# Liquidity, Unemployment, and the Stock Market*

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**Abstract**

Interest-rate spreads and the unemployment rate vary negatively with stock prices. Liquidity plays a role in a Mortensen-Pissarides economy with a twist: households self-insure against preference shocks by accumulating equity claims. Higher stock market valuations relax liquidity constraints, creating an aggregate demand channel that strengthens firms' hiring incentives and the creation of private assets. That is, assets create jobs, and jobs create assets. Quantitatively, a negative shock to stocks decreases the liquidity value of equity and increases unemployment. A "perfect storm" of an increase in risk and a drop in the velocity of publicly-provided assets produces a temporary self-fulfilling crash with high unemployment and low stock prices. Reliance on privately-issued assets heightens fragility.

**JEL Classification:** D82; D83; E40; E50

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## 1. Introduction

Economic turmoil over the past decade, or more, has refocused attention on the interaction between financial and labor markets and the resulting implications for aggregate economic outcomes. Empirical evidence documents that the unemployment rate comoves with stock market capitalization (Farmer (2012), Hall (2017)). This paper's focus is on a model with a liquidity channel that connects labor and financial markets. Inspired by the New Monetarist literature, we present a model that links stock prices and unemployment through the endogenous creation of liquid assets.

*We are greatly indebted to Guillaume Rocheteau for generating many of the ideas and inspiring our work on this project. All errors are our own.

Why focus on a liquidity channel? Recent empirical evidence identifies causal connections between household balance sheets, consumption, and unemployment. Mian, Rao, and Sufi (2013) find a greater decrease in consumption in countries that suffered more adverse shocks to housing values. Mian and Sufi (2012), in turn, estimate that shortfalls of aggregate demand due to the deterioration of U.S. household balance sheets accounted for almost 4 million (65%) of the jobs lost between 2007 and 2009. To this end, Section 5 offers two pieces of evidence in favor of a model with a liquidity role for assets. First, we establish, through an estimated cointegrating vector, a long-run relationship between stock market values, unemployment rates, and liquidity premia. Second, a structural VAR identified with sign restrictions shows that a stock market shock has a strong short-run impact on unemployment and liquidity premia.

In the standard Diamond-Mortensen-Pissarides (DMP) search model, aggregate stock market capitalization is a function of employment and (exogenous) productivity and does not play any causal role in the labor market. Accordingly, we augment the Mortensen and Pissarides (1994) model with a limited commitment problem in the goods market. Households encounter idiosyncratic spending opportunities that, because of their limited ability to commit to repaying unsecured debt, they finance by pledging the value of some assets. Consumers' liquid assets are shares in a mutual fund comprised of stocks and government bonds. In our model, shares in this mutual fund provide an insurance role analogous to capital in Aiyagari (1994).

This simple twist of an otherwise standard model imparts a key macroeconomic role to the stock market. Higher stock market valuations relax consumers' liquidity constraints and raise expenditure, thereby creating an *aggregate demand channel*, which strengthens firms' hiring incentives. The novelty in this paper is that the creation of new firms and jobs, likewise, enhances market capitalization and feeds back into consumer demand. Such strategic complementarities between the labor market and the stock market provide a potential explanation for the observed co-movement between stock prices, unemployment rates, and liquidity premia: jobs create assets, and assets create jobs. However, there is also a countervailing *real interest rate channel*: as real interest rates rise, the expected value of a job falls, and firms post fewer vacancies.

Our paper follows naturally from previous contributions that identify a channel through liquidity to the labor market. Berentsen, Menzio, and Wright (2011) feature an economy with search frictions in both the labor market and a decentralized goods market. Money is the liquid asset. Complementarities arise because vacancy creation eases search frictions in the goods market. Even more closely related is Rocheteau and Rodriguez-Lopez (2014), who also incorporate both labor and goods market search frictions and allow firm equity claims to serve as the liquid asset in over-the-counter transactions. The distinguishing feature in our framework is that firms seek to take advantage of the liquidity constrained expenditure shocks which, in turn, makes firms' revenues depend positively on the aggregate stock market capitalization. Like Rocheteau and Rodriguez-Lopez (2014) there is an endogenous real interest rate channel, but our environment also features a strategic complementarity between labor and goods markets that creates an aggregate demand channel that amplifies shocks. We find this additional channel is important for linking liquidity, unemployment, and stock market capitalization. Thus, the main contribution of this paper is a simple extension of an otherwise standard model that can help explain the observed co-movement of unemployment, stock markets, and liquidity premia.

We begin by studying theoretically the factors that generate the aggregate demand and real interest rate channels. The key ingredients are both the liquidity role of assets as well as incentives for firms to produce in the market with positive economic profits. The paper then uses the model to better understand quantitatively the role liquidity may play in stock and labor markets. We calibrate the model to U.S. data on long-run unemployment rates and the Treasury demand curve. Our estimate of the exposure to expenditure shocks predicts that, on average, 4% of consumers each year find a consumption opportunity that requires secured credit. Moreover, the long-run elasticity of unemployment with respect to stock market values is high. In a counterfactual economy with 10% of consumers facing a liquidity shock in a year, the aggregate demand channel reduces unemployment relative to a Bewley-type economy by over 1 percentage point.

We also make use of the model to study the short-run relationship between liquidity and the economy. Motivated by the aforementioned structural impulse responses, we solve for the economy's response to unanticipated innovations to the marginal utility of consumption (int. margin), the expenditure risk (ext. margin), and public liquidity provision. For the aggregate demand shocks, the model's short-run dynamics are consistent with the estimated impulse response functions. Using counterfactual analysis, we show that the aggregate demand channel produces more persistent and volatile responses to demand shocks. Public liquidity shocks, on the other hand, lead to an attenuated economic response to shocks. A decrease in public liquidity provision reduces real interest rates and, *ceteris paribus*, increases the present value of firm profits. This is offset, though, by the aggregate demand channel which decreases consumption demand with less public liquidity available.

The aggregate demand channel creates a strategic complementarity: high stock market capitalization reduces households' liquidity constraints, increases aggregate demand, and thereby raises consumption prices. Firm revenue increases and spurs job creation, which further propagates the high stock market valuation. A lower need for liquidity, in turn, boosts real interest rates, which dampens firm value and entry. As a final exercise, we show that under a "perfect storm"– a simultaneous increase in liquidity risk, a negative productivity shock, and a large decrease in the velocity of government bonds– a crash state with high unemployment and low stock market capitalization arises as a steady-state and co-exists with a low unemployment/high stock value equilibrium.

## 1.1 Related literature

The framework in our model is inspired by monetary theory, and in particular the class of New Monetarist models that incorporates unemployment and money. The first paper to introduce stock market liquidity into a Lagos-Wright model is Geromichalos, Licari, and Suárez-Lledó (2007). The timing structure of our model comes from Berentsen, Menzio, and Wright (2011). In Berentsen, Menzio, and Wright (2011) households have access to a single liquid asset, fiat money, and trade goods with firms in a decentralized goods market characterized by search frictions. In the Appendix we explicitly show that the set of steady-states in our framework is qualitatively different from the pure currency economy. Several New Monetarist papers emphasize the dual role of assets as collateral. For instance, in Lagos (2010) consumption is financed with loans collateralized by Lucas trees (a real asset) and fluctuations in liquidity premia are shown to be important in explaining the equity premium puzzle.[1] Similarly, in Rocheteau and Wright (2013) the asset is again a Lucas tree, and with endogenous firm entry the model exhibits multiple steady-states and cycles reminiscent of recurring bubbles and crashes. Finally, Lagos and Rocheteau (2008) study the co-existence of money and capital when claims to capital can collateralize consumption.[2]

The theory proposed in this paper is also related to a class of incomplete market models where households hold assets with a precautionary savings motive to insure themselves against idiosyncratic shocks. Most closely related is Aiyagari (1994) where households self-insure by acquiring claims to physical capital. In our model, the risk arises from idiosyncratic spending opportunities, rather than income shocks, and assets are claims on aggregate firm values. Unlike in Aiyagari (1994), where the price of capital is fixed, here the value of firms is endogenous and affects household liquidity. Though the mechanism resembles the financial

[1] The model under consideration here abstracts from features that may capture the equity premium. The households in our model pay a (potentially) substantial liquidity premium on stocks.

[2] Other papers with liquidity and unemployment include Branch, Petrosky-Nadeau, and Rocheteau (2016) and Rocheteau and Bethune (2021).

multiplier of Kiyotaki and Moore (1997), in which asset prices affect collateral and investment, here we focus on how the stock market helps insure households against shocks and thereby boosts job creation.[3]

Though Bewley models have traditionally emphasized income risk, expenditure risk is also substantial. According to the 2015 Federal Reserve Report of Economic Well-Being, $22\%$ of respondents experienced a major out-of-pocket medical expense in the past year. Among these, the median out-of-pocket cost was $\$1,200$ and the mean was $\$2,782$. $27\%$ of respondents reported foregoing major medical care because they could not afford it. Moreover, $55\%$ of respondents indicated that their expenses are roughly the same each month, compared to $67\%$ who say the same about their income. Focusing on expenditure shocks allows us to obtain a highly tractable specification and a liquidity role for assets, which is the main focus of our study.

The current paper is also related to a literature that incorporates financial markets/risk into labor search models. For instance, Krusell, Mukoyama, and Şahin (2010) endogenize income risk through labor market matching and assess the implications for optimal provision of unemployment insurance. Kehoe, Lopez, Midrigan, and Pastorino (2019) generate realistic unemployment fluctuations through volatile, countercyclical risk premia and human capital accumulation on the job. Their findings hold even with a procyclical opportunity cost of employment. We also use preferences that give rise to time-varying interest rates but focus on liquidity premia.

There is also a burgeoning literature that documents and models the link between stock markets, labor markets, and consumption. For example, Majlesi, Di Maggio, and Kermani (2020) use Swedish administrative data to estimate the impact on consumption from stock market returns. They estimate a marginal propensity to consume out of unrealized capital gains of $23\%$ for the bottom half of the wealth distribution and $3\%$ for the top $30\%$. Importantly, those whose liquid wealth is less than 6 months of disposable income–the estimated MPC out of capital gains is nearly $40\%$. Chodorow-Reich, Nenov, and Simsek (2019) use IRS data to impute the county-level stock market return, and then regress employment outcomes on these returns, controlling for county and state-by-quarter fixed effects. They find that a $20\%$ increase in stock market valuations increase aggregate hours by $0.7\%$ and the aggregate labor bill by $1.7\%$. In a two-agent New Keynesian model with geographic heterogeneity

[3]Though Kiyotaki and Moore (1997) features a different kind of aggregate demand channel, there is no interest rate channel. Moreover, productivity fluctuates in that model due to the redistribution of land between farmers and gatherers, but not due to firms accommodating higher demand with fixed inputs. Moreover, the aggregate demand channel generated through expenditure risk can mitigate the unemployment volatility puzzle emphasized by Shimer (2005), and can arise from shocks to either liquidity or productivity.

they estimate a MPC of 3.2 cents per dollar of stock wealth.

## 2. Environment

There is a unit measure of households, composed of one buyer and one worker. Time is discrete and is indexed by $t \in \mathbb{N}$. Each period of time is divided into three stages. The first stage is a frictional labor market where unemployed workers and vacant firms participate in a stochastic matching process. Consumption and production take place in the last two stages. In the second stage, buyers and firms trade consumption goods early in a Walrasian market. In the last stage, buyers and firms have a late opportunity to trade goods and assets and wages are paid. We take the late-consumption good traded in the last stage as the numéraire.

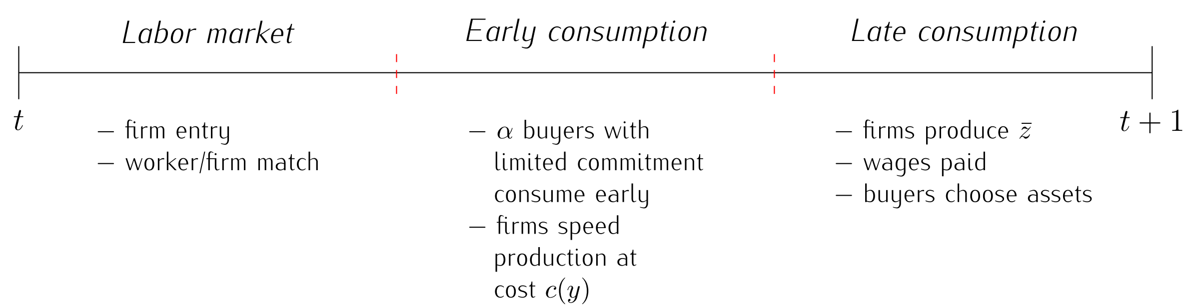


Figure 1: Timing.

The utility of a household is

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\varepsilon_t \upsilon(y_t) + x_t^b + x_t^w\right],$$

where $\beta = (1+\rho)^{-1} \in (0,1)$ is a discount factor, $y_t \in \mathbb{R}_+$ is the buyer's early consumption, $x_t^b \in \mathbb{R}$ is the buyer's late consumption, and $x_t^w \geq 0$ is the worker's late consumption. If $x^b < 0$, then the buyer is self-employed and produces the numéraire good. Because of the linear preferences in terms of the numéraire good, we can either treat the buyer and the worker as distinct agents, or as a joint entity with a consolidated budget constraint and impose conditions on primitives so that $x^b \geq 0$ holds.[4]

[4]The quasi-linearity in preferences keeps the model tractable and, in particular, implies that individual histories in the labor and goods markets are independent of asset holdings made in the third-stage; that is, the equilibrium wealth distribution is degenerate. More general preferences lead to self-insurance against

The utility function for early consumption, $\varepsilon v(y)$, is twice continuously differentiable, strictly increasing, and concave, with $v(0) = 0$, $v'(0) = \infty$, and $v'(\infty) = 0$. The multiplicative term, $\varepsilon$, is an idiosyncratic preference shock that is equal to $\varepsilon = 1$ with probability $\alpha$ and $\varepsilon = 0$ otherwise. These preference shocks correspond to liquidity shocks in the banking literature (e.g., Diamond and Dybvig (1983)) according to which some buyers have the desire for early consumption.

Each firm is a technology to produce $\bar{z}$ units of numeraire with one unit of indivisible labor (one worker) as the only input. Production takes time so that $\bar{z}$ is available in the last stage. The firm can speed up the production process and serve $y$ units of goods to early consumers at cost $c(y)$ in terms of numeraire, where $c' > 0$ and $c'' \leq 0$. Unless stated otherwise, we assume $c(0) = 0$, and $c'(0) = 0$. There is an upper bound $\overline{y}$ such that $\overline{z} = c(\overline{y})$. One can impose conditions on fundamentals that ensures $y \in (0, \overline{y})$, so that the constraint can be ignored. The output in the last stage is $\bar{z} - c(y)$. With probability $\lambda$, the buyer can access intra-period credit. In that case, repayment can be fully enforced. With probability $1 - \lambda$, a firm cannot monitor the buyer.[5]

The convexity of the cost function is crucial to generate the aggregate demand externality. This feature implies that firms price at marginal cost, but above average cost, but in a simple and tractable way that nests, for instance, Rocheteau and Rodriguez-Lopez (2014) as a special case. The convex cost is a standard assumption for multi-product firms (c.f. Pfouts (1961)) and captures that shifting production between goods is costly. The resulting production possibility set is concave and provides a simple formalization that generates pro-cyclical profit margins and firm entry that incentivize firms to enter the early-consumption market (e.g. Anderson, Rebelo, and Wong (2020)). Alternatively, goods market frictions and imperfect utilization of inputs, as by Huo and Ríos-Rull (2015), can make revenue and productivity depend on aggregate demand, but the mechanics are more involved.

In order to hire a worker at time $t$, a firm must advertise a vacant position, which costs $k > 0$ units of the numéraire good at $t - 1$. The measure of matches between vacant jobs and unemployed households in period $t$ is given by $m(s_t, o_t)$, where $s_t$ is the measure of

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employment and expenditure shocks (see Bethune and Rocheteau (2021)). Households have a precautionary demand for assets due to the spending shocks $\varepsilon_t$.

[5]We focus on stock mutual funds, government bonds, and debt obligations as the assets for the following reasons. Stocks are a primitive given the fundamental role of firms in labor search models; government bonds provide a policy instrument and allow the model to fit the Treasury demand curve. Finally, probabilistic access to credit by consumers enables us to index market incompleteness as Guerrieri, Lorenzoni, Straub, and Werning (2021). Though the economy is cashless, Hu and Rocheteau (2013) shows that fiat money is not essential in environments with Lucas trees. In a companion paper we develop an extension to the model that includes fiat money. Introducing money is useful for studying additional relationships, e.g. the Phillips curve, but is not crucial for the issues studied here.

job seekers and $o_t$ is the measure of vacant firms (openings). The matching function, $m$, has constant returns to scale, and it is strictly increasing and strictly concave with respect to each of its arguments. Moreover, $m(0, o_t) = m(s_t, 0) = 0$ and $m(s_t, o_t) \leq \min(s_t, o_t)$. The exit probability out of unemployment for a worker is $e_t = m(s_t, o_t)/s_t = m(1, \theta_t)$ where $\theta_t \equiv o_t/s_t$ is referred to as labor market tightness. The vacancy filling probability for a firm is $q_t = m(s_t, o_t)/o_t = m\left(1/\theta_t, 1\right)$.

Employment (measured after the matching phase at the beginning of the second stage) is denoted $n_t$ and the economy-wide unemployment rate (measured after the matching phase) is $u_t$. Therefore, $u_t + n_t = 1$. An existing match is destroyed at the beginning of a period with probability $\delta$. A worker who loses their job in period $t$ becomes a job seeker in period $t+1$. So, workers who lose their jobs must go through at least one period of unemployment, i.e. $s_{t+1} = u_t$. An employed worker in period $t$ receives a wage in terms of the numéraire good, $w_{1,t}$ in the last stage. An unemployed workers enjoys $w_0$, which represents unemployment benefits and the value of leisure.

There is a fixed supply of one-period real government bonds $A_g$. Each bond issued in the third stage is a claim to one unit of the numeraire in the following period. In the second stage buyers are anonymous and cannot commit to repay their debt. There are perfectly competitive mutual funds which buy stocks and bonds and issue risk-free shares. We let $r_t$ denote the rate of return of such claims from the last stage of $t-1$ to the last stage of $t$. These claims are perfectly diversified and hence free of idiosyncratic risk.[6] Moreover, they can be authenticated and transferred at no cost. Household wealth, $a_{t+1}$, thus comprises shares in mutual funds that acquire existing firms or invest in new firms by creating vacant positions.

## 3. Equilibrium

In the following, we characterize an equilibrium by moving backwards from agents' choice of asset holdings in the last stage, to the determination of prices and quantities for early consumption/production, and finally the entry of firms and the determination of wages in the labor market.

[6] It is standard to think of a firm/job as a short-lived Lucas-type tree. The mutual fund diversifies exposure to the idiosyncratic job destruction. Moreover, holding mutual fund shares abstracts complications in wage bargaining if the worker holds undiversified claims to the firm.

## 3.1 Goods and asset markets

As previously indicated, the lifetime utility of a household is the sum of the lifetime utility of the buyer and the lifetime utility of the worker. Therefore, in the following, we treat separately the two agents composing the households. Let $W_t(\omega_t)$ denote the lifetime *expected* discounted utility of a buyer at the beginning of the last stage with $\omega_t$ units of wealth in terms of the numeraire. Wealth $\omega_t$ is composed of shares of mutual funds net of debt obligations and tax liabilities. Assets $a_{t+1}$ taken into the second subperiod consist solely of mutual funds since we only consider intra-period debt. Similarly, let $V_t(a_t)$ be the buyer's *expected* value function at the beginning of the second stage, before preference shocks for early consumption are realized, where expectations are taken with respect to the distribution over $\varepsilon_t$.[7]

The buyer's problem can be written recursively as

$$W_t(\omega_t) = \max_{x_t, a_{t+1}} \{x_t + \beta V_{t+1}(a_{t+1})\} \quad \text{s.t.} \tag{1}$$

$$a_{t+1} = (1 + r_{t+1})(\omega_t - x_t) \geq 0. \tag{2}$$

From (1), the buyer chooses its consumption, $x_t$, and asset holdings, $a_{t+1}$, in order to maximize its lifetime utility subject to a budget constraint. The budget constraint says that next-period wealth is equal to the current wealth net of consumption capitalized at the gross interest rate, $1 + r_{t+1}$. Or, combining (1) and (2) leads to

$$W_t(\omega_t) = \omega_t + \max_{a_{t+1} \geq 0} \left\{ -\frac{a_{t+1}}{1 + r_{t+1}} + \beta V_{t+1}(a_{t+1}) \right\} \tag{3}$$

From (3), $W_t$ is linear in wealth and $a_{t+1}$ is independent of $\omega_t$. The first order condition for the buyer's problem is:

$$1 = (1 + r_t)\beta V_t'(a_t). \tag{4}$$

The disutility cost of accumulating one unit of wealth in the last stage is equal to one. This investment yields $1 + r$ and is valued according to the buyer's discounted marginal utility of wealth in the early-consumption stage, $\beta V_t'(a_t)$.

[7] The focus of this paper is the mechanics linking liquidity to stock and labor markets. As such, we abstract from aggregate shocks. Without loss of generality for the moment, we assume rational agents have perfect foresight over payoff relevant aggregate variables, and economize on notation by defining $V(\cdot)$ as the expected value function. In subsequent sections, with the possibility of multiple steady-states, there *may* exist sunspot equilibria and rational agents will not have perfect foresight along the now stochastic equilibrium path. Our analysis mentions, but does not focus on, this possibility and it should be understood that $V(\cdot)$ also depends on conditional expectations of future prices and other aggregate variables beyond the household's control.

We now turn to the goods market for early consumers. The expected discounted utility of a buyer at the start of the early-consumption stage holding assets $a_t$ is

$$V_t(a_t) = \alpha\Big[(1-\lambda)\max_{p_t y_t \leq a_t}\{v(y_t) + W(a_t - p_t y_t - \tau_t)\} \\ + \lambda \max_{y_t \geq 0}\{v(y_t) + W(a_t - p_t y_t - \tau_t)\}\Big] + (1-\alpha)W_t(a_t - \tau_t)$$

With probability $\alpha$ the buyer wants to consume early. In that case, the buyer can finance expenditures using intra-period credit with probability $\lambda$, or with assets when payment cannot be enforced. The constraint, $p_t y_t \leq a_t$, captures the inability of buyers to delay settlement. As a result, spending cannot exceed wealth. With probability, $1-\alpha$, the buyer does not want to consume early. In general, the buyer enters the late-consumption stage with $a_t - p_t y_t - \tau_t$ units of wealth, where $\tau_t$ are lump-sum taxes. Using the linearity of $W_t$,

$$V_t(a_t) = \alpha[(1-\lambda)\max_{p_t y_t \leq a_t}\{v(y_t) - p_t y_t\} + \lambda \max_{y_t \geq 0}\{v(y_t) - p_t y_t\}] + a_t - \tau_t + W_t(0). \quad (5)$$

Denote the optimal early consumption under perfect credit $y_t^*$ and without credit $\hat{y}_t$. These quantities satisfy $y_t^* = v'^{-1}(p_t)$ and $\hat{y}_t = \min\{y_t^*, a_t/p_t\}$. If the payment constraint does not bind, then the buyer equalizes marginal utility to price. Otherwise, early consumption equals the buyer's wealth.

The expected revenue of a firm in terms of the numeraire in period $t$ is:

$$z_t = \bar{z} + \max_{y \in [0,\overline{y}]}\{p_t y - c(y)\}$$

Relative to the standard MP model, the novelty is the second term, which represents the firm's profits from selling early. Assuming an interior solution, the optimal supply of goods in the early market is

$$y_t^s = c'^{-1}(p_t) \quad (6)$$

Firms price at marginal cost in the early-consumption market. Market clearing requires

$$n_t y_t^s = \alpha[\lambda y_t^* + (1-\lambda)\hat{y}_t] \quad (7)$$

There is a measure $n_t$ of active firms in the early market, each of which produces $y_t^s$. Household consumption is the sum of purchases by individuals with and without access to credit.

Finally, the buyer's choice of assets is obtained by differentiating (5), plugging into (4), and using $p_t = c'(y_t^s)$:

$$\frac{\rho - r_t}{1 + r_t} = \alpha(1 - \lambda)\left[\frac{v'(\hat{y}_t)}{c'(y_t^s)} - 1\right] \tag{8}$$

The left side of (8) represents the cost of holding the asset, which approximately equals the difference between the rate of time preference and the real interest rate. The right side represents the expected marginal benefit from holding liquid wealth. The expected marginal benefit is the percentage increase of marginal utility with respect to marginal cost multiplied by the probability of having a liquidity shock and not being able to access credit. If buyers are not constrained by their asset holdings in the early-consumption stage, then $r_t = \rho$. Otherwise, $r_t < \rho$.

## 3.2 Labor market

The rate of return from investing in a new firm in the last stage of $t$ is the expected value of the firm in $t+1$, $q_{t+1}J_{t+1}$ relative to the initial investment $k$. Alternatively, one can purchase a share of the mutual fund with rate of return $1 + r_{t+1}$. The no-arbitrage or free entry condition is

$$(1 + r_{t+1})k = q_{t+1}J_{t+1}$$

The value of a firm solves

$$J_t = z_t - w_1 + (1 - \delta)\frac{J_{t+1}}{1 + r_{t+1}} \tag{9}$$

Firms discount the future with the real interest rate, a fact that follows directly from the no-arbitrage condition. The value of a firm equals expected revenue net of the wage plus the expected discounted profits of the job multiplied by the survival probability $1 - \delta$. Market tightness is determined by the free entry condition, $(1 + r_{t+1})k = q_{t+1}J_{t+1}$. Combining free entry with (9) yields the job creation condition:

$$\frac{(1 + r_t)k}{q_t} = z_t - w_1 + (1 - \delta)\frac{k}{q_{t+1}}. \tag{10}$$

Define the compound interest rate $R_{t,j} = (1 + r_t)(1 + r_{t+1}) \cdots (1 + r_{t+j})$. Then one can iterate (10) forward to link job creation to the present discounted value of revenue:

$$\frac{k}{q(\theta_t)} = \sum_{j=0}^{\infty}(1-\delta)^j \frac{z_{t+j} - w_1}{R_{t,j}} \tag{11}$$

Current job creation, and hence market tightness, depend positively on the future stream of liquidity through $z_{t+j}$ and negatively on discount rates $R_{t,j}$. In the second stage, firms evolve according to

$$n_{t+1} = (1-\delta)n_t + m(1, \theta_{t+1})(1-n_t).$$

Among the $n_t$ existing firms in period $t$, a fraction $1-\delta$ survive. The measure of new firms equals the measure of job seekers in $t+1$, $u_t$, multiplied by the job finding probability $e_{t+1} = m(1, \theta_{t+1})$.

## 3.3 Wage determination

Here we describe wage determination. Bargaining over the wage takes place at the beginning of the LM in period $t$ and depends on the surpluses of the workers and firms.

Turning to the second agent in a household, let $U_{1t}$ and $U_{0t}$ be the lifetime expected utility of an employed and an unemployed worker, respectively, at the beginning of the LM in time $t$. The value functions of the workers are

$$\begin{aligned} U_{1t} &= w_{1t} + (1-\delta)\beta U_{1,t+1} + \delta\beta U_{0,t+1} \\ U_{0t} &= w_0 + (1-e_{t+1})\beta U_{0,t+1} + e_{t+1}\beta U_{1,t+1} \end{aligned}$$

The employed worker receives a wage, $w_{1,t}$, and keeps her job in the following period with probability $1-\delta$. The unemployed worker enjoys $w_0$ and finds a job in the following period with probability $e_{t+1}$. Therefore, the utility of a household in the third stage composed of a buyer with $a$ units of wealth and a worker with employment state $e$ is $W(a) + Ue$.

The wage is determined according to generalized Nash bargaining between the worker and firm with a worker bargaining power equal to $\phi$. Appendix (D) shows that

While similar to the Nash wage in a standard Mortensen-Pissarides framework, the Nash bargained wage equation in (**??**) is slightly different because firms discount at rate $1 + r_{t+1}$, given their outside option, and $\beta(1 + r_{t+1}) \leq 1$. The difference manifests in two ways. First, the threat of breakdown in negotiations hits the two parties differently: the worker places (weakly) less value on a future match. Second, the lower discounting by firms generates an additional surplus term, some of which is shared by the worker. If the liquidity constraint does not bind, then $\beta(1 + r_{t+1}) = 1$, and the wage equation is standard, though $z_t$ is now endogenous:

$$w_{1t} = (1 - \phi)w_0 + \phi(z_t + k\theta_t)$$

## 3.4 Equilibrium

The value of buyers' assets in the second stage is the market capitalization of firms plus the total value of government bonds.

$$a_t = n_t J_t + A_g = \frac{n_t(1 + r_t)k}{q_t} + A_g. \tag{12}$$

By market clearing, the total value of the stock market and government bonds equals the value of assets held by buyers when entering the early-consumption stage, $a_t$. Equation (12) implies a positive relationship between stock market capitalization, employment, and interest rates. Combining (6) (7), and (12) allows us to express the price as a function of assets and employment. We define an equilibrium as a bounded sequence, $\{J_t, \theta_t, n_t, p_t, r_t\}_{t=0}^{+\infty}$, that solves:

$$J_t = \frac{(1 + r_t)k}{q(\theta_t)} = \bar{z} + \max_y \{p_t y - c(y)\} - w_1 + (1 - \delta)\frac{J_{t+1}}{1 + r_{t+1}} \tag{13}$$

$$c'^{-1}(p_t) = \frac{\alpha}{n_t}\left[\lambda v'^{-1}(p_t) + (1 - \lambda)\min\left\{v'^{-1}(p_t), \frac{n_t J_t + A_g}{p_t}\right\}\right] \tag{14}$$

$$\frac{\rho - r_t}{1 + r_t} = \alpha(1 - \lambda)\left[\frac{v'\left(\frac{n_t J_t + A_g}{p_t}\right)}{p_t} - 1\right]^+ \tag{15}$$

$$n_{t+1} = (1 - \delta)n_t + m(1, \theta_{t+1})(1 - n_t), \tag{16}$$

for some given $n_0$. Equation (13) determines the value of a firm and market tightness taking the real interest rate and the early-consumption price as given. Equation (14) determines the early-consumption price by market clearing while (15) determines the real interest rate from the buyer's demand for liquid wealth. Equation (16) is the law of motion of employment.

We note briefly that consumption expenditures satisfy $nz$ and output is $nz + kv$. Both these series thus move closely with productivity and employment. To save space, we instead focus the analysis on the labor-market variables, interest rate spreads, and stock market capitalization.

# 4. Deconstructing the model

To better understand the components of the model, we deconstruct it by starting with the textbook Mortensen-Pissarides model and adding one new ingredient at a time. For sake of illustration, we use a continuous-time version of the model that allows us to represent dynamics graphically through phase diagrams.[8] For clarity, we temporarily assume a fixed wage. The remainder of the paper uses the endogenous wage (D).

## 4.1 A Mortensen-Pissarides economy

The Mortensen-Pissarides economy with a single good and frictionless goods market can be obtained by shutting down the idiosyncratic preference shocks, $\alpha = 0$, so that there is no early consumption. In this case, $z_t = \bar{z}$ and $r_t = \rho$ since stocks and bonds provide no liquidity/insurance role. Hence, a change in $A_g$ has no effect on interest rates or output. An equilibrium can be reduced to a pair, $(J_t, n_t)$, which solves

$$(\rho + \delta)J = \bar{z} - w_1 + \dot{J}$$
$$\dot{n} = m\left[1, \theta(J)\right](1 - n) - \delta n,$$

where $\theta(J)$ is the solution to $J = k/q(\theta)$. It is easy to check that there is a unique steady state and, for any initial condition $n_0$, a unique equilibrium corresponding to the saddle path leads to the steady state. Along this equilibrium, $J$ is constant and equal to the discounted sum of the profits, $(\bar{z} - w_1)/(\rho + \delta)$, where the effective discount rate is the sum of the rate of time preference and the depreciation rate. Similarly, market tightness is constant. Graphically, in the left panel of Figure 2, the $J$-isocline is horizontal. The $n$-isocline is upward-sloping since a higher market value of firms induces a higher market tightness, and higher employment at the steady state.

[8] Here we employ the methodology in Choi and Rocheteau (2019). The Appendix presents results on existence and local uniqueness of rational expectations equilibria in the discrete-time version of the model.

## 4.2 Mortensen-Pissarides with early consumption and perfect credit

We reintroduce preference shocks for early consumption by setting $\alpha > 0$ but keep the goods market frictionless by assuming that buyers have access to perfect credit in the early-consumption stage, i.e., $\lambda = 1$. In that case an equilibrium is a list, $\{J_t, p_t, y_t^s, n_t\}$, that solves

$$(\rho + \delta) J = \bar{z} + \max_y \{py - c(y)\} - w_1 + \dot{J} \tag{17}$$

$$v' \left( \frac{ny^s}{\alpha} \right) = p = c'(y^s) \tag{18}$$

$$\dot{n} = m[1, \theta(J)](1-n) - \delta n \tag{19}$$

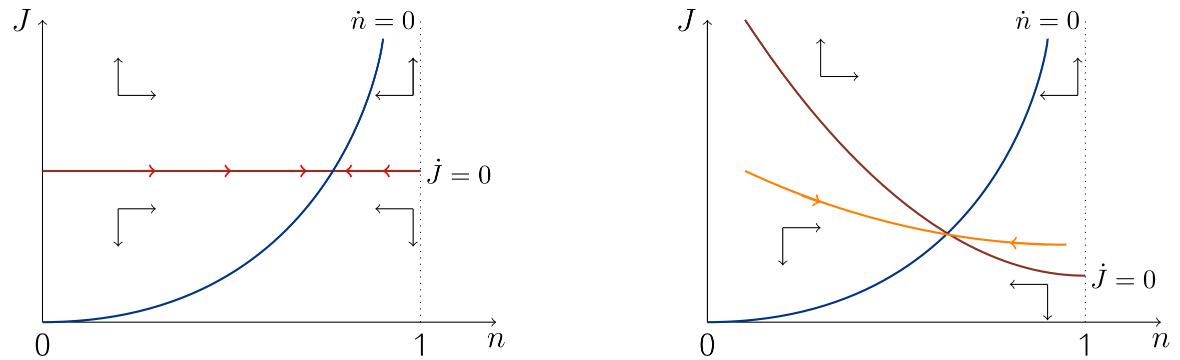
From (18), assuming $c'' > 0$, each firm's early-supply of goods decreases with $n$. As a result, the price of early consumption is a decreasing function of $n$. It implies that the firm's total revenue on the right side of (17) is $z = z(n)$ with $z' < 0$. As there are no liquidity constraints, a change in government bonds $A_g$ has no effect on equilibrium.

The dynamic system, (17)-(19), can be reduced to two ODEs and two unknowns, $J$ and $n$. In the right panel of Figure 2, the $J$-isocline is decreasing in $n$, since higher $n$ means lower early-consumption prices and lower profits. As before, there is a unique steady state and a unique equilibrium starting from any initial condition $n_0$. Along the saddle path trajectory, the value of firms is negatively correlated with $n$. A positive productivity shock that raises $\bar{z}$ shifts the $J$-isocline upward. So the value of firms and market tightness overshoot their steady-state values. As employment increases, $p_t$ decreases which brings $J_t$ and $\theta_t$ back to their steady states.

## 4.3 Mortensen-Pissarides with limited commitment

Households accumulate wealth to self-insure against the idiosyncratic risk of early consumption. To mimic the one-good economy of the Aiyagari model, we impose a linear cost function, $c(y) = y$, so that $p = 1$ and firms are indifferent between producing early or late.[9] For ease of exposition, we also assume $\lambda = 0$. As a result, the marginal product of capital, as captured by $\bar{z} - w_1$, does not depend on market capitalization. An equilibrium can now be reduced to a triple, $(J, r, n)$,

[9] This is the case considered by Rocheteau and Rodriguez-Lopez (2014).




(a) MP model.

(b) 2-good MP model with convex cost.

Figure 2: Phase diagrams: MP models.

$$\begin{aligned} (r+\delta)\,J &= \bar{z} - w_1 + \dot{J} \\ \rho - r &= \alpha\left[v'\left(nJ + A_g\right) - 1\right]^+ \\ \dot{n} &= m\left[1, \theta(J)\right](1-n) - \delta n. \end{aligned} \tag{20}$$

The novelty is equation (20) features an endogenous real interest rate. From (20) one can express $r$ as an increasing function of $nJ + A_g$ and reduce the system to two ODEs. The $J$-isocline, such that $\dot{J} = 0$, is given by $[r(nJ + A_g) + \delta]J = \bar{z} - w_1$. There is a negative relationship between $J$ and $n$. Intuitively, as the measure of firms increases, market capitalization increases for given $J$. As households have more liquidity to finance demand shocks, $r$ rises, which reduces the value of each firm. Thus, an increase in $A_g$ lowers the J-isocline: raising real interest rates $r$, reducing firm value $J$, and hence depressing employment $n$ via a reduced incentive to hire. Let $\bar{M}$ denote the market capitalization above which $r = \rho$. For all $nJ > \bar{M}$, $J$ is constant and equal to $(\bar{z} - w_1)/(\rho + \delta)$. Let $\underline{M}$ denote the market capitalization such that $r = -\delta$, i.e., $v'\left(\underline{M} + A_g\right) = 1 + (\rho + \delta)/\alpha$. As $nJ$ approaches $\underline{M}$, $J$ tends to $+\infty$ and $n$ tends to 0. The $n$-isocline gives a positive relationship between $n$ and $J$. So there is a unique steady state. Moreover, for given $n_0$ the equilibrium is unique. Along this equilibrium $J$ decreases over time and $r$ increases if $n_0$ is less than the steady state.

A positive productity shock moves the $J$-isocline upward. If the initial steady state is such that households are liquidity coinstrained, then $J$ overshoots its steady-state value. As $n$ increases, market capitalization rises as well and the real interest rate decreases, which brings the value of firms back to their steady state.

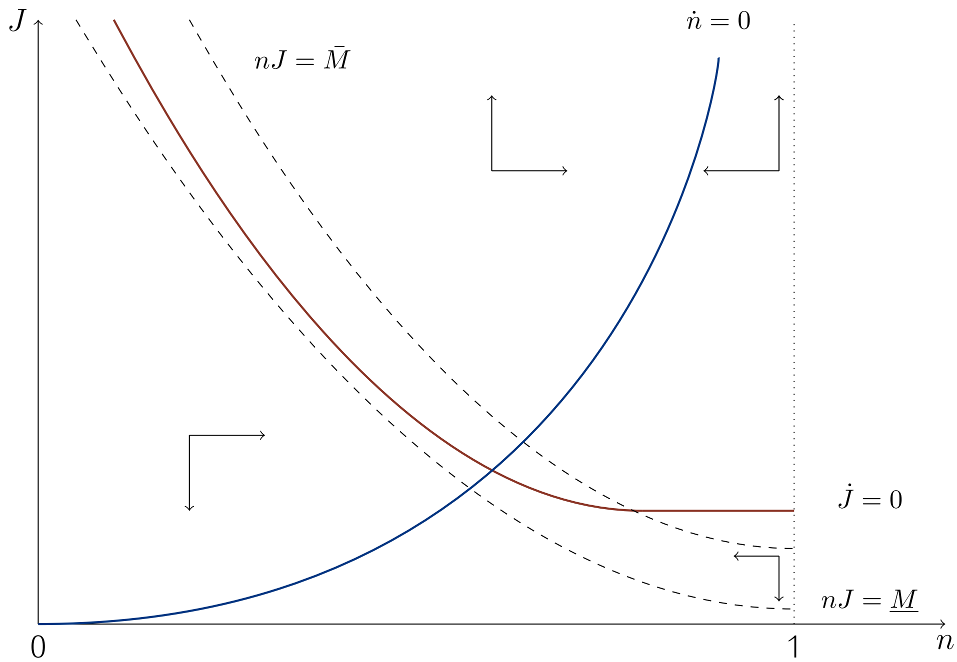



Figure 3: Phase diagram: Bewley-Aiyagari where $c(y) = y$.

## 4.4 The general case

We now combine all the ingredients: (i) households are subject to idiosyncratic preference shocks for early consumption, $\alpha > 0$; (ii) they can access credit with probability $\lambda$; (iii) and the cost of early production, $c(y)$, is strictly convex. The early-consumption price, $p = c'(y^s)$, now depends on households' liquid wealth, thereby providing another channel through which liquid wealth affects firms' revenue.[10]

Appendix E lists the continuous-time equilibrium conditions. We can reduce equilibrium of the continuous-time system to a pair of ordinary differential equations by defining a sequence of functions. Let $y^*(n)$ be the solution to $v'(y^*) = c'(\alpha y^*/n)$, that is, the first-best level of consumption provided no household is constrained. It is easy to check that $y^*$ is a decreasing function of $n$ with $\lim_{n \to 0} y^* = +\infty$ and $\lim_{n \to +\infty} y^* = 0$ Moreover, let $y^c(n, J, A_g)$ solve

$$y^c = \frac{\alpha}{n} \left[ \lambda v'^{-1}(c'(y^c)) + (1 - \lambda) \frac{nJ + A_g}{c'(y^c)} \right]$$

The quantity $y^c$ is the firm supply under the assumption that households lacking in credit spend all their assets. In general, $y^c$ is an increasing function of $J$ and $A_g$ and a decreasing

[10] One further comparison, to a pure currency economy, is presented in the Appendix.

function of $n$. Thus, the buyer's liquidity constraint is more likely to bind if $n$ is low and $J$ is large.

Let $y^s(n, J, A_g) = \min\{y^c(n, J, A_g), (\alpha/n)y^*(n)\}$. We define the price, in turn, as $p(n, J, A_g) = c'[y^s(n, J, A_g)]$. The price is weakly decreasing in $n$ and weakly increasing in $J$ and $A_g$ (an aggregate demand effect). The total revenue of a firm is

$$z(n, J, A_g) = \bar{z} + p(n, J, A_g)y^s(n, J, A_g) - c[y^s(n, J, A_g)].$$

Revenue is weakly decreasing in $n$ and weakly increasing in $J$ and $A_g$. The real interest can also be expressed as a function of $n$ and $J$ as follows:

$$r(n, J, A_g) = \rho - \alpha(1-\lambda)\left[\frac{v'\left(\frac{nJ+A_g}{p(n,J,A_g)}\right)}{p(n, J, A_g)} - 1\right]^+,$$

where $y^b$ is an increasing function of $n, J.$ and $A_g$. So, $r$ is a weakly increasing function of $n, J$, and $A_g$. Using the functions $z(n, J, A_g)$ and $r(n, J, A_g)$ we reduce the dynamic system to two autonomous, nonlinear ODEs:

$$\begin{aligned}
\dot{J} &= \left[r(\overset{+}{n}, \overset{+}{J}, \overset{+}{A_g}) + \delta\right] J + w_1 - z(\overset{-}{n}, \overset{+}{J}, \overset{+}{A_g}) \equiv f(J, n) \\
\dot{n} &= m\left[1, \theta(\overset{+}{J})\right](1-n) - \delta n \equiv g(J, n).
\end{aligned}$$

The right side of the $J$-ODE is monotone increasing in $n$ but can be non-monotone in $J$. As a result, the $J$-isocline can also be non-monotone, with consequences for the multiplicity of steady states and dynamics. An increase in government bonds $A_g$ raises both interest rates and revenue, thus having an ambiguous effect on the J-nullcline.

In Figure 4 we provide a numerical example for the following parameter values: $m(s, o) = s^{\xi}o^{1-\xi}$ with $(1-\xi)/\xi = 0.2$, $c(y) = y^{1.9}/1.9$, $\upsilon(y) = y^{0.5}/0.5$, $\bar{z} - w_1 = -0.5$, $\rho = 0.1$, $\alpha = \delta = 1$, and $A_g = 0$. Note that for this parametrization $\bar{z} - w_1 < 0$, i.e., if the early-consumption opportunities are shut down, then firms make negative profits. This numerical example exhibits multiple active steady states. An equilibrium with high employment, stock market capitalization, and interest rates coincides with an equilibrium with low values for these variables. We remark, though, that with weaker strategic complementarity, the J-nullcline is monotonic, which will be an implication of the calibrated parameters below. Thus, multiplicity of equilibria is a reflection of the strategic complementarity, a key feature of our model, and not a central contribution.

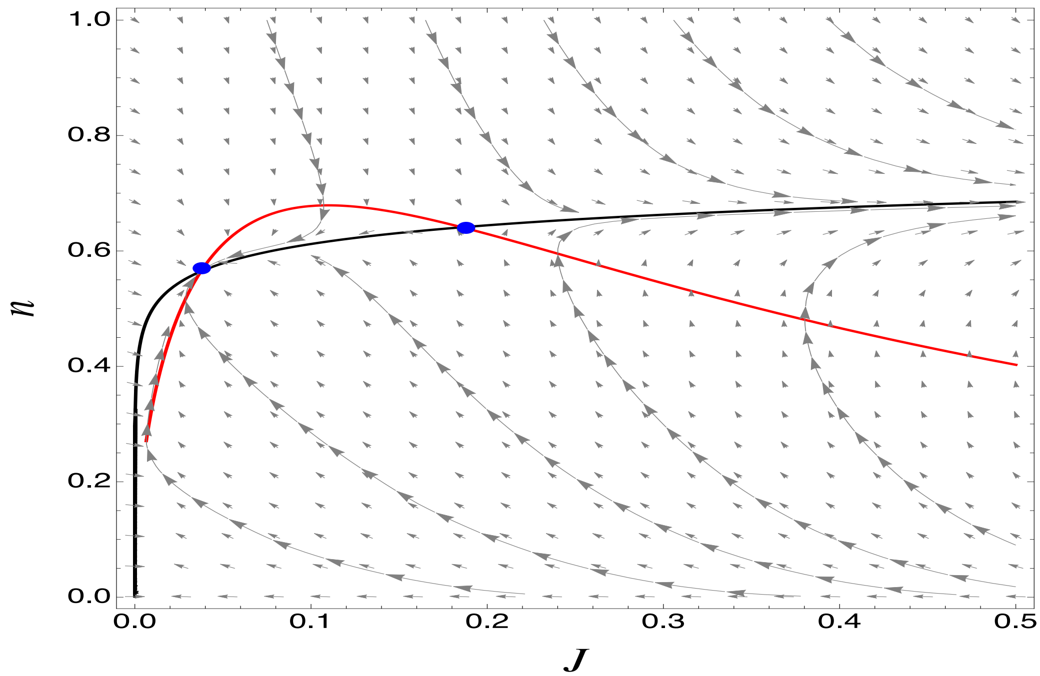



Figure 4: Multiple equilibria: full model.

Multiple steady-states are a special case of the model when strategic complementarity is particularly strong. It turns out, however, that the lower steady-state is indeterminate and can feature a multiplicity of dynamic equilibrium paths to the steady-state and even stochastic sunspot equilibria that remain bounded in a neighborhood of the low employment steady-state. The indeterminacy of this steady-state calls for an equilibrium selection mechanism. Below, when considering a particular counterfactual experiment, we adopt stability under adaptive learning as the selection mechanism (c.f. Evans and Honkapohja (2001)). As we show below, the lower steady-state (if it exists) is unstable under learning but nevertheless exerts an influence on the transition path following a shock.

## 5. Quantitative analysis

We first calibrate the steady-state to the U.S. economy. Then, using that calibration, we explore the role of liquidity via the aggregate demand and real interest rate channels in a long-run equilibrium, and present estimated cointegrating relationships in support of our long-run results. The section then conducts short-term analysis by first presenting evidence on the short-run relationship between the stock market valuation, interest rate spreads, and the unemployment rate. Using the calibrated model, we compute impulse responses to aggregate demand and public liquidity shocks. The section concludes by considering an unlikely shock, which we call a "perfect storm", that can feature a self-fulfilling crash in stock and labor markets.

The exercises in this section focus squarely on the interdependence between liquidity premia, unemployment rates, and stock market values. A full quantitative evaluation of the economy in terms of second moments is beyond the scope of the present paper; a more complete quantitative framework would require additional ingredients such as fixed matching costs (Pissarides (2009)) to address the unemployment volatility puzzle (Shimer (2005)), aggregate shocks to match empirical volatilities, and departing from quasi-linear utility to match wealth distributions.

## 5.1 Calibration

The value $w_0$ encompasses both unemployment benefits and the value of leisure. Hall and Milgrom (2008) use standard assumptions on preferences to impute a replacement ratio $w_0/z = 0.71$, which we adopt. Many articles, including Petrosky-Nadeau and Wasmer (2015), use this specification. The job finding and separation rates, an annual interest rate of $4\%$, the vacancy filling rate of $80\%$, and the Treasury demand as described below. We also set $\lambda = 0.8$ following evidence of the fraction of U.S. households with access to a credit card.[11] Under the parameterized cost function (see below), the convexity of the cost function $\sigma$ equals the price-cost margin: (price-average cost)/average cost. Silva (2019) calculates this measure across a large swath of non-financial Compustat firms between 1964 and 2017 and finds that it ranges from $18\%$ to $30\%$, increasing in recent years.[12] We set $\sigma = 0.2$ as a conservative benchmark.

The matching function is Cobb-Douglas with level parameter A and a matching elasticity $\xi$. Following Pissarides (2009), we target $A$ to match the job finding rate and set $\xi = 0.5$ to target the elasticity of the matching function with respect to unemployment. We obtain the job finding rates $e$ and separation rates $s$ using worker flows as in Shimer (2005). Next-period unemployment satisfies $U_{t+1} = U_t(1-e_t) + U^s_{t+1}$, given newly unemployed $U^s_{t+1}$. We rearrange to isolate $e_t$. Given that job losers have on average half a month to find a new job before being recorded as unemployed, the newly unemployed satisfies $U^s_{t+1} = s_t(1-U_t)(1-$

[11] See, FRB-Atlanta's *Survey of Consumer Payment Choice* (2020).

[12] In terms of Compustat variables, the price-cost margin is (SALES-COGS)/COGS. Please see Silva (2019) for more details.

$1/2)e_t$. Rearranging determines $s_t$.[13] We find the series' means, $\overline{e} = 41.5\%$ and $\overline{s} = 3.10\%$, and also back out the corresponding employment target: $n = \overline{e_t/(s_t + e_t)} = 92.8\%$. The implied unemployment rate exceeds the official unemployment rate but is consistent with the adjustment for marginally attached workers.[14]

For specific functional forms of preferences and technologies, we set

$$\begin{aligned} m(s,o) &= As^{\xi}o^{1-\xi} \\ v(y) &= B \ln y \\ c(y) &= C\frac{y^{1+\sigma}}{1+\sigma} \end{aligned}$$

Appendix H shows that, under these functional forms, $C$ drops out of the equilibrium conditions, so we can set $C = 1$ without loss of generality.

The parameters $k, B, \alpha, w_0$, and $\phi$, the workers' bargaining power, are chosen to simultaneously fit the Treasury demand, employment, market tightness, and the replacement ratio $w_0/z$. The latter is set to 0.71 following Hall and Milgrom (2008). We fit a quadratic polynomial to the same locus of debt-GDP and the interest rate spread examined by Krishnamurthy and Vissing-Jorgensen (2012). We choose the parameters so that the model fits three points of the fitted polynomial and the remaining targets. The three points of the fitted polynomial are 0.2, the mean value, and 0.8. The estimator solves a bounded, nonlinear least squares problem. We convert the model debt-to-GDP to annual by dividing it by 12 and the interest rate spread to annual, by multiplying it by $100 * 12$. Figure 5 shows the original Treasury demand curve together with the fitted quadratic polynomial and model-implied Treasury demand. Note that the model-implied Treasury demand curve from the model fits the quadratic polynomial well.

This approach is analogous to fitting money demand in the New Monetarist literature. One key difference with respect to fitting money demand is that, whereas the nominal interest rate is taken to be an exogenous policy parameter, here the spread is endogenous. In particular, for each candidate level of debt-to-GDP $x$, we impute the corresponding value of $A_g$ and then

[13] We use series on aggregate unemployment (FRED code UNEMPLOY), aggregate employment (CE160V) the the aggregate number employed for less than five weeks (UEMPLT5).

[14] In general, given calibration targets for two variables among the separation rate, job finding rate, and the unemployment rate, the Beveridge curve determines the third. Targeting the job-finding and separation rates results in an unemployment rate slightly higher than the official U-3 measure (FRED UNRATE). However, the U-5 unemployment rate, which takes into account marginally attached workers, is 6.8% between 1994, the first available year, and 2019. Since the unemployment rate in this period is lower than the overall sample, we conclude that the implied model unemployment rate is reasonable.

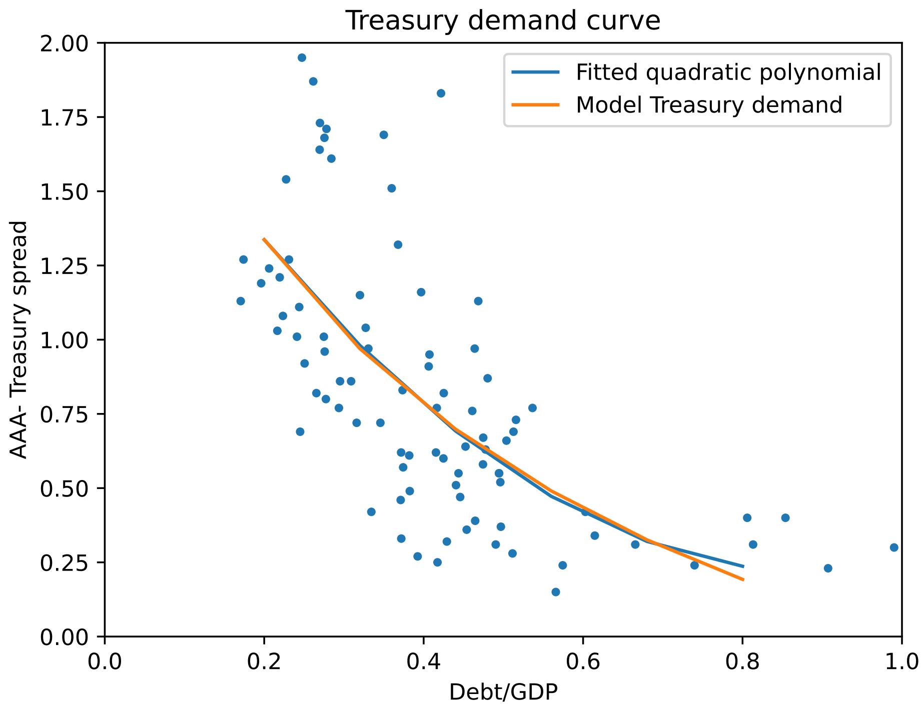



Figure 5: Treasury demand curve. The original data is taken from Krishnamurthy and Vissing-Jorgensen (2012), and coincides with Figure 1 in the article. We fit a quadratic polynomial by least squares, and overlay the Treasury demand curve from the model. Debt-to-GDP is expressed at an annual frequency, and the interest rate spread is expressed in annual percentage units.

solve for the interest rate spread. Throughout, we select the high employment equilibrium (only relevant if multiple steady-states exist for a particular parameter draw in the calibration algorithm).

## 5.2 The aggregate demand and real interest rate channels: the long-run

The model predicts two channels connecting financial, goods, and labor markets: the endogenous real interest rate and the aggregate demand channel. The quantitative analysis begins by quantifying the long-run (steady-state) properties of the model. Then we define a way to decompose the effects of these two channels. Crucially, the decomposition is *not* additive: steady-state firm value depends on productivity and discounting, but each of these components depends on firm value, i.e. $z(n, J)$ and $r(n, J)$.

| Parameter | Values | Calibration Strategy |
|---|---|---|
| $\delta$ | 0.03045 | mean separation rate |
| $\overline{z}$ | 1 | normalization |
| $w_0$ | 0.7186 | replacement ratio |
| $\phi$ | 0.04702 | Treasury demand |
| $\rho$ | 0.002466 | risk free rate |
| $\alpha$ | 0.004449 | Treasury demand |
| $B$ | 16.65 | Treasury demand |
| $\sigma$ | 0.2 | Price to average cost |
| $k$ | 4.413 | consistency with market tightness |
| $A$ | 0.5631 | Job finding rate |
| $\xi$ | 0.5 | Elasticity of matching function with respect to unemployment |
| $A^g$ | 4.529 | Treasury demand |
| $\lambda$ | 0.8 | Fraction of HH with credit access to replace income |

Table 1: Parameterization

### 5.2.1 The long-run equilibrium

We begin by plotting the steady-state of the model with the calibrated parameters in Table 1. As noted in the previous section, there are two endogenous state variables $(J, n)$ and the equilibrium is a sequence $\{J_t, n_t\}$ that satisfy a pair of differerence equations:

$$J_t = f\left(J_{t+1}, n_{t+1}, n_t\right)$$
$$n_{t+1} = g\left(J_{t+1}, n_t\right)$$

A steady-state is $J^* = f(J^*, n^*, n^*), n^* = g(J^*, n^*)$. Figure 6 plots the resulting contour map in $(n, J)$ space.

The model calibrated to U.S. data features a unique steady-state, with a 7.2% unemployment rate. The downward sloping line describes the relationship between $(J, n)$, in a steady-state, prescribed by the firm's Bellman equation. This relationship captures the intuition described in the preceding section: as $n$ increases the price received by firms in the

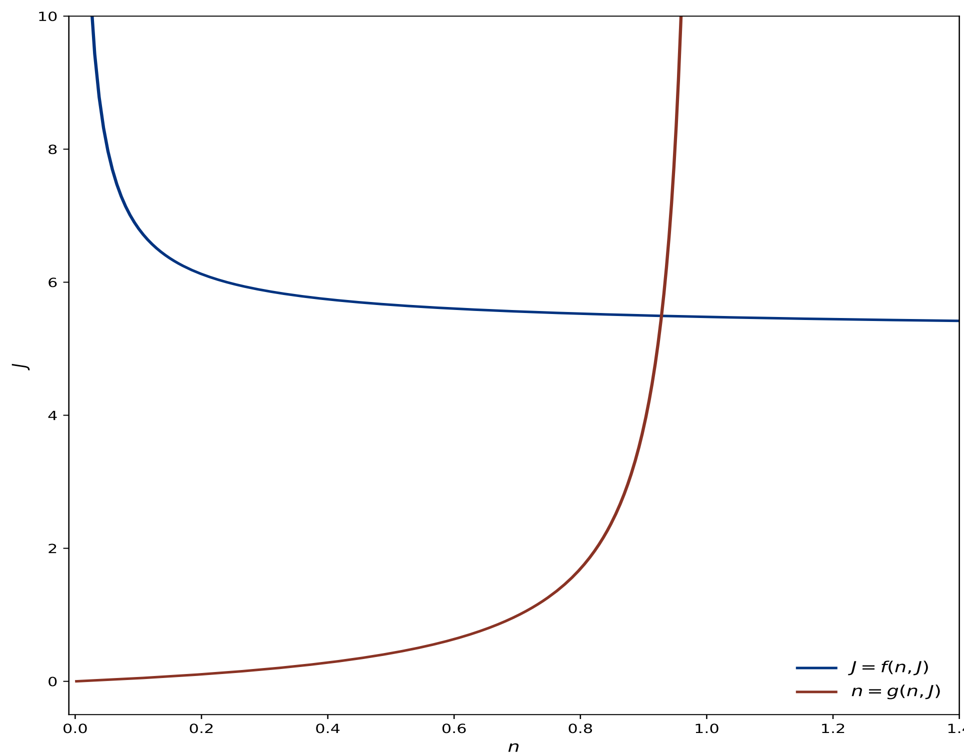



Figure 6: Steady-state at calibrated parameter values

early market decreases and, for each value of $J$, market capitalization increases, which raises the real interest rate and reduces the firm value. There is a unique steady-state, which suggests that the data are not consistent with sufficiently strong complementarities via the aggregate demand channel to generate the multiplicity of equilibria suggested by Figure 4. The $J = f(J, n, n)$ contour is nearly flat near the long-run equilibrium: the calibration suggests that the long-run equilibrium in stock and labor markets is relatively close to the unconstrained equilibrium with the interest rate spread in the long-run equilibrium $\approx 77$bp on an annualized basis. From Figure 6, we can anticipate the calculations to follow which demonstrate that changes in unemployment rates are elastic with respect to stock market capitalization.

### 5.2.2 Aggregate demand v. real interest rate channels

A representative firm's long-run value can be decomposed into 2 terms depending only on aggregate variables (see Appendix for details):

$$\begin{aligned} J &= \frac{1+r}{r+\delta}\left[\bar{z} - w_1 + \left(\frac{\sigma}{1+\sigma}\right)(y^s)^{1+\sigma}\right] \\ &= \underbrace{\frac{1+r}{r+\delta}}_{\text{discounting}} \times \underbrace{\left[\bar{z} - w_1 + \left(\frac{\sigma}{1+\sigma}\right)\left(\frac{\alpha}{n}\right)(\lambda B + (1-\lambda)\, L)\right]}_{\text{profit flow}} \end{aligned} \tag{21}$$

where $L = \min\{B, M + A_g\}$ is effective liquidity, $M \equiv nJ$, $w_1$ is the wage, and $y^s$ are sales in the early-consumption market. The first factor in (21) is a function only of the long-run equilibrium real-interest rate. The second factor comes from the long-run flow profitability of the firm from selling in the early-consumption market. Together, the long-run firm value is the product of the discounting and profit flow components.

An important caveat in interpreting (21) is that each component of the firm value depends on $J$. Higher firm value raises both the profit flow and the interest rate. As a precursor to defining the channels, consider the Bewley version of the model, i.e. $\sigma \to 0$:

$$J = \frac{1+r}{r+\delta}(\bar{z} - w_1)$$

Motivated by Aiyagari (1994), consider a rise in idiosyncratic risk ($\alpha$). Intuitively, stock market capitalization directly affects the real interest rate but does not directly affect contemporaneous revenues: there is an interest-rate channel but not aggregate-demand channel. Now, suppose we evaluate the effects of higher $\alpha$ in the baseline $\sigma = 0.2$. In this case, revenue and productivity also rise with greater expenditure risk. To decompose the aggregate-demand and interest-rate effects under $\sigma = 0.2$, define $J_{r=\rho}$, in which firms are priced *as if* there were no liquidity premium:

$$J_{r=\rho} = \frac{1+\rho}{\rho+\delta}\left[\bar{z} - w_1 + \left(\frac{\sigma}{1+\sigma}\right)\left(\frac{\alpha}{n}\right)(\lambda B + (1-\lambda)L)\right] \tag{22}$$

Given the rise in $\alpha$, define the total effect as the general-equilibrium response of the steady state of the full system (13)-(16). The aggregate-demand channel is the response in which the Firm Bellman is replaced by (22). The interest-rate channel is the gap between the response of the model using $J_{r=\rho}$ and the total effect.

We thereby decompose the long-run (steady-state) aggregate demand and interest rate channels with respect to expenditure shock frequency. Figure 7 plots key equilibrium quantities as a function of the frequency of spending shocks $\alpha$ in each version of the model. The total effect, for a fixed $\alpha$, is the difference between the full model's steady-state values for $M, n, r$ and the corresponding values in the steady-state with $\sigma = 0$, i.e. the Bewley version. The aggregate demand, then, is the difference between the steady-state values in the full model and the counterfactual case with $J_{r=\rho}$, i.e. with discounting at rate $\rho$. This is denoted by the blue shaded region. The real interest rate effect (red region), is the difference between the counterfactual with $J_{r=\rho}$ and the Bewley version with $\sigma = 0$. The parameters are set to the calibrated values. The left panel showcases the stock market capitalization. In the general model, greater expenditure risk increases market capitalization through more firm-entry ($n$) and firm value ($J$). If $\sigma = 0$, there is no effect on firm revenue, so the rise is much less pronounced. The stock market capitalization still rises as lower interest rates boost firm values and encourage job creation. If, instead, there is no liquidity premium on firm profits ($J = J_{r=\rho}$), then $M$ only rises from stronger aggregate demand. The effects on unemployment in the middle panel are a mirror image of those on the stock market capitalization. Evidently, the strategic complementarities between goods and labor markets – that is, with $\sigma > 0$ – magnify the effects of spending shocks vis a vis a Bewley economy.

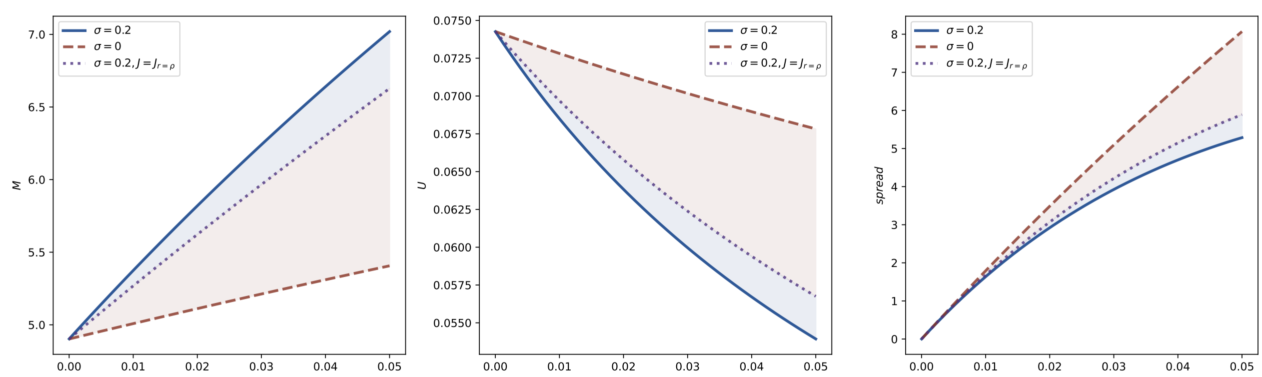



Figure 7: Steady-state values of full model, Bewley-Aiyagari($\sigma = 0$), and model with firms' profits discounted at rate $\rho$. The variables include the stock market capitalization $M$, unemployment $U$, and interest rate spread for different values of $\alpha$. The liquidity premium been annualized and expressed in interest rate form by multiplying by $12 \times 100$. The blue shaded region is the aggregate demand channel, the red region captures the real interest rate effect.

The right panel of Figure 7 plots the associated interest rate spread. When $\alpha = 0$, there is no expenditure risk and $r = \rho$ in each version of the model. For small $\alpha > 0$, households are liquidity constrained and further increases in $\alpha$ raise the liquidity premium. In the Bewley-Aiyagari version, the relationship is approximately linear. In the full version of the model, the large increase in the stock market capitalization loosens consumers' liquidity constraints and makes the spread concave. Shutting down the interest rate channel only mildly dampens the concavity.

The strong strategic complementarities between labor markets and goods markets renders the creation of private liquid assets elastic with respect to spending shocks. Figure 7 highlights the unique implications relative to a Bewley-Aiyagari model and the contribution of liquidity premia of firm values.

## 5.3 Semi-elasticities of the stock-market value and employment

We also gauge the role played by liquidity in stock and labor markets by examining the relationship between the interest rate spread and the variables of interest. In particular, using the calibrated parameter values and the steady-state equations we calculate the semi-elasticities

$$\begin{aligned}
\epsilon_{J,s} \equiv \frac{\partial \log(J)}{\partial s} &= 0.0996 \\
\epsilon_{n,s} \equiv \frac{\partial \log(n)}{\partial s} &= 1.10446 \\
\epsilon_{n,M} \equiv \frac{\partial \log(n)}{\partial M} &= 2.3819
\end{aligned}$$

where $s = \rho - r$ is taken to be fixed, and these semi-elasticities are evaluated at the long-run equilibrium. Thus, in the long-run equilibrium the semi-elasticity of firm-values with respect to the liquidity premium are relatively inelastic and employment/unemployment rates are responsive to liquidity. Our model implies that job creation is quite sensitive to liquidity and financial frictions.

We can provide evidence in favor of the model's empirical implication that unemployment, the stock market, and liquidity premia are related in the long-run. Farmer (2012) provides evidence of a cointegrating relationship between stock market values and the unemployment rate over the period 1959-1979. We revisit this relationship and include the liquidity spread and evaluate over the period 1959-2019. Table 2 presents the findings. First, we fail to reject the null of no cointegration, using the Johansen maximum eigenvalue test, in the case where the coefficient on the liquidity premium is normalized to zero. Second, we reject the null at

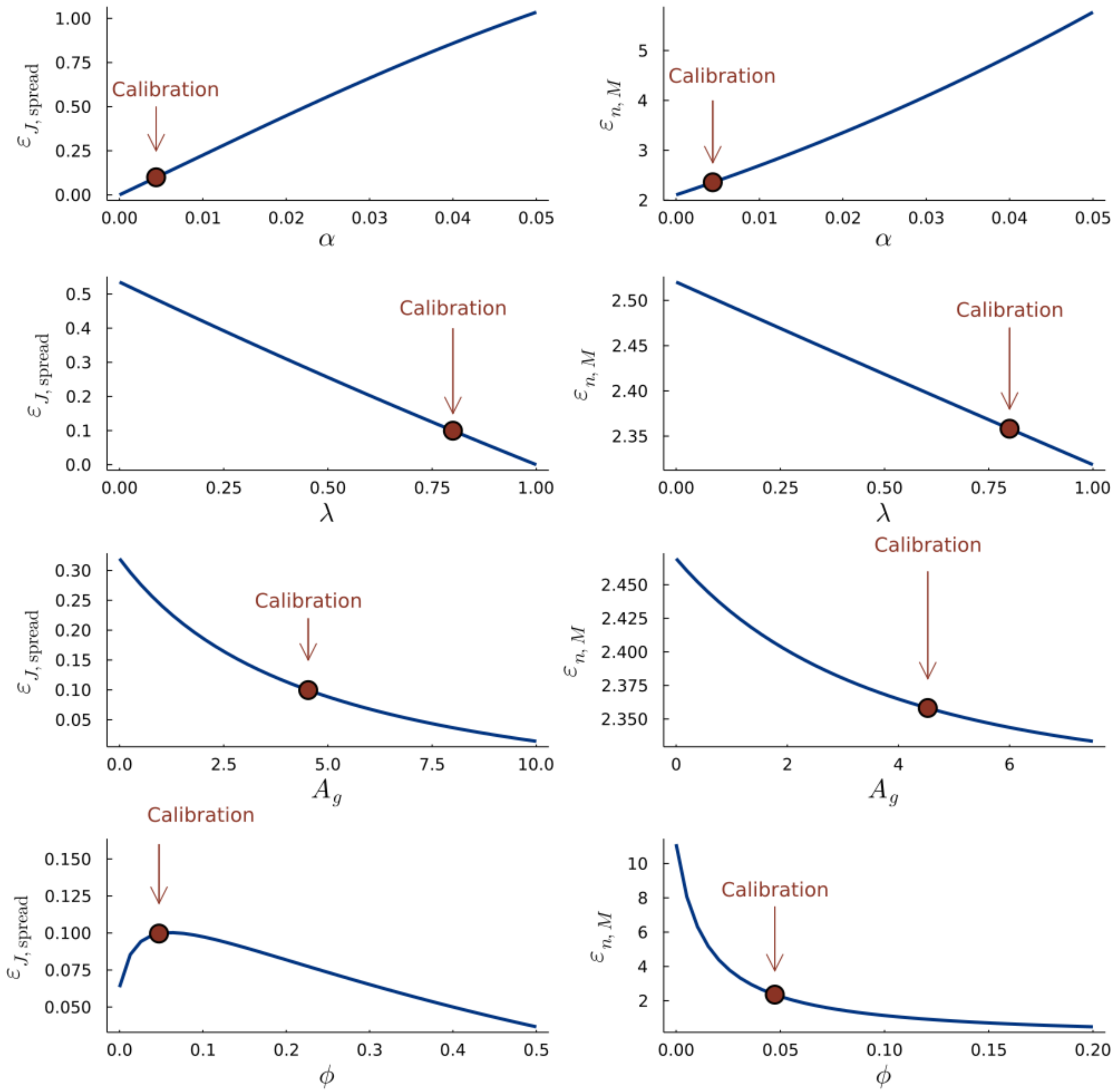



Figure 8: Counterfactuals. Spread semi-elasticities in a long-run equilibrium under a variety of counterfactual scenarios.

the 10% level in favor of a cointegrating relationship between the three variables. Moreover, as implied by the model in a steady-state equilibrium, unemployment is negatively related in the long-run to stock market capitalization and the interest rate spread.

One advantage to working with a calibrated model, consistent with U.S. data, is that we can use counter-factual analysis to better understand the role of liquidity in stock and labor

| Coint. coeff.'s | $\beta_1$ | $\beta_2$ | $\beta_3$ | $J(0)$ |
|---|---|---|---|---|
| Unemp. & Stock only | 1.0* | 0.55 | 0.0* | 3.498 |
| | | (0.138) | | |
| Unemp., Stock, and Spread | 1.0* | 0.60 | 1.4849 | 16.31** |
| | | (0.397) | (0.389) | |

Table 2: Long-run relationship between unemployment, stock market capitalization, and liquidity premia (1959-2019): $\beta_1 u_t + \beta_2 M_t + \beta_3 s_t = 0$. $J(0)$ refers to the Johansen max. eigenvalue test for the null of no cointegrating relationship. $*$ denotes normalization, $**$ is significance at 10% level. $u_t$ is the logit transformation of UNRATE, following Farmer (2012), the other variables are defined in Appendix B.

markets. Figure 8 computes these liquidity semi-elasticities under counterfactual assumptions about the key model parameters $\alpha, \lambda, A_g, \phi$. The top row of Figure 8 plots the spread elasticities for counterfactual values of the expenditure shock probability $\alpha$. The calibrated value of $\alpha$ implies that, on average, 5.3% of households in a year will experience an early-consumption expenditure shock that requires either unsecured or secured credit to finance the purchases. If a greater percentage of households face this expenditure risk, then the spread elasticity for firm-values and employment increase markedly. Similar, but less pronounced results obtain for different values for the fraction with perfect credit ($\lambda$) and public liquidity $A_g$.

## 5.4 The aggregate demand and real interest rate channels: the short-run

While the focus so far has been on the long-run, we now turn to an analysis of the short-run dynamics. We begin by presenting evidence from a structural VAR of the dynamic co-movements between stock market capitalization, interest rate spreads, and unemployment rates. We then solve for the model-implied impulse responses to a variety of economic shocks and compare the equilibrium paths with the empirical counterparts.

### 5.4.1 Empirical impulse responses

To present evidence of the co-movement we estimate the impulse responses to an identified structural stock price shock. A negative shock to the stock market induces less consumption and output. It is therefore natural to identify these shocks with sign restrictions. In general, sign restrictions generate set identification, in which there is a potentially large number of candidate models. Since the unemployment rate is a slow-moving state variable, we also restrict the contemporaneous impact of shocks on unemployment to zero. This assumption is fairly mild, but we nevertheless examine the consequences of relaxing the zero impact response in the appendix.

Table 3 summarizes the baseline identification scheme: The Appendix details the data

| Shock | Stock mkt | Spread | Ind. pr. | Cons. | Unemployment |
|---|---|---|---|---|---|
| Stock market | + | - | + | + | 0 |

Table 3: Identification assumptions. Restrictions only apply at impact.

construction and estimation strategy. Briefly, we apply the Bayesian algorithm developed by Arias, Rubio-Ramírez, and Waggoner (2018). The procedure combines the approach of imposing sign restrictions via the QR decomposition in Rubio-Ramirez, Waggoner, and Zha (2010) and uses an importance sampler to embed zero restrictions. The same data series for stock market capitalization, interest rate spread, and unemployment from the calibration exercise are incorporated into the VAR. We also include variables on aggregate consumption and output; see the Appendix for details.

Figure 9 plots the main piece of evidence linking stock prices, liquidity premia, and unemployment rates. The figure plots point-wise median and 68 percent equal-tailed probability bands for the impulse responses of the interest rate spread, stock prices, unemployment, consumption, industrial production, and vacancies to a negative unit standard-deviation shock to stock prices. The median stock price falls by 2%, and the median interest rate spread increases by 6 basis points. The interest rate spread is relatively persistent in responding to the stock market rise.

#### 5.4.2 Modeling expectations

When computing the transition paths following unanticipated shocks, one must take a stand on how households and firms form expectations. The typical benchmark is to assume rational expectations and solve for the rational expectations equilibrium path given the shocks. However, rational expectations requires agents to perfectly understand the dynamic evolution and behavior, including beliefs and preferences, of all other agents. The present model,

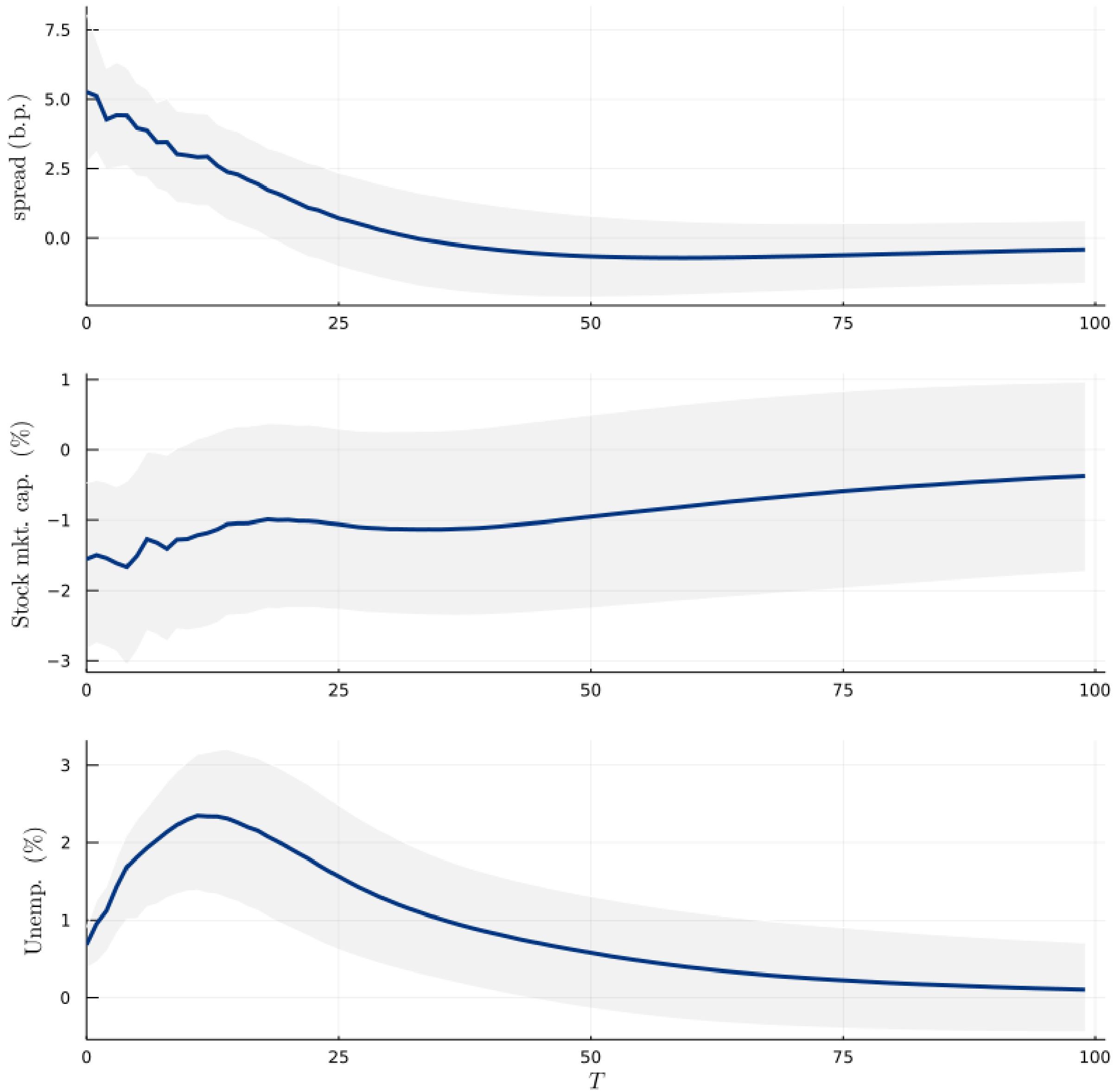



Figure 9: Impulse response to a 1 standard-deviation negative shock to the stock-market valuation. The solid line indicates the point-wise median, and the shaded region represents the 68% probability bands.

lacking aggregate stochastic shocks and featuring a single slow-moving state variable, does not capture the persistence of stock-price movements in the data. Moreover, in the presence of multiple equilibria, the rational expectations hypothesis is silent about equilibrium selection, a particularly salient point when an indeterminate steady-state exists. To address these limitations, we instead formulate an adaptive learning version of the model, following in the footsteps of Marcet and Sargent (1989) and Evans and Honkapohja (2001), among others. The adaptive learning literature replaces rational expectations with a "cognitive consistency principle": agents should be modeled like a good economist who formulates and adapts a well-specified forecasting rule about payoff relevant aggregate variables. To discipline that rule, expectations are typically assumed to come from a model that nests the rational expectations equilibrium. It is well known (e.g. Timmermann (1996), Branch and Evans (2011), Adam, Beutel, and Marcet (2017)) that replacing rational expectations with beliefs generated from a learning rule can substantially improve the empirical performance of otherwise standard asset-pricing models. We similarly adopt the learning approach and compare the findings with the perfect foresight alternative.[15]

In the absence of aggregate shocks, the limit point of a learning process is a steady state. Thus, we adopt a "steady-state" learning rule, as follows. First, we define a temporary equilibrium by taking one step ahead subjective expectations as given and impose market clearing. Without imposing rational expectations, the equations governing a temporary equilibrium are:

$$J_t = z_t - w_t + \frac{(1-\delta)M^e_{t+1}}{n^e_{t+1}\left(1+r^e_{t+1}\right)}$$
$$n_t = (1-\delta)n_{t-1} + A^{1/\xi}\left[\frac{J_t}{k\left(1+r_t\right)}\right]^{(1-\xi)/\xi}\left(1-n_{t-1}\right)$$

and without loss of generality $r^e_{t+1} = r\left(M^e_{t+1}, n^e_{t+1}\right)$. Thus, agents in the economy need only to generate one-step ahead forecasts of the aggregate variables, i.e. the stock market capitalization and the aggregate employment rate. Steady-state learning dictates that those forecasts should come from a geometrically weighted average of past-data, written recursively as

$$\begin{aligned} M^e_{t+1} &= M^e_t + \gamma\left(M_{t-1} - M^e_t\right) \\ n^e_{t+1} &= n^e_t + \gamma\left(n_{t-1} - n^e_t\right) \end{aligned}$$

[15]Other examples of learning in search-theoretic models include Baranowski (2015), Branch, Petrosky-Nadeau, and Rocheteau (2016), and Evans, Evans, and McGough (2021).

The parameter $0 < \gamma < 1$ is called the constant gain coefficient. The dynamic stability of a rational expectations equilibrium depends, in a complicated way, on whether the particular steady-state of interest is determinate or indeterminate and on the size of the gain coefficient $\gamma$. As $\gamma \to 0$, then beliefs are a simple time-average of the states, and in our impulse response experiments this case will be closest to the outcome under rational expectations. For $0 < \gamma < 1$ there is slow adaption to a steady-state that is stable under this learning rule, but larger values of $\gamma$ amplify the reaction of expectations and so are capable of further propagating shocks. This feature will help generate persistence in the model's impulse responses consistent with the data.

While employing a learning process now seemingly emerges from left field, it has several advantages for us and keeping the model tractable. First, by setting $\gamma \to 0$ we can reasonably approximate the rational expectations equilibrium path. Second, learning provides a tailor-made selection device in the presence of multiple equilibria. Third, learning helps generate persistence in the model's impulse responses consistent with the data.

With this definition of subjective beliefs, the temporary equilibrium is generated by a pair of non-linear first-order difference equations. In the calibrated model and counterfactual experiments below, the steady-state is stable under learning for all values of $\gamma$, but in the case of multiple steady-states the lower indeterminate steady-state will be unstable. In the impulse responses that follow we construct short-run responses to unanticipated temporary shocks to the key parameters $B$ (demand for early-consumption goods), $\alpha$ (expenditure shock frequency), and $A_g$ (public liquidity), which decay with a persistence $0 \leq \mu \leq 1$. Beliefs and the economy are initialized at the steady-state. The values of the learning gain $\gamma$ and the persistence of the exogenous shock $\mu$ play important roles in the propagation and persistent impacts of the shocks. We calibrate those values to $\gamma = 0.20$ and $\mu = 0.90$ to provide the closest match between the estimated impulse response in Figure 9 and the model implied response to a demand shock. A gain value of $\gamma = 0.20$ implies agents weight most heavily the most recent 6 months of data. We also show how the results depend on faster/slower learning speeds.

### 5.4.3 Demand shocks

We begin our short-run analysis by considering an unanticipated negative shock to $B$, i.e. the marginal utility from consuming the early good. The scale of the innovation is fixed so that the quantitative impact to stock market capitalization is in line with the estimates in Figure 9. We consider two cases: $\sigma = 0.2$ (calibrated model) and $\sigma = 0$, the counterfactual of the Bewley model without the aggregate demand channel. The difference between the two describes the additional propagation and volatility attributable to the aggregate demand channel (and its general equilibrium impacts on the interest rate channel).

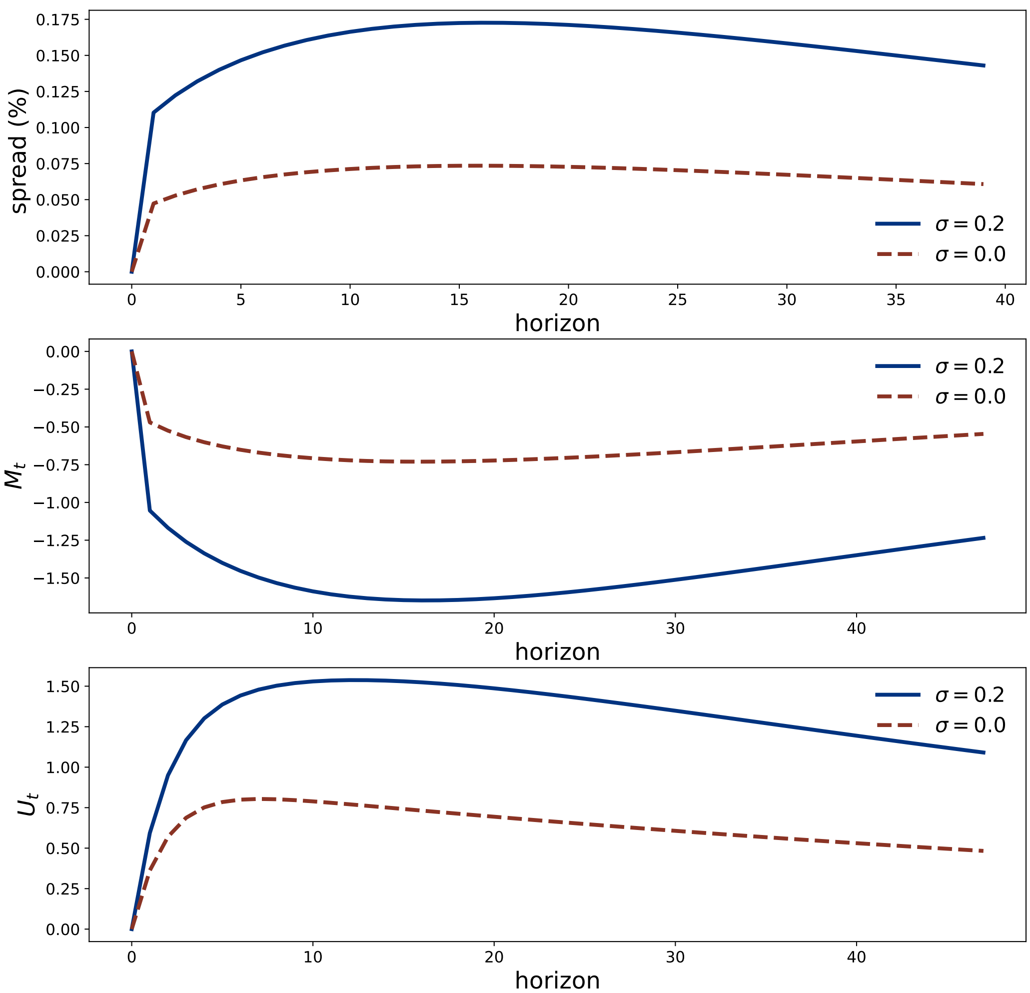



Figure 10: Demand shocks.

The top panel in Figure 10 plots the interest rate spread $\rho - r_t$ across the two cases. The interest rate spread jumps contemporaneously on impact. The greater liquidity spread is equivalent to a decrease in the real interest rate capturing the decrease in aggregate demand producing lower demand for the liquid asset and an increased liquidity premium. The real interest rate is not a state variable and is a function of contemporaneous firm-values and the aggregate employment rate. The middle panel plots the impact on stock market capitalization $M$. Here the impact on stock market values peaks at the same magnitude as the estimated impulse responses, though both the hump-shape and the persistence is stronger in the model than in the data. Finally, the bottom panel plots the unemployment response. The unemployment rate increases with an elasticity similar to the estimated effect.

The short-run dynamics in Figure 10 tell the following story for the joint determination of liquidity, stock market values and unemployment. An exogenous decline in the demand for the early-consumption goods has a large immediate impact on the (subjective) expected value to a firm vacancy. As firm vacancies decrease, the aggregate stock market capitalization declines, and the unemployment rate increases gradually. Beliefs, in turn, react to the new data and firms/consumers expect that next period's stock market values and employment rates will be lower. While the magnitude of the demand shock attenuates in the next period, the expectations-effect and lower demand for liquid assets propagates into a larger decrease in stock market values. With lower stock market capitalization, there is a negative effect on constrained consumers' demand for the good, leading to further reductions in vacancies and unemployment rates. This cycle continues until the demand shock wanes, and beliefs converge back to their (long-run) rational expectations equilibrium values.

Notice also that the aggregate demand channel plays an important role in the propagation of the shock and beliefs. The dashed line in Figure 10 plots the impulse response to the same magnitude shock but in an economy without an aggregate demand channel ($\sigma = 0$). Evidently, the impact on stock market valuations and unemployment rates is roughly half of what it is in the calibrated economy.

### 5.4.4 Expenditure shocks

Figure 10 considered a shock directly to the marginal utility to the consumer from consuming the early good; that is, a shock to the intensive margin of aggregate demand. We now consider an expenditure shock to the frequency of the idiosyncratic preference shock for early consumption. The shock to $\alpha$ is essentially an increase to the number of consumers who would like to consume early; that is, a shock to the extensive margin of demand. Because the calibrated value of $\alpha$ is close to zero, we consider here a 20% innovation to $\alpha$ increasing its value from $0.0044$ to $0.0053$. That is, a temporary increase of about 1% of the population exposed over the course of 12 months to an expenditure shock.

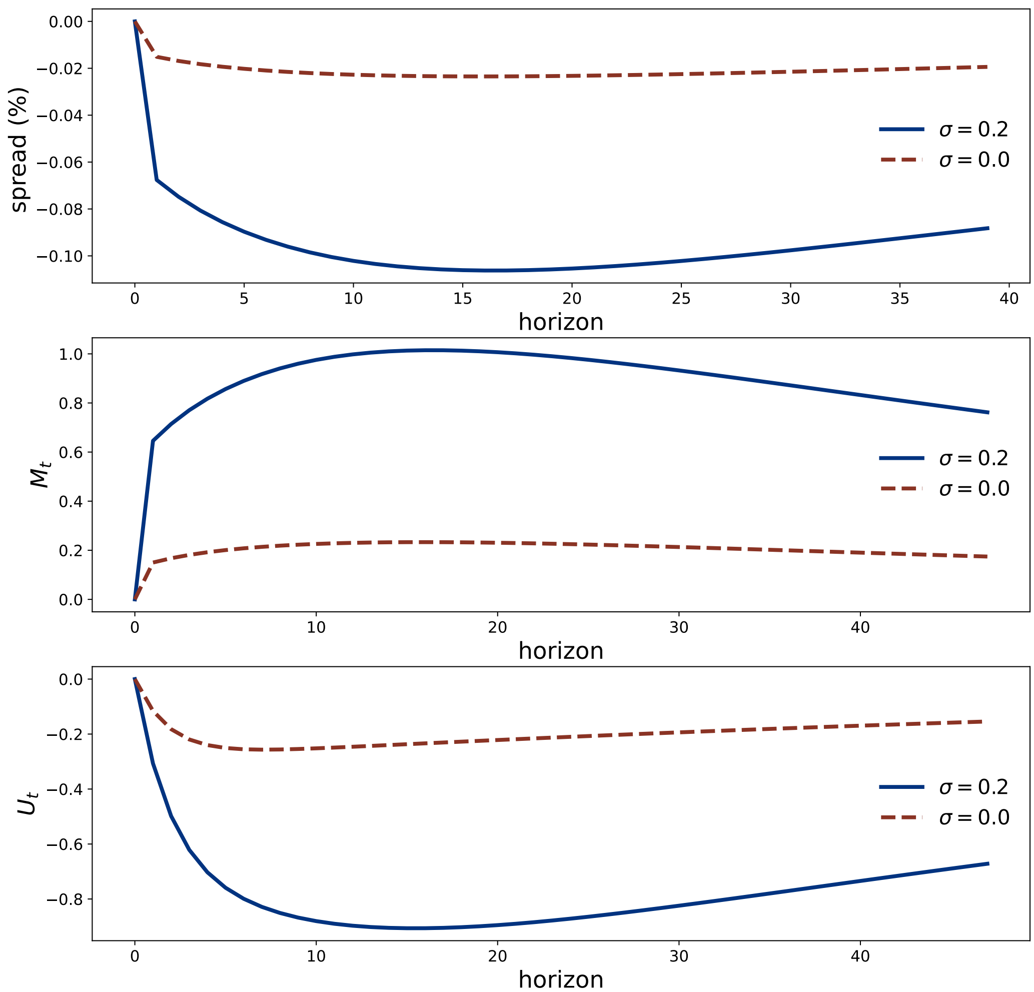



Figure 11: Expenditure shocks.

Figure 11 again plots the short-run dynamics to the interest rate spread, stock market capitalization, and the unemployment rate. The effect of an expenditure shock is roughly the same as the intensive margin shock to $B$. As there is an increase in the number of households seeking to consume early, there is an increase in the demand for the safe asset. Firms become more valuable and post more vacancies, causing the stock market capitalization to increase and the unemployment rate to fall. Comparing the paths to the counterfactual with $\sigma = 0$

illustrates a starker difference here from Figure 10. In the Bewley economy, shocks to $B$ have a stronger impact than the shock to $\alpha$. The expenditure shock without an aggregate demand channel still impacts the present values of firms, and so the stock market capitalization and unemployment rate, it is just the magnitude is much smaller than the intensive margin effect.

#### 5.4.5 Public liquidity shocks

The aggregate demand channel in short-run economic dynamics does not always increase shock propagation and volatility. An example of that is given in Figure 12. Here we consider a contractionary shock to $A_g$, the quantity of government bonds and a proxy for public liquidity provision. The experiment here is to analyze the effects to an economy from a temporary decrease in public liquidity and greater reliance on private liquidity provision.

The middle panel of Figure 12 demonstrates that the impact of a public liquidity shock on stock market values is *greater* in the Bewley economy with $\sigma = 0$. The intuition comes back to the potentially offsetting impacts of the real interest rate and aggregate demand channels. The effect of a decrease in $A_g$ is to decrease the spread/increase the real interest rate. With less public liquidity, there is greater demand for stocks, which increases stock market capitalization and increases vacancy creation, lowering the unemployment rate strongly. In the calibrated model with $\sigma = 0.20$ there is additionally an attenuating effect operating through the aggregate demand channel. Less public liquidity provision decreases the overall market liquidity and, hence, the demand for early consumption.

### 5.5 Counterfactual: a perfect storm

The calibrated model features an important role for liquidity and aggregate demand. However, the overall size of the early-consumption sector, and the liquidity premium, imply that there is not sufficiently strong strategic complementarities to generate multiple steady-state equilibria. In this section, we ask what conditions would be required for there to be a second steady-state with high unemployment and low stock market values? It turns out that what is needed is a constellation of parametric shocks to firm productivity, public liquidity, and the intensive/extensive margins to aggregate demand. If firm productivity and public liquidity are sufficiently low and the intensive/extensive margins to aggregate demand are sufficiently high, then multiple steady-state equilibria can occur with the rest of the parameters held at their calibrated values. We call this situation a "perfect storm." It occurs in the event of a recession, with a greater reliance on private liquidity provision, and elevated uninsured

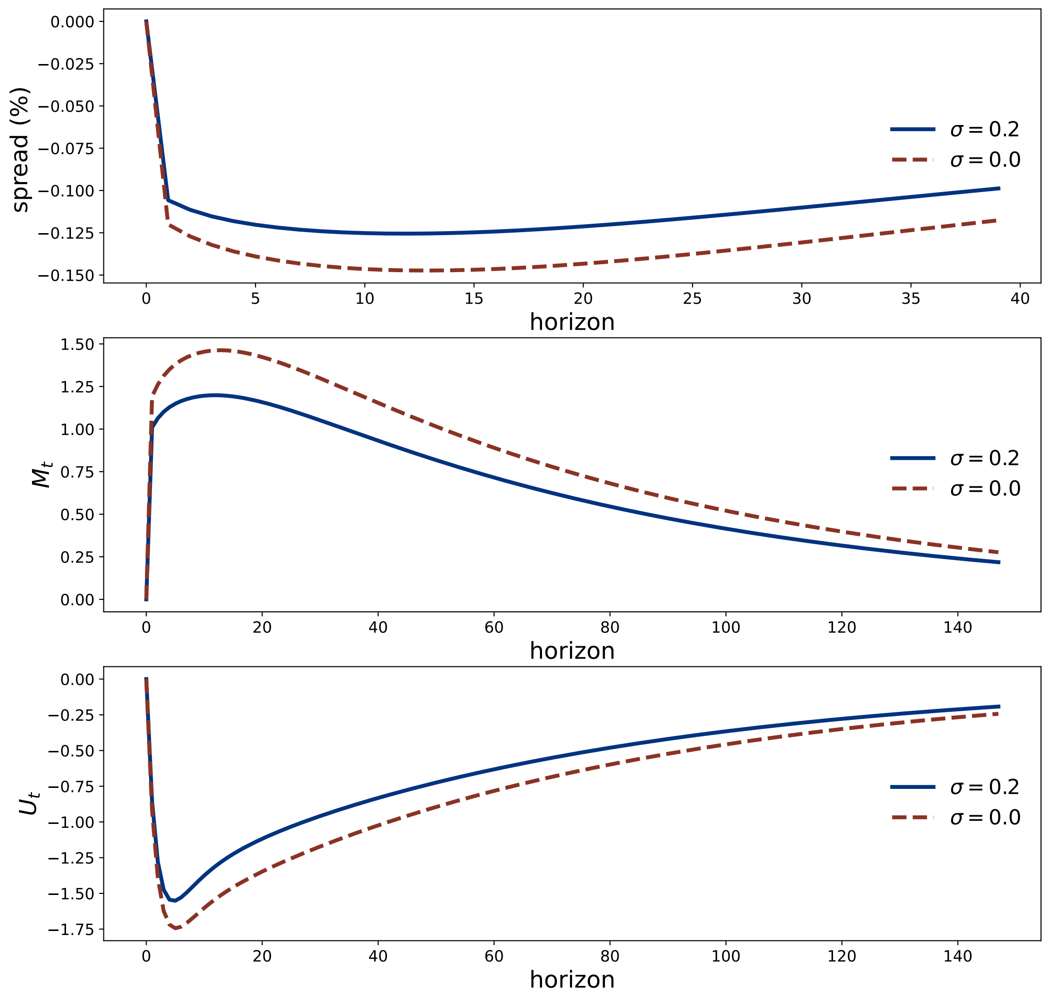



Figure 12: Public liquidity shocks.

expenditure risk.

Figure 13 plots a perfect storm that arises with the parameterization: $\alpha = 0.04, B = 18.0, \bar{z} = 0.8, A_g = 1.25$. In this scenario, the fraction of consumers facing the expenditure risk in a given month jumps an order of magnitude from .4% to 4%, there is a 20% reduction in aggregate productivity, and public liquidity drops roughly by 25%. In this case, now the contour plot for $J = f(n, J)$ is not nearly as close to the unconstrained steady-state,

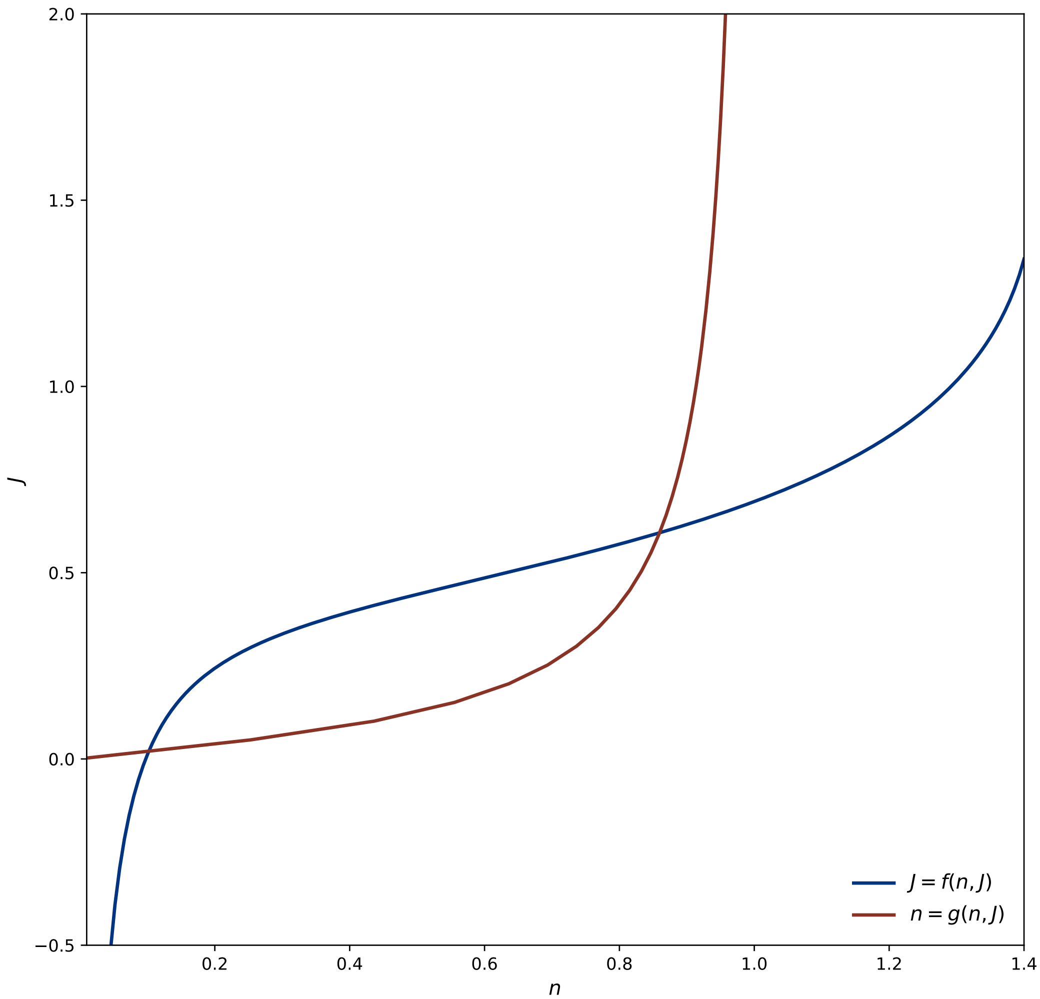



Figure 13: Perfect storm: steady-states.

though equilibrium unemployment is close to its historical average. The strong strategic complementarities imply the existence of another steady-state with unrealistically high levels of unemployment and a near collapse of stock market values.

The perfect storm experiment we are interested in imagines an economy at its usual long-run equilibrium, and then a "shock" hits that brings into existence a secondary steady-state that represents a crash state. However, there are two reasons to not take that lower steady-

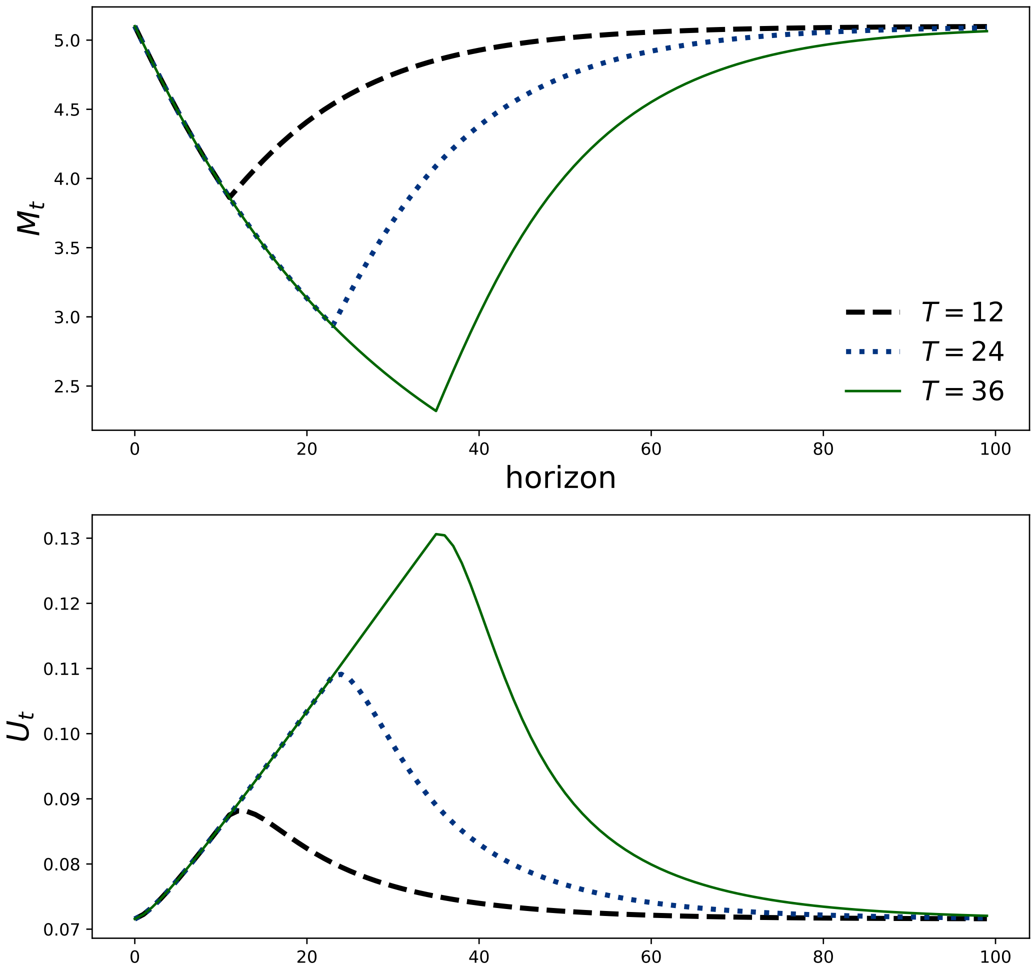



Figure 14: Perfect storm: shock to expectations.

state too seriously as an equilibrium outcome. First, the lower steady-state is very close to autarky and so its empirical relevance is limited. Second, it turns out that the higher steady-state is determinate and the lower steady-state is indeterminate. Under the learning rule we have adopted this implies that the high steady-state will be stable under learning and the low steady-state is unstable. Thus, under a perfect storm experiment the crash state is not a limit point to learning and so would not be observed in the data. However, it turns out that even though the lower steady-state is unstable asymptotically it still has an effect on the transition path back to the steady-state following a shock.

Figure 14 plots the experiment, assuming that the perfect storm will only last for a finite period of time (12, 24, or 36 months) at which point the parameters return to their calibrated values and beliefs will evolve until the economy transitions back to its original equilibrium. In Figure 14 we can see that the pressure exerted by the lower steady-state draws stock market values and unemployment rates towards that crash state. The magnitude of the collapse in stock market prices and employment rates depend on the persistence of the perfect storm. In the case of a 1 year storm, unemployment increases by 2 percentage points and stock market values drop by roughly 20%. If it is a 3 year storm, however, unemployment crosses 13% and stock market values drop by half.

## 6. Conclusion

We have studied the effects of changes in household liquidity constraints on the labor and stock markets. We generalized the Mortensen-Pissarides model along a single dimension: idiosyncratic expenditure risk introduces a limited commitment problem. Households can finance some shocks with unsecured debt but must pledge a mutual fund comprised of stocks and bonds for others. As a result of this single twist, high stock market valuations relax household liquidity constraints and induce firms to create more jobs. The stock market consequently rises further and propagates consumption demand. A novel finding is that these complementarities can produce multiple steady-state equilibria with high employment/high stock market capitalization co-existing with low employment/low stock market values.

We calibrated the model to the long-run properties of the U.S. economy. In the long-run, stock market capitalization is inelastic, and the unemployment rate is quite elastic, with respect to the interest rate spread, which reflects a liquidity premium. The role of liquidity can be split into separate real interest rate and aggregate demand channels, with the latter being novel to the model in this paper. By comparing the calibrated model with a counterfactual economy without the aggregate demand channel, we show strong strategic complementarities at work in determining both stock market values and unemployment rates. We also present evidence on the co-movement between stock markets and unemployment via a structural VAR identified with sign restrictions. Impulse responses from the calibrated model to unanticipated shocks to aggregate demand and public liquidity broadly capture the empirically observed negative co-movement in the short-run between the unemployment rate and stock market valuations.

This framework is a fertile benchmark for introducing money alongside equity. Inflation reduces the rate of return on money and induces substitution into stocks and bonds, with implications for the interest rates, firm values, and unemployment.[16] Another promising extension is to relax quasilinearity and study dynamic equilibria with both expenditure and income risk, where the latter arise endogenously from labor market frictions.

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[16] In related work, Geromichalos, Licari, and Suárez-Lledó (2007) study an environment in which money and a real financial asset compete as media of exchange. Money is valuable if and only if the stock of the financial asset is insufficient to satisfy liquidity needs, and inflation reduces the rate of return on the asset and thereby raises its price. However, this extension would endogenize the stock of the financial, link it to expenditure risk, and thereby affect the shape of the Phillips curve.

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# Appendix: For Online Publication

## A. Data appendix

| Variable | Source |
|---|---|
| Stock market capitalization | Wilshire 5000 (in logs), |
| | FRED code WILL5000INDFC |
| Buffet measure of market capitalizaton | FRED code NCBEILQ027S |
| Nominal wage | A576RC1 |
| Total unemployed | FRED code UNEMPLOY |
| Total employed | FRED code CE160V |
| Average hourly earnings of all employees | Fred code CES0500000003 |
| Average weekly hours of all employees | Fred code AWHAETP |
| Unemployed less than 5 weeks | FRED code UNEMPLT5 |
| Vacancies | https://www.briancjenkins.com/dmp-model |
| Moody's AAA | FRED code AAA |
| Long-term government bonds | FRED code GS20 LTGOVTBD |
| Treasury bonds with 20-year maturities | FRED code GS20 |

Table 4: Data sources used in motivating evidence and model calibration.

## B. Further details on identifying the stock price shock

Each series is monthly from January 1959 to October 2016. The frequency motivates the choice of industrial production in lieu of overall output. Consumption is measured in personal consumption expenditures and is normalized by the population and the consumer price index. Industrial production is also normalized by population. We use the longer horizon of stock price data available from Robert Shiller's webpage and normalize it by the nominal wage. The stock market valuation, industrial production, consumption, and vacancies enter the VAR in log levels.[17]

We apply the algorithm developed by Arias, Rubio-Ramírez, and Waggoner (2018). The authors provide a theory in a Bayesian setting to independently draw from a family of conjugate posterior distributions with sign and zero restrictions. They leverage the fact that a SVAR can be written as a product of the reduced form and a set of orthogonal matrices. There is a conjugate uniform normal inverse Wishart density for the reduced form parameters. There is also a uniform conjugate density over the set of orthogonal matrices conditional on the reduced-form parameters. The method draws from the conjugate uniform-normal-inverse Wishart posterior over the orthogonal reduced-form parameterization and maps the draws into the structural parameterization.[18] The procedure combines the approach of imposing sign restrictions via the QR decomposition in Rubio-Ramirez, Waggoner, and Zha (2010) and uses an importance sampler to embed zero restrictions. Another another common way of imposing restrictions on signs and zeros the penalty function approach by Mountford and Uhlig (2009). However, this approach imposes additional constraints to zeros and signs and thereby distorts inference. For instance, Beaudry, Nam, and Wang (2011) provide use the penalty function approach to argue for the importance of optimism shocks in business cycles, but Arias, Rubio-Ramírez, and Waggoner (2018) show that the results largely depend on the additional constraints. Finally, we take $10^6$ draws of the orthogonal reduced form, each of which consists of the coefficient vector, the covariance-variance matrix, and the orthgonal matrix.

## C. Role of the zero impact response of unemployment for inference

This section relaxes the zero restriction of the response of unemployment in the first month. Table 5 describes the relaxed identification scheme.

Figure 15 examines a shock to the stock market valuation without a zero restriction.

[17] Robert Shiller's webpage, http://www.econ.yale.edu/ shiller/data.htm contains the stock market data. In general, the spurious regression problem does not apply to a VAR with nonstationary series provided there are enough lags. Sufficient lags induce a cointegration relationship, which generates consistent estimates of the impulse response functions, as stressed by Hamilton (1994). Many recent applications of VAR's, such as Arias, Caldara, and Rubio-Ramirez (2019), enter variables in levels.

[18] The procedure thus involves transforming densities from the orthogonal reduced-form parameterization to the structural form and relies on change-of-variable theorems. Baumeister and Hamilton (2015) directly draw on the structural parameterization but require the Metropolis Hastings algorithm to draw from the posterior density.

| Shock | Stock market | Spread | Industrial production | Consumption |
|---|---|---|---|---|
| Stock market | + | - | + | + |

Table 5: Identification assumptions. Restrictions only apply to the impact response.

The median responses are similar, and now it takes about two quarters for the probability bands associated with the unemployment rate to not contain zero. Though relaxing the zero

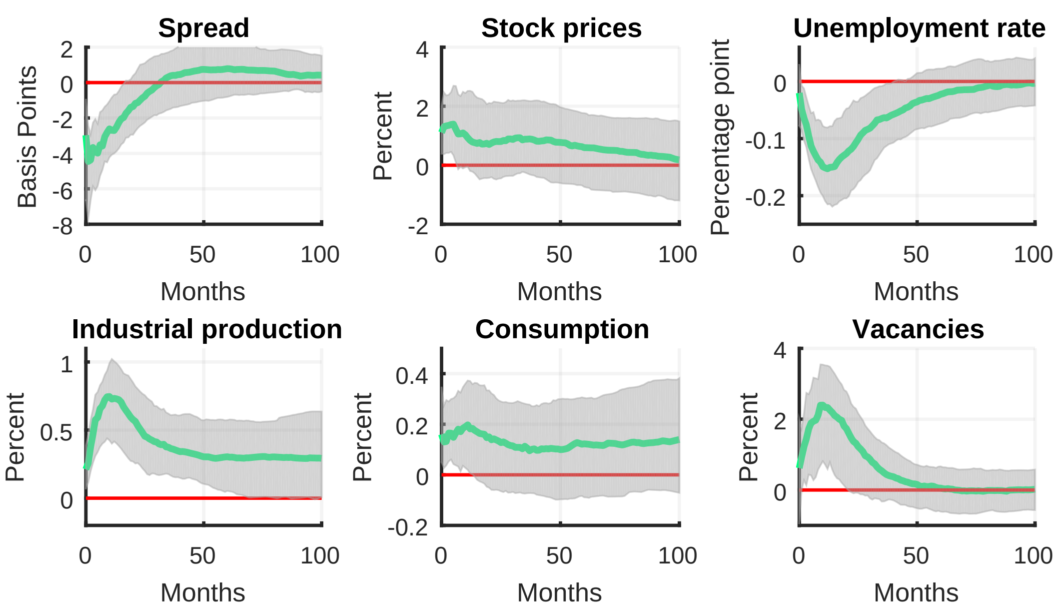


Figure 15: Impulse response to a positive unit standard-deviation shock to the stock-market valuation. The identifying restrictions follow Table 5.

restriction on unemployment increases the area between the probability bands and makes inference less precise, there is still reasonable evidence of a negative relationship between the stock market and unemployment pertaining to the aggregate demand and interest rate channels.

# D. Derivation of wage equation

Let $\tilde{J}_t$ be the corresponding expected present-value of the firm before the labor market opens. For the firms, we have

$$\tilde{J}_t = z_t - w_1 + (1-\delta)\frac{\tilde{J}_{t+1}}{1+r_{t+1}} + \delta\frac{\tilde{J}_{v,t+1}}{1+r_{t+1}}$$
$$\tilde{J}_{vt} = -k + q(\theta_t)\frac{\tilde{J}_{t+1}}{1+r_{t+1}}$$

The free entry condition implies that $\tilde{J}_{vt} = 0$, or that

$$\tilde{J}_{t+1} = \frac{(1+r_{t+1})k}{q(\theta_t)}$$

It follows that $\tilde{J}_t = J_t$, as we previously defined it.

The Nash bargaining problem solves

$$w_{1t} = \arg\max(U_{1t}(w_t) - U_{0t})^{\phi}(J_t(w_t) - \tilde{J}_{vt})^{1-\phi}$$

Using the fact that $w_{1t}$ enters linearly in both $U_1(w_t)$ and $J_t(w_t)$, we have

$$(1-\phi)(U_{1t} - U_{0t}) = \phi J_t$$
$$\Leftrightarrow U_{1t} - U_{0t} = \phi S_t$$

where $S_t = U_{1t} - U_{0t} + J_t$ is the joint surplus. Straightforward calculations lead to the Nash wage expression.

# E. Equilibrium in continuous time

Equilibrium is a path $(J, r, p, n)$ satisfying

$$
\begin{aligned}
(r+\delta)\, J &= \bar{z} + \max_y \{py - c(y)\} - w_1 + \dot{J} \\
c'^{-1}(p) &= \frac{\alpha}{n}\left[\lambda v'^{-1}(p) + (1-\lambda)\min\left\{v'^{-1}(p), \frac{nJ + A_g}{p}\right\}\right] \\
\rho - r &= \alpha(1-\lambda)\left[\frac{v'\left(\frac{nJ+A_g}{p}\right)}{p} - 1\right]^{+} \\
\dot{n} &= m\left[1, \theta(J)\right](1-n) - \delta n.
\end{aligned}
$$

# F. Robustness to constant gain coefficient

The model for belief updating essentially weights past data with geometrically declining weights. Greater values of the gain are more responsive to data innovations and are partially responsible for the persistent and hump-shaped impulse responses. Here we consider the robustness to smaller and larger gain parameters. When $\gamma \to 0$, then the agents essentially believe the economy will return to steady-state in the next period. It turns out that the small gain case closely resembles the dynamics under perfect foresight (rational expectations) and gives a natural comparison point for the effect of learning on the results.

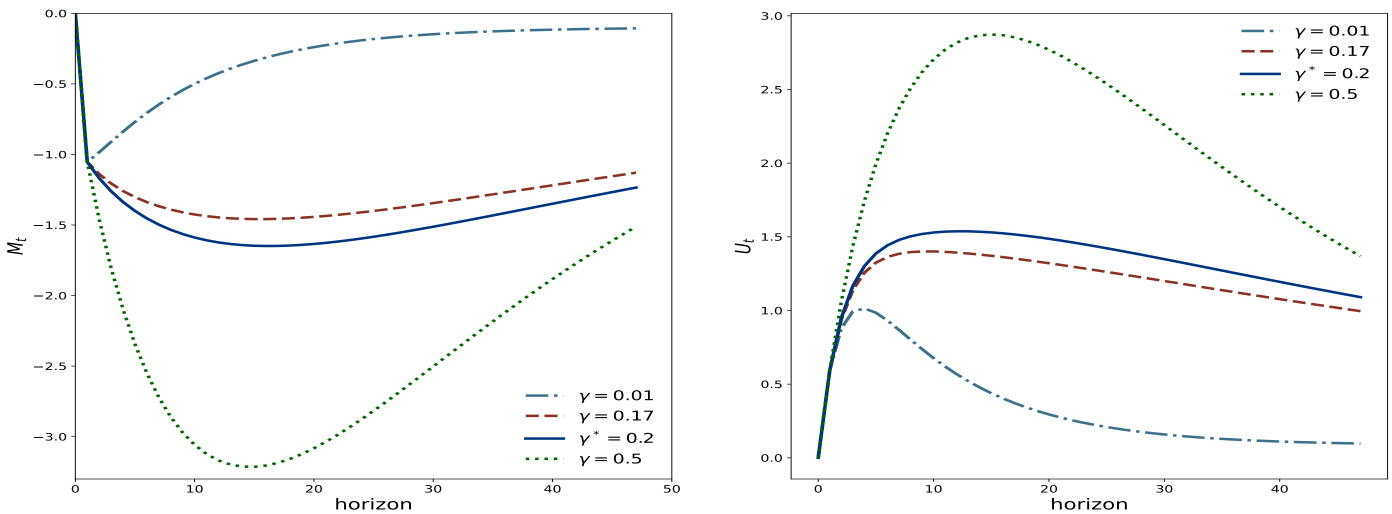


Figure 16: Demand shocks. Robustness to learning speed.

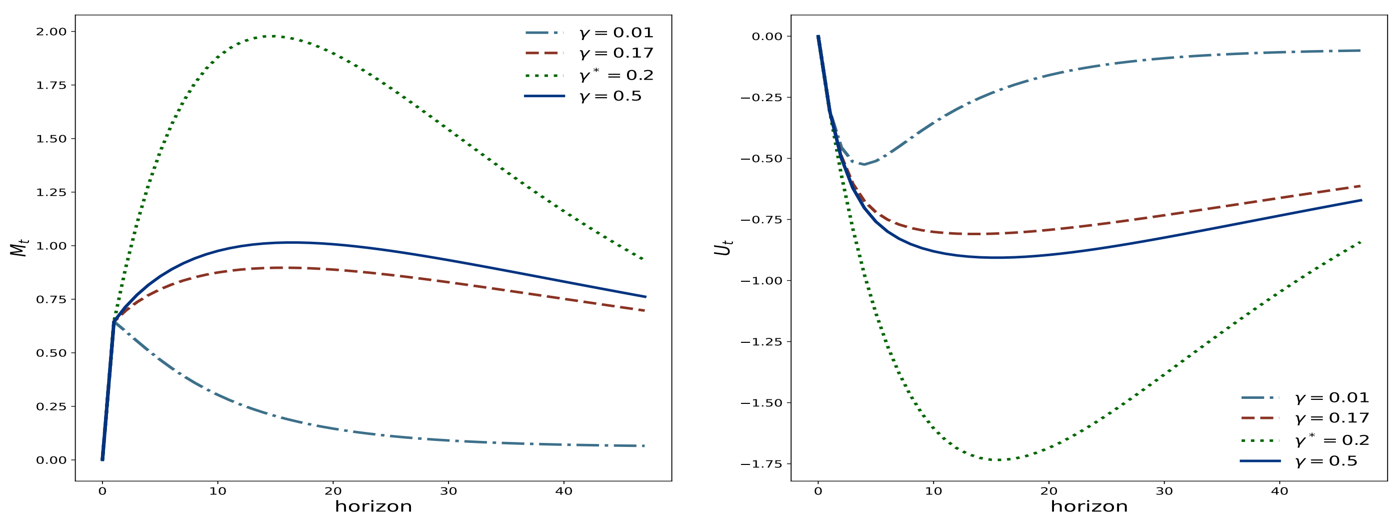


Figure 17: Expenditure shocks. Robustness to learning speed.

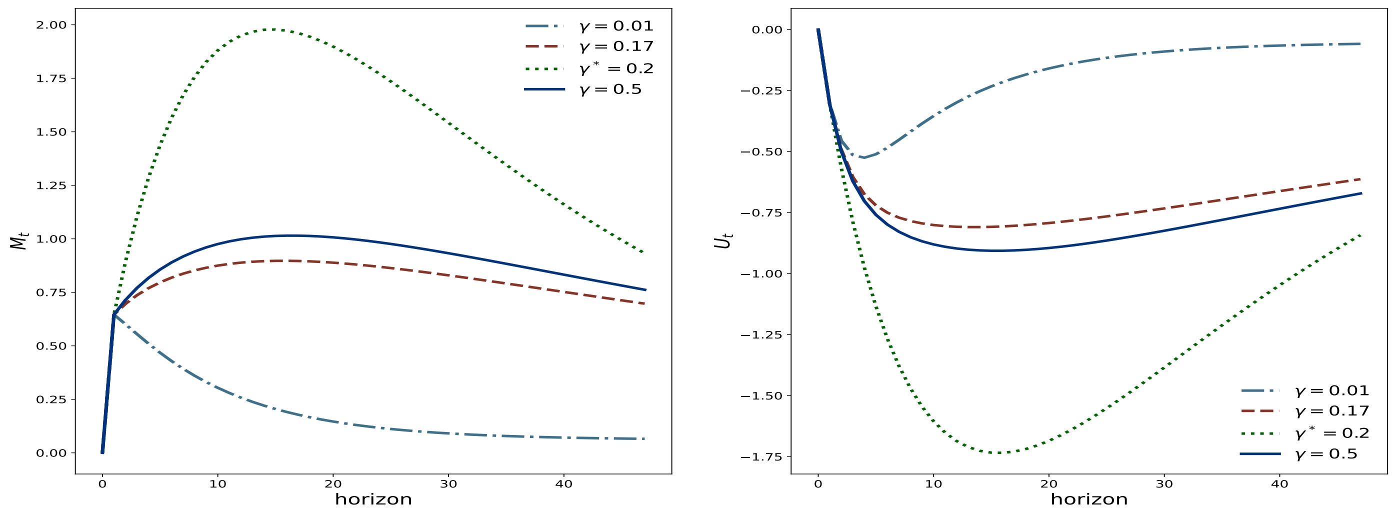


Figure 18: Public liquidity shocks. Robustness to learning speed.

In each of Figures 16-18 we plots the impulse responses to the same demand, expenditure, and public liquidity shocks considered earlier. The solid line in each plot reproduces the earlier impulse response. The other lines correspond to different gain values $\gamma = 0.01, 0.17$ and $0.5$. In each plot the shock propagation, persistence, and hump-shape response is increasing in the learning gain. One interesting difference is in the case of the public liquidity shock (Figure 18) where the larger gain induces a negative root in the dynamics and the transition path features over-shooting of the steady-state equilibrium. In the case of small gains (and, perfect foresight), there is little additional propagation and the economy's responses to the shock decay at the same rate as the underlying shock.

## G. Details on Equilibrium Determinacy

Section 4 presented results on the set of equilibria in a continuous time approximation of the model. While the continuous time formulation enhances tractability, in this Appendix we provide additional details on the set of rational expectations equilibria in the discrete-time version of the model. In the following, we set $v(y) = y^{1-\gamma}/(1-\gamma)$.

## Mortensen-Pissarides Economy

Here we shut-down the early consumption preference shocks: $\alpha = 0$. An equilibrium is a pair $(J_t, n_t)$ that is non-explosive solution to the following pair of non-linear difference equations:

$$J_{t+1} = -\frac{1+\rho}{1-\delta}z + \frac{1+\rho}{1-\delta}J_t$$
$$n_{t+1} = (1-\delta)n_t + \left(\frac{J_{t+1}}{(1+\rho)k}\right)^{\frac{1-\xi}{\xi}}(1-n_t)$$

There is a unique steady-state $\bar{J} = \frac{1+\rho}{\delta+\rho}z$ and $\bar{n} = 1/(1+\delta\left((\delta+\rho)k/z\right)^{(1-\xi)/\xi})$. Moreover, the eigenvalues of the Jacobian matrix, evaluated at the unique steady-state, are

$$\lambda_1 = \frac{1+\rho}{1-\delta}$$
$$\lambda_2 = 1-\delta-\left(\frac{z}{(\delta+\rho)k}\right)^{(1-\xi)/\xi}$$

We can summarize the results as follows.

**Proposition 1** *Let $\alpha = 0$. There exists a unique steady-state. For $z/k < (2-\delta)^{\xi/(1-\xi)}(\delta+\rho)$, there exists a unique perfect foresight equilibrium path to the steady-state for any given $n_0$. Else, there is a degenerate equilibrium with a unique perfect foresight path for $n_0 = \bar{n}$, and no equilibrium otherwise.*

## Perfect Credit

Now we consider the case of a DMP model with two goods, and perfect credit for the early-consumption good. In this case, $r_t = \rho$, and equilibrium is a (non-explosive) solution to the difference equations

$$J_{t+1} = -\frac{1+\rho}{1-\delta}z + \frac{1+\rho}{1-\delta}J_t - \frac{\sigma(1+\rho)}{(1+\sigma)(1-\delta)}\left(\frac{n_t}{\alpha}\right)^{-\frac{\gamma(1+\sigma)}{\sigma+\gamma}}$$
$$n_{t+1} = (1-\delta)n_t + \left(\frac{J_{t+1}}{(1+\rho)k}\right)^{(1-\xi)/\xi}(1-n_t)$$

A steady-state is a pair $(\bar{J}, \bar{n})$ that solves

$$\bar{J} = \frac{1+\rho}{1-\delta} z + \frac{\sigma(1+\rho)}{(1+\sigma)(1-\delta)} \left(\frac{\alpha}{\bar{n}}\right)^{-\frac{\gamma(1+\sigma)}{\sigma+\gamma}}$$
$$\bar{J} = (1+\rho) k \left(\frac{\delta \bar{n}}{1-\bar{n}}\right)^{\xi/(1-\xi)}$$

The first equation is monotonically decreasing in $n$, with $\bar{J} \to \infty$ as $\bar{n} \to 0$, while the second equation monotonically increases with $n$ and features $\bar{J} \to \infty$ as $\bar{n} \to 1$, while also $\bar{J} = 0$ when $\bar{n} = 0$. Thus, there exists a unique steady-state. Similarly computing the eigenvalues of the Jacobian, we can establish the following result.

**Proposition 2** *In the DMP with 2 goods and perfect credit, there is a unique steady-state. Moreover, for $k$ sufficiently large, there exists a unique (non-explosive) perfect foresight path, for any given $n_0$, that converges to the steady state. For $k$ sufficiently small, the steady-state is a source and there exists a degenerate equilibrium with $n_0 = \bar{n}$.*

## Bewley-Aiyagari Economy

Now we consider the Bewley-Aiyagari version of the economy: there's two goods, a limited commitment problem for the early-consumption good, and a linear cost function for the early-consumption good. These assumptions imply that the real interest rate is endogenous, compared to the standard DMP model or the model with perfect credit. However, $py - c(y) = 0$, which is interpreted as the marginal product of a job (asset) is independent of the *market value* of the liquid assets. With these assumptions, an equilibrium is a (non-explosive) solution to the pair of equations

$$J_t = z + (1-\delta) \frac{J_{t+1}}{1+r_{t+1}}$$
$$1 + r_t = \frac{1+\rho}{1 + \alpha \left[(n_t J_t)^{-\gamma} - 1\right]^+}$$
$$n_{t+1} = (1-\delta) n_t + \left[\frac{J_{t+1}\left(1 - \alpha + \alpha (n_{t+1} J_{t+1})^{-\gamma}\right)}{k(1+\rho)}\right]^{(1-\xi)/\xi} (1 - n_t)$$

The first two equations, in turn, can be re-written as

$$J_t = z + \frac{(1-\delta)(1-\alpha)}{1+\rho} J_{t+1} + \frac{(1-\delta)\alpha}{(1+\rho)} n_{t+1}^{-\gamma} J_{t+1}^{1-\gamma}$$

As before, the steady-state can be calculated as a pair $(\bar{J}, \bar{n})$ that solve the equations

$$\bar{J}\left[1-\frac{(1-\delta)(1-\alpha)}{1+\rho}-\frac{(1-\delta)\alpha}{(1+\rho)}\bar{n}^{-\gamma}\bar{J}^{-\gamma}\right]=z$$
$$\left(\bar{J}/k\right)^{(1-\xi)/\xi}\left[\frac{1-\alpha+\alpha\left(\bar{n}\bar{J}\right)^{-\gamma}}{1+\rho}\right]^{(1-\xi)/\xi}=\frac{\delta\bar{n}}{1-\bar{n}}$$

As before, the first equation implies $\bar{J}$ is decreasing in $n$, with $\bar{J}\to\infty$ as $\bar{n}\to 0$. The second equation implies that $\bar{J}$ is increasing in $n$ with $\bar{J}\to\infty$ as $\bar{n}\to 1$ and $\bar{J}=0$ at $\bar{n}=0$. Again, it follows that there exists a unique steady-state.

Expressions for the eigenvalues of the Jacobian are complicated and analytic results are not, in general, available. However, for the following special case we can provide a uniqueness result.

**Proposition 3** *In the Bewley-Aiyagari version of the model, there exists a unique steady-state. Furthermore, for $\xi$ sufficiently large, there exists a unique (non-explosive) perfect foresight equilibrium, for any given $n_0$, that converges to the steady-state.*

## The general case

The equilibrium path is found as the solution to the following equations:

$$J_t=\overline{z}+\frac{\sigma}{1+\sigma}\max\left\{\alpha J_t,(\alpha/n_t)^{\gamma(1+\sigma)/(\gamma+\sigma)}\right\}+\frac{(1-\delta)J_{t+1}}{1+r_{t+1}}$$
$$1+r_t=\frac{1+\rho}{1+\alpha\left[\frac{1}{\alpha^{\sigma(1-\gamma)/(1+\sigma)}n_t^{\gamma}J_t^{(\sigma+\gamma)/(1+\sigma)}}-1\right]^{+}}$$
$$n_{t+1}=(1-\delta)n_t+\left[\frac{J_{t+1}\left(1+\alpha\left[\frac{1}{\alpha^{\sigma(1-\gamma)/(1+\sigma)}n_t^{\gamma}J_t^{(\sigma+\gamma)/(1+\sigma)}}-1\right]\right)}{k\left(1+\rho\right)}\right]^{(1-\xi)/\xi}(1-n_t)$$

Too see this, note that in a constrained equilibrium,

$$py_s-c(y_s)=\frac{\sigma}{1+\sigma}y_s^{1+\sigma}$$

Since $nJ = py_s \frac{n}{\alpha}$, $\alpha J = y_s^{1+\sigma}$, the surplus is $py_s - c(y_s) = \frac{\sigma}{1+\sigma}\alpha J$. In the interest rate equation,

$$\begin{aligned} v'(nJ/p)/p &= (nJ)^{-\gamma}p^{\gamma-1} \\ &= (nJ)^{-\gamma}y_s^{\sigma(\gamma-1)} \\ &= (nJ)^{-\gamma}(\alpha J)^{\sigma(\gamma-1)/(1+\sigma)} \\ &= \frac{1}{\alpha^{\sigma(1-\gamma)/(1+\sigma)}n^{\gamma}J^{(\sigma+\gamma)/(1+\sigma)}} \end{aligned}$$

Define $\bar{n} = J(\bar{J})$ as the implicit steady-state function defined from the firm's profit recursion assuming the liquidity constraint binds. It can be shown that:

1. $\bar{n} \to 0$ as $\bar{J} \to 0$;
2. $\bar{n} \to 0$ as $\bar{J} \to \infty$;
3. if $z < 0$ then $\bar{n} > 0$ for $0 < \bar{J} < \infty$.

These properties imply that the function $J(\bar{J})$ is non-monotonic. However, the second equation, as before features $\bar{n}$ increasing with $\bar{J}$. The non-monotonicity of the firm's steady-state profit equation raises the possibility of multiple steady-state equilibria. Moreover, it is apparent that the slope of the profit functions are complicated expressions which raise the possibility of bifurcations not only in the number of steady-states but also the stability of steady-states.

# H. Results and proofs with specific functional forms

We summarize the equilibrium conditions with the functional forms as specified in Section 5. We define an equilibrium as a bounded sequence, $\{J_t, \theta_t, n_t, p_t, r_t, y_t^c\}_{t=0}^{+\infty}$, that solves:

$$
\begin{aligned}
J_t &= \frac{(1+r_t)k}{q(\theta_t)} \\
J_t &= \bar{z} + C\frac{\sigma}{1+\sigma}(y_t^s)^{1+\sigma} - w_1 + (1-\delta)\frac{J_{t+1}}{1+r_{t+1}} \\
y_t^* &= \left[\frac{B}{C}(n_t/\alpha)^\sigma\right]^{1/(1+\sigma)} \\
y_t^c &= \frac{\alpha}{n_t}\left[\lambda\frac{B}{C(y_t^c)^\sigma} + (1-\lambda)\frac{n_t J_t + A_g}{C(y_t^c)^\sigma}\right] \\
y_t^s &= \min\left\{y_t^c, y_t^*\frac{\alpha}{n_t}\right\} \\
\frac{\rho - r_t}{1+r_t} &= \alpha(1-\lambda)\left[\frac{B}{n_t J_t + A_g} - 1\right]^+ \\
n_{t+1} &= (1-\delta)n_t + m(1,\theta_{t+1})(1-n_t)
\end{aligned}
$$

From the Cobb Douglas matching form, we have $m(1,\theta_t) = A\theta_t^{1-\xi}$ and $q(\theta_t) = A\theta_t^{-\xi}$, so that free entry yields

$$\theta_t = \left(\frac{AJ_t}{(1+r_t)k}\right)^{1/\xi}$$

and the employment law of motion satisfies

$$n_{t+1} = (1-\delta)n_t + A^{1/\xi}\left(\frac{J_{t+1}}{(1+r_{t+1})k}\right)^{\frac{1-\xi}{\xi}}(1-n_t)$$

Households without credit are constrained if the assets $a_t = n_t J_t + A_g$ are such that $r_t < \rho$ or $B/(n_t J_t + A_g) - 1 < 0$. Rearranging, we have

$$nJ + A_g \leq B$$

Plugging in $y^c$, the general supply function is

$$y^s = \left(\frac{\alpha}{n}\right)^{1/(1+\sigma)} \left[\frac{\lambda B + (1-\lambda)\min\{B, nJ + A_g\}}{C}\right]^{1/(1+\sigma)}$$

We can rearrange the Firm Bellman as

$$\begin{aligned} J &= \frac{1+r}{r+\delta}\left[\overline{z} - w_1 + C\left(\frac{\sigma}{1+\sigma}\right)(y^s)^{1+\sigma}\right] \\ &= \frac{1+r}{r+\delta}\left[\overline{z} - w_1 + \frac{\sigma}{1+\sigma}\frac{\alpha}{n}\left(\lambda B + (1-\lambda)L\right)\right] \end{aligned}$$

This equation highlights the aggregate demand channel: higher stock-market values augment revenue and further amplify the stock market. The feedback depends on $\sigma, \alpha$, the availability of credit, the bond supply $A_g$, and the interest rate $r$.

We can also solve for the interest rate directly:

$$r = \frac{\rho + \alpha(1-\lambda) - \frac{\alpha(1-\lambda)B}{nJ+A_g}}{1 - \alpha(1-\lambda) + \frac{\alpha(1-\lambda)B}{nJ+A_g}}$$

We can clearly see that $r$ rises with $nJ + A_g$. Rearranging, we find

$$\frac{1+r}{r+\delta} = \frac{1+\rho}{\rho + \alpha + \delta(1-\alpha) - \frac{\alpha B(1-\delta)}{nJ+A_g}}$$

We can thereby compose the channel on $J$ into the aggregate demand and interest rate channels:

$$\begin{aligned} J = &\frac{1+\rho}{\rho + \alpha(1-\lambda) + \delta(1-\alpha(1-\lambda)) - \frac{\alpha(1-\lambda)B(1-\delta)}{nJ+A_g}} \\ &\times \left[\overline{z} - w_1 + \frac{\sigma}{1+\sigma}\frac{\alpha}{n}\left(\lambda B + (1-\lambda)(nJ + A_g)\right)\right] \end{aligned} \tag{23}$$

subject to the caveat that $n$ also responds to $r$ through free entry. A few special cases are noteworthy. As $\alpha \to 0$, both the interest rate and aggregate demand channels vanish, so that

$$J = \frac{1+\rho}{\rho+\delta}(\overline{z} - w_1)$$

As $\lambda \to 1$, there is no need for liquidity, and

$$J = \frac{1+\rho}{\rho+\delta}\left[\overline{z} - w_1 + \frac{\sigma}{1+\sigma}\frac{\alpha}{n}B\right]$$

As $\sigma \to 0$, the aggregate demand channel vanishes, so that

$$J = \frac{1+\rho}{\rho + \alpha(1-\lambda) + \delta(1-\alpha(1-\lambda)) - \frac{\alpha(1-\lambda)B(1-\delta)}{nJ+A_g}}\left(\overline{z} - w_1\right)$$

which has the quadratic form

$$\psi n J^2 + [A_g\psi - \alpha(1-\lambda)B(1-\delta) - (1+\rho)(\overline{z} - w_1)]J - (1+\rho)(\overline{z} - w_1)A_g = 0$$

where $\psi \equiv \rho + \alpha(1-\lambda) + \delta(1-\alpha(1-\lambda))$.

Consider the special case in which $A_g = 0$. Then (23) becomes linear in $J$, and has the solution

$$J = \frac{(1+\rho)(\overline{z} - w_1 + \frac{\sigma}{1+\sigma}\frac{\alpha}{n}\lambda B) + \frac{\alpha(1-\lambda)B(1-\delta)}{n}}{\psi - (1+\rho)\frac{\sigma}{1+\sigma}\alpha(1-\lambda)} \tag{24}$$

Equation (24) clearly highlights the contribution of $\alpha$ and $\sigma$ to $J$.

Finally, let $\delta \to 1$, so that the interest rate only affects $J$ through $n$. Then we have

$$J = \frac{\overline{z} - w_1 + \frac{\sigma}{1+\sigma}\frac{\alpha}{n}\left(\lambda B + (1-\lambda)A_g\right)}{1 - \frac{\sigma}{1+\sigma}\alpha(1-\lambda)} \tag{25}$$

# I. Comparative statics with specific functional forms

Consider the above functional forms and impose a fixed wage under continuous time. Then the $n$ and $J$ nullclines are

$$J = \frac{1}{r+\delta}\left[\overline{z} - w_1 + \frac{\sigma}{1+\sigma}\frac{\alpha}{n}\left[\lambda B + (1-\lambda)L\right]\right] \tag{26}$$

$$\delta n = A^{1/\xi}\left(\frac{J}{k}\right)^{\frac{1-\xi}{\xi}}(1-n) \tag{27}$$

where

$$r = \rho - \alpha(1-\lambda)\left[\frac{B}{nJ + A_g} - 1\right] \tag{28}$$

We examine comparative statics with respect to $\alpha, B$, and $\lambda$. Note that none of these parameters shift the $n$-nullcline. To evaluate the shift in the J-nullcline, we evaluate the semi-elasticity of $J$ with respect to the parameter, holding $n$ fixed. If there is a rightward shift in the $J$ nullcline, then in equilibrium both $J$ and $n$ must rise, and therefore so must the stock market capitalization $M = nJ$. To this end, we evaluate the change in $J$ from the Firm Bellman holding $n$ fixed.

$$\frac{\partial \log J}{\partial \alpha} = -\frac{1}{r+\delta}\frac{\partial r}{\partial \alpha} + \frac{1}{(r+\delta)M}\frac{\sigma}{1+\sigma}\left[\lambda B + (1-\lambda)L + \alpha(1-\lambda)M\frac{\partial \log J}{\partial \alpha}\right] \tag{29}$$

$$\frac{\partial \log J}{\partial B} = -\frac{1}{r+\delta}\frac{\partial r}{\partial B} + \frac{1}{(r+\delta)M}\alpha\frac{\sigma}{1+\sigma}\left[\lambda + (1-\lambda)M\frac{\partial \log J}{\partial B}\right] \tag{30}$$

$$\frac{\partial \log J}{\partial \lambda} = -\frac{1}{r+\delta}\frac{\partial r}{\partial \lambda} + \frac{1}{(r+\delta)M}\alpha\frac{\sigma}{1+\sigma}\left[B - L + (1-\lambda)M\frac{\partial \log J}{\partial \lambda}\right] \tag{31}$$

We also evaluate the change in $r$ with respect to the parameter for $n$ fixed:

$$\frac{\partial r}{\partial \alpha} = -(1-\lambda)\left[\frac{B}{L} - 1\right] + \frac{\alpha(1-\lambda)BM\frac{\partial \log J}{\partial \alpha}}{L^2} \tag{32}$$

$$\frac{\partial r}{\partial B} = -\frac{\alpha(1-\lambda)}{L} + \alpha(1-\lambda)\frac{BM}{L^2}\frac{\partial \log J}{\partial B} \tag{33}$$

$$\frac{\partial r}{\partial \lambda} = \alpha\left(\frac{B}{L} - 1\right) + \alpha(1-\lambda)\frac{BM}{L^2}\frac{\partial \log J}{\partial \lambda} \tag{34}$$

We can derive the following upon substitution:

$$\frac{\partial \log J}{\partial \alpha}\left(1 + \frac{\alpha(1-\lambda)}{r+\delta}\left(\frac{BM}{L^2} - \frac{\sigma}{1+\sigma}\right)\right) = \frac{1-\lambda}{r+\delta}\left(\frac{B}{L} - 1\right) + \frac{\sigma}{1+\sigma}\frac{\lambda B + (1-\lambda)L}{(r+\delta)M} \tag{35}$$

$$\frac{\partial \log J}{\partial B}\left(1 + \frac{\alpha(1-\lambda)}{r+\delta}\left(\frac{BM}{L^2} - \frac{\sigma}{1+\sigma}\right)\right) = \frac{1}{r+\delta}\frac{\alpha(1-\lambda)}{L} + \alpha\frac{\sigma}{1+\sigma}\frac{\lambda}{(r+\delta)M} \tag{36}$$

$$\frac{\partial \log J}{\partial \lambda}\left(1 + \frac{\alpha(1-\lambda)}{r+\delta}\left(\frac{BM}{L^2} - \frac{\sigma}{1+\sigma}\right)\right) = -\frac{\alpha}{r+\delta}\left(\frac{B}{L} - 1\right) + \alpha\frac{\sigma}{1+\sigma}\frac{B-L}{(r+\delta)M} \tag{37}$$

We can use the relationship $B/L = \frac{\rho+\alpha(1-\lambda)-r}{\alpha(1-\lambda)}$ to simplify

$$-\frac{\alpha}{r+\delta}\left(\frac{B}{L}-1\right)+\alpha\frac{\sigma}{1+\sigma}\frac{B-L}{(r+\delta)M}=\frac{\rho-r}{(r+\delta)(1-\lambda)}\left(\frac{L}{M}\frac{\sigma}{1+\sigma}-1\right) \tag{38}$$

# J. Comparison to a pure-currency economy

In the following we show that the set of steady states arising from our model differs qualitatively from the one of a pure currency economy where fiat money is the only means of payment (Shi (1998) or Berentsen, Menzio, and Wright (2011)).[19] Let $\pi$ denote the growth of the money supply. A steady-state equilibrium of a pure currency economy is a list, $(J, p, y^s, n)$, that solves:

$$\begin{aligned}
(\rho+\delta)\,J &= \bar{z}+\max_y\{py-c(y)\}-w_1\\
p &= c'\left(y^s\right)\\
\rho+\pi &= \alpha\left[\frac{v'\left(ny^s/\alpha\right)}{c'\left(y^s\right)}-1\right]\\
\delta n &= m\left[1,\theta(J)\right](1-n).
\end{aligned}$$

The third equation pins down $y^s$ as a decreasing function of $n$. From the second equation, $p$ is a decreasing function of $n$. From the first equation, $J$ is a decreasing function of $n$. From the Beveridge curve $n$ is increasing with $J$. So the steady state of the pure currency economy is unique. Our economy differs from the pure currency economy in two ways. First, in our model the real interest rate is endogenous and depends on the measure of firms and their valuation. As discussed above, the logic for the determination of the real interest rate is similar to the one in the Aiyagari model. Second, the price of early consumption depends on market capitalization through a limited commitment problem. This channel links asset prices, expenditure, and employment and potentially generates multiple steady states.

[19] The comparison here is not directly to Berentsen, Menzio, and Wright (2011), which includes a frictional goods market where the probability of matching is affected by matching in the labor market.