Lender Concentration and Sudden Stops

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Abstract

This paper studies how lender structure affects credit conditions in open economies via a model with lenders of different sizes. Unlike atomic lenders who foreclose on all collateral when borrowers default, large lenders internalize the pecuniary externality whereby selling seized collateral injects supply and reduces the collateral price. Thus, a more concentrated lender structure alleviates the severity of sudden stops via a higher collateral price, thus demanding less precautionary saving by decentralized borrowers and increasing overborrowing. I document that the lender structure of the external debt of emerging countries is more concentrated than that of advanced countries, and thus emerging countries tend to overborrow. This explains overborrowing alternative to the pecuniary externality of borrowers often highlighted in the literature. Under plausible parameterization, the size of the pecuniary externality internalized by lenders is one-third of that internalized by borrowers. Finally, allowing lender countries to choose lender structure optimally will increase lender concentration, raising debts and lowering borrowers' consumption by 3.5%.

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1 Introduction

It is often argued that the pecuniary externality driven by borrower's decisions causes inefficiency and should be decentralized. However, a pecuniary externality can also be internalized by lenders during collateral foreclosure when borrowers fail to repay debt. Specifically, lenders who own a large share of debt may be reluctant to entirely foreclose on collateral because they understand that selling foreclosed collateral injects asset supply and reduces the collateral price. Thus, a more concentrated lender structure leads to fewer foreclosures and a higher equilibrium collateral price (Favara and Giannetti, 2017). Since the open-economy literature has focused on pecuniary externalities that stem from borrowers' decisions, the aim of this paper is to fill the gap by emphasizing the lender side of external debt. Specifically, I ask how lender concentration affects overborrowing of external debt and how allowing lender countries to optimally choose the lender structure affects borrowers' welfare.

This paper begins by empirically documenting two stylized facts about lender concentration of external debt using the quarterly exposure of individual US banks to the external debts of other countries. First, the lender concentration of the external debt of emerging countries is significantly higher than that of rich countries. Second, lender concentration alleviates sudden stops in terms of the magnitude of current account reversal. These results provide an important theoretical implication: emerging countries tend to overborrow more because a more concentrated lender structure alleviates the severity of sudden stops, thus demanding less precautionary saving.

To analyze the effect of lender concentration on overborrowing, this paper incorporates two new features into a standard SOE-DSGE with a continuum of identical domestic borrowers constrained by an occasionally binding collateral constraint. First, as in practice, borrowers may only consume collateral once debts are repaid. Second, when borrowers do not repay, lenders optimally choose how much collateral to foreclose on. These assumptions contrast with those in the literature on open economy models with collateral constraints, which assume that all goods that serve as collateral can be consumed by agents before the debts are repaid. In those models, agents always borrow less than or equal to the borrowing capacity that is unaffected by foreclosure. Thus, the effects of lenders' decisions on foreclosing collateral are muted.[1] Such effects are especially important for emerging countries

[1] Although Mendoza (2010) argues that a collateral haircut may be viewed as limited enforcement (soft default) when borrowers default, such enforcement is not endogenously determined by lenders.

because they tend to rely more on secured borrowing that involves collateral (Menkhoff et al., 2006).

There are two types of lenders, atomic lenders who take the collateral price as given and one large lender who internalizes the pecuniary externality of foreclosing on collateral. During foreclosure events, atomic lenders seize all collateral and the large lender only seizes a fraction of the collateral. Thus, when the large lender owns a larger share of external debts (i.e., a more concentrated lender structure), the share of collateral seized is lower. This leads to a higher collateral price that incentivizes borrowers to hold less precautionary savings for agents to avoid sudden stops defined as states with a binding collateral constraint. Thus, lender concentration affects credit allocation via states of foreclosure.

The effect of lender concentration in decentralized equilibria differ between the states with a binding and nonbinding collateral constraint. When the collateral constraint is slack, lender concentration increases borrowing because the large lender who forecloses on collateral raises the collateral price. Thus, agents' borrowing increases with lender concentration because future consumption will be supported by a high price in future foreclosure events. That is, the marginal benefit of lowering debt in raising future consumption decreases with lender concentration.

When the collateral constraint is binding, the relationship between lender concentration and borrowing is hump-shaped. Increasing lender concentration from a low level reduces the foreclosure share and raises borrowing capacity in sudden stops, increasing the nontradable price by increasing tradable consumption and reducing nontradable consumption. However, as lender concentration surpasses a certain threshold, the foreclosure share is so low that tradable consumption and the collateral price are extremely high in sudden stops. With a high initial price, when agents raise borrowing in binding states, tradable consumption and the collateral price may significantly increase so that the slackness of the collateral constraint actually decreases. In this case, a lower foreclosure rate driven by a more concentrated lender structure worsens the severity of a sudden stop.

In contrast to the decentralized equilibria, borrowing in the social planner's (SP's) problem tends to be decreasing in lender concentration. Thus, overborrowing that measures the gap between the debt holdings in the decentralized equilibria and the SP's solution is hump-shaped with lender concentration. The normative perspective considered in this model is an SP who faces the same collateral constraint and internalizes the effect of current debt holdings on the future collateral price. In the SP's problem, lender concentration affects borrowing via an additional benefit of lowering borrowing: lender concentration alleviates post-foreclosure sudden stops by lowering the nominal shadow value of the collateral constraint. However, when reducing current debt, the SP lowers the future probability of facing a post-foreclosure binding state and increases the probability of facing a no-foreclosure binding state. Thus, the SP actually benefits less from the alleviation of post-foreclosure sudden stops, which reduces the nominal shadow value compared with agents in the decentralized equilibrium. With a higher nominal shadow value, the SP faces a higher marginal benefit of loosening the collateral constraint and borrows less. This mechanism strengthens as lender concentration increases.

I show that the model in general has multiple equilibria with a binding collateral constraint in states of foreclosure, leading to the discussion of equilibrium selection criteria, which has been emphasized in the literature as a fundamental factor of credit allocation. In Schmitt-Grohé and Uribe (2021), multiple equilibria exist when increasing debt from a binding equilibrium by one unit increases the collateral value by more than one unit such that the collateral constraint may bind at two different levels of debt. They show that while decentralized agents overborrow under an optimistic criterion that chooses a nonbinding equilibrium over binding equilibria, they underborrow under a pessimistic criterion that prefers the worst binding equilibrium with the lowest borrowing because agents hold more precautionary savings than the SP.

This paper adds nuance to how equilibrium selection criterion influences overborrowing. I show that an optimistic criterion is important to maintain a debt level that is high enough to trigger foreclosure, during which lender concentration matters for overborrowing. However, under a pessimistic criterion, agents tend to hold little debt such that foreclosure is not possible, and thus credit allocation is independent of the lender structure. Therefore, the difference between the credit allocation in the equilibrium under optimistic and pessimistic criteria not only stems from the borrower's decisions but also from the lender's decision on foreclosure, which affects the level of precautionary saving.

In contrast to the literature where multiple binding equilibria exist only under a low elasticity of substitution between tradable and nontradable consumption, this paper shows that the above result also holds with a high elasticity of substitution between tradable and nontradable consumption. In the model of Schmitt-Grohé and Uribe (2021), having multiple binding equilibria requires a steep price function of current debt, with which deleveraging causes a significant decline in the collateral value. Thus, deleveraging in a binding equilibrium reduces the slackness of the collateral constraint, and a second binding equilibrium occurs when agents reduce current debt to a level that finally relaxes the collateral constraint. While the literature obtains a steep price function from a low elasticity of substitution between tradable and nontradable consumption, this model obtains it by including the benefit of reducing the future probability of foreclosure as an additional marginal benefit of reducing debt.

While there tend to be multiple binding equilibria in states with no foreclosure, I further show that there exists a unique binding equilibrium in states with foreclosure under certain parameterizations, including the calibrated parameters. This uniqueness simplifies how lender concentration affects overborrowing, as no additional selection criterion is required for states of foreclosure. As long as agents choose high debt that is prone to sudden stops under the optimistic criterion, the relationship between lender concentration and overborrowing is hump-shaped. On the other hand, under a pessimistic equilibrium that fails to trigger foreclosure in the decentralized equilibrium, the effect of lender concentration on overborrowing is fully driven by the SP's debt allocation, which is decreasing in lender concentration. In this case, lender concentration increases overborrowing.

With the theoretical results in hand, I conduct a numerical analysis by calibrating the model to data from Argentina, which is a small open economy prone to sudden stops. In line with the literature, the numerical result shows that overborrowing occurs under the optimistic criterion while decentralized agents underborrow under the pessimistic criterion. The SP's allocation features higher consumption than the two decentralized equilibria due to the probability of a sudden stop.

There are several novel implications from the numerical exercise under different equilibrium selection criteria. First, the result shows that agents fail to repay their debt with a probability of 5.25% under the optimistic criterion. However, agents never fail to repay their debt with low debt holding under the pessimistic criterion. Agents encounter foreclosure with a probability of 0.04% in the SP's solution. Second, the implied foreclosure share of lenders in the SP's problem is 0.85% higher than that in the equilibrium under the optimistic criterion, implying that allowing lenders to internalize the pecuniary externality in a competitive equilibrium with decentralized agents is insufficient to achieve the SP's allocation.

Then, I highlight the numerical importance of lenders in internalizing the pecuniary externality by separately quantifying the magnitude of the pecuniary externality internalized by lenders and borrowers. To this end, I compare the credit allocation in three equilibria. The first equilibrium is the SP's allocation where both lenders and borrowers internalize the pecuniary externality. The second equilibrium considers a decentralized equilibrium where only the large lender, and not the borrowers, internalizes the pecuniary externality. The third equilibrium considers a decentralized equilibrium in which the large lender forecloses on all the collateral like the atomic lenders. This is the case where both borrowers and lenders fail to internalize the pecuniary externality. The result shows that the pecuniary externality internalized by lenders is one-third of the typical pecuniary externality internalized by borrowers.

Next, I numerically study the effect of lender concentration on credit allocation in the decentralized equilibria and the SP's solution. In line with the theoretical predictions, overborrowing has a hump-shaped relation with lender concentration under the optimistic criterion and is increasing in lender concentration under the pessimistic criterion. Under a plausible parameterization with the optimistic criterion, countries with a largest US lender that accounts for less than 73% of total borrowing from the US, such as Brazil and Panama, underborrow. In contrast, countries with a largest share above 73%, such as Argentina and Mexico, overborrow. However, under the pessimistic criterion, underborrowing (i.e., negative overborrowing) exist despite that lender concentration increases overborrowing.

Finally, in light of the discussion since the COVID-19 pandemic on concentrating lender structure akin to the Brady Plan in the late 1980s to combat the increasing coordination problem among dispersed lenders under growing sovereign debt, I consider the welfare implications of allowing lenders to optimally choose the lender structure. To gain higher payment from the seized collateral, the lending countries choose to allocate the largest lender a share that is 10% higher than the empirical concentration, resulting in a foreclosure share that is 15% less than in the data. The severity of sudden stop events under the optimal concentration in terms of the magnitude of deleverage is only one-third of that under the empirical concentration, incentivizing agents under the optimal concentration to borrow 8% more and consume 3.5% less than the case under the empirical concentration.

Related Literature

This paper is related to a large and growing literature on open economy models with pecuniary externalities that can be internalized by borrowers. Selected works include Uribe (2006), Bianchi (2011), Benigno et al. (2013), Benigno et al. (2016), Bianchi and Mendoza (2018), Schmitt-Grohé and Uribe (2018), Jeanne and Korinek (2019), Chi et al. (2021), Schmitt-Grohé and Uribe (2021), and Benigno et al. (2022). This paper complements the literature by incorporating a large lender who also internalizes the pecuniary externality.

Several studies have focused on the relationship between lender concentration and external debt. Fernández and Ozler (1999) empirically find that lender concentration raises the secondary-market prices of external debt. They develop a model where lenders threaten countries with a costly penalty, which becomes more credible as large lenders obtain higher repayment due to higher concentration. Thus, debt repayment and debt prices increase in concentration as more repayments are guaranteed. Using country-level data, Hardy (2019) documents that lender concentration of external debt among cross-country banking systems has been increasing in emerging economies since the Global Financial Crisis (GFC). This paper contributes to the literature by emphasizing the mechanism of foreclosure decisions that affects debt holdings in sudden stops.

Finally, this paper relates to the vast literature on optimal creditor dispersion. Bolton and Scharfstein (1996) analyze the optimal number of creditors by considering the tradeoff in inefficient renegotiation between deterring defaults and incurring costs. Bolton and Jeanne (2009) further study the coordination problem under a dispersed lender structure in the context of renegotiating sovereign defaults. More recently, Zhong (2021) derives optimal lender concentration in a dynamic framework by considering the tradeoff between rollover risk due to coordination problems and the incentive for repayment. This project differs from this literature by emphasizing a novel effect of the lender structure that affects efficiency via pecuniary externalities. Thus, policies such as collective action clauses intended to alleviate the coordination problem of debt restructuring may be insufficient to fully decentralize the impact of the lender structure.

The remainder of this paper is organized as follows. Section 2 presents the empirical patterns of lender structure among emerging and advanced countries and discusses their implications for overborrowing. Then, Section 3 introduces the model and discusses the primary mechanism by which the lender structure affects overborrowing. Section 4 provides numerical analysis of the model and quantifies the effect of the lender structure. Section 5 studies the outcome for the borrowing countries when the lender countries optimally choose the lender structure. Finally, Section 6 concludes the paper.

2 Lender Structure in the Data

This section empirically demonstrates that the lender structure of emerging countries' external debts is more concentrated. Furthermore, lender concentration alleviates sudden stop events in terms of the magnitude of capital reversal. These results lead to a fundamental theoretical implication: large lenders to emerging countries internalize the pecuniary externality, and thus sudden stop events are less severe, entailing less precautionary savings and more overborrowing by emerging countries.

The data on lender concentration come from the Federal Financial Institutions Examination Council's (FFIEC) 009a form that collects the quarterly exposure of individual US banks to the external debts of other countries from 2003Q1 to 2021Q3 (FFIEC, 2019). The exposure to external debt is defined as the sum of the amount of cross-border claims outstanding after mandated adjustments for transfer, amount of foreign office claims on local residents, and the amount of gross claims outstanding from derivative products after mandated adjustments for transfer of exposure. The types of cross-border claims include, but are not limited to, cash, deposit balances held at banks, securities, and loans. Exposure is measured as claims on the basis of the country of residence of the guarantor or collateral provided. This is a useful measure basis because the pecuniary externality internalized by lenders stems from changes in the prices of the underlying collateral, not the price of external debt.

In each quarter, the data provide bank-level exposures in two parts. First, the exposures to any country that exceeds 1 percent of the reporting institution's total assets or 20 percent of its total capital, whichever is less, are fully revealed. Second, for exposure that exceeds 0.75 percent but does not exceed 1 percent of the reporting institution's assets or is between 15 percent and 20 percent of its total capital, whichever is less, the data reports a list of eligible countries and the total exposure to these countries. Since the exact exposure to a country cannot be identified in this case, I only use the second part of the data when there is only one country in the list. The total number of banks included in each quarter ranges from 28 (2019Q1) to 51 (2020Q4), and the average number of banks is 43.4. There are 103

Table 1: Concentrations for lenders of external debts

Borrower	μ_{num}	p^{50}_{num}	σ_{num}	μ_{L1}	p^{50}_{L1}	σ_{L1}	μ_{L3}	p^{50}_{L3}	σ_{L3}
Emerging countries	3.59	4	1.48	0.68	0.69	0.17	0.95	0.97	0.06
Argentina	3.52	3	1.49	0.75	0.74	0.17	0.97	1.00	0.04
Brazil	9.33	9	2.47	0.68	0.69	0.16	0.95	0.97	0.04
Colombia	4.79	5	1.18	0.49	0.47	0.11	0.91	0.94	0.07
Ecuador	3.56	4	1.27	0.62	0.57	0.17	0.95	0.97	0.06
Guatemala	3.59	3	1.48	0.54	0.49	0.20	0.92	1.00	0.11
Israel	1.97	2	0.90	0.75	0.73	0.20	1.00	1.00	0.02
Mexico	8.96	9	2.09	0.74	0.74	0.21	0.93	0.97	0.09
Panama	3.20	3	1.07	0.68	0.70	0.14	0.98	1.00	0.04
Venezuela	6.63	7	1.67	0.38	0.34	0.10	0.78	0.77	0.09
Rich countries	11.81	11.50	2.42	0.38	0.28	0.17	0.74	0.69	0.12
Canada	17.11	16	5.85	0.32	0.23	0.17	0.62	0.55	0.17
France	12.39	12	2.22	0.35	0.27	0.16	0.73	0.68	0.12
Germany	12.99	13	1.96	0.32	0.28	0.11	0.71	0.67	0.13
Japan	11.24	11	4.30	0.41	0.27	0.24	0.75	0.69	0.16
Netherlands	8.56	8	2.61	0.63	0.62	0.18	0.93	0.93	0.06
Singapore	2.08	2	0.83	0.90	0.94	0.13	1.00	1.00	0.01
Switzerland	6.56	6	2.00	0.70	0.69	0.17	0.97	0.99	0.04
United Kingdom	22.28	22	4.42	0.31	0.26	0.15	0.61	0.58	0.12

Notes: This table lists the mean (μ), median (p^{50}), standard deviation (σ) of the quarterly data on the number of lenders (num), share of the top-1 lender ($L1$), and total share of the three largest lenders ($L3$). The moments of emerging and rich countries are the median across countries. The data are a balanced panel ranges from 2003Q1 to 2021Q3. Classification of emerging and rich countries follows Schmitt-Grohé and Uribe (2017). Source: FFIEC 009a and Joint External Debt Hub.

countries that borrow from the banks, and 18 countries borrow every quarter.

Table 1 shows the empirical moments regarding the number of US lenders, the share of the largest lender, and the total share of the three largest lenders. The median number of lenders for emerging countries is 3.59, while the number for rich countries is 11.81. The top lender to emerging countries owns 68% of the total external debt, whereas the share is only 38% for rich countries. The top three lenders own 95% of emerging countries' external debt, while the associated share for rich countries is 74%.[2]

The lender structure of the external debt of emerging counties is significantly more concentrated. Figure 1 shows that the sums of the top-3 lender concentration of emerging and rich countries were initially similar before the GFC, but they become significantly different immediately after the crisis as the lender structure of rich countries concentrated. The average difference in mean lender concentration is 0.14 and this difference is significant with the t-statistic equal to 17.7. The correlation between the annual gross domestic product (GDP) and the top-1 lender's and top-3 lenders' shares are -0.1951 and -0.1959, respectively.

Another observation is that lender structure was particularly dispersed during the crisis for both types of countries. This is in line with the firm-level empirical evidence documented by Farinha and Santos (2002), who show that borrowers with poor performance tend to switch from a single-lender relationship to a multiple-lender relationship. Note that this paper does not focus on explaining the occurrence of the discrepancy in lender structure since the GFC but rather focuses on its implications for the relationship between lender concentration and overborrowing.

A major theoretical prediction of this paper is that lender concentration alleviates the severity of sudden stops. To empirically test this argument, I check whether current account reversal, which is a typical feature of sudden stop events, is less severe under a more concentrated lender structure by running the following difference-in-difference specification at a quarterly frequency:

$$ca_{i,t} = \alpha_0 + \alpha_1 SS_{i,t} + \alpha_2 Concentration_{i,t} + \alpha_3 SS_{i,t} \times Concentration_{i,t} + X_{i,t} + F_i + F_t + \epsilon_{i,t},$$

where $ca_{i,t}$ stands for the log of the current account in USD, $SS_{i,t}$ is a dummy for sudden stop events, and $Concentration_{i,t}$ represents the measures of lender concentration, including the loan amount of the US top-1 lender and the top-3 lenders to the US total lending to country i, denoted as $L_{i,t}^{Top1}$ and $L_{i,t}^{Top3}$. $X_{i,t}$ includes the log of the current account in the last quarter to capture the lagged effect and the log of GDP to control for country size. F_i and F_t represent the country and year-quarter fixed effects, respectively.

[2] Among these countries, the share of external debt associated with US banks to the total cross-border loans from BIS reporting banks ranges from 0.1% to 28.2%. US banks play a particular role in the external debt of Brazil, Mexico, France, Germany, and Japan, in which the share exceeds 10%. For example, the largest US bank that lends to Brazil covers on average 11.8% of the country's external debt, with Citigroup frequently being the top lender.

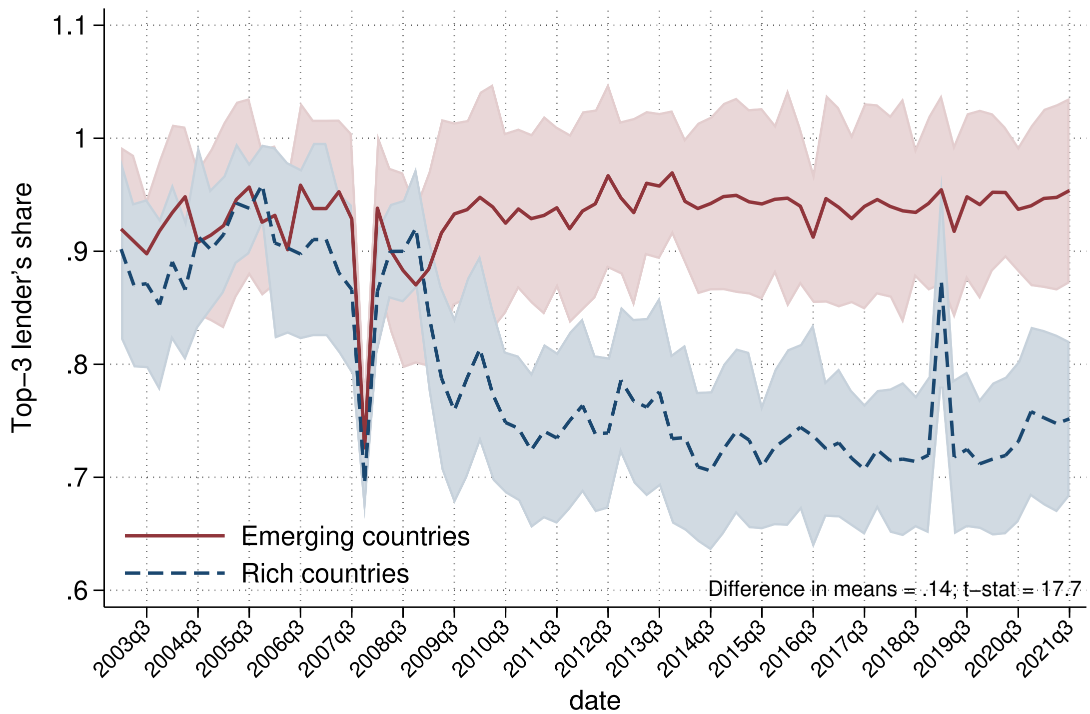

Figure 1: Top-3 lender's shares in rich and emerging countries

Notes: The shaded area indicates a one-standard-deviation band. Source: FFIEC 009a and author's calculation.

The sudden stop dummy is taken from the list of quarterly sudden stop events collected by Eichengreen and Gupta (2016), who set the start of a sudden stop event as the quarter in which capital flows by nonresidents drop below the mean of the past 20 quarters by more than one standard deviation and lasts for more than one quarter. Furthermore, the capital flow in at least one quarter has to be two standard deviations lower than the average. The end date of a sudden stop is defined in two ways: the first is that the event ends when the capital flow exceeds the level one standard deviation lower than the average, and the second is that the event ends when the capital flow rebounds to the mean of the last 20 quarters. In Table 2, columns (1), (2), and (3) adopt the first definition and columns (4), (5), and (6) focus on the second definition.

Table 2 shows that while sudden stop events raise the current account, its magnitude significantly decreases with lender concentration, as shown by the negative and statistically significant α_3. This result holds when using either the share of the top-1 lender or top-3 lenders as the concentration measure and when adopting either the first or the second

Table 2: Lender concentration and severity of sudden stops

	(1)	(2)	(3)	(4)	(5)	(6)
$SS_{i,t}$	0.068	1.013**	4.697***	0.023	0.664	6.767***
	(0.15)	(2.28)	(4.17)	(0.05)	(1.51)	(6.36)
$SS_{i,t} \times L_{i,t}^{Top1}$		−1.732**			−1.175*	
		(−2.61)			(−2.08)	
$SS_{i,t} \times L_{i,t}^{Top3}$			−4.896***			−7.191***
			(−4.42)			(−6.96)
$L_{i,t}^{Top1}$		0.006			0.013	
		(0.03)			(0.05)	
$L_{i,t}^{Top3}$			0.176			0.191
			(0.26)			(0.28)
$log(CA_{i,t-1})$		0.367***	0.366***		0.367***	0.364***
		(9.77)	(9.74)		(9.74)	(9.65)
$log(GDP_{i,t})$		1.243*	1.170**		1.241*	1.173**
		(2.08)	(2.15)		(2.06)	(2.14)
Time FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
N	884	863	863	884	863	863
R^2	0.754	0.802	0.803	0.754	0.802	0.803

Notes: Standard errors in parentheses clustered at the country-year-quarter level. t statistics in parentheses. *$p < 0.1$, ** $p < 0.05$, and *** $p < 0.01$. Data sources: Eichengreen and Gupta (2016), FFIEC 009a, IMF International Financial Statistics, World Development Indicators, and author's calculation.

definition of the sudden stop events. Regarding the other controls, higher past current account predicts a higher present current account. The current account significantly increases with the size of a country measured by GDP.

3 Model

To analyze how the lender structure affects the equilibrium, I extend the representative-agent SOE-DSGE model of Bianchi (2011) by incorporating the lender structure similarly to Favara and Giannetti (2017). The model features a continuum of identical and infinitely lived households of measure unity and two types of foreign lenders: one large lender who provides an exogenous share η of the total loans and atomic lenders who lend out $(1-\eta)$ in aggregate.

3.1 Domestic agents

Domestic agents receive tradable endowments y_t^T and two types of nontradable endowments: collateralizable goods y_t^N, such as plants and machinery, and noncollateralizable goods $\bar{y}_t^N$, such as electricity and water supply.[3] I assume that only y_t^N can serve as collateral and can never be consumed by agents directly unless the collateral is seized and sold by lenders in the domestic market at the same price as $\bar{y}_t^N$.[4] This contrasts with the standard assumption in the literature that collateral can be traded and consumed. Since from the agents' perspective, the only function of y_t^N in period t is to serve as collateral, they will collateralize the entire amount of y_t^N to maximize borrowing capacity for consumption smoothing. Importantly, y_t^N can be only be consumed in period $t+1$ once the loan d_{t+1} is repaid. This assumption is also emphasized by Donaldson et al. (2021), who study the inefficiency of asset allocation when assets are locked in as collateral. Throughout the theoretical analysis and numerical exercise under plausible parameterization, domestic agents are assumed to be borrowers with $d_{t+1} \geq 0$, for all t.

At the beginning of period t, agents receive tradable endowments y_t^T and repay initial borrowing d_t. If $y_t^T < d_t$, agents cannot fully repay back debts, and lenders will waive d_t and make foreclosure decisions on seizing and selling an optimal share of the underlying collateral y_{t-1}^N. Agents are allowed to consume the remaining collateral not foreclosed upon. As will be shown later, when agents default, lenders may not foreclose on all collateral y_{t-1}^N because selling foreclosed collateral increases the supply of nontradable goods and reduces the price. If there is no foreclosure, the full amount of collateral y_{t-1}^N will be consumed by domestic agents in period t.

The agents' optimization problem is given by

$$\max U_0 = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(u\left(c_t\right) + \gamma c_t^C \right) \right],$$

[3] As will become clear later, $\bar{y}_t^N$ is imposed to ensure well-defined nontradable prices in nonbinding states where no collateralized nontradable good is sold.

[4] The intuition for this assumption is that only lenders can utilize collateralizable goods y_t^N to produce noncollateralizable goods $\bar{y}_t^N$ following a linear production function $\bar{y}^N = y^N$. For example, lenders may use collateralizable-nontradable plants and machinery to produce noncollateralizable electricity and water supply, which is nontradable and can be consumed by agents.

subject to the budget constraint

$$c_t^T + p_t c_t^N = y_t^T + \frac{d_{t+1}}{1+r} - d_t(1 - I_t) + p_t \bar{y}_t^N \left(1 - \delta I_t\right), \tag{1}$$

and an occasionally binding collateral constraint

$$d_{t+1} \leq \kappa p_t y_t^N, \tag{2}$$

where d_{t+1} is the debt chosen in period t. p_t is the relative price of nontradable goods and κ is the associated collateral margin, which indicates the borrowing capacity per dollar of collateral. A binding constraint (2) with $d_{t+1} < d_t$ defines a sudden stop event. δ is the coefficient for the output loss of default. c_t aggregates tradable consumption c_t^T and nontradable consumption c_t^N:

$$c_t = \left[a\left(c_t^T\right)^{1-1/\xi} + (1-a)\left(c_t^N\right)^{1-1/\xi}\right]^{1/(1-1/\xi)},$$

where $\xi > 0$ represents the elasticity of substitution between tradable and nontradable goods and $a \in (0, 1)$ is the weight on tradable consumption. r is the constant world interest rate.[5] I_t is a binary variable for foreclosure:

$$I_t = \begin{cases} 0 & \text{if } y_t^T < d_t \\ 1 & \text{if } y_t^T \geq d_t \end{cases}, \tag{3}$$

where lenders foreclose on collateral against a domestic agent when the agent's tradable endowment is insufficient to repay the initial borrowing.

The model distinguishes between a sudden stop event and foreclosure because they are fundamentally different: Sudden stops are episodes in which agents can repay initial debt but are constrained to issue new debt, while foreclosure events are sovereign defaults in which agents fail to repay initial debt. This critical difference is also highlighted by Sánchez et al. (2018), who argue the importance of distinguishing the role of sudden stops in affecting debt maturity in a sovereign default model. Furthermore, modeling foreclosure that is based on initial debt d_t instead of current debt d_{t+1} like sudden stops alleviates the concern of multiple

[5] Unlike the interest rate premium characterized by Eaton and Gersovitz (1981) and extended works, the debt price does not depend on the default probability and lender's risk attitude because debt is collateralized.

equilibria and provides a clean way of identifying the effect of lender concentration. If foreclosure is also modeled as a consequence of a binding collateral constraint determined by d_{t+1}, multiple equilibria may be driven not only by self-fulfilling sudden stops, as emphasized in Schmitt-Grohé and Uribe (2021), but also self-fulfilling foreclosure and their combinations.

To prevent agents from strategically choosing a borrowing level that triggers foreclosure for debt waiver, the coefficient of the output loss δ is assumed to be sufficiently large. Alternatively, strategic defaults can be prevented by a large γ that values remaining consumption of collateral c_t^C. Denote by $c_t^{T,F}$ and $c_t^{T,NF}$ the tradable consumption under foreclosure and no foreclosure, respectively. When $\gamma > \partial U_0/\partial c_t^{T,NF} - \partial U_0/\partial c_t^{T,F}$ for every pair of $\{d_t, d_{t+1}; y_t^T, y_t^N, \bar{y}_t^N, y_{t-1}^N\}$, the planner prioritizes c_t^C because the marginal utility (MU) of c_t^C exceeds the MU of c_t^T that the planner gains from a strategic default. While there is a set of combinations of δ and γ that prevent strategic defaults, I assume that $\gamma = 0$ and δ is calibrated to be large enough to facilitate the analysis.

Agents' consumption depends on the foreclosure decisions of lenders. When the collateral constraint binds, lenders will foreclose on collateral and sell it to domestic agents at the market price p_t. Thus, consumption is given by

$$c_t^N = \zeta_t^* y_{t-1}^N I_t + (1 - \delta I_t)\,\bar{y}_t^N, \tag{4}$$

$$c_t^T = y_t^T + \frac{d_{t+1}}{1+r} - d_t(1 - I_t) - p_t\left(c_t^N - (1 - \delta I_t)\,\bar{y}_t^N\right), \tag{5}$$

$$c_t^C = (1 - \zeta_t^* I_t)\,y_{t-1}^N \tag{6}$$

where $\zeta_t^* = \eta\zeta_t^{L*} + (1-\eta)\,\zeta_t^{A*}$ is the weighted sum of the lenders' foreclosure rates and $\{\zeta_t^{L*}, \zeta_t^{A*}\} \in [0, 1]$ represent the optimal foreclosure rate of nontradable collateral chosen by the large lender and atomic lenders, respectively. For example, $\zeta_t^{L*} = 40\%$ means that the large lender forecloses on 40% of the underlying collateral. Equation (4) states that nontradable consumption equals the seized amount of collateral in $t-1$ plus the noncollateralizable-nontradable endowment in period t. Equation (5) states that when the collateral constraint binds, agents are forced to allocate some resource for tradable consumption to purchasing the nontradable goods seized and sold by lenders. Equation (6) indicates the consumption of the remaining collateral.

The optimality conditions of the competitive equilibrium are given by

$$\lambda_t = \frac{\partial u(c_t)}{\partial c_t^T} \qquad (c_t^T), \tag{7}$$

$$p_t = \left(\frac{1-a}{a}\right)\left(\frac{c_t^T}{c_t^N}\right)^{1/\xi} \qquad (c_t^N), \tag{8}$$

$$\lambda_t = -\beta(1+r)F'(d_{t+1}) + (1+r)\mu_t \qquad (d_{t+1}), \tag{9}$$

and the complementary slackness conditions,

$$\mu_t \left[\kappa p_t y_t^N - d_{t+1}\right] \geq 0, \tag{10}$$

$$\mu_t \geq 0, \tag{11}$$

where

$$F(d_{t+1}) = \int_{\underline{y}^T}^{d_{t+1}} u\left[c_{t+1}^F(y_{t+1}^T)\right] f(y_{t+1}^T|y_t^T)dy_{t+1}^T + \int_{d_{t+1}}^{\bar{y}^T} u\left[c_{t+1}^{NF}(d_{t+1};y_{t+1}^T)\right] f(y_{t+1}^T|y_t^T)dy_{t+1}^T$$

is the expected utility of consumption that aggregates the expected utility in the foreclosure and no-foreclosure states where F stands for foreclosure states in which $y_t^T < d_t$ and NF stands for no-foreclosure states in which $y_t^T \geq d_t$. $c_{t+1}^s = c(c_{t+1}^{T,s}, c_{t+1}^{N,s})$ is the consumption in future state $s \in \{F, NF\}$. λ_t and μ_t are the nonnegative multipliers of equations (1) and (2), respectively. $f(y_{t+1}^T|y_t^T)$ is the conditional probability of tradable endowment bounded within $[\underline{y}^T, \bar{y}^T]$. Equations (7) and (8) are the first-order conditions with respect to tradable and nontradable consumption. Equation (9) equates the marginal benefit that increases agents' current utility and the marginal cost that decreases agents' future utility and tightens the future collateral constraint.

Agents internalize that changing d_{t+1} changes the probability of facing future foreclosure and the expected consumption. To see this, note that $F'(d_{t+1})$ measures the MU of tradable consumption with respect to d_{t+1} and can be decomposed into the following two parts:

$$F'(d_{t+1}) = f(d_{t+1}|y_t^T)\left[\underbrace{u\left[c_{t+1}^F(d_{t+1})\right] - u\left[c_{t+1}^{NF}(d_{t+1};d_{t+1})\right]}_{\text{(a): Change in MU}}\right] + \underbrace{\mathscr{U}_1(d_{t+1},\bar{y}^T) - \mathscr{U}_1(d_{t+1},d_{t+1})}_{\text{(b): Expected MU with no foreclosure}}, \tag{12}$$

in which $\mathscr{U}(d_{t+1}, y_{t+1}^T) = \int u\left[c_{t+1}^{NF}(d_{t+1};y_{t+1}^T)\right] f(y_{t+1}^T)dy_{t+1}^T + \epsilon$ and $\mathscr{U}_1(d_{t+1}, y_{t+1}^T)$ is the MU of tradable consumption with respect to d_{t+1} in no-foreclosure states. Component (a) is the precautionary saving captured by the marginal difference in consumption when agents with given d_{t+1} move from a no-foreclosure state to a foreclosure state. As (a) becomes more negative, the benefit of lowering d_{t+1} increases, implying higher precautionary saving. Component (b) measures the expected MU when there is no foreclosure. (b) is always positive, decreasing in y^T_{t+1} and uncorrelated with η because concentration only matters in states with foreclosure.

How concentration η affects the debt decision depends on how tradable, nontradable, and total consumption under foreclosure, $c^{T,F}_{t+1}$, $c^{N,F}_{t+1}$, and c^F_{t+1}, are affected. Specifically, using equation (12) we have that $\partial F'(d_{t+1})/\partial\eta = f(d_{t+1}|y^T_t)(\partial u[c^F_{t+1}(d_{t+1})]/\partial\eta)$ in which c^F_{t+1} aggregates $c^{N,F}_{t+1}$ and $c^{T,F}_{t+1}$. While $c^{N,F}_{t+1} = \zeta^*_{t+1} y^N_t + (1-\delta)\,\bar{y}^N_{t+1}$ always decreases with η because, as will be shown later, ζ^*_{t+1} is decreasing in lender concentration, the sign of $\partial c^{T,F}_{t+1}/\partial\eta$ depends on states. If η lowers nontradable consumption, raising the nontradable price, then the available resources for tradable consumption may increase. When the increase in tradable consumption dominates the decrease in nontradable consumption, η increases c^F_{t+1}, and thus $\partial F'(d_{t+1})/\partial\eta > 0$. The decentralized debt level is increasing in η because the marginal benefit of lowering debt characterized by the right-hand side of equation (9) is decreasing in η. This theoretical prediction is in line with the numerical analysis shown later in Figure 4.

Now, we are prepared to analyze the foreclosure decisions of lenders.

3.2 Foreign lenders

This subsection derives the foreclosure decisions of the two types of lenders, atomic lenders and the large lender, when borrowers cannot repay their debt ($d_t > y^T_t$). The only difference between the two types of lenders is that atomic lenders take the collateral price as given, while the large lender internalizes that her own foreclosure decision affects supply of nontradable goods, influencing the following collateral price:

$$p_t \triangleq \begin{cases} p^{NF}_t = \left(\frac{1-a}{a}\right)\left(\frac{c^T_t}{\bar{y}^N_t}\right)^{1/\xi} & \text{if } y^T_t \geq d_t \\ p^F_t = \left(\frac{1-a}{a}\right)\left(\frac{c^T_t}{(1-\delta)\bar{y}^N_t + \zeta^*_t y^N_{t-1}}\right)^{1/\xi} & \text{if } y^T_t < d_t \end{cases}, \tag{13}$$

where p^{NF}_t and p^F_t are the nontradable price in states with no foreclosure and with foreclosure, respectively.

Several observations emerge from the price function (13). First, the foreclosure share lowers p_t^F by reducing tradable consumption and increasing nontradable consumption. However, from equation (5) foreclosure may also raise the price when a significantly large initial debt d_t is foregone. Second, p_t^F decreases with the weighted sum of foreclosure rates ζ_t^*.

3.2.1 Atomic lenders

Taking p_t^N as given, the atomic lenders seize collateral to maximize the payoff:

$$\max_{\zeta_t^A} \zeta_t^A p_t y_{t-1}^N,$$

where the optimal foreclosure rate ζ_t^{A*} is always 1, meaning that atomic lenders foreclose on all collateral.

3.2.2 The large lender

Considering the price function (13), the large lender chooses the foreclosure rate ζ_t^L to maximize the following payoff taking as given domestic agents' decisions $\{c_t^T, d_{t+1}\}$ and the foreclosure decision of atomic lenders $\zeta_t^{A*} = 1$:

$$\max_{\zeta_t^L} \zeta_t^L \left(\frac{1-a}{a}\right) \left(\frac{c_t^T}{(1-\delta)\,\bar{y}_t^N + [\eta_t \zeta_t^L + (1-\eta_t)]\, y_{t-1}^N}\right)^{1/\xi} y_{t-1}^N. \tag{14}$$

The foreclosure rate ζ_t^L affects the payoff in two opposite ways. On the one hand, it increases the payoff by directly raising the seized share. On the other hand, it reduces the payoff because it lowers the nontradable price when the lender sells seized nontradable goods to domestic agents. The resulting optimal foreclosure rate is then given by

$$\zeta_t^{L*} = \frac{(1-\delta)\,\frac{\bar{y}_t^N}{y_{t-1}^N} + (1-\eta)}{\eta} \left(\frac{\xi}{1-\xi}\right), \tag{15}$$

When $\xi < 1$, as is standard in the literature, ζ_t^{L*} is decreasing in η because lender concentration strengthens the price decline that results from foreclosures. When the size of the large lender is large enough (i.e., η is large enough), the optimal foreclosure rate of the large lender will be less than one, so that $\zeta_t^{L*} < \zeta_t^{A*} = 1$ and $\zeta_t^{*'}(\eta) < 0$. In this case, a more concentrated lender structure helps maintain the collateral price and borrowing capacity in sudden stop events. Note that ζ_t^{L*} is decreasing in $y_{t-1}^N/\bar{y}_t^N$, which indicates the share of supply of the nontradable good controlled by the foreclosure decision relative to exogenous nontradable supply. Moreover, ζ_t^{L*} is increasing in ξ because the price of nontradables declines by less as nontradable consumption increases. Equivalently, plugging (15) into $\zeta_t^* = \eta\zeta_t^{L*} + (1-\eta)$, we observe that lender concentration $\eta = 1-(1-\xi)\zeta_t^* + \xi(1-\delta)\bar{y}_t^N/y_{t-1}^N$ is decreasing in ζ_t^* when $\xi < 1$. The result can be summarized by the following lemma.

Lemma 1. *(Foreclosure decisions) Atomic lenders foreclose on all their collateral, and the large lender's foreclosure rate decreases with lender concentration η when $\xi < 1$.*

With the foreclosure decisions of lenders, we have that $p_t \in \{p_t^F, p_t^{NF}\}$ is a convex function of d_{t+1}, as summarized by Lemma 2. In what follows, I assume an empirically plausible $\xi \in (0,1)$ so that the collateral price is always convex.

Lemma 2. *(Convex collateral price) With $c_t^T > 0$ and $\xi \in (0,1)$, we have that $\partial p_t/\partial d_{t+1} > 0$ and $\partial^2 p_t/\partial d_{t+1}^2 > 0$ for $p_t \in \{p_t^F, p_t^{NF}\}$.*

Proof: See Appendix A.1.

As will be shown in the following section, the convexity of the collateral price matters for the uniqueness of the equilibrium, which is fundamental to the effect of lender concentration on overborrowing.

3.3 Competitive equilibrium

This section shows that the lender structure crucially determines whether agents overborrow by affecting the severity of sudden stops, depending on the equilibrium selection criterion. Distinguishing equilibrium selection criteria is important because this section shows that the model in general features multiple equilibria. Lender concentration only matters for debt decisions under the criterion where a foreclosure is possible. I further show in this section that, once there is a foreclosure, the binding equilibrium is unique. This uniqueness greatly simplifies how lender concentration affects overborrowing: As long as foreclosure is possible, lender concentration raises overborrowing when it significantly alleviates sudden stop events.

The timing of the competitive equilibrium can be summarized as follows:

1. Period t begins. Agents receive y_t^T to repay the initial debt d_t.
2. If $y_t^T < d_t$, agents cannot fully repay their loans. In this case, proceed to step 3. If $y_t^T \geq d_t$, proceed to step 4.

3. Lenders process foreclosure by seizing a total share ζ_t^* of y_{t-1}^N and waiving d_t. Agents consume the remaining collateral not seized, i.e., $c_t^C = (1 - \zeta_t^*)$. Proceed to Step 5.

4. Agents fully repay d_t and consume all collateral $c_t^C = y_{t-1}^N$ (as no collateral is seized.) Proceed to Step 5.

5. Agents receive $\{\bar{y}_t^N, y_t^N\}$ and pledge y_t^N as collateral.

6. Lenders sell seized collateral if there is any. Agents choose d_{t+1} and c_t^T under the equilibrium price p_t. If a foreclosure happened, we have that $p_t = p_t^F$, $c_t^T = y_t^T + \frac{d_{t+1}}{1+r} - p_t^F \left(c_t^N - \bar{y}_t^N\right)$, and $d_{t+1} \leq \kappa p_t^F y_t^N$. Otherwise, $p_t = p_t^{NF}$, $c_t^T = \frac{d_{t+1}}{1+r} + \left(y_t^T - d_t\right)$, and $d_{t+1} \leq \kappa p_t^{NF} y_t^N$.

7. Period $t+1$ begins. Agents receive y_{t+1}^T to repay the initial debt d_{t+1}.

The competitive equilibrium is defined as follows:

Definition 1. (Competitive Equilibrium) A competitive equilibrium is a set of processes c_t^T, p_t, d_{t+1}, and λ_t satisfying equations (1)-(11) for $t \geq 0$, given exogenous processes y_t^N, $\bar{y}_t^N$, y_t^T, I_t and the initial condition $d_{-1} > 0$.

A sudden stop is then defined as follows:

Definition 2. (Sudden-stop equilibrium) A sudden-stop equilibrium is a set of the processes c_t^T, p_t, d_{t+1}, and λ_t satisfying equations (1)-(6) for $t \geq 0$, $d_1 < d_0$, where equation (2) binds.

The model is subject to multiple equilibria, which calls for equilibrium selection criteria. To see this, I first assume that the model starts from a no-foreclosure deterministic economy where $y_t^T = y^T$ and $\bar{y}_t^N = \bar{y}^N$. Furthermore, $y_t^N = y^N$ for all t. To find the initial steady state, note that the level of debt at which the collateral constraint in a steady-state equilibrium binds is given by

$$\tilde{d} = \kappa \left(\frac{1-a}{a}\right) \left(\frac{y^T - \frac{r}{1+r}\tilde{d}}{\bar{y}^N}\right)^{1/\xi}, \tag{16}$$

which is on the downward-sloping curve that indicates the no-foreclosure-steady-state collateral value $\kappa p_t^{NF}(d_{t+1} = d_t) y_t^N$ in Figure 2. For a given y_t^T any $d_t > y_t^T$ represented by the dash-dotted curve cannot be supported as a steady state-equilibrium as agents default. The

45-degree line represents the left-hand side of equation (2). It follows that any debt that satisfies $d_t = d_{t+1} < \tilde{d}$ and $d_t \leq y_t^T$ represents a candidate for the no-foreclosure steady-state equilibrium.

Given initial debt d_t, point A is a no-foreclosure steady-state equilibrium in which equation (2) is slack because the right-hand side of equation (2) in no-foreclosure states, represented by the red solid line, is above the 45-degree line. In addition to A, there exist sudden-stop equilibria, as shown in the bottom panel of Figure 2, which zooms in on the lower-left corner of the top panel. When there is no foreclosure, points B and C are two equilibria in which equation (2) binds. When there is foreclosure, the corresponding collateral value is shown by the dashed (solid) blue line when the large lender owns 72.5% (90%) of total loans, resulting in a binding equilibrium at point D (E). The two shares of the large lender assumed here are plausible: the maximum empirical holdings of the top-1 lender across countries is 90%, and the extreme concentration associated with a 100% foreclosure rate of the large lender is 72%.

This model in general has multiple binding equilibria in no-foreclosure states, and thus equilibrium selection criteria in these states are needed. To see this, denote $S(d_{t+1}) = \partial p_t^{NF}(d_{t+1})\kappa y_t^N / \partial d_{t+1}$ as the slope of the collateral value with respect to d_{t+1} and $\hat{d}$ as the debt level that satisfies $S(\hat{d}) = 1$. The criterion to determine the number of binding solutions is summarized by the following lemma:

Lemma 3. *(Uniqueness of the no-foreclosure binding equilibrium) If $y_t^T = d_t$, there exists a unique binding equilibrium. If $y_t^T > d_t$ and $\xi \in (0, 1)$,*

(i) there exist two binding equilibria when $p_t^{NF}(\hat{d})\kappa y_t^N < \hat{d}$,

(ii) there exists one binding equilibrium when $p_t^{NF}(\hat{d})\kappa y_t^N = \hat{d}$, and

(iii) there exists no binding equilibrium when $p_t^{NF}(\hat{d})\kappa y_t^N > \hat{d}$.

Proof: See Appendix A.2.

Thus, when the collateral constraint binds in no-foreclosure states, the parameterization that guarantees a unique binding equilibrium is a knife-edge case. The selection criterion is especially important when the two binding equilibria in case (ii) have different relationships between lender concentration and borrowing. For example, if the lowest possible y_t^T lies within the corresponding debt levels of points B and C, then the criterion that favors point C is not subject to foreclosure, and thus borrowing is independent of lender concentration.

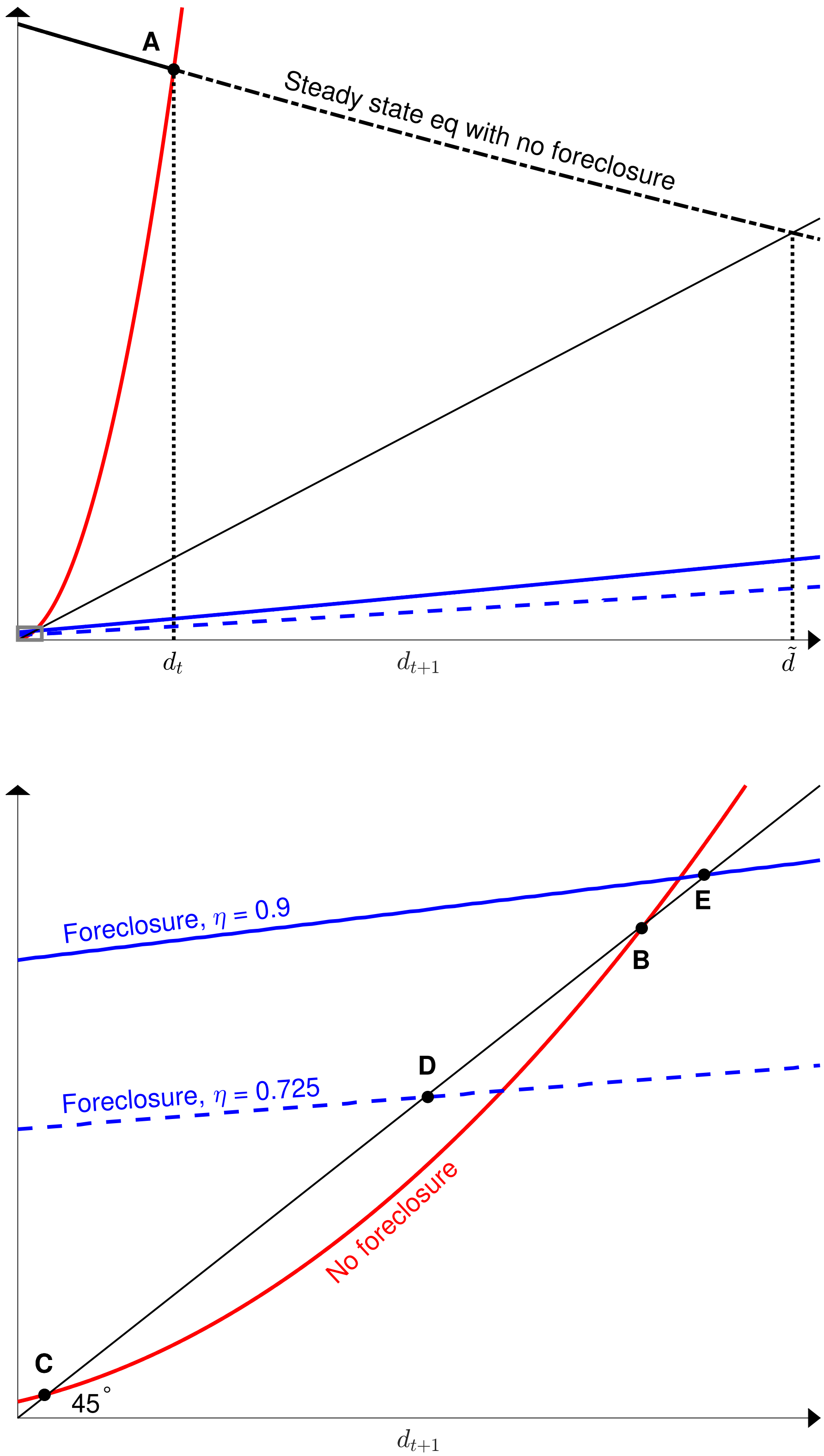

Figure 2: Multiple equilibria and lender concentration

However, in foreclosure states, the binding equilibrium is unique within a much wider set of parameterizations with a sufficiently large foreclosure share $\zeta_t^* > \underline{\zeta}_t$. The reason is that d_{t+1} increases the collateral price by less under a higher foreclosure share because nontradable consumption crowds out tradable consumption. Thus, the slope of p_t^F with respect to d_{t+1} is flat enough that it intersects with the 45-degree line only once, as shown in Figure 2. Lemma 4 summarizes this result. Under the parameters calibrated in Section 4 or used in Figure 2, all pairs of states satisfy condition (A.1) that guarantees the uniqueness of the post-foreclosure binding equilibrium.

Lemma 4. *(Uniqueness of the post-foreclosure binding equilibrium) There exists a unique post-foreclosure binding equilibrium when* $\zeta_t^* > \underline{\zeta}_t$.

Proof: See Appendix A.3.

With the uniqueness result of the post-foreclosure binding equilibrium in hand, we can then study how lender concentration affects credit allocations in this equilibrium. Since the large lender limits foreclosure and protects the collateral price, a foreclosure may not necessarily lead to a sudden-stop equilibrium when $d_{t+1}^{F*} \geq d_t$, where d_{t+1}^{F*} is the debt level in a post-foreclosure binding equilibrium. To ensure that the post-foreclosure binding equilibrium is a sudden stop, the foreclosure share should be high enough that buying foreclosed collateral significantly crowds out agents' tradable consumption and forces deleveraging by lowering the collateral price.

Furthermore, when the foreclosure share is large enough, lender concentration alleviates the sudden stop in terms of the magnitude of decline in the price, that is, $dp_t^{F*}/d\zeta_t^* < 0$. The intuition is that higher concentration reduces the foreclosure share, raising the price by limiting the decline in tradable consumption and the increase in nontradable consumption. However, when the foreclosure share is small, initial consumption is high enough that increasing the foreclosure share only leads to a limited decline in prices. Thus, agents may actually increase d_{t+1} in a binding equilibrium when the collateral value increases by more than one unit under one unit increase in d_{t+1}. In this case, the foreclosure share raises the price in the binding equilibrium. That is, lender concentration that reduces the foreclosure share actually worsens the sudden stops. Thus, the relationship between the severity of sudden stops and lender concentration is nonmonotonic, in line with the numerical result shown later. Lemma 5 summarizes that a post-foreclosure binding equilibrium is a sudden stop whose severity is increasing (decreasing) in the foreclosure share (lender concentration) when ζ_t^* is large enough, that is, when η is small enough.

Lemma 5. *(Concentration and post-foreclosure sudden stops) If* $\zeta_t^* > \frac{1}{1+r}\frac{\kappa y_t^N}{y_{t-1}^N}$*, we have that* $d_{t+1}^{F*} < d_t$*,* $\frac{dd_{t+1}^{F*}}{d\zeta_t^*} < 0$*, and* $\frac{dp_t^{F*}}{d\zeta_t^*} < 0$*.*

Proof: See Appendix A.4.

While the literature shows that deleveraging in no-foreclosure states is frequently triggered by boom-bust cycles, a post-foreclosure deleveraging ($d_{t+1}^{F*} < d_t$) is also prone to boom-bust cycles (i.e., y_{t-1}^N/y_t^N is large), but for a different reason. In standard models with no foreclosure, boom-bust cycles trigger sudden stops by incentivizing agents to hold high d_t in good times that lowers initial wealth in bad times. However, p_t^F is independent of d_t in foreclosure states because of debt waiver. Instead, boom-bust cycles trigger sudden stops in foreclosure states when foreclosing on a sizable amount of collateral secured significantly lowers the price of tradable goods and tightens the collateral constraint. This implies a positive correlation between the occurrence of sudden stops and sovereign defaults, which is also empirically documented by Sánchez et al. (2018). In line with the empirical analysis, the calibrated simulation of this model predicts that the correlation equals 0.94.

The key theoretical result of this section is that the debt level in a decentralized equilibrium depends on the equilibrium selection criterion and lender concentration.

3.4 Social planner's allocation

While the decentralized agents above take the price as given and fail to internalize the effect of individual debt decision on the price, the SP internalizes this externality. This subsection studies an SP who directly chooses debt subject to the collateral constraint but allows the goods market to clear in a competitive way. The Ramsey optimal allocation is characterized by the following recursive problem:

$$V(b, y) = \max_{d', c^T} u(c(c^T, c^N) + \beta E_{y'|y} V(b', y'),$$

subject to

$$c^T = y^T + \frac{d'}{1+r} - d(1-I) - \left(\frac{1-a}{a}\right)\left(\frac{c^T}{c^N}\right)^{1/\xi}\left(c^N - (1-\delta I)\,\bar{y}^N\right), \tag{17}$$

$$c^N = \zeta^* y^N I + (1 - \delta I)\,\bar{y}^N,$$

$$d' \leq \kappa \left(\frac{1-a}{a}\right)\left(\frac{c^T}{c^N}\right)^{1/\xi} y^N. \tag{18}$$

The first-order conditions in sequential form for the SP are given by the price function (13) and the following equations:

$$\lambda_t^{SP} = \frac{\partial u\,(c_t)}{\partial c_t^T} + \mu_t^{SP}\frac{\partial(\kappa p_t y_t^N)}{\partial c_t^T}, \tag{19}$$

$$\lambda_t^{SP} = \beta\,(1+r)\left[-F'(d_{t+1}) - \frac{\kappa\partial E(\mu_{t+1}^{SP} p_{t+1} y_{t+1}^N)}{\partial d_{t+1}}\right] + \mu_t^{SP}, \tag{20}$$

$$\mu_t^{SP}\left[\kappa p_t y_t^N - d_{t+1}\right] \geq 0, \tag{21}$$

$$\mu_t^{SP} \geq 0, \tag{22}$$

where μ^{SP} and λ^{SP} indicate the shadow values of equations (17) and (18).

The fundamental difference between the first-order conditions of the competitive equilibrium and the SP's solution is driven by the fact that the SP internalizes changes in the collateral value. Such a difference only arises when the future collateral constraint binds with positive probability. From equations (19) and (20), the Euler equation for consumption when the current collateral constraint is not binding ($\mu_t^{SP} = 0$) is

$$\frac{\partial u\,(c_t)}{\partial c_t^T} = \beta\,(1+r)\left[-F'(d_{t+1}) - \frac{\kappa\partial E(\mu_{t+1}^{SP} p_{t+1} y_{t+1}^N)}{\partial d_{t+1}}\right], \tag{23}$$

which equates the marginal cost with the marginal benefit of lowering a unit of debt. The first term in brackets is future marginal utility and the second term is the benefit of loosening the future collateral constraint. Comparing equation (23) with the agents' Euler equation characterized by equations (7) and (9), we observe that the SP has an additional marginal benefit of lowering d_{t+1}, $-\kappa\partial E(\mu_{t+1}^{SP} p_{t+1} y_{t+1}^N)/\partial d_{t+1}$. Agents overborrow (underborrow) when this additional marginal benefit is positive (negative).

Since endogenous variables in the no-foreclosure states are affected by $y_t = \left[\bar{y}_t^N, y_t^T\right]$ and d_t, and those in the foreclosure states are only affected by d_t, I denote the endogenous variable x in post-foreclosure and no-foreclosure binding equilibrium in $t+1$ with endowment $y_{t+1}^T = y^T$ and initial debt $d_{t+1} = d$ by $x_{t+1}^{SP,F*}(y^T)$ and $x_{t+1}^{SP,NF*}(d; y^T)$, respectively. The exogenous variable $\bar{y}_t^N$ is ignored because it does not determine foreclosure. It follows that overborrowing is given by

$$-\frac{\partial E(\mu_{t+1}^{SP} p_{t+1})}{\partial d_{t+1}} = -f(d_{t+1}|y_t^T)\left[\mu_{t+1}^{SP,F*}(d_{t+1})p_{t+1}^{F*}(d_{t+1}) - \mu_{t+1}^{SP,NF*}(d_{t+1};d_{t+1})p_{t+1}^{NF*}(d_{t+1};d_{t+1})\right] - \tilde{\mathscr{M}}_1, \tag{24}$$

scaled by the collateral value κy^N, where

$$\mathscr{M}(d_{t+1}; y_{t+1}^T) = \int p_{t+1}^{NF}(d_{t+1}; y_{t+1}^T)\mu_{t+1}^{SP,NF}(d_{t+1}; y_{t+1}^T) f(y_{t+1}^T|y_t^T) dy_{t+1}^T$$

is the no-foreclosure expected nominal shadow value of the collateral constraint. $\tilde{\mathscr{M}}_1 \triangleq \mathscr{M}_1(d_{t+1}; \bar{y}_{t+1}^T) - \mathscr{M}_1(d_{t+1}; d_{t+1})$ measures the no-foreclosure expected marginal change in the nominal shadow value with respect to debt. The first component on the right-hand side of equation (24) measures the changes in the priced shadow value when the economy moves from a no-foreclosure binding state to a post-foreclosure binding state under a given d_{t+1}. If this gap is negative, it means that the post-foreclosure binding state yields a lower marginal benefit of loosening the collateral constraint when lowering d_{t+1}. In this case, overborrowing is higher than in the model without foreclosure because the SP now reduces debt not only for the marginal benefit of loosening the collateral constraint but also to avoid future foreclosure that reduces this marginal benefit.

Lender concentration affects overborrowing via two opposite effects. Since η only matters in foreclosure states, it affects overborrowing only via $\mu_{t+1}^{SP,F*}(d_{t+1})p_{t+1}^{F*}(d_{t+1})$ so that equation (24) can be rewritten as

$$-\kappa y^N \frac{\partial E(\mu_{t+1}^{SP} p_{t+1})}{\partial d_{t+1}\partial\eta} = -\kappa y^N f(d_{t+1}|y_t^T)\left[p_{t+1}^{F*}\frac{\partial \mu_{t+1}^{SP,F*}}{\partial\eta} + \mu_{t+1}^{SP,F*}\frac{\partial p_{t+1}^{F*}}{\partial\eta}\right]. \tag{25}$$

The two opposite effects refer to $p_{t+1}^{F*}(\partial\mu_{t+1}^{SP,F*}/\partial\eta)$ and $\mu_{t+1}^{SP,F*}(\partial p_{t+1}^{F*}/\partial\eta)$ that tend to have opposite signs. When η raises the post-foreclosure binding price p_{t+1}^{F*}, under low η (high ζ^*) that satisfies Lemma 5, the shadow value $\mu_{t+1}^{SP,F*}$ that measures the tightness of the collateral constraint declines. When the effect of a negative $p_{t+1}^{F*}(\partial\mu_{t+1}^{SP,F*}/\partial\eta)$ dominates a positive $\mu_{t+1}^{SP,F*}(\partial p_{t+1}^{F*}/\partial\eta)$, η alleviates post-foreclosure sudden stops by lowering the nominal shadow value of the collateral constraint, $p_{t+1}^{F*}\mu_{t+1}^{SP,F*}$, leading to more overborrowing.

The intuition for how lender concentration increases overborrowing in equation (25) is the following. When lowering d_{t+1}, the SP lowers the probability of facing a post-foreclosure binding state and raises the probability of facing a no-foreclosure binding state in period $t+1$. Thus, the SP actually benefits less from the alleviation of post-foreclosure sudden stops that reduces the shadow value of the constraint. Since the shadow value of the constraint measures the marginal benefit of loosening the constraint by reducing d_{t+1}, an SP who faces a higher shadow value of the constraint borrows less. That is, when η alleviates post-foreclosure sudden stops, a higher η demands less precautionary saving to avoid sudden stops. On the other hand, when η worsens sudden stops and leads to a higher nominal shadow value of the collateral constraint $p^{F*}_{t+1}\mu^{SP,F*}_{t+1}$, overborrowing decreases with η.

The relationship between lender concentration and overborrowing is also determined by the selection criterion of the decentralized equilibrium. Under criterion (C) that chooses point C over A and B in Figure 2, the debt level in the binding equilibrium is independent of η because debt is too low to trigger foreclosure. In this case, the magnitude of overborrowing will be fully driven by the response of the SP's allocation to changes in η. While equation (9) shows that agents' debt in the decentralized equilibrium is increasing in η if $\partial F'(d_{t+1})/\partial\eta > 0$, debt holdings in the SP's solution can be decreasing in η if the overborrowing term (25) is significantly positive to outweigh $-\partial F'(d_{t+1})/\partial\eta$. In this case, overborrowing increases with η under criterion (C), as shown later in Figure 4.

This section shows that whether lender concentration raises or reduces overborrowing requires quantifying the magnitude of the changes in p^{F*}_{t+1} and $\mu^{SP,F*}_{t+1}$. This leads us to the calibrated numerical exercise in the next section.

4 Quantitative Analysis

Following Bianchi (2011) and Chi et al. (2021), I assume that the exogenous endowment vector $y_t = \left[y^T_t, \bar{y}^N_t\right]'$ follows an AR(1) process, $logy_t = \alpha logy_{t-1} + \epsilon_t$, where $\epsilon_t = \left[\epsilon^T_t\ \epsilon^N_t\right]'$ follows a bivariate normal distribution featuring zero mean and a variance-covariance matrix $V = [0.0022, 0.0016; 0.0016, 0.0017]$. The estimated AR(1) coefficient $\alpha = [0.9010, 0.4950; -0.4530, 0.2250]$. The transition probability matrix of the endowment vector is estimated via the approach in Schmitt-Grohé and Uribe (2014). Following Bianchi (2011), I set $\sigma = 2$, $a = 0.31$, $\xi = 0.83$ and $r = 0.04$. The collateralizable non-tradable endowment y^N is normalized to 1.

The baseline model assumes that $\eta = 0.74$, which is the median concentration of the largest US lender of Argentina's external debt across quarters. The remaining coefficients β, κ, and δ are calibrated to the three empirical moments of Argentina: (1) the debt-to-output ratio, (2) the probability of financial crises, and (3) the average share of mortgages foreclosed on by US financial institutions.[6] Due to data limitations, I do not calibrate the model to the average share of secured external debt. The underlying assumption of calibrating the model to the average share of foreclosed mortgages is that US financial institutions internalize the price change when foreclosing on secured external debt in the same way as they do with mortgages.

With the calibrated model in hand, I first analyze overborrowing by comparing debt decisions and allocations in the competitive equilibrium under optimistic and pessimistic selection criteria and the SP's problem. I denote the optimistic equilibrium selection criterion by criterion (A), under which decentralized agents select a d_{t+1} that satisfies equations (1)-(11) and the collateral constraint is not binding for every state $(y_t^T, \bar{y}_t^N, d_t)$. If no such d_{t+1} exists, agents choose the binding equilibrium with the higher d_{t+1} if there are multiple binding equilibria. I denote the pessimistic equilibrium selection criterion by criterion (C), under which decentralized agents choose the binding equilibrium with the lowest d_{t+1} such as point C in Figure 2. Under criterion (C), agents only choose the nonbinding equilibrium when there exists no binding equilibrium.

The left panel of Figure 3 plots the policy function of debt. In line with the literature, the policy function of debt is nonmonotonic. It increases with the initial debt when the future collateral constraint never binds. Then, the slope of the policy function decreases when the future collateral constraint binds with a positive probability, as borrowers reduce debt to avoid future sudden stops. As in Schmitt-Grohé and Uribe (2021), while agents under criterion (A) tend to borrow more than the level chosen by the SP, they underborrow under criterion (C) because of self-fulfilling crises that demand precautionary saving when initial debt is high.

Another feature of the policy function is that having multiple binding equilibria leads to a hike in current debt in criterion (A) and the SP's solution when the initial debt approaches the foreclosure threshold, represented by the solid horizontal line. In these states, the price

[6] Unlike the standard approach in the literature on sovereign defaults, the coefficient of output loss here is not calibrated to the frequency of defaults or average output loss conditional on financial autarky for two reasons. First, the foreclosure in this model is not endogenously determined by a government. Second, agents will not be excluded from the credit market after foreclosure.

Table 3: Calibration

Parameter	Value	Description
σ	2.00	Parameter of CES aggregator
a	0.31	Weights on tradables in CES aggregator
ξ	0.83	Elasticity of substitution between c^T and c^N
r	0.04	World interest rate
y^N	1	Collateralizable non-tradable endowment
η	0.74	Median top-1 concentration of emerging countries
δ	1.1600	Coefficient of output loss
κ	0.4525	Collateral margin of nontradable goods
β	0.8281	Subjective discount factor
	Discretization of State Space	
n_{y^T}	13	Number of equally-spaced grid points for lny^T
$n_{\bar{y}^N}$	13	Number of equally-spaced grid points for $ln\bar{y}^N$
n_d	800	Number of equally-spaced grid points for d_t
$[ln\underline{y}^T, ln\overline{y}^T]$	$[-0.1093, 0.1093]$	Range for logarithm of tradable endowment
$[ln\underline{\bar{y}}^N, ln\overline{\bar{y}}^N]$	$[-0.1328, 0.1328]$	Range for logarithm of nontradable endowment
$[\underline{d}, \overline{d}]$	$[0, 1.1]$	Debt range
Model	Data	Calibration target
0.2823	0.2900	Mean debt-to-output ratio
0.0558	0.0550	Probability of financial crises
0.7469	0.7470	Average foreclosure rate ζ_t

Notes: The empirical debt-to-output ratio and probability of financial crises are from Bianchi (2011). The average foreclosure rate is the share of mortgages that are ever foreclosed on between 2007 and 2010 calculated by Favara and Giannetti (2017).

function is convex enough that the collateral constraint binds at two different levels of current debt. The SP and the agents under criterion (A) then choose the binding equilibrium with the higher current debt. A sudden jump in the policy function is also found by Schmitt-Grohé and Uribe (2021).

However, unlike the literature in which initial debt reduces current wealth and the borrowing capacity in the binding states, the policy function in this model is flat in binding states with foreclosure because any initial debt obligation, regardless of its level, will be waived. In these binding states the borrowing capacity is the same across the three equilibria.

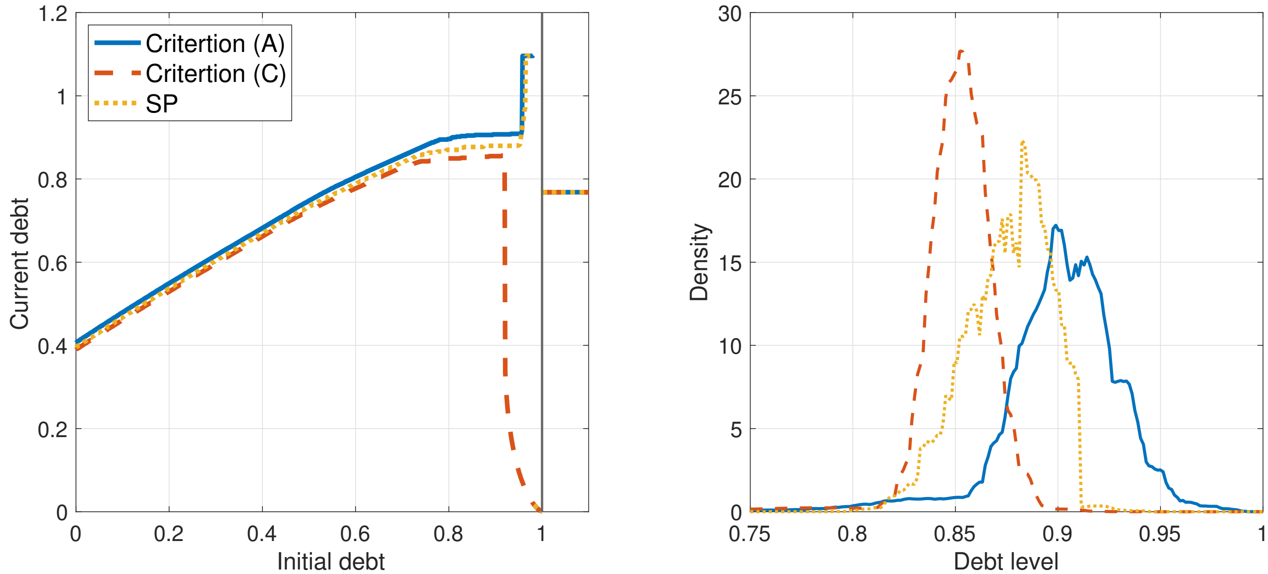

Figure 3: Policy functions and unconditional distributions of debt

Notes: The left panel plots the policy function under the means of exogenous y_t^T and $\bar{y}_t^N$. The vertical solid line represents $y^T = 1$. Following Schmitt-Grohé and Uribe (2021), densities in the right panel are smoothed by averaging the densities of grid points d_{i-20} to d_i for $i = 21, ..., 800$. The models are simulated for one million periods where the first decile of periods are dropped.

The right panel of Figure 3 shows that overborrowing occurs when simulating the competitive equilibrium under criterion (A), and overborrowing is overturned when applying criterion (C). This result is in line with the finding of Schmitt-Grohé and Uribe (2021). Nevertheless, the authors show that having multiple binding equilibria in states of no foreclosure requires low $\xi(= 0.5)$, while this model continues to have multiple binding equilibria under a high ξ. In the model of Schmitt-Grohé and Uribe (2021), ξ has to be low enough that the price function (8) is sensitive enough to changes in d_{t+1} at point B, generating a second binding equilibrium C when agents further decrease d_{t+1} to some point where deleveraging shrinks the slackness of the collateral constraint. However, the price function of this model is steep due to the marginal benefit of lowering d_{t+1} that includes the benefit of reducing the future probability of foreclosure. Thus, by Euler condition (9), the marginal cost of lowering d_{t+1}, $\partial u(c_t)/\partial c_t^T$, will also be larger than that in the model with no foreclosure, implying a steeper price function due to the lower c_t^T under the given c_t^T.

Table 4 shows the simulated results of competitive equilibria and the equilibrium of the SP's problem. Similar to the numerical findings in literature, overborrowing under the

Table 4: Simulated results of the equilibrium solutions

	SP	CE_a	CE_c	CE_a^f
Mean debt-to-output ratio	0.2796	0.2823	0.2646	0.2760
Median debt-to-output ratio	0.2789	0.2841	0.2694	0.2824
Mean debt	0.8755	0.8884	0.8332	0.8709
Mean debt in sudden stops	0.7342	0.6038	0.1906	0.4147
Mean debt in normal times	0.8757	0.9052	0.8460	0.8976
Median debt	0.8797	0.9045	0.8522	0.9031
Mean price	2.1391	2.1540	2.1477	2.1609
Mean price in sudden stops	1.6222	1.3335	0.4210	0.9155
Mean price in normal times	2.1399	2.2025	2.1819	2.2337
Mean consumption	0.9908	0.9766	0.9882	0.9853
Foreclosure probability	0.0004	0.0525	0	0.0507
Sudden stop probability	0.0016	0.0558	0.0195	0.0552
Mean ζ_t^{L*} among foreclosure	0.7150	0.7235	NaN	1.0000

Notes: SP stands for the allocation of the SP's solution. CE_a stands for the competitive equilibrium under criterion (A); CE_c stands for the competitive equilibrium under criterion (C); CE_a^f denotes the competitive equilibrium under criterion (A) with full foreclosure ($\zeta_t^{*L} = \zeta_t^{*A} = 1$). Other parameters of CE_a^f follow Table 3. The debt-to-output ratio is defined as $d_{t+1}/(y_t^T + p_t y_t^N)$. Simulated moments are calculated from the last 1 million periods of a simulation of 1.1 million periods.

optimistic criterion is 0.52% in terms of the median debt-to-output ratio, and underborrowing under the pessimistic criterion is 0.95%. Defining overborrowing by the mean debt-to-output ratio or mean or median debt yields similar results. In terms of the magnitude of deleveraging and the decline in price, sudden stops are more severe under the pessimistic criterion than under the optimistic criterion. Among all equilibria, the SP's solution has the lowest sudden stop probability with the mildest decline in debt and price in sudden stops, leading to the highest mean consumption.

A novel feature of the numerical analysis is that the foreclosure share in the SP's problem is lower than that in the decentralized equilibrium, meaning that internalizing pecuniary externality by the existing lender structure is insufficient to achieve the planner allocation. Specifically, the mean foreclosure share of the large lender ζ_t^{L*} in CE_a is 0.85% higher than that in SP. The intuition is that borrowing in CE_a is on average higher, and thus agents fail to repay debt even when the exogenous nontradable supply $\bar{y}_t^N$ is not extremely low. According to the large lender's foreclosure share in equation (15), the ratio of nontradable supply controlled by the lender's foreclosure decision $\bar{y}_t^N$ regarding noncollateralizable nontradable goods y^N in CE_a is relatively low. In those states, the large lender realizes that the pecuniary externality of her own decision is smaller and thus forecloses on more collateral than the average level during foreclosure in the SP's solution.

An immediate question is then whether the pecuniary externality internalized by lenders is quantitatively important compared with that internalized by borrowers. To this end, I consider a decentralized equilibrium under selection criterion (A), denoted as CE_a^f, in which the large lender does not internalize changes in the collateral price and chooses to foreclose on all collateral ($\zeta_t^{L*} = 1$) like the atomic lenders.[7] In this equilibrium, both lenders and borrowers fail to internalize the pecuniary externality. Compared with CE_a, which is the case in which only the large lender internalizes the pecuniary externality, sudden stops under full foreclosure are more severe, incentivizing agents to borrow less and thus leading to a lower foreclosure probability and sudden stop probability. The SP's solution is the case in which both the large lender and borrowers internalize the pecuniary externality.

The relative size of the pecuniary externality internalized by lenders to the pecuniary externality internalized by borrowers is significant. Comparing CE_a and CE_a^f with SP, we observe that allowing the large lender to internalize the pecuniary externality widens overborrowing from 0.35% to 0.52% because sudden stops in CE_a are less severe than those in CE_a^f in terms of the magnitude of declines in the collateral price and debt holdings, thus requiring less precautionary saving by agents. Since lowering ζ_t^{L*} is similar to raising η in reducing the aggregate foreclosure share $\zeta_t^* = \eta\zeta_t^{L*} + (1-\eta)$, this result is in line with the finding in Section 3.3 that raising η increases overborrowing by alleviating the severity of sudden stops. The pecuniary externality internalized by lenders is -0.17% ($= 0.35\% - 0.52\%$), whose absolute value is one-third of the pecuniary externality internalized by borrowers, which is simply the overborrowing in CE_a equal to 0.52%.

[7] An alternative decentralized equilibrium with $\zeta_t^{L*} = 1$ under criterion (C) generates an identical solution to CE_c because agents hold too little debt to encounter any foreclosure, and thus the lender's foreclosure decisions do not matter.

4.1 The effect of lender concentration on overborrowing

One of the primary goals of this paper is to understand how lender concentration affects overborrowing. Figure 4 numerically shows that the relationship between overborrowing and concentration depends on the selection criterion of the competitive equilibrium. Under the optimistic criterion (A), the relationship between lender concentration and overborrowing is hump-shaped. Lowering lender concentration below a certain threshold overturns overborrowing. Under the pessimistic criterion (C), overborrowing increases with lender concentration.

To understand the effect of lender concentration on overborrowing, let us begin by studying debt holdings in the two decentralized equilibria under different selection criteria. First, under criterion (A), the mean debt holding first increases and then decreases with η. As noted in Section 3, this implies that $F'(d_{t+1})$ in equation (9) first increases and then slightly decreases with η. This hump-shaped relationship is driven by debt in sudden stops, in which η only alleviates sudden stops and raises borrowing capacity when η is below a certain threshold provided in Lemma 5. One can show that every state satisfies the condition in Lemma 5 when $\eta < 0.74$. However, when η goes above 0.76, most of the states violate this condition. Second, under criterion (C), debt holding is independent of changes in η because agents hold too little debt to encounter foreclosure due to the precautionary saving motive, as shown by the flat debt level in middle-left panel and the zero foreclosure probability shown in the lower-right panel.

On the other hand, debt in the SP's allocation decreases with η, as shown by the dash-dotted line. As explained in Section 3.4, this implies that η alleviates post-foreclosure sudden stops in terms of the nominal shadow value of the collateral constraint $p_{t+1}^{F*}\mu_{t+1}^{SP,F*}$. Thus, reducing d_{t+1} lowers the foreclosure probability and the magnitude of the reduction of the expected nominal shadow value by η, leading to a higher marginal benefit of loosening the collateral constraint and lower debt holdings. Comparing debt allocations among the three equilibria, we have that overborrowing is hump-shaped under criterion (A) and decreasing in η under criterion (C).

Incorporating a pecuniary externality internalized by lenders overturns the standard overborrowing results of emerging countries. While the decentralized agents always underborrow under the pessimistic criterion due to precautionary saving, they may overborrow or un-

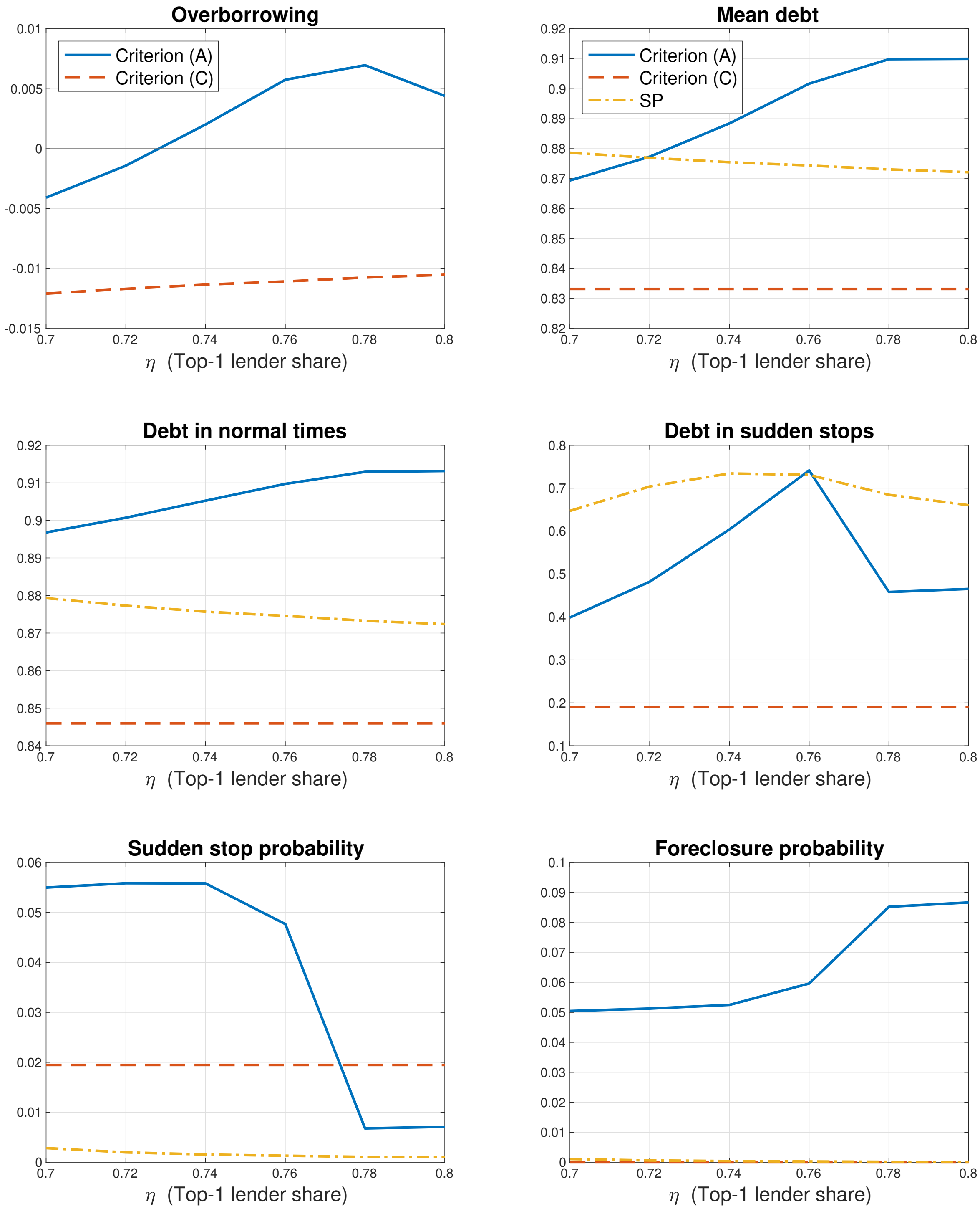

Figure 4: Overborrowing and lender concentration

Notes: This figure plots the simulated mean of counterfactuals under different η values. Other parameters follow the values in Table 3. The models are simulated for 1.1 million periods where the first 0.1 million periods are dropped.

derborrow under the optimistic criterion, depending on lender concentration. Under the baseline parameterization, emerging countries with $\eta < 0.73$, such as Brazil and Panama, underborrow regardless of the equilibrium selection criterion. However, emerging countries with $\eta > 0.73$, such as Argentina and Mexico overborrow under criterion (A) and underborrow under criterion (C).

Another observation from the two bottom panels is that while lender concentration that boosts borrowing capacity leads to, on average, a higher foreclosure probability, it may reduce the sudden stop probability because a higher collateral price in a foreclosure state prevents a binding collateral constraint.

5 Optimal lender concentration

Since lender concentration affects the returns of each lender, the lending countries have an incentive to maximize returns by optimally choosing the concentration. An example of this type of policy is the Brady plan in the 1980s where the US bought back sovereign bonds of emerging countries via US Treasuries bonds, thus concentrating the lender structure to solve the coordination problem among lenders. Such a policy exercise has been recently emphasized by the World Bank (2022) due to the growing accumulation of external debt during the COVID-19 pandemic. While lender concentration raises the collateral price in bad times due to limited foreclosure, it may also incentivize domestic borrowers to borrow more, leading to a higher probability of foreclosure and a binding collateral constraint. Thus, whether allowing the lending country to optimally set lender concentration benefits or harms the domestic borrowers requires numerical analysis. This section provides the results of this exercise.

I consider a planner of the foreign lending country who maximizes lenders' profit, taking as given the foreclosure decisions of atomic lenders, $\zeta_t^{A*} = 1$, and the large lenders given by equation (15). The foreign planner's maximization problem is given by

$$\max_{\eta_t} \zeta_t^*(\eta) p_t^F(\eta_t) y_{t-1}^N + \pi_{t+1} d_{t+1} + (1 - \pi_{t+1}) \zeta_{t+1}^* p_{t+1}^F y_t^N,$$

where $\pi_{t+1} = Pr\left(d_{t+1} \leq y_{t+1}^T\right)$ is the probability that the borrowers fully back the debt, and the first component of the objective is the foreclosure value. The second and third components are the expected payment under foreclosure and no foreclosure in period $t+1$, respectively. Similar to the maximization problem considered in Subsection 3.4, in which the domestic planner chooses the debt while taking lender's foreclosure decisions and lender concentration as given, I assume that the foreign planner also takes the borrower's debt decision d_{t+1} as given.[8]

The above maximization problem can then be simplified as maximizing $\zeta_t^*(\eta_t)p_t^F(\eta_t)y_{t-1}^N$ by choosing η_t, which yields the following first-order condition:

$$\frac{d\zeta_t^*}{d\eta_t}\left(p_t^F + \frac{\partial p_t^F}{\partial \zeta_t^*}\zeta_t^*\right) = 0, \tag{26}$$

under which the optimal foreclosure rate under the optimal concentration η_t^* is $\zeta_t^*(\eta_t^*) = -p_t^F/(\partial p_t^F/\partial \zeta_t^*)$, which is positive under the condition in Lemma 5 that guarantees that $\partial p_t^F/\partial \zeta_t^* < 0$.[9] In this case, lender concentration lowers profits by incentivizing the large lender to foreclose on less collateral, but it also raises profits by increasing the collateral price under foreclosure. Under a given d_{t+1}, the weighted sum of the optimal foreclosure decisions ζ_t^* and p_t^F can be jointly solved by equations (13) and (26). Then, the state-contingent optimal concentration η_t^* can be derived from $\zeta_t^* = \eta_t^*\zeta_t^{L*} + (1-\eta_t^*)$ and the corresponding large lender 's foreclosure decision (15) that replaces η with η_t^*.

The goal here is to understand how allowing the optimal concentration to be set by the foreign planner affects the allocation of different equilibria. Replacing the empirical concentration $\eta = 0.74$ with η_t^*, I solve and simulate the equilibria of the domestic planner's problem and the decentralized equilibria under optimistic and pessimistic criteria. The parameters are identical to those in Table 3 except for two new values, $[\delta, \xi] = [0.65, 0.4]$,[10] that calibrates the equilibrium with the optimal lender concentration under criterion (A) to match the empirical debt-to-output ratio. While the most plausible calibration is not obvious

[8]The foreign planner does not internalize how η_t will affect the borrowing decision d_{t+1}. However, if the foreign planner considers how η_t affects domestic borrowing decisions via the effect on p_t, concentration η_t will influence the expected debt repayment in $t+1$ via d_{t+1}, depending on the probability of foreclosure. Conditional on having no foreclosure in $t+1$, η_t affects future repayment $\pi_{t+1}d_{t+1}$. Note that while $(1-\pi_{t+1})\,p_{t+1}^F y_t^N$ is not affected by η_t, the expected gain $(1-\pi_{t+1})\,p_{t+1}^F y_t^N$ will be affected by η_{t+1}, which can be discretely set by the foreign planner.

[9]$\zeta_t^*(\eta_t^*)$ maximizes profit $\zeta_t^*(\eta_t)p_t^F(\eta_t)y_{t-1}^N$ because the second-order condition,

$$\frac{d^2\zeta_t^*}{d\eta_t^2}\left(p_t^F + \frac{\partial p_t^F}{\partial \zeta_t^*}\zeta_t^*\right) + \frac{d\zeta_t^*}{d\eta_t}\left(\frac{\partial p_t^F}{\partial \zeta_t^*}\frac{d\zeta_t^*}{d\eta_t} + \frac{\partial^2 p_t^F}{\partial \zeta_t^{*2}}\frac{d\zeta_t^*}{d\eta_t}\zeta_t^* + \frac{\partial p_t^F}{\partial \zeta_t^*}\frac{d\zeta_t^*}{d\eta_t}\right),$$

is negative, as $\partial^2 p_t^F/\partial \zeta_t^{*2} < 0$.

[10]$\xi = 0.4$ lies within the bound of the empirical estimates by Akinci (2011).

in this case because lender concentration may not be optimally chosen in reality, matching the debt-to-output ratio provides a counterfactual analysis of optimal lender concentration when foreign lenders takes as given the borrower's debt decisions that generate a debt share of a similar size as in the data.

Table 5 compares the simulated results under fixed and optimal lender concentration. The new set of calibrated parameters produces similar implications for the borrowing decisions of agents and the SP as the baseline parameters in Section 4: Under a fixed η $(= 0.74)$, the decentralized agents overborrow under the optimistic criterion (A) and underborrow under the pessimistic criterion (C). With lower levels of debt, the equilibria under criterion (C) feature a lower foreclosure probability and sudden stop probability.

Several unique observations emerge from Table 5. First, under criterion (A), η_t^* in foreclosure states is, on average, 10% larger than η. Thus, the large lender under η_t^* will foreclose on less collateral because the foreclosure decision will more significantly affect the collateral price. On average, the large lender under η_t^* will foreclose on 39.4% of seized collateral, which is approximately 15% less than that under η. Under criterion (C), average lender concentration and the foreclosure share are not available because there is no foreclosure when the agent's debt holding is low.

Second, under criterion (A), replacing η with η_t^* raises the agent's debt holdings and the degree of overborrowing. This is because lender concentration makes future binding states less severe, as a higher collateral price generates larger borrowing capacity, with which domestic agents borrow more. Under criterion (A) and the SP's problem with η, we observe that the debt holdings in sudden stops decline by 27.1% $(1 - 0.6324/0.8676)$ and 23.4% $(1 - 0.6367/8317)$, respectively. However, with η_t^*, the debt holding in sudden stops only declines by 8.9% $(1 - 0.8378/0.9198)$ under criterion (A). With more borrowing, the agent's consumption under η_t^* is 3.5% lower than that under η. While the optimal concentration generates higher borrowing that leads to a higher foreclosure probability, it does not necessarily lead to a higher probability that the collateral constraint binds because, in foreclosure states, the borrowing capacity under η_t^* is higher, making the collateral constraint less likely to bind.

Finally, similar to the results in Figure 4, under criterion (C), whether the lender concentration is optimal does not change the allocation. This is because the foreclosure probability is zero, and thus concentration will not affect allocation via foreclosure.

Table 5: Simulated results under fixed and optimal lender concentration

	SP	Criterion (A)		Criterion (C)	
	fixed η_t	fixed η_t	optimal η_t	fixed η_t	optimal η_t
Mean debt-to-output ratio	0.2423	0.2487	0.2816	0.1878	0.1878
Median debt-to-output ratio	0.2549	0.2648	0.2927	0.1941	0.1941
Mean debt	0.8107	0.8204	0.9064	0.5883	0.5883
Mean debt in sudden stops	0.6367	0.6324	0.8378	0.1560	0.1560
Mean debt in normal times	0.8317	0.8676	0.9198	0.6030	0.6030
Median debt	0.8426	0.8811	0.9169	0.6099	0.6099
Mean consumption	1.0044	0.9974	0.9698	0.9898	0.9898
Foreclosure probability	0.1250	0.1075	0.1977	0	0
Sudden stop probability	0.1076	0.2007	0.1641	0.0330	0.0330
Mean η_t among foreclosure	0.7400	0.7400	0.8422	NaN	NaN
Mean ζ_t^{L*} among foreclosure	0.5363	0.5401	0.3913	NaN	NaN
Mean ζ_t^* among foreclosure	0.6568	0.6597	0.4871	NaN	NaN

Notes: The debt-to-output ratio is defined as $d_{t+1}/(y_t^T + p_t y_t^N)$. Simulated moments are calculated from the last 1 million periods of a simulation of 1.1 million periods. Fixed η_t stands for $\eta_t = 0.74$, and optimal η_t stands for $\eta_t = \eta_t^*$.

6 Conclusion

This paper studies the effect of lender concentration on countries' external debt via the pecuniary externality internalized by large lenders. This mechanism is motivated by the empirical fact that the external debt of emerging countries tends to have a more concentrated lender structure, which alleviates the severity of sudden stops.

With the empirical facts in hand, this paper develops a model that incorporates the influence of lender concentration via lenders' foreclosure decisions. The theoretical results show that how lender concentration affects overborrowing depends on whether lender concentration alleviates sudden stops in states of foreclosure. If it does, overborrowing increases with lender concentration because the planner benefits less from the alleviation of sudden stops when lowering debt and therefore faces a higher marginal benefit of loosening the collateral constraint and borrows less. The relationship between lender concentration and overborrowing also depends on equilibrium selection criteria because the model in general features multiple equilibria.

Quantitative analysis shows that the relationship between lender concentration and overborrowing is hump-shaped under the optimistic equilibrium selection criterion that triggers foreclosure. Lender concentration alleviates the sudden stop and increases overborrowing when the ratio of debt owned by the largest lender to the total debt is below 0.78. Countries underborrow (overborrow) when the share of the largest lender is below (above) 0.73. Finally, I show that lender countries have an incentive to concentrate their lender structure to increase their payoff. Consequently, borrower countries feature more overborrowing that decreases consumption.

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Appendix

A Proofs

A.1 Proof of Lemma 2

The derivative of p_t^F with respect to d_{t+1} under foreclosure is given by

$$\frac{dp_t^F}{dd_{t+1}} = \frac{p_t^F}{\xi c_t^{T,F}} \left(\frac{1}{1+r} - \zeta_t^* y_{t-1}^N \frac{dp_t^F}{dd_{t+1}} \right) \Rightarrow \frac{dp_t^F}{dd_{t+1}} > 0,$$

where $c_t^{T,F} = y_t^T + d_{t+1}/(1+r) - p_t^F \zeta_t^* y_{t-1}^N$ is tradable consumption under foreclosure. If $\xi \in (0,1)$, we have that

$$\frac{d^2 p_t^F}{dd_{t+1}^2} = \left(\frac{1}{\xi} - 1\right) \frac{p_t^F}{\xi} \left(\frac{\xi c_t^{T,F}}{p_t^F} + \zeta_t^* y_{t-1}^N \right)^{-1} \left(\frac{1}{c_t^{T,F}} \right)^2 \left[\frac{1}{(1+r)} - \zeta_t^* y_{t-1}^N \frac{dp_t^F}{dd_{t+1}} \right]^2 > 0.$$

A.2 Proof of Lemma 3

If $y_t^T = d_t$, we have that $S(0) = 0$ and $p_t^{NF}(0)\kappa y_t^N$. Since p_t^{NF} is increasing and convex in d_{t+1}, there exists only one intersection other than the point where $c_t^T = 0$ and $d_{t+1} = 0$. If $y_t^T > d_t$, we have that $S(0) > 0$ and $p_t^{NF}(0)\kappa y_t^N > 0$. Thus, there exists only one equilibrium when $p_t^{NF}(d_{t+1})\kappa y_t^N$ is tangent to the 45-degree line, implying that $p_t^{NF}(\hat{d})\kappa y_t^N = \hat{d}$. If $p_t^{NF}(\hat{d})\kappa y_t^N < \hat{d}$, the slope is flat enough that there exist two equilibria. If $p_t^{NF}(\hat{d})\kappa y_t^N > \hat{d}$, the slope is too steep to cross the 45-degree line.

A.3 Proof of Lemma 4

Denote by $\bar{d}$ the natural debt limit. Since $p_t > 0$ with c_t^T and $c_t^N > 0$ and that p_t is increasing and convex in d_{t+1}, the following condition

$$\kappa p_t^F(\bar{d}) y_t^N < \bar{d}, \tag{A.1}$$

guarantees the uniqueness of a binding solution in states of foreclosure when $d_t < \bar{d}$. Note that $\kappa p_t^F(\bar{d}) y_t^N$ is given by

$$\kappa y_t^N\left(\frac{1-a}{a}\right)\left(\frac{y_t^T+\frac{\bar{d}}{1+r}-p_t^F(\bar{d})\zeta_t^* y_{t-1}^N}{(1-\delta)\bar{y}_t^N+\zeta_t^* y_{t-1}^N}\right)^{1/\xi} < \kappa y_t^N\left(\frac{1-a}{a}\right)\left(\frac{y_t^T+\frac{\bar{d}}{1+r}}{(1-\delta)\bar{y}_t^N+\zeta_t^* y_{t-1}^N}\right)^{1/\xi}.$$

Thus, a sufficient condition for $\kappa p_t^F(\bar{d})y_t^N < \bar{d}$ in states of foreclosure where $y_t^T < d_t$ is

$$\zeta_t^* > \left(y_t^T+\frac{\bar{d}}{1+r}\right)\left(\frac{\frac{1-a}{a}\frac{\kappa y_t^N}{y_{t-1}^N}}{\bar{d}}\right)^{\xi} - \frac{(1-\delta)\bar{y}_t^N}{y_{t-1}^N} \triangleq \underline{\zeta}_t.$$

A.4 Proof of Lemma 5

If the collateral constraint binds after foreclosure, we have that

$$c_t^{T*} = y_t^T + \left(\frac{1}{1+r} - \frac{\zeta_t^* y_{t-1}^N}{\kappa y_t^N}\right) d_{t+1}^{F*} > 0.$$

Since in states of foreclosure

$$d_t > y_t^T > -\left(\frac{1}{1+r} - \frac{\zeta_t^* y_{t-1}^N}{\kappa y_t^N}\right) d_{t+1}^{F*},$$

implying that $d_t > d_{t+1}^{F*}$ if

$$\zeta_t^* > \frac{\kappa}{(1+r)}\frac{y_t^N}{y_{t-1}^N}. \tag{A.2}$$

When the constraint binds following foreclosure, the collateral price satisfies

$$p_t^{F*} = \left(\frac{1-a}{a}\right)\left[\frac{y_t^T + p_t^{F*}\left(\frac{\kappa y_t^N}{1+r} - \zeta_t^* y_{t-1}^N\right)}{\bar{y}_t^N + \zeta_t^*(\eta) y_{t-1}^N}\right]^{1/\xi}.$$

Thus,

$$\frac{dp_t^{F*}}{d\zeta_t^*} = \frac{y_{t-1}^N}{\xi}\frac{p_t^{F*}}{y_t^T + p_t^{F*}\left(\frac{\kappa y_t^N}{1+r} - \zeta_t^* y_{t-1}^N\right)}\left[-p_t^{F*} + \frac{dp_t^{F*}}{d\zeta_t^*}\left(\frac{\kappa}{1+r}\frac{y_t^N}{y_{t-1}^N} - \zeta_t^*\right) - \left(\frac{ap_t^{F*}}{1-a}\right)^{\xi}\right],$$

in which $dp_t^{F*}/d\zeta_t^* < 0$ when

$$\frac{1}{\xi}\frac{p_t^{F*}\left(\frac{\kappa y_t^N}{1+r}-\zeta_t^* y_{t-1}^N\right)}{y_t^T+p_t^{F*}\left(\frac{\kappa y_t^N}{1+r}-\zeta_t^* y_{t-1}^N\right)}<1.$$

Note that condition (A.2) guarantees that $dp_t^{F*}/d\zeta_t^* < 0$.