TFP growth slowdown and the Japanese labor market in the 1990s

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Abstract

Esteban-Pretel, Julen, Nakajima, Ryo, and Tanaka, Ryuichi—TFP growth slowdown and the Japanese labor market in the 1990s

Unemployment in Japan nearly tripled during the 1990s. Underlying this upsurge lie an increase in the probability of workers to lose their jobs and a decrease in the probability that the unemployed find jobs. This paper analyzes the sources responsible for these labor market changes in Japan in the decade of the 1990s. We build, calibrate, and simulate a neo-classical growth model with search frictions in the labor market. Using actual TFP data, the model is able to reproduce the path of unemployment and the job flows, as well as that of output. We find it to be the decrease in productivity, coupled with the reduction in hours worked, which curtails the profits of firms, inducing a drop in employment and an increase in unemployment. J. Japanese Int. Economies 24 (1) (2010) 50–68. University of Tokyo, Japan; Yokohama National University, Japan; Tokyo Institute of Technology, Japan.

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1. Introduction

The 1990s in Japan have been labeled the Lost Decade. It was a time when output per capita grew at an average of 0.5%, far below the average of the previous decade, 3.2%, and that of the US during the

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same period, 2.6%. At the same time, the labor market suffered one of the worst periods in recent Japanese history. Workers were fired in record numbers, and unemployment reached a historical high of 5.4% in 2002, more than 2.5 times the level in 1990. Underlying this substantial increase in unemployment lie a decrease in the probability of unemployed workers to find jobs, and an increase in that of employed workers losing their jobs. This rise in unemployment has been a matter of great concern for economists and policy makers.

The aim of this paper is to explain the behavior of the Japanese labor market during the Lost Decade by investigating the causes of the increase in unemployment and the pattern of the flow of workers during the 1990s. To do so, we build, calibrate and simulate, using actual data, a neo-classical growth model with search frictions in the labor market. We use a deterministic general equilibrium framework based on the traditional growth model of Cass–Koopmans, with capital and labor as inputs of production, and we impose a search and matching labor market, in the style of Mortensen and Pissarides (1994).1 We also assume a government which taxes labor and capital income and spends its revenue in its own consumption. The calibration of the model parameters is performed to match the empirical evidence from Japan prior to the Lost Decade. We solve and simulate, using a two-boundary problem solution method, the transition path of the economy from an initial steady state, assumed to be 1990, to a new steady state far in the future. The driving force behind the switch in steady state is the drop in TFP growth. We also impose the empirically observed reduction in hours of work2 and the rise in government expenditure over the 1990s. To the best of our knowledge, this is the first paper that uses actual TFP data fed into a model with labor search frictions in order to account for the transition path of unemployment and other labor market variables between two steady states: this is one of the contributions of this paper.

We find that our model is capable of reproducing the increase in the unemployment rate from 1990 to 2002. The forces responsible for this increase, according to our simulations, are the drop in TFP growth and the decrease in hours of work. Each of these two factors contributes equally to the increase in unemployment by the year 2002, which implies that technology change alone explains around 50% of the movements in the labor market. The model not only accounts for the change in the level of unemployment between 1990 and 2002, it closely tracks the path of this variable during the transition. The model is also capable of reproducing the paths of the flows of workers in and out of unemployment, as well as the behavior of output observed in the data. Finally, we show that our results are robust to alternative specifications for the future path of TFP.

The intuition why the model can replicate the empirical path of output and the labor market variables is that the slowdown of TFP growth reduces the stationary level of productivity, which together with the fact that firms cannot work their employees longer due to the exogenous decrease in hours lowers the profits of the firm. This makes it more difficult for firms to continue the current matches, and increases the probability of job loss for the worker. At the same time, it discourages the posting of new vacancies, which reduces the probability of finding new jobs for each unemployed worker. The overall reduction in employment and the decrease in productivity accounts for the drop in output.

Our research is related to the strand of literature that offers explanations of the poor performance of the Japanese economy during the Lost Decade. One of the best known explanations is found in Hayashi and Prescott (2002). They show that the slowdown of TFP can account for the drop in detrended output and the behavior of other macroeconomic variables, such as the capital–output ratio and the after-tax return. An alternative hypothesis which has attracted attention is the so-called zombie lending explanation. Peek and Rosengren (2005) and Caballero et al. (2008) show how the misallocation of resources to unproductive (zombie) firms, following the financial bubble burst at the turn of the decade, could explain the downturn of the economy. Another explanation, put forward by Krugman (1998) and used in Eggertsson and Woodford (2003), is that the Japanese economy fell into a liquidity trap, which prevented the monetary authority from acting in an effective manner against the recession which began with the financial crisis. These papers have focused on explaining the performance of the Japanese economy measured by several of the main macroeconomic variables, such as output or the

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1 This type of non-Walrasian labor market allows the model to display involuntary unemployment in equilibrium.
2 As is explained in the empirical evidence section, during the 1990s the Japanese government introduced changes in regulations that effectively imposed a reduction of the length of the work week from 6 to 5 days.
capital–output ratio. However neither these nor other papers have tried to theoretically explain the reasons behind the increase in the unemployment rate and the evolution of the labor market flows during the 1990s.

Of the previous papers, Hayashi and Prescott (2002) is the closest to ours, since we focus on the TFP explanation to account for the Japanese labor market changes over the Lost Decade. We use this hypothesis since we believe that a decade-long recession, such as the one analyzed here, is deeply rooted in real factors. However, we do not discount the value of the other explanations, and in fact Fukao and Miyagawa (2008) show that the misallocation of resources between 1996 and 2002 can account for around 5% of the decline in Japanese TFP. Since the focus of this paper is not on the causes underlying the decline in TFP but the effects of this decline on the economy, we follow Hayashi and Prescott (2002) and assume exogenous TFP movements.

Our paper is also related to the growing literature which embeds a search and matching labor market in a general equilibrium framework. Pioneering in this literature are the works by Merz (1995) and Andolfatto (1996). Papers in this area have focused primarily on business cycle fluctuations, but have been under scrutiny following the criticisms raised by Shimer (2005) and Hall (2005). The former study shows that the textbook search and matching model cannot replicate the empirically observed volatility of unemployment using reasonable movements in productivity. Recent work by Hagedorn and Manovskii (2008), however, has shown that under certain parameterizations of the model, wages become very insensitive to changes in productivity, and unemployment becomes more responsive to such shocks. Although our paper focuses on medium-term changes in the economy and not business cycle fluctuations, we provide a discussion on the reasons for the success of our model in generating movements in unemployment. The two main reasons are the size of the productivity movements and the existence of capital in our model. The inclusion of capital, which is essential in a growth model, makes wages more insensitive to productivity changes, as in Hagedorn and Manovskii (2008), even though we do not use their calibration but rather that of Shimer (2005). The intuition is that capital rents make firms’ profits smaller, and this implies that the difference between the after non-labor cost flow profits for the firm and the flow value of unemployment is very small. This makes wages more insensitive, and unemployment more responsive to productivity movements.

The remainder of the paper is organized as follows. Section 2 briefly explains the experience of the Japanese economy in the 1990s, putting emphasis on the behavior of the labor market variables. Section 3 builds the theoretical model. Section 4 describes the calibration of the model parameters and the exogenous variables. Section 5 displays the simulation results. It also presents a robustness test of the results to alternative specifications of the future path of TFP. In Section 6, we discuss how our results relate to the Shimer–Hall critique. Finally, Section 7 summarizes and concludes the paper.

2. The Japanese Economy in the 1990s

During the 1990s, Japan experienced its worst economic times since World War II. Hayashi and Prescott (2002) report the basic facts about the Japanese economy over this period. As they show, detrended GNP decreased over the decade, and by 2000 its level was less than 90% of what it had been in 1990. This drop in detrended output was accompanied by an increase in the capital output ratio and a decrease in the after-tax return on capital. The focus of this paper is on unemployment and the underlying movements in the labor market, hence we now explain the characteristics of these variables during this turbulent decade.
Fig. 1 presents the evolution of the main labor market variables from 1990 to 2002.\(^6\)^\(^7\) As we can see in panel (a), the unemployment rate almost tripled in this period, increasing from 2.1% to a maximum of 5.4% in 2002. This increase is even more striking when compared to the previous unemployment rate maximum in the post-war era, which was 2.8% in 1988. Although not shown in Fig. 1, labor force participation was fairly stable, around 63%, between 1990 and 1996, decreasing to around 61% by the turn of the decade. These numbers imply that during the 1990s, the total number of workers, the employment to population ratio, and the employment rate all decreased.

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\(^6\) The source of the data is the same as in Hayashi and Prescott (2002), except for variables not studied in their study. In particular, for the labor market variables, data on unemployment and the flow probabilities comes from the Labor Force Survey, whereas hours per worker are taken from the establishment survey conducted by the Ministry of Welfare and Labor (Maigetsu Kinro Tokei Chosa). As in Hayashi and Prescott (2002), we consider the hours series for establishments with 30 or more employees.

\(^7\) We focus on the period between 1990 and 2002, since after 2002 the TFP stabilizes, and labor market variables either stabilize, as for instance hours of work do, or reverse their trend, as is the case for unemployment. Hence, since the focus of this paper is on the Lost Decade, 2002 seems the natural terminal point for the analysis.
In order to better understand the forces responsible for the changes in unemployment, it is useful to analyze the flows of workers in the labor market. Fig. 1b shows that over the 1990s, the quarterly probability for a worker to leave unemployment (left axis) decreased from 41% to 27%. At the same time, the probability of losing a job (right axis) increased from 0.8% to 1.9% from 1990 to 2002. However, it is interesting to note that, although the probability of finding a job decreased and that of losing a job increased, the total number of workers who both found and lost jobs in this period increased (Fig. 1c). That is, unemployment increased despite the fact that the number of workers moving out of unemployment and into employment increased over time. The reason is that the total number of workers losing their jobs was higher, therefore raising unemployment.

Simultaneous to the increase in unemployment, the government instituted several policies to reduce the number of hours worked. Some of these policies included the creation of new national holidays and the closure of government offices on Saturdays, which effectively reduced the length of the work week from 6 to 5 days for many workers. The evolution of hours is shown in panel (d) of Fig. 1, where we can see that weekly hours of work dropped from 43 to 38 in the course of the decade.

The increase in unemployment and decrease in hours worked was not paralleled by a decrease in detrended wages until the latter part of the decade. Fig. 1e shows how detrended real wages decreased to below 90% of the 1990 level by the end of the decade, and declined further with the turn of the century.

As stated in the introduction, the hypothesis of this paper is that the force behind the increase in unemployment and the other labor market movements, as well as the decrease in detrended output, is the slowdown in TFP growth, with the extra push provided by the government imposed decrease in hours worked. Fig. 1f shows the evolution of TFP since 1980. We can see that technology grew at a healthy rate during the 1980s, 1.9%, but this growth slowed considerably during the Lost Decade, when it averaged 0.3%. It seemed to recover after 2002, and averaged 1.5% from that year until 2006.

In summary, the 1990s in Japan saw a decrease in detrended output, an increase in unemployment due to a greater increase in the number of workers losing their jobs than those finding a new one, although both series increased, a decrease in hours worked due to changes in regulations, and a drop in detrended wages after the first half of the decade. We now proceed to explain the model used to try to account for these facts.

3. The model

The model is a discrete time neo-classical growth model, in the style of Cass–Koopmans, with three types of infinitely lived agents: consumers/workers, firms, and government. There is only one good, which is produced using capital and labor and sold by the firms to the consumers. The labor market is modeled in the style of the search and matching literature with endogenous job destruction. The replacement of the traditional Walrasian labor market for one with search frictions allows the model to display involuntary unemployment in equilibrium, essential to the focus of this paper. Furthermore, by assuming endogenous job destruction, we analyze the ability of the model to reproduce the increase in the rate of job destruction observed in the data over the Lost Decade.

We follow Hayashi and Prescott (2002) and assume that there is no uncertainty in terms of the aggregate exogenous variables in the economy, although the model still displays uncertainty at the...
firm and worker levels. We model individual uncertainty in terms of an idiosyncratic shock to the match, which is responsible for the heterogeneity across matches. However, this individual uncertainty disappears when the model is aggregated before solving it. In the following, we explain the decision problem faced by each of the agents and the definition of the equilibrium.

3.1. Household

We follow the literature on models with labor market frictions in a general equilibrium framework and assume that all of the individuals in the economy belong to a big family or household. This assumption delivers perfect self-insurance for the individuals in the family, and simplifies the problem by not requiring keeping track of employment and wealth distributions. The household owns the capital, which it rents to firms, and also earns income from the wages collected by the members who are employed and from the unemployment benefits or home production of the unemployed members. Both capital and labor income are taxed by the government. The household decides every period on the levels of consumption and savings, which maximizes its life-time utility.

Therefore, the household chooses \( \{ C_{t,i}, K_{t,i+1} \}_{t=0}^{\infty} \) to max

\[
\sum_{i=0}^{\infty} \beta^i u(C_{t,i})
\]

subject to

\[
C_{t,i} + K_{t,i+1} = (1 - \tau_n)W_{t,i} + \Pi_{t,i} + (1 - \delta)K_{t,i} + r_{t,i}K_{t,i} - \tau_k(r_{t,i} - \delta)K_{t,i}
\]

\[
+ (1 - n_{t,i})b_{t,i}z_{t,i} - T_{t,i},
\]

given \( K_0, \)

for \( i = \{0, \ldots, \infty\}, \) where \( \beta \in (0, 1) \) is the discount rate; \( C_i \) the consumption level of the household; \( K_i \) the total capital in the economy; \( W_t \) the total amount of wages paid to the employed workers; \( \Pi_t \) the total profits of the firms, which are distributed back to the household, since it ultimately owns them; \( r_t \) the rental rate of capital, and \( \delta \) its depreciation rate; \( \tau_n \) the labor income tax; \( \tau_k \) the capital income tax, where only the return net of depreciation is taxed; \( b_t \) the amount of home production or unemployment benefits collected by the unemployed workers; \( n_t \) the number of employed workers, and since population is normalized to one, and there are no workers out of the labor force, \( u_t = 1 - n_t \) the number of unemployed workers; \( z_t \) a variable that grows at the average growth rate of technology along the balance growth path, and it multiplies \( b_t \) and other stationary parameters throughout the model so that their importance does not decay as the economy grows; finally, \( T_t \) is the total amount of lump sum taxes collected by the government.

This problem yields the traditional consumption-Euler equation, which shows how the individual is indifferent between saving or consuming and extra unit in equilibrium:

\[
u'(C_t) = \beta[(1 - \tau_k)(r_{t-1} - \delta)]u'(C_{t-1}).
\]

3.2. Labor market

The labor market is modeled in the style of Mortensen and Pissarides (1994), where there exist search frictions, and workers and firms try match and form employment relationships. Firms produce using capital, labor and available technology, and matches are destroyed endogenously as an optimal decision by the firm and worker.

Employment relationships are of one worker and one firm, and matching occurs randomly according to a constant returns to scale matching function, \( m(u_t, \nu_t) \), where \( u_t \) is total unemployment and \( \nu_t \) is the number of vacancies. We define the market tightness as the ratio of vacancies to unemployed workers, \( \theta_t \equiv \frac{\nu_t}{u_t} \), and further define the probability that a firm meets a worker in any given period as \( q_x(\theta_t) \equiv m \left( \frac{\nu_t}{u_t}, 1 \right) \). Similarly, the probability that a worker matches with a firm is \( \theta_t q_x(\theta_t) \).

We model the endogenous destruction by assuming that productive firms need to pay, on top of the labor and capital costs, a non-productive intermediate input cost \( x_t \), which is idiosyncratic to each
match. The firm-specific intermediate input cost is independent and identically distributed across firms and time, with distribution function \(G: [x_{min}, x_{max}] \rightarrow [0, 1]\). A new idiosyncratic cost is drawn every period by existing matches, and if the cost is too high it may be beneficial for the firm and the worker to discontinue the employment relationship. The value of \(x_t\) which dissolves the match is denoted by \(x_t\). The probability of job destruction is therefore \(1 - G(x_t)\).

In a productive employment relationship, the firm produces output according to a constant returns to scale production function which has hours and capital as inputs. While capital is a choice variable for the firm, hours are not, and they are assumed to be exogenous to the firm and the worker.\(^1\) The production function of the individual firm is \(y_t = A_f(k_t, h_t)\), where \(A_f\) is total factor productivity (TFP), and \(y_t, k_t,\) and \(h_t\) are, respectively, output, capital and hours per worker. Hence, \(y_t\) and \(k_t\) are related to aggregate output and capital according to the following equations\(^\text{13}\):

\[
Y_t = n_t y_t \quad \text{and} \quad K_t = n_t k_t.
\]

The timing of the model is as follows. At the beginning of every period, the level of technology of the economy is revealed, and every matched firm draws an idiosyncratic cost. These two variables determine the number of productive and unproductive matches for the period. After destruction takes place, the levels of employment and unemployment are determined. At that point production starts at firms, and vacancies and unemployed workers try to meet in the labor market. At the end of the period, wages are paid and the firm’s profits are distributed to the household, which pays taxes and decides how much to consume and how much to save.

### 3.2.1. The firm’s problem

Firms post vacancies in the labor market and, when matched with a worker, implement optimal production plans in order to maximize their profits. Posting vacancies has a flow cost for the firm of \(\phi\), which is multiplied by the growing component of technology, \(z_t\), so that it does not disappear in the long run. If the firm is matched, and the idiosyncratic cost is low enough, below \(x_t\), the firm obtains the value of being filled in the following period, otherwise it remains as a vacancy. \(V_t\) and \(J_t(x_t)\) denote the values, measured in terms of consumption, of having a vacancy opened and of a match for a firm with the idiosyncratic cost \(x\), which hires a worker. The value of a vacancy is

\[
V_t = -\phi z_t + \beta_t \left[ q(\theta_t) \int_{x_{min}}^{x_{max}} J_{t+1}(z_{t+1}) dG(z_{t+1}) + (1 - q_t(\theta_t)G(x_{t+1}))V_{t+1} \right],
\]

where \(\beta_t = \beta_t(G(x_{t+1}))\) is the factor used by firms and workers to discount the future, since firms are ultimately owned by the household.

Free entry of firms is assumed in equilibrium, which implies that firms enter the market up to the point where the value of posting an extra vacancy is zero. Therefore, in equilibrium, we find that the value of a vacancy is

\[
0 = -\phi z_t + \beta_t q(\theta_t) \int_{x_{min}}^{x_{max}} J_{t+1}(z_{t+1}) dG(z_{t+1}).
\]

The value for the filled firm is

\[
J_t(x_t) = A_f(k_t, h_t) - r_t k_t - x_t z_t - w_t(x_t) h_t + \beta_t \int_{x_{min}}^{x_{max}} J_{t+1}(z_{t+1}) dG(z_{t+1}).
\]

\(^1\) We make this assumption to be consistent with the evidence explained in Section 2 which indicates that the decrease in hours observed in Japan over the 1990s was mostly due to changes in regulations by the government, rather than a voluntary reduction in hours on the part of firms or workers. This relationship between the change in labor laws and the decline in hours is already stated in Hayashi and Prescott (2002, pp. 209–210). Nevertheless, it could be argued that part of the decline in hours might also be the result of the increase in part-time jobs in Japan over the period of study. The rise in part-time employment would have reduced the average hours of work in the economy, even if every worker did not reduce his work hours. However, micro data analysis using the Employment Status Survey shows that the decline in hours per week occurred across the board for all types of workers (full-time, part-time, dispatch,…).

\(^\text{13}\) As is shown later, every firm chooses the same amount of capital, and hence produces the same quantity of output.
The interpretation of the previous equation is as follows. During the current period, given the firm’s idiosyncratic cost, $x_t$, the firm produces output and pays wages, the rental cost of capital and such intermediate inputs, $x_t$. In the following period, if the idiosyncratic cost is below the threshold, the match is still productive, with a value of $J_{t+1}(x_{t+1})$, otherwise the match is destroyed and it becomes a vacancy, which has value zero.

The firm chooses capital $k_t$ to maximize the present discounted value of being filled, which implies that it will rent capital to the point where its rental cost equals its marginal product,

$$r_t = A d f_t(k_t, h_t).$$

(8)

The optimal condition for capital determination, Eq. (8), implies that in equilibrium, every firm chooses the same amount of capital and therefore produces the same quantity of output.

We can now define the total profits of the firms, which are transferred to the household, as

$$P_t = n_t A f_t(k_t, h_t) - r_t n_t k_t - W_t - x_t^r z_t - v_t z_t,$$

where $x_t^r$ is the total amount of intermediate input costs paid by the firms. Total wages paid to the workers, $W_t$, are defined as the average wage, conditional on working, times the number of employed workers:

$$W_t = n_t \frac{1}{G(x_t)} \int_{x_{\text{min}}}^{x_t} w_t(x_t) h_t dG(x_t).$$

(9)

3.2.2. The worker’s problem

Consider now the problem for the worker. We denote by $U_t$ and $N_t(x_t)$, measured in terms of consumption, the values of being unemployed and of being matched with a firm with the idiosyncratic cost $x_t$.

An unemployed worker obtains $b_t$ units of consumption while unemployed. $b_t$ can be interpreted as home production, unemployment benefits, or simply the value of leisure in units of consumption. If the worker matches with a firm, which happens with probability $q_t q(\theta_t)$, and the idiosyncratic cost for the firm is below the threshold, $x_{t+1}$, he becomes a productive worker in the following period. If he does not enter into an employment relationship with a firm, he remains unemployed. Hence, the value of being unemployed at period $t$ is:

$$U_t = b_t z_t + \beta_t \left[ \theta_t q(\theta_t) \int_{x_{\text{min}}}^{x_{t+1}} N_t(x_{t+1}) dG(x_{t+1}) + (1 - \theta_t q(\theta_t) G(x_{t+1})) U_{t+1} \right].$$

(9)

As in the case of the firm, the value of a match for a worker is a function of the idiosyncratic shock $x_t$. The value of employment for a worker is composed by the after-tax wage and the continuation value, which is the value of being employed if the match is not destroyed, or the value of being unemployment if the idiosyncratic cost is too high.

$$N_t(x_t) = (1 - \tau_t w_t(x_t) h_t + \beta_t \left[ \int_{x_{\text{min}}}^{x_{t+1}} N_t(x_{t+1}) dG(x_{t+1}) + (1 - G(x_{t+1})) U_{t+1} \right].$$

(10)

3.2.3. Surplus, bargaining, wages, and destruction threshold

When an employment relationship takes place, it creates a surplus which is shared between the firm and the worker. The surplus of the match is defined as the sum of the values of a filled job for a firm and a worker minus their outside options, which are the value of a vacancy and the value of unemployment, respectively. Since there is free entry of firms, the expression for the surplus is $S_t(x_t) = J_t(x_t) + N_t(x_t) - U_t$.

Wages are chosen as the Nash solution to the following bargaining problem, where $\eta$ is the bargaining power of the worker:

$$\max_{W_t(x_t)} (N_t(x_t) - U_t)^\eta (J_t(x_t) - V_t)^{1-\eta}.$$
The previous problem delivers the sharing rule for the surplus, and it implies that both firm and worker receive a constant fraction of the surplus. Due to the existence of labor taxes, the fraction differs from the bargaining power, and the optimal sharing rules are:

\[ N_t(x_t) - U_t = \eta \frac{1 - \tau_n}{1 - \eta \tau_n} S_t(x_t) \quad \text{and} \quad J_t(x_t) = (1 - \eta) \frac{1}{1 - \eta \tau_n} S_t(x_t). \] (11)

Combining the previous two expressions with Eqs. (7)–(10), the surplus for a match can be expressed as:

\[ S_t(x_t) = A f(k_t, h_t) - r_t k_t - x_t z_t - \tau_n w_t(x_t) h_t - b_z t + \beta_t \left(1 - \theta_t q(\theta_t) \eta \frac{1 - \tau_n}{1 - \eta \tau_n} \int_{x_{\text{min}}}^{x_t} S_{t+1}(x_{t+1}) dG(x_{t+1}). \right) \] (12)

The expression for the wage is:

\[ w_t(x_t) h_t = \eta A f(k_t, h_t) - r_t k_t - x_t z_t + \theta_t \phi z_t + (1 - \eta) \frac{b_z t}{1 - \tau_n}. \] (13)

The worker is compensated for a proportion \( \eta \) of the production of the firm, and for a measure of the saved cost of searching for new matches. He is also compensated for a fraction \( (1 - \eta) \) of the for-gone home production, adjusted by taxes.

An employment relationship is terminated when the idiosyncratic intermediate input cost to the firm is so high that it drives the surplus to zero.\(^{14}\) This determines the threshold cost above which both firm and worker agree to dissolve the match and search for better options. Using Eq. (12) and equating it to zero, we obtain the expression for the threshold:

\[ \hat{x}_t z_t = A f(k_t, h_t) - r_t k_t - \tau_n w_t(\hat{x}_t) h_t - b_z t + \beta_t \left(1 - \theta_t q(\theta_t) \eta \frac{1 - \tau_n}{1 - \eta \tau_n} \int_{x_{\text{min}}}^{x_t} S_{t+1}(x_{t+1}) dG(x_{t+1}). \right) \] (14)

### 3.2.4. Evolution of unemployment

Finally, given this timing explained earlier and normalizing the labor force to unity, the evolution of unemployment is characterized by the following equations:

\[ u_t = [1 - \theta_{t-1} q(\theta_{t-1}) G(\hat{x}_t)] u_{t-1} + [1 - G(\hat{x}_t)] n_{t-1} \] (15)

\[ n_t = 1 - u_t, \] (16)

where \( \theta_{t-1} q(\theta_{t-1}) G(\hat{x}_t) \) is the fraction of workers who found a successful match the previous period and \([1 - G(\hat{x}_t)]\) is the proportion of employed workers who lost their jobs.

### 3.3. Government

The government taxes the household and uses the revenues to finance its expenditures on goods and services, which are assumed to have no productive or utility use. We express government spending as a fraction of aggregate output, where \( \psi_t \) is the share of these expenditures in output. The government budget constraint, which is balanced every period, is:

\[ \psi_t Y_t = \tau_n W_t + \tau_r (r_t - \delta) K_t + T_t \] (17)

\(^{14}\) As can be seen from the sharing rules in Eq. (11), when the surplus is zero, the value for the match for both the firm and the worker are also zero. Hence, the decision to dissolve a match is shared by the firm and worker, which is standard in search and matching models in the style of Mortensen and Pissarides (1994).
3.4. Equilibrium

A perfect foresight competitive equilibrium, given the path of TFP and hours \( (A_t, h_t) \), a government policy \( \{\tau_n, \tau_k, \psi_t, T_t\} \) and \( K_0 \), is a set of prices \( \{r_t, w_t(x_t)\} \) and allocations \( \{Y_t, K_{t+1}, C_t, k_t, n_t, u_t, v_t, \theta_t, x_t\} \) which satisfy that: (i) agents optimize, i.e. the household’s optimal condition (3), the value functions in the labor market (6)–(10), the capital rental optimal condition (8), and the optimal surplus sharing rule (11) are satisfied; and (ii) markets clear for consumption goods, \((1 - \psi_t) Y_t = C_t + K_{t+1} - (1 - \delta) K_t - (1 - n_t) b z_t + \phi z_t v_t + n z_t x_t^2\); capital, Eq. (4); and labor, Eqs. (15) and (16); (iii) the government has a balanced budget, as in Eq. (17).

In order to solve the model, we rewrite the equilibrium conditions in terms of stationary variables. These non-growing variables are obtained by dividing each by \( z_t \), which grows at the rate \( \frac{g}{1 + z} \), the average growth rate of the TFP factor, \( A_t^{\frac{g}{1 + z}} \), in the balance growth path.\(^{15}\) The steady state of this economy is a perfect foresight stationary equilibrium in which all detrended variables remain constant across time. The definition of this perfect foresight stationary equilibrium can be found in Appendix A.

4. Parametrization and exogenous variables

We now proceed to explain the method used to parametrize the model, and we later explain the simulation technique, along with the assumptions related to the exogenous variables of the model.

4.1. Calibration

We choose functional forms which are standard in the literature and then calibrate the parameters of the model to match the empirical evidence for Japan in 1990, which is assumed to be the initial steady state in our simulations. We set the length of the period to one quarter.

The utility function is assumed to be logarithmic, \( u(C_t) = \log(C_t) \). The discount factor, \( \beta \), is calibrated using the stationary Euler Eq. (19), shown in Appendix A, to match the quarterly capital–output ratio of Japan in 1990, 7.82, and set to \( \beta = 0.9957 \).

The labor tax rate, \( \tau_n \), was fairly stable over the 1990s, and the capital income tax, \( \tau_k \), fluctuated only slightly over the decade. We set these rates to the average from 1990 to 1996 using Mendoza et al. (1994) extended data from Enrique Mendoza’s website.\(^{16}\) These tax rates are set to \( \tau_n = 0.28 \) and \( \tau_k = 0.44 \).\(^{17}\)

The production function of the firms is assumed to have a standard Cobb-Douglas form, \( f(k_t, h_t) = k_t^{\alpha} h_t^{\beta} \). We set the share of capital rents in output, \( \alpha \), to 0.383. The depreciation rate of capital, \( \delta \), is set to 0.028. These two numbers are estimated extending the data used in Braun et al. (2006b) and they are used to calculate the TFP of the economy. The initial level of technology, \( A_0 \), is normalized to unity, and the average and long-run growth rate of TFP, \( \gamma \), is assumed to be 1.5%, which is the average in the data from 2002 to 2006.

In the labor market, we assume that the bargaining power of the worker, \( \eta \), is 0.5, as has become standard in the literature. The matching function in the labor market is Cobb-Douglas, \( m(u_t, v_t) = \mu u_t^{\xi} v_t^{\frac{\eta}{1 - \xi}} \). We follow what is standard in these types of models and set the elasticity of matching with respect to unemployment, \( \xi \), to 0.5. The idiosyncratic cost to the firm is drawn from an exponential distribution, \( x \sim \frac{1}{\mu} e^{-\frac{x}{\mu}} \), which only requires to calibrate one parameter, the mean of the distribution, \( \mu \). Hence, \( x_{\text{min}} = 0 \) and \( x_{\text{max}} = \infty \). \( \chi \) is jointly calibrated with the scaling parameter in the matching function, \( \mu \), and the cost of posting a vacancy, \( \phi \), to match the unemployment rate

\(^{15}\) \( \gamma \) is the average growth rate of TFP, \( A_t \), along the balance growth path.

\(^{16}\) The extended data only reports taxes up to 1996. Enrique Mendoza’s website is http://www.econ.umd.edu/~mendoza.

\(^{17}\) While not shown in the results, when we perform the simulations allowing the capital income tax (the only one that seemed to change over this period) to vary as it does in the data, the simulations do not vary significantly, which implies that the results are robust to our assumption of constant capital tax rates.
and probability of leaving unemployment in Japan in 1990, 0.021, and 0.42, respectively, and a market tightness of unity.\(^\text{18}\) We set these parameters to \(v = 0.0407\), \(l = 0.42\), and \(/ = 0.037\).

We assume that the value of leisure, home production, or unemployment benefit, \(b_t\), is a fraction \(\lambda\) of the output that the average worker would produce in the firm, or what is the same in standard search and matching models, the marginal product of one worker. We set \(\lambda = 0.4\), which is the value used in Shimer (2005). This value of \(\lambda\) implies a replacement ratio in terms of the average wage in the model of 0.68, which is consistent with the replacement ratio in Japan of between 60% and 80% of the last wage received.\(^\text{19}\)

The values for the model parameters are summarized in Table 1.

### 4.2. Simulation technique and path of exogenous variables

The model presented above is simulated by assuming that the economy transitions from its position in 1990, which is assumed to be an initial steady state, to a new steady state at a point far enough in the future.\(^\text{20}\) The length of time in the model simulations is one quarter, but the data is later aggregated by year, since we are interested in the long-run transition of the economy and not in its short-term fluctuations. The simulation is deterministic, and the agents know the path of the variables that change exogenously over time.

The data on TFP growth rate, hours per worker and government expenditure, is an extended version of the data set of Braun et al. (2006a).\(^\text{21}\) The path of these exogenous variables in the data is as follows:

- **Growth rate of technology, \(\gamma_t\):** The path of TFP growth rates, \(\gamma_t\), is calculated from the data as the period by period change, i.e. \(e^{\gamma_t} = \frac{A_t}{A_{t-1}}\), from 1990 to 2002 and then remains constant at the long-run value, \(\gamma\).
- **Hours of work, \(h_t\):** These decrease from the initial level of 44 to the final level of 38 over the sample period, following the path shown in Fig. 1d. They remain constant after the last period at the final level.
- **Government expenditure share in output, \(\psi_t\):** This variable fluctuates between 12% and 14% over the studied period. The actual path of government purchases as a share of output is shown in Fig. 5 of Hayashi and Prescott (2002). We assume that it remains constant at the final level after the last period.

### 5. Simulation results

The results of the simulations are shown in Figs. 2 and 3. These figures show that the model is successful in replicating the path observed in the data from 1990 to 2002 for the labor market variables, as well as that of output.

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\(^\text{18}\) As explained in Shimer (2005), changing the value of the market tightness only rescales the value of \(\mu\), leaving everything else unchanged.

\(^\text{19}\) Although not detailed in this paper for the sake of brevity, the results shown in the Section 5 are robust to changes in \(\eta\) and \(\zeta\) within standard values (0.4–0.6). They are also robust to reducing the replacement ratio, \(\lambda\), below the benchmark 0.4–0.3 and even 0.2. However, the model calibration does not allow us to increase the value of \(\lambda\) much higher than the benchmark. The reason is the existence of capital and endogenous job destruction in the model, which, given the other parameter values, implies that raising the outside option of the worker too greatly prevents the existence of matches with surplus above zero.

\(^\text{20}\) The model is simulated using the equations shown in Appendix A, the parameters from Section 4.1 and the paths of exogenous variables explained in this subsection. The simulations were performed using the Dynare package for Matlab, version 3.065.

\(^\text{21}\) We are thankful to Nao Sudou for his help on the extension of the data. The original and extended data sets can be found in the research section of Julen Esteban-Pretel’s website <http://www.e.u-tokyo.ac.jp/~julen>.
Fig. 2. Data and simulation’s output, unemployment rate and wages.

Fig. 3. Data and simulation’s flows in and out of unemployment.
Fig. 2a shows that the decrease in TFP growth and the decline in hours are the main causes of the sharp increase in the unemployment rate in Japan over the 1990s. The model can also replicate well the actual path of unemployment over the decade. The intuition for the success of the model is as follows. The drop in productivity growth produces a decline in the detrended productivity level, which along with the fact that firms cannot work their employees longer due to the exogenous decrease in hours decreases the profits of the firm. Productive firms face higher incentives to discontinue matches, which increases the probability for workers to lose their jobs, as shown in Fig. 3b. At the same time, potential entrants expect lower future profits, and therefore fewer vacancies are posted, which with the increase in unemployment reduces the probability for non-working individuals to find jobs, as shown in Fig. 3a. The model is able to capture the movements of these two variables in the data, although the drop in the probability of finding a job in the model is not as pronounced as in the data. When looking at the fraction of workers who find and lose jobs every period, the model tracks the increase in both variables well, as seen in Fig. 3c and d. Hence, as is the case in the data, the model produces an increase in unemployment due to a greater increase in the number of workers losing their jobs than of those finding employment.

The behavior of detrended output over the 1990s, which is already well explained by Hayashi and Prescott (2002), is shown in Fig. 2b. We can see that the inclusion of search frictions in the labor market does not worsen the ability of the model to track the path of output, at which it does a good job. However, it is worth noting one variable, the behavior of which our model is not able to replicate although that of Hayashi and Prescott (2002) can, namely the capital–output ratio. This variable is not shown in the figures, but increases in the data from 7.82 to 9.66, while in the model it does not increase, since the long-run TFP growth remains constant throughout the simulation.

In the data, detrended wages decreased over the course of the decade by more than 15%. The model is able to reproduce the level drop in wages, but it initially decreases too quickly, while in the data it took 5–6 years to begin dropping. The model’s immediate decrease in wages is caused by the reduction in the value of the outside option for the workers due to the increase in unemployment (the labor market becomes less tight), which lowers the outside option of the worker and decreases the wages. One possible explanation for the fact that wages in the model decrease faster than in the data during the first half of the decade could be that wages in Japan are not as flexible as is implied by the Nash bargaining assumption of the model. It is also possible that the bargaining power of the workers, which in some respects serves as a proxy for the power of the unions, decreased as the recession continued over the years. Perhaps workers were forced to accept lower wage increases after it was clear that Japan was facing a severe recession. While wage setting arrangements different from the one implied by Nash bargaining, or changes in bargaining power of workers, are possible in our model, we abstract from these here, since we want to understand how much can be explained by the standard search and matching model embedded into the Neo-classical growth framework.

Although not shown in the figures, it is worth stating that each of the two sources of variation, TFP growth decline and drop in hours worked, contribute about half of the change in unemployment in our simulation. This implies that the drop in growth rate of TFP alone is not sufficient to generate the observed changes in the data in our model. The decline in hours imposed by the government also plays a crucial role.

5.1. Robustness to the future path of TFP

The simulation results shown above are obtained under the assumption that the level of the exogenous variables remains constant at the 2002 level after that year. While this assumption is

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22 Despite the limited role of government expenditures in the simulation results, they are included in the model for two reasons. The first is for consistency between the model and the data variables, since output in the data is the net of government purchases. The second is that Hayashi and Prescott (2002) use government expenditures in their model, and therefore, by including them here we can make clear comparisons of our results with theirs for the non-labor market variables.

23 These explanations could also potentially help to further reconcile the model with the data in terms of the job finding rate.

24 This assumption is standard in papers with this type of simulation technique, such as Hayashi and Prescott (2002, 2008) and Chen et al. (2006).
consistent with the data up to the year 2006, if Japan were to stay in the balance growth path implied by the final TFP growth, there would be a permanent divergence between the Japanese and the US economies. Although there are studies which rationalize the existence of differences in the levels of TFP across economies, such as Parente and Prescott (1994), the notion of these two economies diverging permanently, although also maintained in Peek and Rosengren (2005), can be thought of as unappealing.

In this section, we check the robustness of our results to an alternative specification of the future path the TFP. Here we assume that TFP is as observed in the baseline simulation until 2006, and then starts returning to the pre-1990s balance growth path. We assume that it takes 10 years to reach that path. In other words, TFP grows at a rate of 3.72% from 2006 to 2016 while transitioning back to the original balance growth path, and then remains constant at 1.9% after reaching it in 2016.25

Fig. 4 shows the evolution of the unemployment rate in the data and in the simulation under the two specifications for the future path of TFP, the benchmark and the alternative. The first thing to note is that in both cases the model does a good job of replicating the path of unemployment in the 1990s. Therefore, the results are robust to changes in the assumed future path of TFP.

The second point of note is that under both specifications, the model predicts that the unemployment rate should have continued to increase beyond the level of 2002, whereas the data shows that unemployment started to decrease after that year. Therefore, it seems that something took place in the Japanese economy which reversed the increase in unemployment in 2002 and is not explained by TFP changes. Ohtake (2004) and Yamakawa (2001) describe several measures taken by the Japanese government after 1998 in order to try to control the rise in unemployment. These measures include economic package incentives to revitalize employment and changes in the labor law to flexibilize the labor market. Among the labor law changes, the most important are the revision of the Labor Standards Law, the Worker Dispatch Law, and the Employment Security Law. These amended laws meant a structural change to the labor market, which allowed firms to hire workers in more flexible circumstances, such as through employment placement services or as dispatched workers, and may have

25 Under this specification, we could consider the period between 1990 and 2016 as a very long cycle, or a medium-term cycle, rather than a change in the balance growth path.
increased the incentive for firms to hire workers and eventually have contributed to the reduction in unemployment. A detailed investigation is needed to conclude whether or not the cause of the decrease in unemployment after 2002 was a structural change in the labor market; given that the purpose of this paper is to account for the increase in the unemployment rate in Japan over the 1990s, we leave this issue for future research.

6. Discussion

As explained in the introduction, the search and matching models have been under recent scrutiny following the critiques of Shimer (2005) and Hall (2005). In particular, Shimer (2005) shows that under a calibration similar to ours, the basic search and matching model is not able to reproduce the behavior of unemployment and vacancies over the business cycle for reasonable movements in productivity. Specifically, it is the volatility of these variables that the model has difficulty replicating. As shown in the previous section, our model is able to reproduce the movements in unemployment over the 1990s in Japan. Although our model does not consider business cycle frequencies but medium-term changes, it is worth analyzing why it succeeds in replicating the changes in unemployment.

The main reason that our model can replicate the increase in unemployment in Japan over the 1990s is that there are two sources of fluctuations. Changes in TFP account for half of the increase in the unemployment rate, whereas the other half is due to the decrease in hours. In this sense, our results also indicate the inability of the standard search and matching model to fully account for observed changes in unemployment only through TFP changes. However, in our simulations, productivity movements explain 50% of the change in unemployment, whereas in Shimer (2005), productivity shocks only explain around 5% of the volatility of unemployment. There are two main reasons for this higher explanatory power in our model. The first is the size of the fluctuations in productivity. TFP growth declined in Japan for over a decade. This growth decline had an effect of the path of TFP equivalent to a 11% decline in its stationary level. This drop in detrended productivity is much larger than the fluctuations observed at business cycle frequencies, and especially than the 2% volatility of US productivity reported by Shimer (2005). The second reason is the existence of capital and labor taxes, which make wages less responsive to productivity shocks through the same mechanism put forward by Hagedorn and Manovskii (2008), and in turn makes unemployment more sensitive to productivity movements. Let us explain this mechanism further.

Hagedorn and Manovskii (2008) show that if the difference between the productivity of the worker and the flow value of unemployment \((p - z)\) in their model) is very low, then wages become very insensitive to productivity changes, and the vacancy posting behavior of firms becomes more responsive to such movements. They claim that calibrating the flow value of unemployment to be 40% of the productivity of the worker, as in Shimer (2005), is too low, and in their preferred calibration they set that fraction to be 95.5%, i.e. \(\frac{z}{p} = 0.955\). Under their preferred calibration, they show that the basic search and matching model is capable of reproducing the observed volatility of unemployment in the US for reasonable movements in productivity.27 In our model, given the existence of capital28 and taxes, the equivalent ratio to \(\frac{z}{p}\) from the basic text-book model is different. Here, firms need capital to produce output and must pay for its rental cost. These costs reduce the flow profits for the firm by a percentage equal to a share of capital rents in output, \(r = 38\%\). The existence of labor taxes, \(\tau_n\), implies that the worker gets a fraction \(\frac{1 - r}{1 - \tau_n}\) of the forgone flow value of unemployment, higher than \(1 - \eta\) as in the basic search and matching model. Therefore, if we look at our stationary wage equation, (25), the equivalent ratio to \(\frac{z}{p}\) in our model is

\[
\frac{b}{Ak^h(1-r) - rk - nx^a}.
\]

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26 Similar labor market changes have been argued as being responsible for the decrease in unemployment for European countries with high unemployment rates (i.e. Siebert, 1997 and Blanchard and Wolfers, 2000).

27 See Hagedorn and Manovskii (2008) for a more detailed explanation of the mechanism and the intuition behind this result.

28 We also have an additive intermediate input cost, but it represents less than 3% of the flow profit of the average firm.
Having capital rents and taxes in the model implies that even though we follow Shimer (2005) and set the stationary flow value of unemployment to be 40% of the productivity of the worker,29 the equivalent ratio to \( z \) in our model, Eq. (18), is 94.75%. Hence, in our simulations, as in Hagedorn and Manovskii (2008), unemployment is much more responsive to movements in productivity than in Shimer (2005), which is why we can account for 50% of the movements in the unemployment rate.30

Finally, although not shown in the figures for the sake of brevity, we have performed an alternative simulation of the model without capital and using the appropriate measure of productivity,31 and have found that the model accounts only for 10% of the change in unemployment by the year 2002. This provides further evidence for the premise that it is the existence of capital which is responsible for the better performance of our model compared to the basic textbook version.

7. Conclusions

During the 1990s, Japan suffered the biggest increase in unemployment of the post-war era. Underlying this rise in unemployment was an increase in the destruction of jobs relative to job creation. In this paper, we build, calibrate and simulate a neo-classical growth model with search frictions in the labor market to try to account for the changes seen in the Japanese unemployment rate and the flows of workers during the Lost Decade. In our simulations, we feed the path of actual TFP, hours of work, and government expenditures from the data into the model, to not only assess its ability to replicate the medium-run changes in the economy, but also the actual path of the variables.

We find that the slowdown in TFP growth, together with the decrease in hours worked, can explain the changes observed in the labor market. The reason for this is that the drop in TFP growth, which reduces detrended TFP, together with the fact that firms cannot work their employees for more hours, induces a reduction in firms’ profits in the model. This in turn makes it harder for firms to keep workers and to hire new ones, increasing the probability for workers to lose their jobs and reducing the probability for unemployed workers to find jobs. In the model, these changes induce increased unemployment and reduce output, as in the data.

Finally, we should note that while this paper studies the Japanese experience over the Lost Decade, there is nothing specific to Japan or this period in our model and methodology. Hence, our framework can be used to analyze other episodes of medium-run changes in the labor market and other macroeconomic variables.

Acknowledgment

The authors wish to thank Toni Braun, Junichi Fujimoto, Fumio Hayashi, Hidehiko Ichimura, Daiji Kawaguchi, Keisuke Otsu, Yasuyuki Sawada, Katsuya Takii and the seminar participants at RIETI and GRIPS for helpful comments and discussions. We are also grateful to Nao Sudou for his help in constructing part of the data used in this paper, and to Sachiko Kuroda for sharing her data on worker flows. The authors would also like to thank two anonymous referees for their comments and suggestions.

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29 As explained in the calibration, setting \( b = 0.4 \cdot \bar{A}(k, h) \) implies a replacement ratio of 68%, which is consistent with Japanese data.

30 Note that in our model the movement in productivity accounts for 50% of the change in unemployment, whereas in Hagedorn and Manovskii (2008) productivity shocks produce slightly more unemployment volatility than in the US data. The reason that productivity changes only account for 50% of the unemployment changes is twofold. First, there is no study that has analyzed whether the “Shimer-Hall” puzzle applies to the Japanese economy. Therefore, there is no evidence of how much of the cyclical volatility of Japanese unemployment can be accounted for by technology shocks using the standard search and matching model. Second, since we are looking at medium term changes in the economy, and not at fluctuations around a steady state, we assume that over time, the flow value of unemployment also changes when productivity varies, as can be seen in Eq. (26). This assumption is consistent with the empirical evidence for Japan, where the replacement ratio decreased over the Lost Decade. This implies that in our model, wages are not as insensitive as in Hagedorn and Manovskii (2008), where the flow value of unemployment is constant. However they are much more insensitive than in Shimer (2005) for the reasons explained above; our model therefore accounts for more than Shimer, but less than Hagedorn and Manovskii.

31 If capital is not present in the model, the empirically relevant measure of productivity is no longer TFP but output per unit of labor input, i.e. \( \frac{Y}{L} \). The results of this alternative simulation are available from the authors upon request.
Appendix A. Stationary equilibrium

Define \( \gamma_t \) as the growth rate of TFP, i.e. \( e^{\gamma_t} = \frac{A_t}{A_{t-1}} \), and the TFP factor as \( A_t^{1-k_t} \). To obtain the stationary equilibrium we use \( z_t \), which is a variable growing at the average growth rate of the TFP factor along the balance growth path, \( \frac{g_t}{r_t} \). The growing variables in the economy can be rendered stationary by deflating them by \( z_t \) as follows:

\[
\bar{Y}_t = \frac{Y_t}{z_t}, \quad \bar{K}_t = \frac{K_t}{z_t}, \quad \bar{C}_t = \frac{C_t}{z_t}, \quad \bar{k}_t = \frac{k_t}{z_t}, \quad \bar{T}_t = \frac{T_t}{z_t}.
\]

Since we assume that technology is labor augmenting, and given the functional form of the production function, \( f(k_t, h_t) = k_t^a h_t^{1-a} \), we define detrended TFP as \( \bar{A}_t = \frac{A_t}{z_t} = \bar{A}_{t-1} e^{(\gamma_{t-1})} \). In the simulations, we normalize the initial value of technology to unity, i.e. \( A_0 = \bar{A}_0 = 1 \). Finally, denote the average stationary wage, conditional on being productive, as \( \bar{w}_t = \frac{1}{e^x_{z_t}} \int_{x_{tmin}}^{x_t} w_t(x_t) dG(x_t). \)

We can define a perfect foresight stationary competitive equilibrium, for a given path of exogenous of TFP growth rate and hours \( \{\gamma_t, h_t\}_{t=0}^{\infty} \), and government policy \( \{\tau_n, \tau_k, \psi_t\}_{t=0}^{\infty} \), as a set \( \{r_t, \bar{w}_t, \bar{Y}_t, \bar{K}_{t+1}, \bar{C}_t, \bar{k}_t, n_t, u_t, v_t, \theta_t, x_t^g, b_t, \beta_t, T_t\}_{t=0}^{\infty} \) which satisfy the following equations:

- **Euler equation:**
  \[
  1 = \beta \left( 1 + (1 - \tau_k)(r_{t+1} - \delta) \right) \frac{\bar{C}_{t+1}}{\bar{C}_t} e^{\frac{\gamma_t}{r_t}}. \tag{19}
  \]

- **Aggregate resource constraint:**
  \[
  (1 - \psi_t) \bar{Y}_t = \bar{C}_t + \bar{K}_{t+1} e^{\frac{\gamma_t}{r_t}} - (1 - \delta) \bar{K}_t - (1 - n_t)b_t + \phi v_t + n_t x_t^g. \tag{20}
  \]

- **Aggregate output:**
  \[
  \bar{Y}_t = \bar{A}_{t-1} e^{(\gamma_{t-1})} n_t k_t^a h_t^{1-a}. \tag{21}
  \]

- **Aggregate capital:**
  \[
  \bar{K}_t = n_t k_t. \tag{22}
  \]

- **Optimal capital rental choice for the firm:**
  \[
  r_t = \alpha \bar{A}_{t-1} e^{(\gamma_{t-1})} \bar{k}_t^{a-1} h_t^{1-a}. \tag{23}
  \]

- **Average intermediate input cost in productive matches:**
  \[
  x_t^g = \frac{1}{G(x_t)} \int_{x_{tmin}}^{x_t} x_t dG(x_t) \tag{24}
  \]

- **Optimal wages:**
  \[
  \bar{w}_t k_t = \eta \left[ \bar{A}_{t-1} e^{(\gamma_{t-1})} \bar{k}_t^a h_t^{1-a} - r_t \bar{k}_t - n_t x_t^g + \phi \theta_t \right] + (1 - \eta) \frac{b_t}{1 - \tau_n}. \tag{25}
  \]

- **Value of home production/leisure:**
  \[
  b_t = \lambda \bar{A}_{t-1} e^{(\gamma_{t-1})} \bar{k}_t^a h_t^{1-a} \tag{26}
  \]

- **Creation condition:**
  \[
  0 = -\phi + \beta \mu t^{\theta_t} \left( \frac{1 - \eta}{1 - \eta \tau_n} e^{\frac{\gamma_t}{r_t}} G(x_{t+1}) (x_{t+1} - x_t^g) \right). \tag{27}
  \]
• Destruction condition:
  \[ \dot{x}_t = A_{t-1} e^{(\alpha - \gamma_1)h_{t-1}} - r_k h_t - \tau_n w_t h_t - b_t + \beta_t \left( 1 - \mu \right) \left[ 1 - \left( 1 - \tau_n \right) e^{\gamma_1 G(x_{t+1})} (x_{t+1} - x_t^*) \right]. \]
  (28)

• Evolution of unemployment and employment:
  \[ u_t = [1 - \mu (1 + G(x_t))] u_{t-1} + [1 - G(x_t)] n_{t-1}, \]
  \[ n_t = 1 - u_t, \]
  (29)
  (30)

• Market tightness:
  \[ \theta_t = \frac{\theta_t}{u_t}. \]
  (31)

• Stochastic factor:
  \[ \beta_t = \beta_t \frac{C_t}{C_{t+1}} e^{\gamma_1 G(x_{t+1})}. \]
  (32)

• Government budget constraint:
  \[ \psi_t \dot{Y}_t = \tau_n w_t n_t + \tau_k (r_t - \delta) \bar{K}_t + \bar{T}_t. \]
  (33)

Appendix B. Supplementary material


References