for } j = (k+1), ..., N \text{ and } \forall s_{j-1} \in \Omega_s. \tag{23}$$

Note that the conditions (21) and (22) are relaxed by the subsidy term g, whereas the conditions (23) are not. Moreover, no matter which agent $k \geq 2$ receives the subsidy g, a marginal increase in this subsidy leads to a positive change equal to ρ in the right-hand side of condition (21). Similarly, no matter which agent $k \geq 3$ receives the subsidy g, a marginal increase in the subsidy leads to the following positive change in the right-hand side of condition (22):

$$\rho \cdot \Pr\left[\rho \cdot \left(g + \underline{d}_j\right) \ < \ v_{j-1} - \underline{v}_{j-1} \ \middle|\ \underline{s}_{j-1}\right]. \tag{24}$$

When $g < \overline{d}_j - \underline{d}_j$ for all j, the pass-through of resources in a debt chain implies that a subsidy provided to agent k affects the efficient renegotiation condition for agent $(k-2)$ in just the same way as a subsidy to the direct borrower $(k-1)$ would. Yet by providing the subsidy to agent $k=N$, all conditions are relaxed by the maximum amount attainable with a given subsidy, providing the maximum expansion of the default-free set Ψ. $\square$

While even in models without strategic renegotiation providing subsidies to a struggling business could prevent default by upstream lenders, our model highlights that subsidies can also relax the efficient-*renegotiation* conditions of upstream lenders, without affecting downstream borrowers. The results reveal that providing a subsidy to a downstream borrower has a larger effect on the set of chains that become default-free, compared to providing it to an upstream borrower. Moreover, the optimal renegotiation channel featured in our model highlights that government subsidies can prevent default even when the amount injected is not large enough to make up for a borrower's maximum possible shortfall. The reason for this is that the lender might optimally respond to the associated change in the distribution of this shortfall by more efficiently renegotiating agent j's liabilities. In the following, we further investigate the forces determining how potent a given subsidy can be in averting default waves.

Discussion: Multiplier effects of subsidies in the presence of renegotiation. Consider the effect that a subsidy has for the first borrower, agent 2, in the case where all other downstream lending relationships are already efficiently renegotiated in equilibrium. Absent renegotiation between agents 1 and 2, the maximum shortfall that agent 2 may experience is then equal to $\overline{d}_2 - \underline{d}_2$. To ensure that agent 2 also does not default in equilibrium, the government generally does not have to provide a subsidy equal to that maximum shortfall. Rather, it suffices to provide a subsidy in the amount of

$$g_{min} = \min\left(\frac{1}{\rho \cdot f_2(\underline{v}_2)} - \underline{d}_2, \overline{d}_2 - \underline{d}_2\right), \tag{25}$$

which accounts for the fact that it may be sufficient to ensure that the efficient renegotiation condition (21) holds with equality. The ratio of the minimum subsidy to the maximum shortfall, that is:

$$\frac{\text{Minimum subsidy required to rule out default}}{\text{Maximum shortfall absent renegotiation}} = \min\left[\frac{\frac{1}{\rho \cdot f_2(\underline{v}_2)} - \underline{d}_2}{\overline{d}_2 - \underline{d}_2}, 1\right], \tag{26}$$

highlights that subsidies are particularly effective in the presence of renegotiation when agent 2 is more likely to obtain a low cash-flow realization (a high value of $f_2(\underline{v}_2)$) and when default is particularly inefficient (a high value of ρ). In those cases, the government has to provide a smaller subsidy to rule out default, once the endogenous private renegotiation decisions are taken into account.

While an equivalent closed-form expression is not available for the renegotiation decisions involving agents $j \geq 3$, we can define g_{min} in those cases as follows:

$$\begin{aligned} g_{min} = \min\{g \geq 0 : 1/f_j(\underline{v}_j) \ \leq \ \mathbb{E}[\min(\rho \cdot (g + \underline{d}_j)\ ,\ v_{j-1} - \underline{v}_{j-1}) \mid s_{j-1}] \forall s_{j-1} \in \Omega_s, \\ \text{or } g \geq \overline{d}_j - \underline{d}_j\}. \end{aligned} \tag{27}$$

Note that an increase in g is less effective in relaxing the efficient renegotiation constraints associated with downstream lenders (relative to those pertaining to agent 1), since those agents' existing debt and associated limited liability reduces the extent to which they internalize inefficiencies. Moreover, ceteris paribus (assuming identical $f_j(\cdot)$ functions), downstream constraints are more binding since downstream borrowers collect less ($\underline{d}_{j+1} < \underline{d}_j$). Thus, in that case, the downstream constraints are more likely to be binding and require the highest subsidy values to ensure a default-free debt chain.

The following numerical example further illustrates the effects of subsidies in the case of binomially distributed asset values, highlighting the difference between maximum shortfalls and the minimum subsidies that are required to avoid default in a debt chain.

Numerical example. Consider a government intervention where the government gives a subsidy $g_3 = \$20K$ to agent 3 to help it meet its financial obligations. Note that when agent 3's asset value is low (i.e., when $v_3 = \$100K$) the subsidy $g_3 = \$20K$ is insufficient to allow agent 3 to make its debt payment $\overline{d}_3 = \$125K$. However, in that case, agent 2's efficient-renegotiation condition after observing the bad signal s_2 is satisfied:

$$\frac{1-F_3(\underline{v}_3)}{F_3(\underline{v}_3)}(\overline{d}_3-(\underline{v}_3+g_3)) = \left(\frac{1-0.5}{0.5}\right)(\$125K-(\$100K+\$20K)) = \$5K$$
$$\leq \ \mathbb{E}\left[\min\left(\rho\left(\underline{v}_3+g_3\right),\ v_2-\underline{v}_2\right) \mid s_2\right] = 0.25\cdot 0.6\cdot \$120K + 0.75\cdot \$0 = \$18K. \tag{28}$$

Agent 2 is then willing to renegotiate agent 3's debt to a new default-free level $\underline{d}_3 = \underline{v}_3 + g_3 = \$120K$. Moreover, agent 1 is also willing to renegotiate agent 2's debt since its efficient-renegotiation condition is satisfied:

$$\frac{1-F_2(\underline{v}_2)}{F_2(\underline{v}_2)}\left(\overline{d}_2-(\underline{v}_2+\underline{v}_3+g_3)\right) = \left(\frac{1-0.5}{0.5}\right)(\$325K-\$220K) = \$105K$$
$$\leq \rho\cdot(\underline{v}_2+\underline{v}_3+g_3) = 0.6\cdot \$220K = \$132K. \tag{29}$$

Overall, while a subsidy of \$20K is not enough to enable agent 3 to fulfill its existing liabilities of \$125K with probability 1, it is enough to incentivize agent 2 to renegotiate agent 3's debt from $\overline{d}_3 = \$125K$ to its new default-free level $\underline{d}_3 = \underline{v}_3 + g_3 = \$120K$. And while the subsidy is far from covering agent 2's shortfall of $\overline{d}_2 - \underline{v}_2 - \underline{v}_3 = \$125K$ when both asset values happen to be $v_j = \$100K$, it is enough to convince agent 1 to renegotiate agent 2's debt from $\overline{d}_2 = \$325K$ to its new default-free level $\underline{d}_2 = \underline{v}_2 + \underline{v}_3 + g_3 = \$220K$. In both renegotiation decisions, the default-free debt level (i.e., the minimal value of the borrower's assets) is increased by the subsidy, which contributes to making efficient debt renegotiation more attractive to the lender. This example thus illustrates that relatively small subsidies to downstream agents can incentivize upstream lenders to renegotiate down their borrowers' liabilities to avoid default.

4.2 Mandated Debt Reductions

We now turn our attention to how government interventions targeting the private bargaining process can be effective in preventing default waves. Eliminating a lender's bargaining power by mandating a debt reduction can also incentivize downstream lenders to renegotiate the liabilities owed to them to default-free levels. As observed above, the fact that agents $j > 1$ owe debt to a lender with bargaining power weakens their incentives to efficiently renegotiate down the debt of their downstream borrowers. As a result, if the government were to forgive or reduce part of the debt an agent owes, it would relax this agent's efficient-renegotiation conditions with downstream borrowers.

Formally, suppose that after a mandated debt reduction, agent $(j-1)$ owes a face value $\hat{d}_{j-1}$ that can be fully repaid even after efficiently renegotiating agent j's debt, that is:

$$\hat{d}_{j-1} \leq \underline{v}_{j-1} + \underline{d}_j. \tag{30}$$

We obtain the following corollary revealing how the efficient-renegotiation conditions depend on the value of $\hat{d}_{j-1}$.

Corollary 2. *When expecting efficient renegotiation by downstream lenders (i.e., $d_i = \underline{d}_i$ for all $i > j$), agent $(j-1)$ optimally renegotiates down agent j's debt in the binomial case if:*

$$\frac{1-F_j(\underline{v}_j)}{F_j(\underline{v}_j)}\left(\overline{d}_j - \sum_{i=j}^{N}\underline{v}_i\right) \leq \mathbb{E}\left[\min\left(\rho\cdot\sum_{i=j}^{N}\underline{v}_i\ ,\ v_{j-1}+\sum_{i=j}^{N}\underline{v}_i - \hat{d}_{j-1}\right)\ \middle|\ s_{j-1}\right] \quad \forall s_{j-1}\in\Omega_s, \tag{31}$$

and in the continuous case if:

$$\frac{1-F_j(\underline{v}_j)}{f_j(\underline{v}_j)} \leq\ \mathbb{E}\left[\min\left(\rho\cdot\sum_{i=j}^{N}\underline{v}_i\ ,\ v_{j-1}+\sum_{i=j}^{N}\underline{v}_i - \hat{d}_{j-1}\right)\ \middle|\ s_{j-1}\right] \quad \forall s_{j-1}\in\Omega_s. \tag{32}$$

Corollary 2 reveals that the efficient-renegotiation conditions are relaxed by a reduction of agent $(j-1)$'s liabilities $\hat{d}_{j-1}$. Thus, forgiving agent $(j-1)$'s debt to agent $(j-2)$ might incentivize agent $(j-1)$ to renegotiate down agent j's debt and avoid default, which then incentivizes agent j to do the same with agent $j+1$'s debt and so on.

The impact of this type of intervention on upstream lenders greatly depends on what happens to agent $(j-2)$. If the government simply reduces the amount that is transferred from agent $(j-1)$ to agent $(j-2)$, this intervention reduces agent $(j-3)$'s incentives to efficiently renegotiate how much agent $(j-2)$ owes. By reducing how much agent $(j-2)$ collects from agent $(j-1)$, the government effectively lowers how much agent $(j-2)$ and all upstream agents can pay without defaulting. It thus makes efficient renegotiation behavior less attractive for upstream lenders. A poorly designed intervention can therefore lead to higher default risk in the debt chain. As a result, debt reduction policies that do not involve subsidies for the lenders become more effective if the targeted liabilities are owed to lenders that are still expected to have their own (upstream) liabilities renegotiated down after the intervention, or lenders that have low levels of liabilities (like agent 1, who has none). If on the other hand, the government lowers agent $(j-1)$'s debt owed to agent $(j-2)$ but also gives agent $(j-2)$ the difference between the renegotiated debt amount without intervention and the new debt amount, then the efficient-renegotiation conditions of upstream lenders is unchanged by the intervention.[11] This intervention relaxes downstream lenders' efficient-renegotiation conditions without affecting upstream lenders'.

Numerical example. Consider a government policy that reduces agent 2's debt to agent 1 from $\overline{d}_2 = \$325K$ to $d_2 = \$175K$. In this case, the lower level of debt entices agent 2 to renegotiate agent 3's debt from $\overline{d_3} = \$125K$ to $\underline{d}_3 = \$100K$, even after observing the bad signal s_2, since:

$$\begin{aligned} &\frac{1-F_3(\underline{v}_3)}{F_3(\underline{v}_3)}(\overline{d}_3 - \underline{v}_3) = \left(\frac{1-0.5}{0.5}\right)(\$125K - \$100K) = \$25K \\ &\leq \mathbb{E}\left[\min\left(\rho \underline{v}_3 \ , \ v_2 + \underline{v}_3 - d_2\right) \mid s_2\right] = 0.25 \cdot 0.6 \cdot \$100K + 0.75 \cdot \$25K = \$33.75K. \end{aligned} \tag{33}$$

Thus, by effectively forgiving part of agent 2's debt, the government is incentivizing agent 2 to renegotiate down agent 3's debt and avoid default. In a debt chain, upstream debt reductions can incentivize downstream lenders to renegotiate their borrowers' liabilities to default-free levels.

Discussion: Debt reductions vs. subsidies. As shown above, a government can help eliminate default waves in a debt chain by providing subsidies to a subset of borrowers or by mandating that their liabilities be reduced. These two policies might look similar at first as they both reduce the gap between a targeted borrower's assets and liabilities. A subsidy reduces this gap by increasing the targeted borrower's assets, whereas a debt reduction lessens the liabilities. However, our

[11] One way of compensating a lender for a debt reduction is through a tax credit provided by the government, as was proposed by Greenwood and Thesmar (2020) at the onset of the COVID-19 pandemic.

analysis shows that these policies affect renegotiation outcomes differently when the targeted credit relationship is part of a debt chain. First, subsidies relax the efficient-renegotiation conditions of upstream lenders, whereas mandated debt reductions can relax the efficient-renegotiation conditions of downstream lenders. Which credit relationships in a chain are most likely to exhibit inefficient renegotiation outcomes should thus inform the choice and design of government interventions (see our earlier discussion on multiplier effects in Section 4.1). Second, comparing conditions (32) and (22) shows that mandated debt reductions also differ from subsidies in how they affect renegotiation within a given credit relationship. A subsidy increases the costs a lender faces when its borrower defaults. In contrast, a mandated debt reduction increases the information rent a lender might lose by defaulting on its own liabilities. Which of these two costs of inefficient renegotiation needs to be amplified to ensure efficient renegotiation should thus inform the choice and design of government interventions.

4.3 Early vs. Late Renegotiation

In Section 3.3, we discussed the insight that a lender's information at the time of renegotiation affects its incentives to efficiently renegotiate its borrower's liabilities. In particular, if at the renegotiation stage an agent knows that its future asset value will be low, a lenient renegotiation strategy generates little equity value. Thus, the agent might be better off gambling for a positive profit by keeping its borrower's debt at a higher level.

In contrast, if each agent in the chain has only imprecise information about its future asset value, beliefs are more optimistic than they would be under the worst possible ex-post scenario. As a result, relative to this agent-specific worst-case scenario, each agent has stronger incentives to follow a lenient renegotiation strategy with its own borrower. Since incentives to be lenient are non-linear in an agent's asset value, obtaining efficient renegotiation outcomes in *all* states of the world (i.e., a default-free equilibrium) is easier to achieve when renegotiation occurs before agents have had the opportunity to acquire precise information about their individual asset values.

An immediate implication of this channel is that the timing of the renegotiation process is an important determinant of inefficiencies, as it affects the quality of information available to agents.

Formally, we obtain the following result highlighting how late renegotiation and associated precise information can in fact eliminate default-free equilibria altogether.

Corollary 3. *Suppose $\overline{d}_j > \underline{d}_j$ for some $j \in \{2, ..., N\}$. If the renegotiation date $(t = 1)$ is immediately followed by the payment date $(t = 2)$ such that agents have perfect information about their own date-2 asset values at the time of renegotiation ($t = 1$), a default-free equilibrium does not exist.*

Under the conditions laid out in Corollary 3, an agent $(j-1)$ may already know during the renegotiation process (at $t = 1$) that his date-2 asset value is the worst-possible realization, $v_{j-1} = \underline{v}_{j-1}$, implying that the right-hand-sides of the conditions (8) and (18) take the value zero. Thus, these conditions cannot be satisfied in all states of the world, ruling out the existence of an equilibrium where default does not occur on the equilibrium path. When knowing that $\underline{v}_{j-1}$ is realized, an agent knows that a default-free renegotiation strategy will generate zero equity value and is thus better off taking a harder stance by keeping agent j's debt level at the initial level $\overline{d}_j$. In contrast, if renegotiation occurs sufficiently early, so that no agent might already know with certainty that it received the lowest possible asset value, default-free equilibria do exist.

In practice, agents are likely to know less about how an economic shock will affect their financial conditions if the renegotiation takes place right after the shock hits. As a result, a government policy that promotes early renegotiation after a large economic shock could be socially desirable.[12] The following numerical example further illustrates the benefits of early renegotiation.

Numerical example. We now revisit the numerical example from Section 3 and analyze how reducing the strength of agent 2's private signal, perhaps by initiating the renegotiation process earlier, would affect its incentives to renegotiate with its borrower. For now, assume that agent 2 expects its existing debt to agent 1 to be renegotiated to its efficient, default-free level $\underline{d}_2 = \underline{v}_2 + \underline{v}_3 = \$200K$. Consider a stark situation where the government can force agent 2 to decide whether to renegotiate agent 3's liabilities from $\overline{d}_3 = \$125K$ to $\underline{d}_3 = \$100K$ before agent 2 has collected any

[12]See Agarwal et al. (2017) for an example of a government policy intervention that incentivized debt renegotiation following the 2008/09 financial crisis, i.e., the 2009 Home Affordable Modification Program.

private information about the value of its endowment asset v_2. Thus, just like other agents, agent 2 believes that its endowment asset is equally likely to be worth $\overline{v}_2 = \$250K$ and $\underline{v}_2 = \$100K$. (Clearly, the benefits of early renegotiation apply more generally to cases for which the lender's private information is not completely eliminated by accelerating the timing of the renegotiation.) Using these unconditional probabilities, agent 2's efficient-renegotiation condition is satisfied:

$$\begin{aligned} & \frac{1-F_3(\underline{v}_3)}{F_3(\underline{v}_3)}(\overline{d}_3 - \underline{v}_3) = \left(\frac{1-0.5}{0.5}\right)(\$125K - \$100K) = \$25K \\ \leq \ & \mathbb{E}\left[\min\left(\rho \underline{v}_3 \ , \ v_2 - \underline{v}_2\right)\right] = 0.5 \cdot 0.6 \cdot \$100K + 0.5 \cdot \$0 = \$30K. \end{aligned} \tag{34}$$

Unlike in the original example from Section 3, agent 2 is now willing to renegotiate agent 3's debt to its default-free level $\underline{d}_3 = \$100K$.

Note, however, that early renegotiation does not change agent 1's willingness to renegotiate down agent 2's debt as agent 1's private information does not enter its efficient-renegotiation condition. Thus, other interventions like those we analyzed above are needed to incentivize agent 1 to provide concessions. Yet, the example illustrates how a government can reduce some of the inefficiencies associated with defaults by designing policies aimed at accelerating when renegotiation among debt-chain members occurs.

5 Robustness

In this section, we discuss the robustness of our key insights to alternative types of default costs, asset value dependence, renegotiation decisions in the presence of default risk on the equilibrium path, and alternative network structures such as debt trees.

5.1 Borrower-Specific Default Costs

In our baseline model, we assumed that the only inefficiency associated with default emanates from liquidation costs that reduce the value of the assets a lender can recover from a borrower. The parameter ρ was used to capture these proportional deadweight costs associated with default.

Going beyond these costs, it is plausible that borrowers also internalize a subset of the inefficiencies triggered by default. For example, a defaulting borrower might experience a loss of reputation, which can affect its future labor market outcomes and limit its access to capital markets for future projects. In this section, we highlight that introducing these types of default costs does not alter our model's key insights.

Formally, suppose each borrower internalizes a non-pecuniary fixed cost equal to $\phi > 0$ upon default. In this case, borrower j agrees to pay its debt if $d_j \le v_j + d_{j+1} + \phi$, that is, the introduction of borrower-specific costs makes defaulting less attractive for the borrower. As a result, relative to our baseline model, the lender can choose a higher debt level without triggering default. Moreover, borrowers' default costs increase the default-free debt level for each credit relationship, which is now given by:

$$\underline{d}_j \equiv \sum_{i=j}^{N} \underline{v}_i + (N+1-j)\phi. \tag{35}$$

The conditions ensuring that the renegotiation offers by agents $(j-1) = 1, \ldots, (N-1)$ yield a default-free equilibrium outcome for the whole debt chain then take the following form in the case of continuous distributions:

$$\frac{1-F_j(\underline{v}_j)}{f_j(\underline{v}_j)} \le \mathbb{E}\left[\min\left(\rho \cdot \left(\sum_{i=j}^{N} \underline{v}_i + (N-j)\phi\right) + \phi \ , \ v_{j-1} - \underline{v}_{j-1}\right) \,\middle|\, s_{j-1}\right]. \tag{36}$$

As to be expected, this condition reduces to our previous condition (18) when we set $\phi = 0$.

Whereas the costs considered here increase the renegotiated face values, the default costs internalized by the creditor, as captured by the parameter ρ in our baseline model, do not. The reason for this difference is that in a default-free equilibrium, an agent j's borrower, agent $(j+1)$, is collecting the full face value from its borrower, agent $(j+2)$, so default costs are not incurred in equilibrium. Yet, the marginal borrower type (and the associated renegotiated face value) is increased when default costs are internalized by the debtor, as it is then willing to pay more to avoid incurring these additional default costs.

These results highlight that borrower-specific default costs increase the default-free debt levels and loosen lenders' efficient-renegotiation conditions, yet they do so without qualitatively impacting our key insights.

5.2 Asset Value Dependence

In our baseline model, endowment asset values were independently distributed across agents as of date $t = 1$ (while still allowing for an aggregate shock that hit before date $t = 1$ and shaped the distributions $F_j(v_j)$). Thus, at that time, agent $(j-1)$ did not use its signal realization s_{j-1} to update the distribution of agent j's asset value, $F_j(v_j)$. In contrast, if agent $(j-1)$'s signal was also informative about agent j's asset value, due to a dependence between asset values, the distribution $F_j(v_j)$ would be replaced by the updated distribution $F_j(v_j|s_{j-1})$. Moreover, if the lower bound of the support of v_j was still $\underline{v}_j$ under this updated distribution, then the default-free debt level $\underline{d}_j$ would stay the same as in the baseline model, and agent $(j-1)$'s efficient-renegotiation condition under the binomial distribution (i.e., the analogue of condition (8)) would be:

$$\frac{1-F_j(\underline{v}_j|s_{j-1})}{F_j(\underline{v}_j|s_{j-1})}\left(\overline{d}_j - \sum_{i=j}^{N}\underline{v}_i\right) \leq \mathbb{E}\left[\min\left(\rho \cdot \sum_{i=j}^{N}\underline{v}_i\ ,\ v_{j-1}-\underline{v}_{j-1}\right)\ \middle|\ s_{j-1}, v_j = \underline{v}_j\right]. \tag{37}$$

This result highlights that positively correlated asset values would partially mitigate the effect that bad signals have on a lender's renegotiation tradeoff. Whereas a bad signal s_{j-1} reduces the information rents agent $(j-1)$ expects to earn on the default-free path (as pointed out in our baseline analysis), it also increases the probability that agent j defaults if $d_j > \underline{d}_j$. In sum, while introducing dependence between asset values enriches the role of signals in our model, it does not alter the main takeaways of our baseline analysis.

5.3 Default Risk on the Equilibrium Path

In our baseline analysis, we analyzed the conditions under which lenders' private renegotiation decisions lead to default-free debt chains on the equilibrium path. These conditions allowed us to characterize which economic forces and policies support efficient outcomes. Moreover, focusing on the conditions for default-free equilibria facilitated the tractability of our analysis, which involves N strategic, privately informed agents. Specifically, we did not have to keep track of the plethora of cases that exist when defaults occur on the equilibrium path. Yet, one may wonder how our insights are effected when socially efficient outcomes cannot be achieved, which might be the case when government interventions are constrained after large economic shocks. In this section, we discuss how the forces highlighted in our baseline analysis are robust to such cases.

Formally, suppose agent $(j-1)$ expects to owe a face value d_{j-1} to its lender and is considering whether to make a concession to agent j by lowering the face value of the debt contract to $d_j < \overline{d}_j$. Unlike in our baseline analysis, agent $(j-1)$ may not expect that agent j will collect $\underline{d}_{j+1}$ from agent $(j+1)$ with probability 1. Correspondingly, we denote the *stochastic* transfer from agent $(j+1)$ to agent j as $\tilde{d}_{j+1}$. This transfer might be equal to the (potentially renegotiated) value of the debt that agent j collects in case of full repayment, but it might also be less, in case of default.

We define the variable $x_j \equiv v_j + \tilde{d}_{j+1}$ and the associated CDF $H_j(\cdot)$. The marginal debtor type is now given by $x_j^* = d_j$, which is the agent j that would be just indifferent between defaulting and not defaulting. Agent $(j-1)$ then chooses the x_j^* that maximizes the equity value:

$$\begin{aligned}\Pi_{j-1}(x_j^*) \equiv\ & \Pr[x_j < x_j^*] \cdot \mathbb{E}\left[\max\left(v_{j-1} + (1-\rho)x_j - d_{j-1}\ ,\ 0\right)\ \mid s_{j-1}, x_j < x_j^*\right] \\ &+\ \Pr[x_j \geq x_j^*] \cdot \mathbb{E}\left[\max\left(v_{j-1} + x_j^* - d_{j-1}\ ,\ 0\right)\ \mid s_{j-1}\right]. \end{aligned} \tag{38}$$

The marginal benefit of increasing x_j^* can be expressed as follows:

$$\begin{aligned}\Pi'_{j-1}(x_j^*) = (1 - H_j(x_j^*)) \cdot \Pr[v_{j-1} \geq d_{j-1} - x_j^* \mid s_{j-1}] - \frac{dH_j(x_j^*)}{dx_j^*} \\ \cdot \mathbb{E}[\max(v_{j-1} + x_j^* - d_{j-1}\ ,\ 0) - \max(v_{j-1} + (1-\rho)x_j^* - d_{j-1}\ ,\ 0) \mid s_{j-1}]. \end{aligned} \tag{39}$$

Note that this marginal benefit may be infinite, in case the distribution $H_j(\cdot)$ features point masses, which is generally the case when there is default on the equilibrium path. As such, marginal optimality conditions are generally not sufficient conditions, but equation (39) nonetheless illustrates the generic tradeoff agent $(j-1)$ faces.

In particular, this equation is the analogue of equation (19), which applied when default did not occur on the equilibrium path. The first term on the right-hand side represents the probability that neither agent $(j-1)$ nor agent j defaults. For agent $(j-1)$ to benefit from an increased face value d_j, agent j has to actually make the requested payment, and agent $(j-1)$ needs to be able to make its debt payment to agent $(j-2)$ to avoid default. The second term on the right-hand side represents the negative impact of increasing agent j's debt on the probability that this agent fully repays its debt multiplied by the expected loss to agent $(j-1)$ from agent j's default. Consistent with our baseline analysis, the agent makes optimal renegotiation decisions by trading off the amount being collected in case of repayment with the probability that its borrower defaults. Moreover, agent $(j-1)$ does not internalize the inefficiencies of its borrower defaulting on a high face value whenever agent j's default also pushes agent $(j-1)$ into default.

While this tradeoff is consistent with the one featured in our baseline analysis, the tractability of the analysis when default occurs on the equilibrium path is impeded by the fact that the term $H_j(x_j^*)$ depends on agent $(j+1)$'s stochastic transfer $\tilde{d}_{j+1}$. Absent default on the equilibrium path, $\tilde{d}_{j+1}$ is constant and equal to $\underline{d}_{j+1}$, which denoted in our baseline analysis the renegotiated face value associated with no default by agent $(j+1)$. Thus, the effect of increasing the marginal debtor type v_j^* on the probability of repayment was simply $-f_j(v_j^*)$. In contrast, in the case considered here, the distribution of $\tilde{d}_{j+1}$ accounts for all the possible combinations of default outcomes for every one of agent j's downstream borrowers (i.e., from agent $(j+1)$ to agent N). Thus, agent $(j-1)$'s decision to renegotiate not only depends on the distribution of v_j, like in the baseline analysis, but also on the distribution functions and debt face values associated with the $(N-j)$ downstream credit relationships. This complexity impedes general analyses of debt chains with any N agents and implies that one has to resort to special examples for full solution characterizations. In contrast, our baseline analysis provided a full characterizations of the conditions under which default does not occur on the equilibrium path and allowed us to highlight essential forces in agents' renegotiation tradeoffs.

5.4 Debt Trees

In our baseline model, we analyzed the renegotiation behavior of agents that are part of a debt chain, where each lender has one borrower. In reality, however, the network of credit relationships might feature some lenders deciding on how to renegotiate with multiple borrowers. We now show that, if we consider such "debt trees," the forces at play in our baseline model still remain relevant.

Formally, suppose agent $(j-1)$ owes an amount $\underline{d}_{j-1}$ to another agent and considers renegotiating the liabilities of its M borrowers, agents $j_m \in \{j_1, j_2, \ldots j_M\}$. If we assume independent binomial distributions for simplicity, agent $(j-1)$ decides whether to keep the face value of each borrower j_m at $\overline{d}_{j_m}$ or to renegotiate it to a default-free level $\underline{d}_{j_m}$. In contrast to our baseline model, agent $(j-1)$ now has to make M renegotiation decisions, which involves comparing agent $(j-1)$'s equity value for every possible combination of renegotiation strategies with its borrowers $j_m \in \{j_1, j_2, \ldots j_M\}$ (i.e., a total of 2^M possible combinations in the binomial case). Yet, it is possible to explore the economic forces and tradeoffs in this setting by zooming in on the decision to renegotiate j_m's liabilities when every other liability in the economy, including those of agents $j_{m'}$ where $m' \neq m$, are expected to be renegotiated to their respective default-free levels. Extending our notation, we then can show that agent $(j-1)$ prefers renegotiating all of its borrowers' liabilities to a default-free level over renegotiating those of all agents except j_m as long as:

$$\frac{1-F_{j_m}(\underline{v}_{j_m})}{F_{j_m}(\underline{v}_{j_m})}\left(\overline{d}_{j_m}-\underline{d}_{j_m}\right) \leq \mathbb{E}[\min(\rho \cdot \underline{d}_{j_m}\ ,\ v_{j-1}-\underline{v}_{j-1}) \mid s_{j-1}] \ \ \forall s_{j-1} \in \Omega_s. \tag{40}$$

Comparing this condition to the efficient-renegotiation condition in debt chains (i.e., condition (8)), reveals that the strategic decision whether to renegotiate with one specific borrower in order to avoid default in a debt tree (or in many other types of networks) features economic forces and tradeoffs that are consistent with those of our baseline analysis.

6 Conclusion

When an economy is exposed to a large shock such as the ongoing COVID-19 pandemic or the most recent financial crisis, many businesses struggle to fulfill their existing financial obligations, especially so if these businesses are interconnected via debt chains. To analyze the effectiveness of private and public interventions aimed at avoiding large-scale default waves, we develop a tractable model of strategic renegotiation in a debt chain. Our model illustrates how private renegotiation decisions are generically interrelated: a lender's willingness to provide concessions to its borrower depends on how it expects its own liabilities to be renegotiated. Whereas a tough renegotiation strategy may be privately optimal for the lender, it may create negative externalities to renegotiation efforts elsewhere in the chain. In fact, an unaccommodating renegotiation strategy by one lender in a chain can trigger tough renegotiations and increased default probabilities throughout the whole chain.

Our policy analysis reveals how government subsidies to downstream borrowers do not only improve the recipients' ability to make payments, but they also further incentivize upstream lenders to renegotiate debts to default-free levels. Accounting for the recursivity of the optimal renegotiation decision of each agent, we show that awarding relatively small subsidies to downstream borrowers can be highly effective in preventing default waves compared to awarding the same subsidies to upstream borrowers. We also examine how forgiving a struggling borrower's debt or backing it to prevent default can further incentivize downstream lenders to efficiently renegotiate the debt of their borrowers. Finally, we highlight that facilitating early debt renegotiations after a large shock tends to increase incentives for providing concessions, thus reducing default risk. In sum, our analysis not only sheds light on the implications of different types of government interventions but also reveals which members of a debt chain should be targeted to maximize the effectiveness of a given policy intervention.

Appendix: Derivations Omitted from Main Text

A Optimality Conditions with Continuous Distributions

In this section, we derive the optimality conditions in a more general version of our model in which a borrower internalizes some of the default costs, substantiating our discussion on this issue in Section 5. Specifically, we consider a setting in which a borrower incurs a loss equal to ϕ when defaulting.

Suppose that agent j always collects a face value d_{j+1} from its borrower, agent $(j+1)$. Agent j, in turn, does not default on an offer d_j when:

$$v_j + d_{j+1} - d_j \geq -\phi. \tag{A1}$$

Agent $(j-1)$ chooses a marginal debtor type $v_j^* = d_j^* - d_{j+1} - \phi$ to maximize its expected payoff (note that the new proposed face value is then: $d_j^* = v_j^* + d_{j+1} + \phi$):

$$\begin{aligned}
&\Pi_{j-1}(v_j^*) \\
=& \int_{\underline{v}_{j-1}}^{\overline{v}_{j-1}} \int_{\underline{v}_j}^{v_j^*} \max(v_{j-1} + (1-\rho)(v_j + d_{j+1}) - d_{j-1} \ , \ -\phi) \cdot f_j(v_j) \cdot f_{j-1}(v_{j-1}|s_{j-1}) dv_j dv_{j-1} \\
&+ (1 - F_j(v_j^*)) \cdot \mathbb{E}[\max(v_{j-1} + v_j^* + d_{j+1} + \phi - d_{j-1} \ , \ -\phi) \mid s_{j-1}],
\end{aligned} \tag{A2}$$

reflecting that agent $(j-1)$ gets a signal s_{j-1} on its income realization v_{j-1} and can predict the renegotiation offer d_{j-1} from agent $(j-2)$. To derive first-order conditions, we compute the following derivates of terms in equation (A2):

$$\begin{aligned}
&\frac{\partial \int_{\underline{v}_{j-1}}^{\overline{v}_{j-1}} \int_{\underline{v}_j}^{v_j^*} \max(v_{j-1} + (1-\rho)(v_j + d_{j+1}) - d_{j-1} \ , \ -\phi) f_j(v_j) dv_j f_{j-1}(v_{j-1}|s_{j-1}) dv_{j-1}}{\partial v_j^*} \\
=& f_j(v_j^*)\, \mathbb{E}[\max(v_{j-1} + (1-\rho)(v_j^* + d_{j+1}) - d_{j-1} \ , \ -\phi) \mid s_{j-1}],
\end{aligned} \tag{A3}$$

and

$$\frac{\partial\,\mathbb{E}[\max(v_{j-1}+v_j^*+d_{j+1}+\phi-d_{j-1}\ ,\ -\phi)\mid s_{j-1}]}{\partial v_j^*}$$
$$\begin{aligned}
&=\mathbb{E}[\mathbf{1}_{\{(v_{j-1}+v_j^*+d_{j+1}+\phi-d_{j-1})\geq-\phi\}}\mid s_{j-1}]\\
&=\Pr[v_{j-1}\geq d_{j-1}-d_{j+1}-v_j^*-2\phi\mid s_{j-1}].
\end{aligned}\tag{A4}$$

Using these results, we can write the marginal net-benefit of increasing v_j^* as follows:

$$\begin{aligned}
\Pi'_{j-1}(v_j^*)=&f_j(v_j^*)\cdot\mathbb{E}[\max(v_{j-1}+(1-\rho)(v_j^*+d_{j+1})-d_{j-1}\ ,\ -\phi)\mid s_{j-1}]\\
&-f_j(v_j^*)\cdot\mathbb{E}[\max(v_{j-1}+v_j^*+d_{j+1}+\phi-d_{j-1}\ ,\ -\phi)\mid s_{j-1}]\\
&+(1-F_j(v_j^*))\Pr[v_{j-1}\geq d_{j-1}-d_{j+1}-v_j^*-2\phi\mid s_{j-1}].
\end{aligned}\tag{A5}$$

A necessary condition for an equilibrium in which agent j does not default is:

$$\Pi'_{j-1}(\underline{v}_j)\leq 0,\tag{A6}$$

that is, $v_j^*=\underline{v}_j$ is the optimal choice for agent $(j-1)$. This condition can be rewritten as:

$$\begin{aligned}
\frac{1-F_j(\underline{v}_j)}{f_j(\underline{v}_j)}\leq&\frac{\mathbb{E}[\max(v_{j-1}+\underline{v}_j+d_{j+1}+\phi-d_{j-1}\ ,\ -\phi)\mid s_{j-1}]}{\Pr[v_{j-1}\geq d_{j-1}-d_{j+1}-\underline{v}_j-2\phi|s_{j-1}]}\\
&-\frac{\mathbb{E}[\max(v_{j-1}+(1-\rho)(\underline{v}_j+d_{j+1})-d_{j-1}\ ,\ -\phi)\mid s_{j-1}]}{\Pr[v_{j-1}\geq d_{j-1}-d_{j+1}-\underline{v}_j-2\phi|s_{j-1}]}.
\end{aligned}\tag{A7}$$

For a default-free equilibrium to exist, this condition has to hold for all possible signals agent $(j-1)$ might receive, $s_{j-1}\in[\underline{s}_{j-1},\bar{s}_{j-1}]$.

Renegotiated debt values in a default-free equilibrium. The renegotiated face values in a default-free equilibrium then can be written in a recursive form as follows:

$$d_j=\underline{d}_j\equiv\underline{v}_j+d_{j+1}+\phi,\tag{A8}$$

assuming that we have $\overline{d}_j \geq \underline{d}_j$ for all j. Otherwise, if $\overline{d}_j < \underline{d}_j$, the offer will be just matching the previous offer, that is, $d_j = \overline{d}_j$. If $\overline{d}_j \geq \underline{d}_j$ for all j, we obtain the following explicit formulae:

$$d_N = \underline{v}_N + \phi, \tag{A9}$$

$$d_{N-1} = \underline{v}_{N-1} + d_N + \phi = \underline{v}_{N-1} + \underline{v}_N + 2\phi, \tag{A10}$$

$$d_{N-2} = \underline{v}_{N-2} + d_{N-1} + \phi = \underline{v}_{N-2} + \underline{v}_{N-1} + \underline{v}_N + 3\phi, \tag{A11}$$

$$d_j = \sum_{i=j}^{N} \underline{v}_i + (N - j + 1) \cdot \phi. \tag{A12}$$

Note that the borrower-specific default costs ϕ enter these debt values, whereas the proportional default costs captured by ρ do not. The reason for this is that in a default-free equilibrium, an agent j's borrower, agent $(j+1)$ is collecting the full face value from its borrower, agent $(j+2)$ (remember that default costs do not apply in equilibrium). Yet, the marginal borrower type (and the associated debt value) can be increased by the default cost ϕ in excess of the collateral, since a borrower is willing to pay that extra cost to avoid default.

Suppose that the following default-free face values are charged in equilibrium:

$$\underline{d}_{j-1} = \sum_{i=j-1}^{N} \underline{v}_i + (N - j + 2) \cdot \phi, \tag{A13}$$

$$\underline{d}_{j+1} = \sum_{i=j+1}^{N} \underline{v}_i + (N - j) \cdot \phi, \tag{A14}$$

which requires that the initial face value satisfies: $\overline{d}_j \geq \underline{d}_j$. Note that:

$$\begin{aligned} d_{j+1} - d_{j-1} &= \left[\sum_{i=j+1}^{N} \underline{v}_i + (N - j) \cdot \phi \right] - \left[\sum_{i=j-1}^{N} \underline{v}_i + (N - j + 2) \cdot \phi \right] \\ &= - \underline{v}_{j-1} - \underline{v}_j - 2\phi. \end{aligned} \tag{A15}$$

Using this result, we can now simplify the following terms entering our key efficiency condition (A7):

$$
\begin{aligned}
&\mathbb{E}[\max(v_{j-1}+\underline{v}_j+d_{j+1}+\phi-d_{j-1}\ ,\ -\phi)\mid s_{j-1}]\\
=&\mathbb{E}[\max(v_{j-1}+\underline{v}_j+\phi-\underline{v}_{j-1}-\underline{v}_j-2\phi\ ,\ -\phi)\mid s_{j-1}]\\
=&\mathbb{E}[\max(v_{j-1}-\underline{v}_{j-1}-\phi\ ,\ -\phi)\mid s_{j-1}]\\
=&\mathbb{E}[v_{j-1}\mid s_{j-1}]-\underline{v}_{j-1}-\phi,
\end{aligned}
\tag{A16}
$$

$$
\begin{aligned}
&\Pr[v_{j-1}\geq d_{j-1}-d_{j+1}-\underline{v}_j-2\phi|s_{j-1}]\\
=&\Pr[v_{j-1}\geq \sum_{i=j-1}^{N}\underline{v}_i+(N-j)\cdot\phi-\left(\sum_{i=j+1}^{N}\underline{v}_i+(N-j)\cdot\phi\right)-\underline{v}_j|s_{j-1}]\\
=&\Pr[v_{j-1}\geq \sum_{i=j-1}^{N}\underline{v}_i-\sum_{i=j}^{N}\underline{v}_i|s_{j-1}]\\
=&\Pr[v_{j-1}\geq \underline{v}_{j-1}]\\
=&1,
\end{aligned}
\tag{A17}
$$

$$
\begin{aligned}
&\mathbb{E}[\max(v_{j-1}+(1-\rho)(\underline{v}_j+d_{j+1})-d_{j-1}\ ,\ -\phi)\mid s_{j-1}]\\
=&\mathbb{E}[\max(v_{j-1}+(1-\rho)\underline{v}_j-\rho d_{j+1}+d_{j+1}-d_{j-1}\ ,\ -\phi)\mid s_{j-1}]\\
=&\mathbb{E}[\max(v_{j-1}+(1-\rho)\underline{v}_j-\rho d_{j+1}-\underline{v}_{j-1}-\underline{v}_j-2\phi\ ,\ -\phi)\mid s_{j-1}]\\
=&\mathbb{E}[\max(v_{j-1}-\underline{v}_{j-1}-\rho\underline{v}_j-\rho d_{j+1}-2\phi\ ,\ -\phi)\mid s_{j-1}],
\end{aligned}
\tag{A18}
$$

and

$$\begin{aligned}
&\mathbb{E}[v_{j-1}|s_{j-1}] - \underline{v}_{j-1} - \phi - \mathbb{E}[\max(v_{j-1} - \underline{v}_{j-1} - \rho\underline{v}_j - \rho d_{j+1} - 2\phi\ ,\ -\phi) \mid s_{j-1}] \\
=&\mathbb{E}[v_{j-1} - \underline{v}_{j-1} - \phi - \max(v_{j-1} - \underline{v}_{j-1} - \rho\underline{v}_j - \rho d_{j+1} - 2\phi\ ,\ -\phi) \mid s_{j-1}] \\
=&\mathbb{E}[\min(v_{j-1} - \underline{v}_{j-1} - \phi - (v_{j-1} - \underline{v}_{j-1} - \rho\underline{v}_j - \rho d_{j+1} - 2\phi), v_{j-1} - \underline{v}_{j-1} - \phi + \phi) \mid s_{j-1}] \\
=&\mathbb{E}[\min(v_{j-1} - v_{j-1} + \rho(\underline{v}_j + d_{j+1}) + \phi, v_{j-1} - \underline{v}_{j-1}) \mid s_{j-1}] \\
=&\mathbb{E}[\min(\rho(\underline{v}_j + d_{j+1}) + \phi, v_{j-1} - \underline{v}_{j-1}) \mid s_{j-1}]. \qquad \text{(A19)}
\end{aligned}$$

Using these simplifications, we can rewrite condition (A7) as follows:

$$\frac{1 - F_j(\underline{v}_j)}{f_j(\underline{v}_j)} \leq \mathbb{E}[\min(\rho \cdot (\underline{v}_j + d_{j+1}) + \phi, v_{j-1} - \underline{v}_{j-1}) \mid s_{j-1}]. \qquad \text{(A20)}$$

Since we imposed the standard regularity condition that the hazard rate $\frac{f_j(v_j)}{1-F_j(v_j)}$ is increasing on the support $[\underline{v}_j, \overline{v}_j]$, this condition is sufficient for the global optimality of agent $(j-1)$'s renegotiation strategy when it expects all other lenders to renegotiate their respective borrower's debt to its default-free level.

Policy: Mandated debt reductions. We start again with our general condition for a default-free equilibrium:

$$\begin{aligned}
\frac{1 - F_j(\underline{v}_j)}{f_j(\underline{v}_j)} \leq & \frac{\mathbb{E}[\max(v_{j-1} + \underline{v}_j + d_{j+1} + \phi - d_{j-1}\ ,\ -\phi) \mid s_{j-1}]}{\Pr[v_{j-1} \geq d_{j-1} - d_{j+1} - \underline{v}_j - 2\phi | s_{j-1}]} \\
& - \frac{\mathbb{E}[\max(v_{j-1} + (1-\rho)(\underline{v}_j + d_{j+1}) - d_{j-1}\ ,\ -\phi) \mid s_{j-1}]}{\Pr[v_{j-1} \geq d_{j-1} - d_{j+1} - \underline{v}_j - 2\phi | s_{j-1}]}. \qquad \text{(A21)}
\end{aligned}$$

Suppose that we start with a debt level d_{j-1} such that

$$d_{j-1} - d_{j+1} - \underline{v}_j - 2\phi < \underline{v}_{j-1}, \qquad \text{(A22)}$$

or equivalently:

$$\underline{v}_{j-1} + \underline{v}_j + d_{j+1} + \phi - d_{j-1} > -\phi. \tag{A23}$$

Then the initial condition can be written as:

$$\frac{1-F_j(\underline{v}_j)}{f_j(\underline{v}_j)} \leq \mathbb{E}[(v_{j-1} + \underline{v}_j + d_{j+1} + \phi - d_{j-1}) \mid s_{j-1}] \tag{A24}$$

$$- \mathbb{E}[\max(v_{j-1} + (1-\rho)(\underline{v}_j + d_{j+1}) - d_{j-1} \ , \ -\phi) \mid s_{j-1}], \tag{A25}$$

which further simplifies to:

$$\frac{1-F_j(\underline{v}_j)}{f_j(\underline{v}_j)} \leq \mathbb{E}[\min(\rho \cdot (\underline{v}_j + d_{j+1}) + \phi \ , \ v_{j-1} + \underline{v}_j + d_{j+1} - d_{j-1} + 2\phi) \mid s_{j-1}]. \tag{A26}$$

We can take the derivative of the right-hand side with respect to d_{j-1} and get:

$$\begin{aligned} & -\Pr[v_{j-1} + \underline{v}_j + d_{j+1} - d_{j-1} + 2\phi < \rho \cdot (\underline{v}_j + d_{j+1}) + \phi] \\ = & -\Pr[v_{j-1} < d_{j-1} - (1-\rho) \cdot (\underline{v}_j + d_{j+1}) - \phi]. \end{aligned} \tag{A27}$$

Note that we had assumed to begin with that:

$$\underline{v}_{j-1} > d_{j-1} - (\underline{v}_j + d_{j+1}) - 2\phi. \tag{A28}$$

Thus, as long as:

$$d_{j-1} \in (\underline{v}_{j-1} + (1-\rho)(\underline{v}_j + d_{j+1}) + \phi \ , \ \underline{v}_{j-1} + (\underline{v}_j + d_{j+1}) + 2\phi), \tag{A29}$$

this probability is strictly positive. That is, a *decrease* in d_{j-1} loosens the condition for agent $(j-1)$ to pick a renegotiated debt level that leads to no default.

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