How Costly to Sell a Company? A Structural Analysis of Takeover Auctions*

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Abstract

To explain why sellers in takeover auctions restrict bidders' entry, we quantify economic costs incurred when inviting an additional bidder. Our auction model allows bidders to discount their synergy values when other industry players gain access to confidential information about the target company. We identify the model primitives, allowing for unobserved heterogeneity, as confidential information is latent. We obtain the posterior of the primitives using a sample with 287 M&A deals of U.S. public companies. The unobserved heterogeneity explains around 30 percent of the premium variation. We find a considerable economic cost of running a takeover auction. First, information costs are heterogeneous across industries, ranging from 2% to 38%. Second, operating costs are 3 to 9 times higher than the reported accounting fees. However, we do not find evidence that an alternative mechanism would generate higher premiums than the sealed-bid auction, justifying the current mechanism.

JEL classification: C57, D22, D44, G34

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1 Introduction

The literature on mergers and acquisitions (M&A) has reported that sellers of takeover auctions routinely restrict bidders' entry. The seminal article, Boone and Mulherin (2007), documents that about half of the company sales in their sample invite one acquirer, and even when they do more than one, they allow only a few. This observation contradicts the received theory: competition raises the seller's (expected) revenue.[1] We explain the entry regulation by the seller's costs to conduct a takeover auction.

We study two distinctive economic costs discussed in the literature: *operating cost* and *information cost*. First, the operating cost is directly associated with planning, carrying out, and concluding a takeover process. That includes not only accounting costs such as advisory fees for investment banks and law firms but also other opportunity costs resulting from disruption of business, negative impacts on the employee morale, and missing out on business opportunities; see Rosenbaum and Pearl (2009) and DePamphilis (2018). These opportunity costs are naturally target-specific, privately known to the seller, and have not been formally measured or documented.

Second, the information cost *indirectly* arises from bidders' evaluation of the target company and their bidding behavior. Hansen (2001) notes that bidders discount their synergy values when competitors have access to the target's confidential information. Since bidders are often industry rivals, customers, or suppliers who strategically interact in their daily business, they can exploit this information against the acquirer—even after they lose the auction. As a result, when competitors participate in the auction, bidders lower their valuations, reducing their bids and ultimately decreasing the seller's revenue. This revenue loss constitutes the information cost. Confidential information remains critical both during and after the takeover auction, a distinctive feature of takeover auctions.

While the information cost has been mostly discussed in the field of corporate finance, e.g., Boone and Mulherin (2007); Rogo (2014); Schlingemann and Wu (2015), legal studies of corporate takeovers also echo this concept through their term *"legitimate proprietary concerns"* referring to the risks of sensitive information being disclosed to industry players. Specifically, the Delaware Court of Chancery often indicates that a full-blown auction may not be desirable, as the cost could outweigh the benefits.[2] For instance, in the case of *Lear*

[1] Bulow and Klemperer (1996) shows that no bargaining power is as valuable to the seller as attracting one extra bona fide bidder. This claim can fail in various situations, e.g., auctions with almost common value in Klemperer (1998) and auctions with a voluntary entry in Li and Zheng (2009). The takeover auction, however, does not fit into those frameworks; see sections 2 and 3.

[2] Delaware is the leading jurisdiction for publicly traded corporations listed on U.S. stock exchanges, with more than half choosing to incorporate there. It is also the top choice for out-of-state incorporations, where companies headquartered in one state incorporate in another. See for more information https:

Corp. Shareholder Litigation, 926 A.2d 94, 119, the target company rejected a potential acquirer due to the risk of losing an initial bidder or receiving a lower offer if additional bidders were involved; see Sautter (2013) for more examples. Among such cases, concerns over the potential loss of competitive information are frequently cited as a key issue.

While it is evident that sellers limit bidder participation due to economic costs, no study has formally measured these costs or examined their economic implications. This paper addresses that gap.

To quantify information costs, we develop an optimal bidding strategy that allows bidders to discount their synergy values following their beliefs about how many competitors have access to the target's confidential information. Although a takeover process involves multiple stages, the seller shortlists a few bidders for the final stage, where they gain access to the most critical data about the target. At this stage, bidders conduct comprehensive due diligence and submit sealed bids without knowing their competitors' identities or offers.[3] Reflecting this key institutional feature, we develop a sealed-bid (first-price) auction model in which shortlisted bidders receive private signals, update their beliefs about the number of competitors, and discount their synergy values accordingly.

Interpreting bid data as equilibrium outcomes, we then establish the identification of the auction model, allowing for unobserved heterogeneity conditional on observed auction characteristics. Unobserved heterogeneity should be critical in this study because confidential information is, by nature, latent. We use the deconvolution method of Kotlarski (1966) and Krasnokutskaya (2011) to identify the distribution of unobserved heterogeneity by within-auction bid variation.[4] After separating out unobserved heterogeneity, following the strategy of Guerre, Perrigne, and Vuong (2000, 2009), we identify the model primitives, including distributions of private signals and discounting factors by exploiting cross-auction bid variation and the exogenous variation of the number of potential bidders.

We analyze a sample of 287 takeover auctions of U.S. public companies from 2000 to 2008. Using a Bayesian method, we account for unobserved heterogeneity and explore the posterior distribution of structural parameters through a Markov chain Monte Carlo (MCMC) algorithm. From the posterior, we find that sellers invite fewer bidders when signal distri-

//corplaw.delaware.gov/facts-and-myths.

[3]From our conversation with an experienced investment banker specializing in M&A, we learned that it is common practice for the seller to withhold information about their rivals, such as their identities and offers. Strong evidence of this can be found in many SEC (Security and Exchange Commission) filings; see section 3 for more details. Moreover, our empirical findings in section 5 suggest that withholding information about the competition can align with the seller's interests, leading to higher premiums.

[4]Li and Vuong (1998) has first introduced Kotlarski (1966) to identify models with measurement error. Using this technique, Li, Perrigne, and Vuong (2000) identifies a class of auctions with conditionally independent private values, and Krasnokutskaya (2011) identifies auction models with unobserved heterogeneity.

butions are more dominant, and bidders heavily discount their synergy values, highlighting the critical role of value discounting in takeover auctions. Given the high heterogeneity of takeover auctions in the sample, we illustrate prediction quality using five target companies—one from each industry—whose attributes are closest to their industry medians. For these industry-median targets, the posterior accurately predicts observed premiums (winning bids) with a small standard deviation. We also find that unobserved heterogeneity accounts for 26∼32% of the premium variation, highlighting the importance of unobserved heterogeneity in empirical analyses of takeover auctions.

With the posterior distribution, we conduct counterfactual analysis to quantify information and operating costs incurred by the seller. First, the predictive premium with value discount is first-order stochastically dominated by the predictive premium under the hypothetical scenario without value discount. The discrepancy between the predictive premiums with and without value discounts allows us to measure information costs. We find that the information costs vary across industries, ranging from 2 to 38 percent of the premium. Second, we also find that the lower bound of operating costs is three to nine times higher than the documented advisory fees (0.84% of the transaction value; see Hunter and Jagtiani (2003)). Finally, despite such substantial economic costs the seller bears in the current mechanism, our counterfactual analysis also reveals that the predictive premium under the ascending auction or negotiation would be lower than the current premium in many cases, and not clearly higher in others. This justifies the current use of the sealed-bid auction.

The main contribution of this paper is to quantify the information cost and operating cost incurred by the seller in takeover auctions. Many studies in corporate finance use the concept of information cost to explain other stylized features in M&A transactions. For example, Boone and Mulherin (2007) finds that negotiations and auctions yield similar transaction premiums and suggests that the presence of information cost may explain the choice of sale mechanism. In addition, Schlingemann and Wu (2015) finds that the likelihood of choosing auction over negotiation decreases with R&D intensity and explains this by the hypothesis that R&D intensity may serve as a proxy for the cost of disclosing proprietary information. However, these studies take a reduced-form approach, which cannot measure the information cost, as it is structural in nature and requires formulating bidding behavior.

A few articles develop a structural approach but study other aspects of takeover competitions. Taking the pool of acquirers as given, Gorbenko and Malenko (2014) estimates an auction model to learn how strategic and financial bidders evaluate target companies and find that different targets appeal to different groups of bidders. Gentry and Stroup (2019) models potential acquirers' entry decision, i.e., a decision on whether to sign confidentiality agreements and investigates how the pre-entry uncertainty on the target value affects the bidders' entry and bidding behavior. Recently, Allen, Clark, Hickman, and Richert (2023) studies takeover auctions that allocate failed banks to a solid financial institute, focusing on bidders' uncertainty about the scoring rule and multiple bids for hedging the associated risk.

Unlike those papers, we consider a setting where the seller restricts bidders' entry into the final competition to improve his revenue. We examine how the seller's shortlisting process influences bidding behavior, from which we recover the structural costs incurred by the seller. Moreover, none of the previous empirical studies formally consider unobserved target heterogeneity. Confidential information plays a crucial role in our analysis, as it drives the information cost yet remains unobservable. This paper incorporates unobserved target heterogeneity into our econometric analysis and demonstrates its economic significance in the study of M&A transactions.

In addition, Kang and Miller (2023) finds that many government procurement contracts use negotiations with one seller due to buyer solicitation costs, offering insights through a seller menu design approach. Garrett, Ordin, Roberts, and Serrato (2023) also observes similar patterns of one or few bidders competing in bond auctions and model bidders' entry costs to explain limited participation. In both cases, the concept of information costs is irrelevant, as losing bidders do not exploit critical information about the auctioned item afterward in a strategic environment.

Finally, the existing theoretical research primarily resides on the idea that information acquisition is costly for bidders; see, for example, Ye (2007), Quint and Hendricks (2018), and Lu and Ye (2018). Ye (2007) finds that bidders' entry cost cannot explain why the seller of a company uses an indicative bidding stage to shortlist bidders into the final bidding competition. Quint and Hendricks (2018) shows that in a particular scenario, where the indicative bids sort bidders into a finite number of groups, the seller can use indicative bids to shortlist bidders optimally. Lu and Ye (2018) develops an optimal two-stage mechanism where bidders must incur information acquisition costs to discover their values. These studies acknowledge the seller's active role in shortlisting actual bidders in the takeover process, but they do not consider the costs incurred by the seller.

The next section outlines a typical takeover process, based on which section 3 develops our economic model. Section 4 discusses data, identification, and inference, section 5 presents empirical results and counterfactual analysis, and section 6 concludes the paper, followed by an appendix collecting all proofs. The supplementary material (Kim and Zheng, 2025) offers additional details about computation and data collection.

Table 1: Summary of Takeover Process

	Description	Remarks
Stage 1	The seller contacts a set $\mathcal{N}_1$ of prospective buyers with a delivery of a teaser and confidentiality & standstill agreements (C&S agreements).	Cursory info. is released.
Stage 2	A subset $\mathcal{N}_2 \subset \mathcal{N}_1$ of prospective buyers in stage 1 signs the C&S agreements to enter stage 2 and obtain confidential information memorandum (CIM) to conduct the preliminary due diligence.	Preliminary info. in CIM is released.
Stage 3	A subset $\mathcal{N}_3 \subset \mathcal{N}_2$ of prospective buyers submits indications of interest after the preliminary due diligence based on CIM obtianed in stage 2. The buyers who indicate their interest are *potential* bidders.	$N := \lvert\mathcal{N}_3\rvert$.
Final stage (focus)	The seller *invites* a subset $\mathcal{N}_4 \subset \mathcal{N}_3$ of potential bidders to the final round of comprehensive due diligence. After the comprehensive due diligence, the *actual* bidders submit *sealed bids* by the due date. The seller and the winner in this stage sign the M&A agreement and publicly announce the deal, which concludes the private takeover.	$n := \lvert\mathcal{N}_4\rvert$. Comprehensive information is released.

2 Takeover process

A takeover process consists of multiple stages, each with its own objective and releasing different amounts of information on the target. This section outlines a typical takeover process. As we shall discuss briefly, bidders have access to confidential information and offer bids in the final stage, where the information cost arises to the seller. The institutional features of the final stage motivate the specification of the auction model in section 3.

A corporate takeover is initiated either by a target company considering the sale of its enterprise as a strategic alternative or by an acquiring company launching an unsolicited inquiry; see Gorbenko and Malenko (2024). Once a takeover begins, the target hires an investment bank for financial advice and retains a law firm for legal counsel – we call the target and its advisors as the seller in this paper. Then, a takeover typically follows the multiple stages; see also Table 1.

Stage 1: Delivery of a teaser

The seller contacts prospective buyers, delivering a teaser and confidentiality and standstill agreements.[5] For public companies, *Regulation Fair Disclosure concerns* govern the content

[5]The confidentiality agreement governs how buyers can use the information obtained, and the standstill agreement precludes the prospective buyer from making unsolicited offers or purchasing the target's shares, etc., in a specified period of time.

of the teaser; see Hansen (2001).[6] The teaser contains cursory information. The contacted prospective buyers may not even know the target's identity before signing the agreements because revealing it may constitute selective disclosure of material information only to those contacted buyers.

Stage 2: Preliminary due diligence

Prospective buyers, if interested, sign the confidentiality and standstill agreements to obtain the Confidential Information Memorandum (CIM). The seller prepares a CIM to encourage prospective buyers to submit indications of interest (stage 3 below). Therefore, the CIM often describes the target company in the best possible light. The CIM includes an overview of the target company, product and service summary, revenue profiles, data on employee and customer body, key financials (history and projections), management structure, etc. Examples of CIM are available on the internet.

Stage 3: Indications of interest

After reviewing the CIM, if further interested, a prospective buyer submits an indication of interest, which is not legally binding. The indication specifies indicative purchase prices (typically in a range) and form of consideration (cash vs. stock mix), assumptions to arrive at the indicative purchase price, information on financing sources, treatment of management and employees, and conditions to signing and closing. We consider the buyers indicating their interest as *potential bidders*.[7]

Final stage: Comprehensive due diligence & final bidding

After reviewing the indications of interest, the seller *invites* a few potential bidders to the final round of comprehensive due diligence. The shortlisting decision also considers factors such as transaction speed, fulfillment of fiduciary duties, business disruption, and confidentiality concerns. Notably, concerns about the dissemination of confidential information are unique to takeover auctions, which is the focus of this study. For example, in the sale of Spinnaker Exploration Company, the seller's financial advisor Randall & Dewey suggested that the company adopt a "*limited marketing approach as it minimized the exposure of Spinnaker's*

[6] The Regulation Fair Disclosure mandates that all publicly traded companies "may properly share material nonpublic information with outsiders, for legitimate business purposes, when the outsiders are subject to duties of confidentiality." See `https://www.sec.gov/rules/final/33-7881.htm`

[7] The definition of *potential bidders* varies to serve specific research agendas. For example, Gentry and Stroup (2019) refers to prospective buyers contacted in stage 1 as potential bidders to model their decisions on whether to sign the confidentiality agreement, i.e., the decision to enter stage 2. Our definition of *potential bidders* serves our purpose of understanding the bidders' behavior in the final stage.

sensitive confidential information to a smaller group of competitors" because "*the likely buyers for Spinnaker were all competitors.*"[8]

We emphasize that the seller restricts bidders' entry to the final stage. The *actual bidders*—those invited to this stage—gain access to the most confidential materials through a series of on-site visits and multiple consultations with target management. The seller is obligated to disclose all relevant data to these bidders. While the seller extensively communicates with each actual bidder, he keeps them uninformed of other bidders' identities or tentative offers for competitive and legal reasons; see Gorbenko and Malenko (2014). Following the due diligence, the actual bidders submit their final offers in the form of a sealed bid by the pre-specified due date. The seller then analyzes those bids and selects the winner to work on the final definitive agreement, which includes the purchase price, method of payment, fiduciary-out provisions, etc. Upon signing the M&A agreement, the seller and the winner publicly announce the deal, concluding the private takeover process.

We study this stage because it is where the most critical information is released; thereby, information cost arises. We shall develop an auction model for the final stage in section 3.[9]

3 Model

For empirical analysis of takeover auction data, this section introduces an auction model to formulate bidding behavior in the final stage. Section 3.1 explains the auction format, section 3.2 specifies the synergy values, and section 3.3 models bidders' perception on the number of actual bidders. Then, section 3.4 investigates bidders' strategies.

3.1 Sealed-bid auction

We develop a model of sealed-bid (first-price) auctions due to the institutional feature that in the final stage bidders submit sealed bids, the highest bidder wins, and the winner pays her own bid. Recently, Allen, Clark, Hickman, and Richert (2023) and Gorbenko and Malenko (2024) consider sealed-bid auctions for takeover sales. Nevertheless, since the corporate finance literature has employed other allocation mechanisms, we still elaborate on our mod-

[8]The *italic* texts are extracted directly from the *DEFM14A* document filed by Spinnaker to SEC on November 10, 2005. (Kim and Zheng, 2025) shows a detailed description of the takeover deal.

[9] The M&A agreement *can* be challenged by another industry player, triggering a public battle. This occurs 4 percent of the time (Moeller, Schlingemann, and Stulz, 2007), and the original winner wins with 70.7% of chance (Betton, Eckbo, and Thorburn, 2009). That is, the public battle replaces the winner with 1.2 percent of the time, in which case the seller compensates the original winner following the M&A agreements that they signed. Another stream of literature studies the public competition; see, for example, Betton, Eckbo, and Thorburn (2009) and Dimopoulos and Sacchetto (2014).

eling choice.

The entire takeover process consists of multiple stages as outlined in section 2, and formulating the whole process can be daunting. The previous articles have focused on a certain stage by a mechanism that fits their choice. For example, a bargaining model may be used if the seller contacts only one prospective buyer from the beginning; see Hoffmann and Vladimirov (2024). An ascending auction would better approximate the public battle after the final stage; see Footnote 9 and references therein. Unlike those previous articles, we study the final stage where bidders submit sealed bids.

The final stage still has some components resembling negotiation, as the seller interacts with each bidder. The nontechnical monograph, Subramanian (2020), uses the term *negoti-auction*, describing that bidders are "fighting on two fronts", i.e., across the table with the seller and on the same side of the table with other bidders. However, Bulow and Klemperer (1996) argues that the competition between bidders dominates the seller's bargaining power. Following the prediction, we use an auction model to formalize the bidders' competition, abstracting away any marginal influence from the bargaining-like elements.

Another concern is that when the seller communicates with multiple bidders, the seller might be able to leverage his bargaining power by informing bidders of the outstanding offer or other bidders' identities, inducing an environment strategically equivalent to an ascending auction. We also rule out this possibility because the seller cannot release information about the potential competition. Even if the seller attempts to do so, bidders cannot verify the seller's message about other bidders. Thus, such attempts are not credible.

This claim accords with many M&A cases in the SEC (U.S. Security and Exchange Commission) filings. For example, in some cases, bidders were revising their bids, but all were below the outstanding offer. In other cases, the only actual bidder bids more than the market value of the company and even raises her own bid, as if there are other bidders.[10] Subramanian (2020) also gives anecdotes about how sellers try to form a false impression about competition among the bidders, for example, declining access to the data room when available, or leaving empty pizza boxes around the conference room. All of this suggests that bidders are uncertain about the competition.

Based on the institutional details as outlined, we build a first-price auction model, where bidders are uncertain about the competition but observe their own synergy values of merging with the target company. Our modeling choice is similar to Allen, Clark, Hickman, and Richert (2023), which employs a first-price auction model with an uncertain number of bidders to study the bidding stage of the takeover process in the U.S. bank industry.

[10]The supplementary material (Kim and Zheng, 2025) shows an example, the Spinnaker case, where the highest bidder raised her own bid, which was already the highest even before she updated it.

3.2 Value discount

The seller restricts bidders' participation to the final stage because bidders may lower their bids if their rivals also learn confidential information. To consider this feature, we allow bidders to discount their synergy values depending on the number, n, of actual bidders. We build a baseline model and allow it to depend on auction heterogeneity to facilitate our empirical analysis. Formally, let $y_t \in \mathbb{R}$ summarize the heterogeneity of auction $t \in \{1, \ldots, T\}$. The actual bidders commonly observe y_t when they bid, but the researcher has partial information on it. Thus, we specify y_t to depend on a vector $x_t \in \mathbb{R}^{d_x}$ of d_x attributes in the dataset and a scalar $\varepsilon_t \in \mathbb{R}$ summarizing unobserved heterogeneity. Among others, x_t includes the target's book value, $\texttt{size}_t$, and the number N_t of potential bidders, and ε_t may contain some confidential information if affecting all bidders' synergy value in the same way.

Bidder i in auction t also learns an unscaled private signal $v_{it}^u \geq 0$ in the final stage. The signal v_{it}^u represents the idiosyncratic business compatibility of bidder i with target t, which may depend on the target's confidential information. It can also reflect the bidder's opportunity costs of merging with the company, which again depend on idiosyncratic factors, such as strategic enterprise plans, business backlog, recruitment schemes, and production efficiency. Hence, we consider the signal v_{it}^u as private information. If bidder i *knew* that n_t bidders are competing in the final stage, she would form her unscaled synergy value as

$$w_{it}^u := \exp(y_t)(\texttt{size}_t + \delta(n_t) v_{it}^u),$$

where $\delta(n_t) \in (0, 1]$ denotes the discounting factor. Even if there is no positive business compatibility, i.e., $v_{it}^u = 0$, the target company with $y_t = 0$ must still be valued by every bidder at its book (liquidation) value. This specification allows the synergy value to depend on auction characteristics, y_t, and each bidder's private signal, v_{it}^u.

Bidder i would not discount her synergy value if she were the sole bidder but would apply a greater discount as n_t increases.

Assumption 1. $\delta(1) = 1$ and $\delta(n) > \delta(n + 1) \geq 0$ for all integers $n \geq 1$.

To develop the baseline model, we normalize the synergy value by the company's book value. That is, bidder i's (normalized) synergy value is now given as

$$w_{it} = \frac{w_{it}^u}{\texttt{size}_t} = \exp(y_t)(1 + \delta(n_t) v_{it}), \tag{1}$$

where $v_{it} = v_{it}^u / \texttt{size}_t \in [0, \overline{v}]$, (normalized) private signal, for which we maintain the following assumption.

Assumption 2. For each auction $t \in \{1, \ldots, T\}$, conditional on $(n_t, x_t, \varepsilon_t)$, the private signals are independently and identically distributed as $F_v(\cdot|n_t)$, i.e.,

$$v_{1t}, \ldots, v_{n_t t}|n_t, x_t, \varepsilon_t \overset{iid}{\sim} F_v(\cdot|n_t), \tag{2}$$

which has density $f_v(\cdot|n_t)$, differentiable and strictly positive on the support $[0, \overline{v}]$.

Potential bidders estimate their synergy values before accessing the target's critical information, and some submit indications of interest (Section 2). The seller evaluates demand for the target based on these indications and the economic costs of running the auction. Subsequently, the seller invites the most qualified n_t bidders. *Consequently*, the signal distribution (2) is indexed by the number n_t of invited bidders. However, this should not imply that the seller can directly manipulate the signal distribution by selecting n_t.

Under Assumption 2, private signals are independent across bidders, as they reflect each bidder's unique compatibility with the target and the opportunity costs of the takeover. However, synergy values (1) remain correlated through auction attributes. Assumption 2—which posits that signals are identically distributed—aligns with the institutional reality that actual bidders are unaware of each other and compete against an identical 'average' bidder. The zero lower bound ensures that private signals can only increase synergy values above the liquidation value, as bidders with negative compatibility would not have progressed to the final stage.

3.3 Bidder's perception about competition

We use a probability distribution to represent actual bidders' beliefs about the competition level. Since the beliefs potentially depend on auction heterogeneity and private signals, we start with the conditional distribution function, $p(n|x_t, \varepsilon_t, v_{it})$ for $n \in \{1, \ldots, N_t\}$.

Assumption 3. (i) $p(n|x, \varepsilon, v) = p(n|x, v)$ and (ii) $p(n|x, v) \propto p(n|x)f(v|n)$.

Assumption 3(i) implies that bidders do not update their beliefs about the competition level based on ε_t. This assumption is required for econometric identification, but we still justify it by the fact that bidders cannot discern how ε_t influences the seller's shortlisting rule because ε_t, by definition, is not recorded in any public data. Assumption 3(ii) states Bayes rule. Its implication as an assumption is that bidders start with identical ex-ante beliefs, $p(n|x)$, coherently update their beliefs after learning their idiosyncratic private signals, v, forming ex-post asymmetric beliefs, $p(n|x, v)$, which they use to determine their bids. Bidders can infer $p(n|x)$ from data on past M&A deals as we demonstrate in section 4, and then apply $p(\cdot|x_t, v_{it})$ to calculate their optimal bids in auction t.[11]

The ex-ante belief, $p(n|x)$, is a *reduced-form* parameter, as it reflects the statistical relationship between n and x without explicitly modeling the seller's optimal shortlisting rule. An alternative approach would explicitly model the seller's decision-making process, enabling bidders to infer the level of competition based on the seller's choices. However, such a model contradicts institutional practice, as actual bidders cannot observe the factors influencing the seller's shortlisting decisions. These factors encompass bidders' indications of interest and the seller's opportunity costs of pursuing alternative business strategies. Neither these factors nor a complete set of potential bidders are recorded in public data. Thus, any alternative approach would rely on a hypothetical and empirically unverifiable shortlisting rule. In contrast, our reduced-form approach is robust to different shortlisting rules while effectively serving our objective of measuring the seller's economic costs.

As emphasized above regarding Assumption 2, the seller determines n_t based on the demand for the target company. However, the use of a probability function, $p(n|x, v)$, in this paper to represent bidders' perception of competition should not be interpreted as the seller randomly selecting n_t actual bidders from the pool of potential bidders

3.4 Bidding strategy

The competition environment in the final stage induces a game with incomplete information. For that game, we maintain the following assumption.

Assumption 4. In auction $t \in \{1, \ldots, T\}$, every actual bidder is risk-neutral and maximizes her expected utility following a symmetric bidding strategy and having (x_t, ε_t) and $\{p(n|x_t), \delta(n), F_v(v|n)\}_{n=1}^{N_t}$ as common knowledge.

We specify the bidding strategy in an auction with heterogeneity y (or (x, ε)) as $\exp(y)(1+\beta(v))$ for private signal $v \in [0, \overline{v}]$ and optimal mapping $\beta(v)$ with $\beta' > 0$. Let $F_{v,n}(v) := F_v(v|n)^{n-1}$. Under Assumptions 1∼4, a bidder with private signal v in an auction with

[11] It is consistent with the popular methods of valuation, i.e., Comparable companies analysis (CCA) conducted by the investment banks in the analysis of M&A cases; see Eaton, Guo, Liu, and Officer (2022).

characteristics (x, ε) would choose

$$\arg\max_u \sum_{n=1}^{N_t} p(n|x,v) F_{v,n}(u) \{ \underbrace{\exp(y)[1+\delta(n)v]}_{=w,\text{ synergy value in (1)}} - \underbrace{\exp(y)[1+\beta(u)]}_{=\text{ bid if signal is } u} \}$$
$$= \arg\max_u \underbrace{\sum_{n=1}^{N_t} p(n|x,v) F_{v,n}(u)[\delta(n)v - \beta(u)]}_{=U(\beta(u),v;x)}, \quad (3)$$

if all other actual bidders follow $\beta(\cdot)$. The objective function on the left-hand side is bidder i's expected utility when she behaves as if her private signal is u, and the equality holds because multiplying the objective function by a positive constant, $\exp(-y)$, does not change the solution. We let $U(\beta(u), v; x)$ denote the expected utility multiplied by $\exp(-y)$. To obtain the first-order condition, we take a derivative of $U(\beta(u), v; x)$ with respect to u,

$$\sum_{n=1}^{N} p(n|x,v) \{ f_{v,n}(u)[\delta(n)v - \beta(u)] - \beta'(u) F_{v,n}(u) \} = E_{p(n|x,v)} \Psi(n,u,v) \quad (4)$$

where $f_{v,n} := F'_{v,n}$, $\Psi(n,u,v) = f_{v,n}(u)[\delta(n)v - \beta(u)] - \beta'(u)F_{v,n}(u)$, which is the marginal net gain of choosing u for each n, and the expectation operator indicates that the probability measure $\{p(n|x,v)\}$ integrates n out. A rational bidder chooses u to equate (4) to zero. Since $\beta(\cdot)$ is optimal, (4) is zero for $u = v$, resulting in the first-order condition, $E_{p(n|x,v)}\Psi(n,v,v) = 0$. That is, a rational bidder would choose a bid at which the expected marginal net gain is zero. To represent the condition in the form that we can solve for $\beta(\cdot)$, we define

$$Q_N(v;x,\delta) := \frac{\sum_{n=1}^N p(n|x,v) f_{v,n}(v)\delta(n)}{\sum_{n=1}^N p(n|x,v) F_{v,n}(v)} \text{ and } R_N(v;x) := \exp\left(\int_0^v Q_N(\alpha;x,1)d\alpha\right).$$

Note that $R_N(v;x)$ is the integrating factor of the differential equation for β, induced by the first-order condition. Using $Q_N(v;x,\delta)$ and $R_N(v;x)$, we rewrite the first-order condition as

$$R_N(v;x)Q_N(v;x,\delta)v = R_N(v;x)\beta'(v) + R_N(v;x)Q_N(v;x,1)\beta(v)$$
$$= \frac{\partial}{\partial v} R_N(v;x)\beta(v).$$

Observe that R_N appears redundant on both sides around the first equality, but the second equality holds due to $\frac{\partial}{\partial v}R_N(v;x) = R_N(v;x)Q_N(v;x,1)$. By applying the boundary condition, $\beta(0) = 0$, and using $R_N(0; x) = 1$, we have

$$\beta_N(v; x, \delta) := \frac{1}{R_N(v; x)} \int_0^v \alpha R_N(\alpha; x) Q_N(\alpha; x, \delta) d\alpha, \tag{5}$$

where we notify that the solution depends on (N, x, δ). We can also use $\beta_N(v; p(\cdot|x), \delta)$ to emphasize that β_N depends on x only through $p(\cdot|x)$. The strategy (5) plays a critical role in empirical analysis. In particular, we invert (5) at every observed bid to back out latent signals, which is necessary because signal densities, $\{f_v(\cdot|n)\}$, construct a likelihood function. Our method then evaluates the likelihood function at many candidate parameter values throughout the inference procedure. So, it is critical that (5) characterizes an equilibrium. For empirical analysis, we numerically verify that $U(\beta_N(u; x, \delta), v; x)$ is maximized at $u = v$ for all v and for all auctions in our sample for every parameter with a positive density under the posterior.

Without additional assumptions, it is difficult to formalize the existence of equilibrium. In particular, the second-order condition is hard to verify, for which we need derivatives of $p(n|x, v)$ with respect to v, but $p(n|x, v) = \frac{p(n|x) f_v(v|n)}{\sum_{m=1}^N p(m|x) f_v(v|m)}$ depends on v through many *densities* $\{f_v(\cdot|n)\}$ appearing on both numerator and denominator in the summation operator. The derivatives of densities can take any sign. Moreover, the probabilities $\{p(n|x, v)\}$ themselves enter the numerator and denominator of Q_N in summation operators, and Q_N again goes into R_N. Finally, the winning probability in (3) that integrates n out is inseparable from the contingent payoffs that depend on n. The complicated payoff structure makes it difficult to find a useful sufficient condition for equilibria; see (Athey, 2001). Nevertheless, our investigation reveals an implication of (5) when it characterizes an equilibrium.

Proposition 1. *Under Assumptions 1∼4, if* (5) *characterizes an equilibrium, we have*

$$C_{p(n|x,v)}\{s(v|n), \Psi(n, v, v)\} \geq 0, \tag{6}$$

where $C_{p(n|x,v)}$ *denotes the covariance and* $s(v|n) := \frac{d}{dv} \log f(v|n)$ *the score function of* $f(v|n)$.

The score $s(v|n)$ represents how quickly v becomes plausible for each n. Therefore, (6) has an intuitive interpretation that whenever the marginal net gain Ψ with a particular n is positive (negative), that n should come with a higher (lower) chance in equilibrium.

Now, we conclude this section by formalizing an equilibrium with additional assumptions, although our empirical method considers the general setting above, numerically verifying equilibria under every relevant parameter value. We consider bidders who do not update their beliefs about the competition by private values, i.e., $p(n|x, v) = p(n|x)$; n and v might be independent conditional on n, or bidders cannot upate their beliefs. We define

$$\omega_n(v;x) := \frac{p(n|x)F_{v,n}(v)}{\sum_{m=1}^{N} p(m|x)F_{v,m}(v)} \text{ and } \beta_n(v) := v - \int_0^v \frac{F_{v,n}(\alpha)}{F_{v,n}(v)} d\alpha.$$

The weight $\omega_n(v;x)$ is the probability that the auction has n bidders when $n-1$ have signals below v, and $\beta_n(v)$ is the equilibrium bidding strategy in the sealed-bid auction where the number n of bidders is common knowledge among bidders.

Proposition 2. *Under Assumptions 1∼4, if $p(n|x,v) = p(n|x)$, (5) simplifies to*

$$\beta_N(v;x,\delta) = \sum_{n=1}^{N} \omega_n(v;x)\delta(n)\beta_n(v), \tag{7}$$

and it characterizes a symmetric Bayesian Nash equilibrium if strictly increasing.

The bidding strategy (7) is similar to that in Harstad, Kagel, and Levin (1990). Proposition 2 shows that the bidding strategy is a weighted average of discounted standard bidding strategies. The strict monotonicity of $\beta_n(v)$ does not guarantee that (7) is also strictly increasing because $\{\omega_n(v;x)\}_{n=1}^{N}$ also depend on v with $1 = \sum_{n=1}^{N} \omega_n(v;x)$, i.e., if $\omega_n(v;x)$ increases in v for some n, some other components must decrease. Here, we offer a sufficient condition for the strict monotonicity of (7). Jeong and Kim (2024) shows that if $F_{v,m}$ is stochastically dominant over $F_{v,n}$ in reverse hazard rate, i.e., $f_{v,n}(v)/F_{v,n}(v) < f_{v,m}(v)/F_{v,m}(v)$ for all $v \in (0, \overline{v})$, we have $\beta_n(v) < \beta_m(v)$. We also use the reverse hazard rate dominance.

Proposition 3. *The bidding strategy (7) is strictly increasing if $F_{v,n}$ is stochastically dominant over $F_{v,n+1}$ in reverse hazard rate for all $n \in \{1, 2, \ldots, N-1\}$.*

4 Data, identification, and inference

4.1 Data

We collect data on the number of bidders at each stage of the takeover process and the submitted bids from SEC filings.[12] We then augment the data with the target and deal

[12] Since Boone and Mulherin (2007), SEC filings have been an important data source in the literature on M&A. These filings provide detailed documentation of deal backgrounds, as required by SEC regulations. Deal backgrounds are documented across multiple SEC filings. When multiple bidders are involved, the Definitive Proxy Statement (DEFM14A) must disclose the competing offers, the board's deliberations, and the rationale behind selecting the final deal. While other filings—such as PREM14A, PREM14C, DEFM14C, DEFS 14A, Schedule 14D-9, and Form S-4—may also contain relevant details about the merger. These filings are publicly available at `http://www.sec.gov/edgar/searchedgar/companysearch.html`.

attributes from the Center for Research in Security Prices (CRSP) and Compustat.[13] Our sample consists of 287 takeover deals between 1 Jan 2000 and 6 Sep 2008 with the following criteria.

(i) The target is a publicly traded non-financial US company.

(ii) The winning bid is made in cash only.

(iii) The winning bidder obtains 100% of target shares after the transaction.

(iv) The deal is not a spin-off, recap, self-tender, exchange offer, repurchase, minority stake purchase, acquisition of remaining interest, or privatization.

(v) The deal is fulfilled by an auction, i.e., at least two buyers submit indications of interest.

(vi) Deal background is available in SDC platinum, SEC Edgar filings, or Merger Metrics.

(vii) Quarterly financial data on the target are available in Compustat.

Conditions (i)-(iv) ensure takeover deals and bids are comparable. First, we exclude M&A deals in the financial sector because the valuation of financial firms differs from that of non-financial firms. For example, high leverage, which is normal for financial firms, indicates great distress for non-financial firms.[14] Second, the winning payment is cash only so that the deal value is certain. This excludes other payment arrangements whose values depend on unobserved factors. For instance, in a "stock-for-stock" merger, the deal value depends on unobserved winner's characteristics and thus cannot be reliably compared to a cash-only deal or other bids involving payments in stocks. Third, the winner eventually owns 100 percent of the target, so it is a full-scale merger rather than an equity investment. Finally, we exclude deals with motivations other than a business combination. Condition (v) reflects the focus of this paper. The others are associated with data availability.

Structural variables from SEC filings For each takeover deal in the sample, we collect additional information from the background sections in the SEC filings. In particular, we collect the number N of potential bidders in stage 3 and the number n of actual bidders in the final stage (see Table 1), and complement the data with all reported losing cash-only bids from the SEC documents.[15]

[13]We thank Alexander Gorbenko and Andrey Malenko for sharing their data used in Gorbenko and Malenko (2014) for our early draft. Following their instructions for data collection, we collect the explanatory variables from the commercial databases of Compustat and CRSP by a license at the University of Queensland. Then, we collect additional information from SEC filings. See also (Kim and Zheng, 2025).

[14]For this reason, it is a common practice in corporate finance to separate financial firms (SIC codes 6000-6999) from other firms. For example, see Fama and French (1992).

[15]SDC (Securities Data Company) platinum provides the final deal value (winning bid) for every auction, and SEC filings often document other losing bids. We exclude fourteen anomalous losing cash-only bids

Control variables from Compustat and CRSP We use the set of control variables of Gorbenko and Malenko (2014). They are target-specific variables from Computstat, including `size` (asset book value), `leverage` (debt book value ÷ equity market value), `Q-ratio` ([equity market value + debt book value] ÷ asset book value), `cash flow` (over the last four quarters), `cash` (cash, short-term investments, and marketable securities), `R&D` (expense on R&D), `intangibles` (intangible assets accounting measure), and Fama-French five-industry classification, and economy-wide variables from CRSP, including `market return` (S&P 500 cumulative return over a year before the announcement date) and `credit spread` (Moody's Baa bond rate preceding the announcement date minus the ten-year Treasury bond rate on the announcement date).[16]

Finally, our sample consists of $T = 287$ takeover auctions with a total of 327 observed bids and 193 auctions with winning bids only. Table 2 reports descriptive statistics of our sample. Panel A confirms the common practice of entry restriction with invitation rates ranging from 62 to 74 percent and substantial takeover premiums across the industries. Panel B shows the control variables outlined above. The target firms in the `hightech` and `health` industries appear to be growth companies, as indicated by their high `Q-ratio`, low `leverage`, and large `cash` balance. They also stand out in `R&D` spending and `intangible` assets. The vector x_t for auction $t \in \{1, \ldots, T\}$ includes a constant, four industry indicators (except for `others`), all variables in Panel B in Table 2, and the number N_t of potential bidders.

4.2 Identification

We study the identification of the model primitives, $\{F_v(\cdot|n), \delta(n)\}_{n=1}^{\max\{N\}}$, and the distribution, $F_\varepsilon(\varepsilon_t|x_t)$, of unobserved heterogeneity using the sample of takeover auctions, $\{b^0_{1t}, \ldots, b^0_{m_t t}, x_t, n_t\}_{t=1}^{T}$, where $\{b^0_{it}\}$ are observed bids. The sample has the winning bid, say b^w_t, for all T auctions but may not have some losing bids; so, $m_t \leq n_t$.[17] We may use a generic notation, $\mathcal{P}(A|B)$, for the conditional joint distribution of A given B, for random

from the sample because they are not comparable to other bids for various reasons: (1) bids are subject to conditions, (2) bidders cannot provide sufficient financing proofs, (3) bidders quit after winning the auction, etc. In some of those cases, losing cash bids are above the winning bids.

[16]The five-industry classification is defined by companies' four-digit SIC code, and details can be found in Kenneth French's website, https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/det_5_ind_port.html. We encode `size` in $ million, and normalize `cash flow`, `cash`, `R&D` and `intangibles` by `size`. Standard filters are applied to exclude unreasonable values of these covariates that are likely coding errors. Specifically, we exclude observations with market leverage below zero and above 100 percent, `Q-ratio` in excess of 10, cash flow in excess of 10, and negative cash.

[17]Losing bids are omitted mainly for the following reasons: (a) a bidder was invited but did not submit a final bid; (b) a submitted bid is not comparable to the winning bid, e.g., not a cash-only bid; and (c) a losing bid is simply not reported in the SEC filings.

Table 2: Descriptive Statistics

	All	Consumer	Manufacturing	HighTech	Health	Others
Panel A						
N (potential)	4.67 [3]	5.14 [4]	6.22 [6]	4.29 [3]	3.50 [3]	4.80 [4]
n (actual)	2.70 [2]	2.77 [2]	3.06 [2]	2.63 [2]	2.45 [2]	2.71 [2]
n/N	0.68 (0.29)	0.62 (0.29)	0.62 (0.32)	0.70 (0.28)	0.74 (0.29)	0.70 (0.32)
Premium	1.34 (0.29)	1.27 (0.30)	1.22 (0.21)	1.38 (0.31)	1.37 (0.30)	1.40 (0.28)
Panel B						
Size ($ million)	659.011	719.599	731.599	569.504	435.818	975.482
	[183.717]	[287.475]	[161.577]	[101.391]	[238.712]	[380.549]
	(2345.077)	(1400.810)	(1898.607)	(3103.150)	(528.451)	(2449.334)
Leverage	0.154	0.229	0.190	0.071	0.129	0.274
	[0.045]	[0.128]	[0.086]	[0.002]	[0.016]	[0.206]
	(0.219)	(0.265)	(0.209)	(0.134)	(0.220)	(0.257)
Q-ratio	1.530	1.138	1.283	1.526	2.695	1.167
	[1.257]	[0.973]	[1.265]	[1.325]	[2.009]	[0.913]
	(1.170)	(0.618)	(0.505)	(0.995)	(1.987)	(0.805)
Cash Flow	0.021	0.070	0.111	-0.026	-0.023	0.050
	[0.074]	[0.083]	[0.115]	[0.052]	[0.053]	[0.075]
	(0.259)	(0.101)	(0.081)	(0.350)	(0.238)	(0.207)
Cash	0.247	0.098	0.090	0.375	0.306	0.184
	[0.183]	[0.062]	[0.042]	[0.340]	[0.255]	[0.073]
	(0.234)	(0.091)	(0.131)	(0.223)	(0.253)	(0.236)
R&D	0.016	0.001	0.004	0.030	0.027	0.001
	[0.000]	[0.000]	[0.000]	[0.025]	[0.015]	[0.000]
	(0.032)	(0.005)	(0.007)	(0.040)	(0.036)	(0.005)
Intangibles	0.145	0.094	0.086	0.187	0.170	0.131
	[0.058]	[0.036]	[0.030]	[0.114]	[0.088]	[0.000]
	(0.188)	(0.145)	(0.137)	(0.193)	(0.220)	(0.205)
Credit Spread	0.022	0.021	0.022	0.022	0.024	0.022
	[0.019]	[0.018]	[0.020]	[0.018]	[0.020]	[0.019]
	(0.007)	(0.006)	(0.006)	(0.006)	(0.009)	(0.006)
Market Return	0.062	0.069	0.087	0.051	0.051	0.071
	[0.095]	[0.087]	[0.107]	[0.089]	[0.108]	[0.106]
	(0.131)	(0.134)	(0.113)	(0.134)	(0.154)	(0.113)
#. Observations	287	57	36	113	40	41

This table reports descriptive statistics: mean [median], and (standard deviation) of the sample across five industries as classified by Fama and French (1997). `Consumer` includes consumer durables, nondurables, wholesale, retail, and some services (laundries, repair shops). `Manufacturing` includes manufacturing and energy. `HighTech` includes business equipment, telephone, and television transmission. `Health` includes healthcare, medical equipment, and drugs. The `Others` includes mining, construction, construction materials, transportation, hotels, business services, and entertainment. The sample contains 287 takeover auctions that took place between January 1, 2000 to September 6, 2008.

quantities, A and B, when $\mathcal{P}(A|B)$ is not of direct interest.

We directly identify the reduced-form distributions, $\mathcal{P}(x_t)$, $p(n_t|x_t)$, and $\mathcal{P}(b^0_{it}|x_t, n_t)$ from the sample; among them, section 3.3 discusses $p(n_t|x_t)$. To identify model primitives, we assume that the bids are equilibrium outcomes, i.e., bidders all follow the optimal strategy

(5). Then, we can write the observed bids as

$$b_{it}^0 = \exp(y_t)(1 + b_{it}) = \exp(y_t)(1 + \beta_{N_t}(v_{it}; \{p(n|x_t), \delta(n)\}_{n=1}^{N_t}))$$

for some latent private signal v_{it}, where $b_{it} := \beta_{N_t}(v_{it}; \{p(n|x_t), \delta(n)\}_{n=1}^{N_t})$. Conditional on auction-specific observables (x_t, n_t), the private signal v_{it} and unobserved heterogeneity ε_t are independent under Assumptions 2 and 3, implying conditional independence of y_t and b_{it}. Then, we identify $\mathcal{P}(y_t|x_t, n_t)$ and $\mathcal{P}(b_{it}|x_t, n_t)$ from $\mathcal{P}(b_{it}^0|x_t, n_t)$ and the location of b_{it} given by $0 = \beta_{N_t}(0; \{p(n|x_t), \delta(n)\}_{n=1}^{N_t})$, and using observations with $m_t \geq 2$; see Kotlarski (1966), Li and Vuong (1998), and Krasnokutskaya (2011). Since $\mathcal{P}(y_t|x_t, n_t) = \mathcal{P}(y_t|x_t)$ under Assumptions 2 and 3, we identify $\mathcal{P}(x_t, y_t) = \mathcal{P}(x_t)\mathcal{P}(y_t|x_t)$.[18]

Assumption 5. (i) $y_t = x_t'\gamma + \varepsilon_t$ with $\gamma \in \mathbb{R}^{d_\gamma}$, (ii) $E[x_t\varepsilon_t] = 0$, and (iii) $E[x_t x_t']$ is full-rank.

That is, we consider ε_t as the error in the linear regression of y_t on x_t, x_t is exogenous, and $E[x_t x_t']$ is invertible. Although we do not observe y_t, we have identified $\mathcal{P}(x_t, y_t)$ and, therefore, can pin down $\gamma = E[x_t x_t']^{-1}E[x_t y_t]$ under Assumption 5. Then, we identify $\mathcal{P}(\varepsilon_t|x_t) = \mathcal{P}(y_t - x_t'\gamma|x_t)$. We state this result as a lemma.

Lemma 1. *Under Assumptions 2, 3, and 5, $F_\varepsilon(\varepsilon|x)$ is identified.*

Now, we shall identify $\{F_v(\cdot|n), \delta(n)\}_{n=1}^{\max\{N\}}$ from $\{\mathcal{P}(b|x, n)\}_{n=1}^{\max\{N\}}$. Under Assumption 2, the variation of x does not affect the distribution of v (conditional on n), but it shifts around the bid distribution, $\mathcal{P}(b|x, n)$. We exploit the exogenous variation of x to identify $\{F_v(\cdot|n), \delta(n)\}_{n=1}^{\max\{N\}}$. In particular, we use the variation of the number N of potential bidders in the proof, following the convention of the literature; see Guerre, Perrigne, and Vuong (2009), Gentry and Li (2014), and Aryal, Grundl, Kim, and Zhu (2018) among others. Finally, we introduce a mild condition for the discount factor $\delta(\cdot)$.

Assumption 6. For $n \in \{2, 3\}$, $\delta(n) := 1 - \sum_{q=1}^{d_\zeta}(n-1)^q\zeta_q$ with $d_\zeta \in \mathbb{N}_{++}$.

Assumption 6 governs only $(\delta(2), \delta(3))$, leaving $\delta(n)$ for $n \geq 4$ unspecified. Even for $(\delta(2), \delta(3))$, the restriction is weak as polynomials are flexible with a moderate d_ζ.

Proposition 4. *Under Assumptions 1∼4, and 6, $\{F_v(\cdot|n), \delta(n)\}_{n=1}^{\max\{N\}}$ are identified by $\{\mathcal{P}(b|x, n)\}_{n=1}^{\max\{N\}}$.*

[18] Under the stated assumptions, for each n, we have $p(n|x, \varepsilon, v) = p(n|x, v) \Longrightarrow \int p(n|x, \varepsilon, v)f(v|n)dv = \int p(n|x, v)f(v|n)dv \Longrightarrow p(n|x, \varepsilon) = p(n|x) \Longleftrightarrow n \perp \varepsilon|x$. So, $\mathcal{P}(y|x, n) = \mathcal{P}(y|x)$, as (x, ε) determine y.

4.3 Inference methods

4.3.1 Perception of competition

We consider binomial distribution functions to model bidders' perception of competition,

$$p(n|x,\eta) := \begin{pmatrix} N-1 \\ n-1 \end{pmatrix} \Phi(x'\eta)^{n-1} \left[1-\Phi(x'\eta)\right]^{N-n} \mathbb{1}(n \in \{1,\dots,N\}), \tag{8}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function (CDF) of the standard normal distribution, $\mathcal{N}(0,1)$, and $\eta \in \mathbb{R}^{d_\eta}$ is the vector of unknown coefficients. For an auction with any value of x, then, we predict the probability of n by

$$p(n|x) = \int_\eta p(n|x;\eta)\pi_\eta(\eta|\boldsymbol{n},\boldsymbol{x})d\eta, \tag{9}$$

where $\boldsymbol{n} := \{n_t\}_{t=1}^T$, $\boldsymbol{x} := \{x_t\}_{t=1}^T$, and $\pi_\eta(\eta|\boldsymbol{n},\boldsymbol{x}) \propto \pi_\eta(\eta)\prod_{t=1}^T p(n_t|x_t;\eta)$ is the posterior distribution integrating out η.[19] We use (9) as bidders' common ex-ante beliefs about the level of competition in what follows.

4.3.2 Structural parameters

We postulate that the omitted losing bids are lower than the winning bid because the target board of directors has the fiduciary duty to accept the superior offer, and, therefore, the cash values of the losing bids, even if not recorded, should be lower than the observed winning bid. Given our economic model in section 3, then, we can write the likelihood as

$$\ell(\boldsymbol{b}|\boldsymbol{n},\boldsymbol{x},\boldsymbol{\varepsilon},\boldsymbol{\theta}) = \prod_{t=1}^T \left\{ G(b_t^w|n_t,x_t,\varepsilon_t,\boldsymbol{\theta})^{n_t-m_t} \prod_{i=1}^{m_t} g(b_{it}^0|n_t,x_t,\varepsilon_t,\boldsymbol{\theta}) \right\}, \tag{10}$$

where $\boldsymbol{b} := \left\{b_{1t}^0,\dots,b_{m_t t}^0\right\}_{t=1}^T$, $\boldsymbol{\varepsilon} := \{\varepsilon_t\}_{t=1}^T$, G is the bid CDF, g is the bid density, and $\boldsymbol{\theta}$ collects all the model parameters. We shall elaborate on $\boldsymbol{\theta}$. For given observables, $(\boldsymbol{b},\boldsymbol{n},\boldsymbol{x})$, (10) is a function of the latent components $(\boldsymbol{\varepsilon},\boldsymbol{\theta})$. Let $f_\varepsilon(\varepsilon_t|x_t,\boldsymbol{\theta})$ be the density of ε_t conditional on x_t, $f_\varepsilon(\boldsymbol{\varepsilon}|\boldsymbol{x},\boldsymbol{\theta}) := \prod_{t=1}^T f_\varepsilon(\varepsilon_t|x_t,\boldsymbol{\theta})$, and $\pi_{\boldsymbol{\theta}}(\boldsymbol{\theta})$ be the prior over the parameter space, $\boldsymbol{\Theta}$. Then, the posterior density of $(\boldsymbol{\varepsilon},\boldsymbol{\theta})$ can be written as

$$\pi_{\boldsymbol{\varepsilon},\boldsymbol{\theta}}(\boldsymbol{\varepsilon},\boldsymbol{\theta}|\boldsymbol{b},\boldsymbol{n},\boldsymbol{x}) \propto \pi_{\boldsymbol{\theta}}(\boldsymbol{\theta})f_\varepsilon(\boldsymbol{\varepsilon}|\boldsymbol{x},\boldsymbol{\theta})\ell(\boldsymbol{b}|\boldsymbol{n},\boldsymbol{x},\boldsymbol{\varepsilon},\boldsymbol{\theta}). \tag{11}$$

[19] For sampling η from the posterior, we employ a flat prior, i.e., $\pi_\eta(\eta) \propto 1$, and implement a standard Metropolis-Hastings algorithm. We also consider the Logit specification in $p(n|x,\eta)$ in place of $\Phi(\cdot)$. The results are similar.

Discount rate We specify the discount rate as

$$\delta_j(n, \boldsymbol{\theta}) = \exp(-(n-1)\xi_j), \tag{12}$$

where $j \in \{1, 2, \ldots, 5\}$ indicates the five industries and $(\xi_1, \ldots, \xi_5) \in \mathbb{R}^5_+$ are included in $\boldsymbol{\theta}$. We may suppress j when our discussion about δ does not require δ to be industry-specific. The specification (12) with the exponent function is compatible with Assumptions 1 and 6. In particular, the specification in Assumption 6 can match with (12) for $\delta(2)$ and $\delta(3)$ for any $\xi_j \geq 0$. What (12) restricts beyond Assumption 6 is the pattern of $\delta(n)$ for $n \geq 4$. We take a parametric specification for $\delta(n)$ here because there are few auctions with large n in the data, and the inference on $\delta(n)$ for large ns would be imprecise if unspecified.

Bid distribution We parametrize $F_v(\cdot|n, \boldsymbol{\theta})$ by the scaled beta distribution instead of a nonparametric distribution because our model has many unknown coefficients for a relatively small sample. The beta distribution can still take many different shapes depending on its two shape parameters. We specify the mean and variance of the beta distribution as

$$\mu_v(n, \lambda_\mu) := \frac{\exp\left(\sum_{q=1}^{d_\mu} \lambda_{\mu,q} n^{(q-1)}\right)}{1 + \exp\left(\sum_{q=1}^{d_\mu} \lambda_{\mu,q} n^{(q-1)}\right)} \text{ and } \sigma_v^2(n, \lambda_\sigma) := \exp\left(\sum_{q=1}^{d_\sigma} \lambda_{\sigma,q} n^{(q-1)}\right), \tag{13}$$

where $\lambda_\mu := (\lambda_{\mu,1}, \ldots, \lambda_{\mu,d_\mu})$ and $\lambda_\sigma := (\lambda_{\sigma,1}, \ldots, \lambda_{\sigma,d_\sigma})$ are included in $\boldsymbol{\theta}$. We use the polynomials of n in (13) because economic theory is silent on how n and v are associated. We specify the mean and variance, as they are interpretable, and yet uniquely determine the two scale parameters, say $(\alpha_{1,v}(n, \lambda), \alpha_{2,v}(n, \lambda))$, for all $\lambda := (\lambda_\mu, \lambda_\sigma)$. Then, we let $F_v(\cdot|n, \boldsymbol{\theta})$ be the CDF of the signals, $(v_1, \ldots, v_n)$, that follow $\overline{v} \times \mathcal{B}eta(\alpha_{1,v}(n, \lambda), \alpha_{2,v}(n, \lambda))$.

Now, the bid distribution G is determined by the signal distribution F_v, bidding strategy β_N, and the synergy specification (1). Since β_N is strictly increasing, by change of variables, we obtain the CDF of the bid distribution,

$$G(b_{it}^0|n_t, x_t, \varepsilon_t, \boldsymbol{\theta}) = F_v\left(\beta_{N_t}^{-1}\left(\frac{b_{it}^0}{\exp(x_t'\gamma + \varepsilon_t)} - 1 \middle| \{p(n|x_t), \delta(n; \xi)\}_{n=1}^{N_t}, \boldsymbol{\theta}\right) \middle| n_t, \boldsymbol{\theta}\right). \tag{14}$$

The bid density $g(b_{it}^0|n_t, x_t, \varepsilon_t, \boldsymbol{\theta})$ is then a derivative of (14) with respect to b_{it}^0. Then, we can construct (10). Note that $g(b_{it}^0|n_t, x_t, \varepsilon_t, \boldsymbol{\theta})$, is zero outside the implied support,

$$[\exp(x_t'\gamma + \varepsilon_t), \ \exp(x_t'\gamma + \varepsilon_t)(1 + \beta_{N_t}(\overline{v}|\{p(n|x_t), \delta(n; \xi)\}_{n=1}^{N_t}, \boldsymbol{\theta})], \tag{15}$$

which depends on $(\boldsymbol{\theta}, \varepsilon_t, x_t)$ for each auction $t \in \{1, \ldots, T\}$. When the support depends on the parameters to estimate like (15), the Bayesian estimator is econometrically efficient, whereas the MLE loses its efficiency; see Hirano and Porter (2003). Moreover, the Bayesian method provides a framework that naturally handles the unobserved heterogeneity via data augmentation. For those reasons, we develop a Bayesian method for inference.

However, the parameter-dependent support can make computation difficult for a likelihood-based approach. For example, it can be tricky to select an initial point for $(\boldsymbol{\varepsilon}, \boldsymbol{\theta})$ with a nonzero likelihood, required for starting the computation. Even if the algorithm starts with a nonzero likelihood, the algorithm must consider the restrictions imposed on the space for the latent components $(\boldsymbol{\varepsilon}, \boldsymbol{\theta})$ at each iteration. Otherwise, the algorithm might have to reject many candidate parameters if they give zero likelihood after spending much time computing equilibria. For this reason, it is critical to devise an econometric specification to minimize such computational difficulties arising from the theoretical restrictions.

Auction heterogeneity We mitigate the aforementioned difficulties by choosing a certain parametric family for the distribution of y_t along with a specific prior. In particular, we specify the distribution of y_t as

$$y_t | x_t, \boldsymbol{\theta} \sim \mathcal{N}\left(x_t' \gamma, h_\varepsilon^{-1}\right) \tag{16}$$

where $h_\varepsilon > 0$ is the normal distribution's precision (inverse of the variance).[20] Here, $(\gamma, h_\varepsilon) \subset \boldsymbol{\theta}$. The normal distribution is fully supported over $\mathbb{R}$, but the upper bound of (15) restricts the support of the posterior of $y_t (= x_t' \gamma + \varepsilon_t)$. However, that restriction applies for each auction t separately from other auctions, and y_t is one dimensional, so sampling y_t from the posterior is straightforward. After we draw y_t for all t, we draw the high-dimensional γ without any restrictions.[21] Moreover, when we have the Normal-Gamma prior for (γ, h_ε), the sampling of (γ, h_ε) is computationally efficient due to the conjugacy of the prior.

Prior We employ the Normal-Gamma prior for (γ, h_ε),

$$h_\varepsilon \sim \mathcal{G}a\left(\frac{\alpha_{1,\varepsilon}}{2}, \frac{\alpha_{2,\varepsilon}}{2}\right) \text{ and } \gamma | h_\varepsilon \sim \mathcal{N}\left(\mu_\gamma, (h_\varepsilon H_\gamma)^{-1}\right), \tag{17}$$

[20] The Bayesian literature often characterizes the second moment of the normal distribution by its precision to avoid the complicated presentation of the posterior derivation. In particular, when one uses the variance as usual, the posterior of γ could have the inverse of some function of the inverse of the variance, which does not happen when the precision takes the role instead.

[21] Alternatively, one may draw ε_t under the implied restrictions. Then, the algorithm must also draw γ under the implied restrictions. Because γ appears in every auction and each auction provides additional restrictions on γ, the implications of (15) on the high-dimensional coefficient, γ, would be complicated with many restrictions. The algorithm to explore the posterior would be computationally inefficient.

where the gamma distribution $\mathcal{G}a$ is characterized by its shape and *rate* parameters, and the multivariate normal distribution $\mathcal{N}$ by its mean vector and variance-covariance matrix. Let $y = (y_1, \ldots, y_T)'$, $X = [x_1, \ldots, x_T]'$, and $\widehat{\gamma} = (X'X)^{-1}X'y$. Then, the posterior of (γ, h_ε) is conjugate with the same structure as (17) with the updated parameters,

$$\widehat{\alpha}_{1,\varepsilon} = \alpha_{1,\varepsilon} + T + 1 \text{ and } \widehat{\alpha}_{2,\varepsilon} = \alpha_{2,\varepsilon} + (\gamma - \mu_\gamma)' H_\gamma (\gamma - \mu_\gamma) + (y - X'\gamma)'(y - X'\gamma), \tag{18}$$

$$\widehat{H}_\gamma = H_\gamma + X'X \text{ and } \widehat{\mu}_\gamma = \widehat{H}_\gamma^{-1} \left[H_\gamma \mu_\gamma + (X'X)\widehat{\gamma} \right]. \tag{19}$$

Then, most software packages have built-in random number generators for the normal and gamma distributions. Thus, sampling (γ, h_ε) is computationally efficient. In the following sections, we report the posterior analysis, for which the prior has a minimal influence on the empirical results.[22]

Outline of MCMC To explore the posterior (11) of $(\boldsymbol{\theta}, \boldsymbol{\varepsilon})$, we employ a standard MCMC algorithm (a Gibb sampler) that recursively draws three blocks of the latent components: (γ, h_ε), (ξ, λ), and $\{y_t\}$. For (γ, h_ε), we exploit the normal-gamma (17) conjugacy with (18) and (19). For (ξ, λ), we use a standard Metropolis-Hastings algorithm, for which we tune the proposal function by Haario, Saksman, and Tamminen (2001) during the burn-in phase. Finally, we simultaneously draw y_t for each auction t from a one-dimensional truncated normal, where the truncation arises due to (15).

Inference For any measurable function, $c(\boldsymbol{\varepsilon}, \boldsymbol{\theta})$, of interest, its posterior prediction is

$$E[c(\boldsymbol{\varepsilon}, \boldsymbol{\theta}) | \boldsymbol{b}, \boldsymbol{n}, \boldsymbol{x}] = \int c(\boldsymbol{\varepsilon}, \boldsymbol{\theta}) \pi_{\boldsymbol{\varepsilon}, \boldsymbol{\theta}}(\boldsymbol{\varepsilon}, \boldsymbol{\theta} | \boldsymbol{b}, \boldsymbol{n}, \boldsymbol{x}) d(\boldsymbol{\theta}, \boldsymbol{\varepsilon}). \tag{20}$$

Depending on the definition of $c(\boldsymbol{\varepsilon}, \boldsymbol{\theta})$, (20) can be the Bayesian prediction on an object of interest (e.g., density function and revenue function) or a summary (e.g., posterior standard deviation) of uncertainty around the prediction. When (20) does not have a closed-form expression, we evaluate (20) using the MCMC sample $\{\boldsymbol{\varepsilon}^{(s)}, \boldsymbol{\theta}^{(s)}\}_{s=1}^S$ of size S; Tierney (1994) shows $S^{-1} \sum_{s=1}^S c(\boldsymbol{\varepsilon}^{(s)}, \boldsymbol{\theta}^{(s)}) \xrightarrow{a.s} E[c(\boldsymbol{\varepsilon}, \boldsymbol{\theta}) | \boldsymbol{b}, \boldsymbol{n}, \boldsymbol{x}]$, as S grows.

[22] We use $(\alpha_{1,\varepsilon}, \alpha_{2,\varepsilon}) = (2, 0.2)$, implying the prior mean and variance of h_ε are 10 and 100, respectively, and μ_γ is a vector of zeros and H_ε^{-1} is the identity matrix divided by ten so that the prior variance of each element of γ is 100. Our choice of the prior parameter is diffuse relative to the variation of (γ, h_ε) under the posterior. For the other parameters (ξ, λ), we use the flat prior over the theoretically relevant parameter space, e.g., $\xi \geq 0$ and (ξ, λ) for which $\beta(\cdot | \{p, d(\cdot; \xi)\}, N, \lambda)$ is strictly monotone.

Table 3: Posterior of Coefficients for $p(n|x, \eta)$

	Probit Invitation Rate			Logit Invitation Rate		
	posterior mean	standard deviation	95% credible interval	posterior mean	standard deviation	95% credible interval
Consumer	-1.047	0.350	[-1.742, -0.362]	-1.758	0.573	[-2.884, -0.640]
Manufacture	-0.860	0.350	[-1.567, -0.185]	-1.454	0.580	[-2.578, -0.287]
HighTech	-0.762	0.344	[-1.453, -0.118]	-1.306	0.575	[-2.431, -0.195]
Health	-0.933	0.394	[-1.713, -0.171]	-1.569	0.648	[-2.856, -0.298]
Others	-0.851	0.363	[-1.561, -0.147]	-1.427	0.593	[-2.600, -0.285]
Size	0.159	0.034	[0.092, 0.223]	0.264	0.058	[0.149, 0.380]
Leverage	0.202	0.209	[-0.225, 0.612]	0.291	0.339	[-0.362, 0.951]
Q-ratio	0.139	0.061	[0.022, 0.258]	0.219	0.099	[0.030, 0.414]
Cash Flow	-0.540	0.331	[-1.208, 0.075]	-0.860	0.544	[-2.038, 0.134]
Cash	-0.158	0.263	[-0.671, 0.361]	-0.254	0.438	[-1.119, 0.611]
R&D	1.230	1.634	[-2.015, 4.377]	2.507	2.816	[-2.838, 8.185]
Intangibles	-0.809	0.251	[-1.312, -0.312]	-1.319	0.408	[-2.144, -0.535]
Credit Spread	16.636	9.384	[-1.338, 35.119]	28.723	15.824	[-1.635, 60.068]
Market Return	0.727	0.459	[-0.167, 1.606]	1.220	0.749	[-0.289, 2.640]
N of potential bidders	-0.072	0.015	[-0.101, -0.043]	-0.117	0.024	[-0.165, -0.072]

This table reports the posterior mean, standard deviation, and the (2.5, 97.5) percentiles (i.e., a 95 percent credible interval) for each element in η; see (8) for the specification of $p(n|x, \eta)$. The left (right) panel shows the results with the probit (logit) specification. The sample covers January 1, 2000 to September 6, 2008.

5 Empirical results

This section presents the posterior distribution and conducts posterior predictive analysis for several representative target companies. Then, it performs the counterfactual analysis to measure information and operating costs and examine alternative mechanisms.

5.1 Perception of competition

The left panel of Table 3 shows the posterior mean, standard deviation, and the (2.5, 97.5) percentiles (i.e., a 95 percent credible interval) for each element in η. Among the explanatory variables, `size` and `Q-ratio` are positively associated with the number of actual bidders, but `intangible` is negatively associated. In particular, `intangible` assets represent an accounting measure of confidential items, such as patents and trade secrets; thus, a higher value of intangible assets suggests the importance of confidential information.[23] Target companies with more valuable confidential information tend to invite fewer bidders.

[23]For example, it is well known that Coca-Cola has its own formula for its products, which is confidential. Its market value is recorded in intangible assets.

When we consider the posterior mean as an 'estimator' in the frequentist sense, the Bernstein and von Mises theorem argues that the Bayesian estimator is asymptotically equivalent to the maximum likelihood estimator (MLE).[24] Hence, for example, one may regard the posterior mean as the MLE and the 95 percent credible interval as an asymptotic 95 percent confidence interval. Note that for the aforementioned variables, `size`, `Q-ratio`, and `intangibles`, their 95 percent credible intervals do not include zero. Therefore, one may consider the estimates statistically significant at the five percent level in the frequentist sense.

In this reduced-form analysis, we are interested in the posterior predictive mass function (9) for each auction $t \in \{1, \ldots, T\}$. Figure 1(a) shows that (9) predicts the observed n_t by about 60 percent of the time for the auctions with $N_t = 2$ in our data. For this exercise, we compute the probability that (9) picks the observed number n_t of actual bidders for an auction with (n_t, x_t) and then we integrate x_t out for its empirical distribution. Similarly, the other panels in Figure 1 show the distribution of the prediction error, $n - n_t$, for larger Ns, where the prediction appears fairly unbiased. We use $p(n|x_t)$ for all $t \in \{1, \ldots, T\}$ and allow bidders to update it to $p(n|x_t, v_{it})$ for computing the optimal bids (5).

As a robustness check, we consider the Logit invitation rate in $p(n|x, \delta)$ in place of the Probit invitation rate. Table 3 also shows, in its right panel, the posterior of the coefficients, where the signs and significance of the Bayesian estimates are similar to the Probit case. Furthermore, the predictive error probabilities of the Logit case are essentially identical to the Probit case in Figure 1, and, thereby, we avoid repeating the same figure here. We consider the predictive distribution (9) with the Probit invitation rate as the bidders' perception of competition before learning their private signals for our empirical analysis.

5.2 Model Primitives

5.2.1 Posterior Distribution

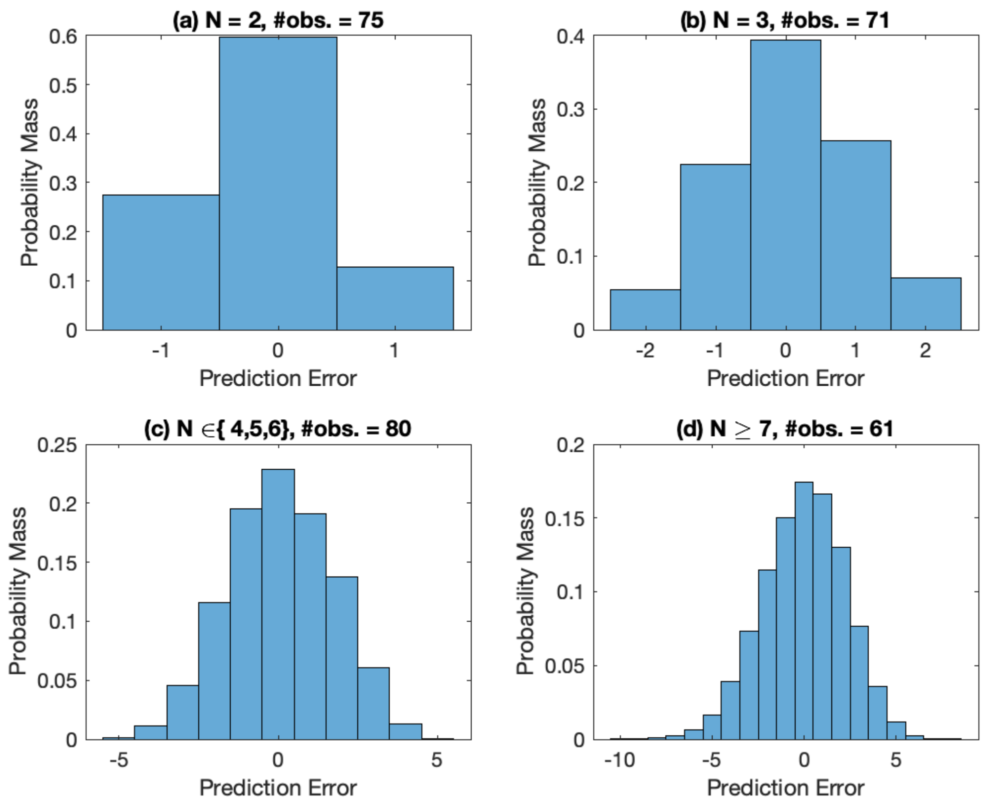
Table 4 summarizes the marginal posterior distribution for each element in γ, which appears in $y_t = x_t'\gamma + \varepsilon_t$, by the posterior mean, standard deviation, and several percentiles. The percentiles offer the median and alternative credible intervals, e.g., the 2.5^{th} and 97.5^{th} percentiles form a 95 percent posterior credible interval. Our empirical method runs for every $\overline{v}$ in a discretized interval between 1.5 and 4 with the grid size of 0.1, and we report the empirical results for $\overline{v} = 2.5$ that minimizes the Bayesian information criterion.[25]

The negative effect of `size` has a clear prediction, i.e., the posterior puts most probability mass on the negative values. A large size can be negatively associated with bidders' value

[24] See Chapter 10 of van der Vaart (1998).

[25] In principle, we could include $\overline{v}$ in $\boldsymbol{\theta}$ and compute its posterior along with other latent components. However, we find that this significantly increases computing time, making it impractical in practice.

Figure 1: Posterior Predictive Analysis on the Number of Actual Bidders



Panel (a) shows the probability mass function that is proportional to $\sum_{t:N_t=2} p(n-n_t|x_t)$, where (n_t, N_t, x_t) are observed in the sample. Panels (b), (c), and (d) similarly for different N_ts as indicated.

(per unit size) if the technology exhibits decreasing returns to scale or if bidders' economic cost of financing for the cash bid gets higher with the company's size. From the negative sign on `Q-ratio`, we can infer that the companies on sale have a large debt ratio, which should adversely be related to the bidders' evaluation. Moreover, the negative coefficient of `market return` suggests that when the market return is high, bidders would have better business opportunities, raising the opportunity costs of acquiring the target. Finally, the positive coefficients of `R&D` and `intangibles` suggest that technological enhancement would generate synergy values.

Table 5 offers the posterior distribution for (λ, ξ) in the same format as Table 4. The negative posterior mean of $\lambda_{\mu,2}$ suggests that the takeover auctions with smaller synergy values invite more bidders to the final stage, and the negative $\lambda_{\sigma,2}$ also suggests that those takeover deals tend to have a smaller variance; see (13) for the specification of $F_v(\cdot|n, \boldsymbol{\theta})$.

Table 4: Posterior Distribution of γ; $y_t = x_t'\gamma + \varepsilon_t$

	Posterior Mean	Standard Deviation	Posterior Percentiles 2.5%	5%	50%	95%	97.5%
Consumer	0.337	0.067	0.207	0.227	0.337	0.448	0.469
Manufacture	0.427	0.069	0.293	0.315	0.427	0.540	0.563
HighTech	0.304	0.069	0.170	0.191	0.304	0.417	0.438
Health	0.495	0.074	0.350	0.374	0.495	0.617	0.640
Others	0.316	0.078	0.163	0.188	0.317	0.446	0.470
Size	-0.031	0.009	-0.048	-0.045	-0.031	-0.017	-0.014
Leverage	0.066	0.060	-0.052	-0.033	0.066	0.164	0.183
Q-ratio	-0.047	0.011	-0.069	-0.065	-0.047	-0.028	-0.024
Cash Flow	-0.005	0.049	-0.102	-0.086	-0.005	0.076	0.091
Cash	0.034	0.067	-0.099	-0.078	0.034	0.144	0.165
R&D	0.861	0.342	0.189	0.298	0.862	1.422	1.526
Intangibles	0.027	0.068	-0.107	-0.085	0.027	0.140	0.161
Credit Spread	0.053	0.534	-0.991	-0.827	0.051	0.929	1.100
Market Return	-0.287	0.092	-0.468	-0.439	-0.287	-0.135	-0.105
N of potential bidders	-0.019	0.005	-0.029	-0.027	-0.019	-0.010	-0.008
$\sigma_\varepsilon = \sqrt{V(\varepsilon)}$	0.173	0.008	0.157	0.159	0.172	0.187	0.190

Table 4 summarizes the marginal posterior distribution for each element in γ by the posterior mean, standard deviation, and a few percentiles. The percentiles offer the median and alternative credible intervals, i.e., the 2.5^{th} and 97.5^{th} (5^{th} and 95^{th}) percentiles form a 95 (90) percent credible interval.

Figure 2 visualizes the implications of λ by plotting the predictive signal densities (solid) across different numbers of actual bidders. The figure also plots the point-wise 2.5^{th} and 97.5^{th} percentiles (dashed), which form a 95 percent credible band. We report the predictive mean, variance, and skewness on each diagram.

We find that auctions with one bidder have values seven percent higher than those with two bidders ($1.789/1.657 = 1.074$) and two-bidder auctions five percent higher than three-bidder auctions. This pattern suggests that the sellers invite fewer bidders when expecting bidders would have higher synergy values.

The lower block of Table 5 presents the posterior distribution for each element of ξ in the discount factor (12). Table 6 provides the economic interpretation of the predictive ξ on the idiosyncratic elements v_{it} of the synergy value. For a takeover auction in the Consumer industry, for example, each bidder lowers her idiosyncratic element by 26 percent if there is one rival bidder, 45 percent if two, and so on. We find that the information discount is particularly pronounced in the manufacturing and health industries.

Table 5: Posterior Distribution of λ and ξ in $f(\cdot|n,\lambda)$ and $\delta(n|\xi)$

	Posterior Mean	Standard Deviation	Posterior Percentiles 2.5%	5%	50%	95%	97.5%
Value density, $f(\cdot\|n)$							
$\lambda_{\mu,1}$	-0.500	0.294	-1.104	-1.005	-0.493	-0.042	0.005
$\lambda_{\mu,2}$	-0.311	0.134	-0.536	-0.522	-0.309	-0.103	-0.063
$\lambda_{\mu,3}$	0.024	0.013	0.000	0.003	0.024	0.045	0.047
$\lambda_{\sigma,1}$	-3.607	0.374	-4.246	-4.202	-3.616	-2.961	-2.848
$\lambda_{\sigma,2}$	-0.092	0.158	-0.406	-0.361	-0.080	0.150	0.182
$\lambda_{\sigma,3}$	0.006	0.013	-0.017	-0.015	0.005	0.028	0.031
Discount rate, $\delta(n\|\xi_j)$							
Consumer, ξ_1	0.303	0.064	0.179	0.195	0.303	0.402	0.418
Manufacture, ξ_2	0.882	0.119	0.647	0.676	0.915	1.053	1.074
HighTech, ξ_3	0.117	0.043	0.044	0.053	0.113	0.194	0.210
Health, ξ_4	0.725	0.136	0.446	0.490	0.745	0.961	0.982
Others, ξ_5	0.036	0.024	0.003	0.005	0.031	0.082	0.092

Table 5 summarizes the marginal posterior distribution for each element in (λ, ξ) by the posterior mean, standard deviation, and a few percentiles. The percentiles offer the median and alternative credible intervals, i.e., the 2.5$^{\text{th}}$ and 97.5$^{\text{th}}$ (5$^{\text{th}}$ and 95$^{\text{th}}$) percentiles form a 95 (90) percent credible interval.

5.2.2 Predictive Analysis

We can conduct the predictive analysis for every auction t in the sample and any out-of-sample auction if we observe its attributes x. For illustration, we select five representative auctions in the data, one from each industry, whose observed characteristics are closest to their industry median values. We call those five auctions the industry-median target companies. For those companies, we first obtain the posterior predictive densities of the transaction premium, which is the highest-order statistic of the predictive bid density,

$$\int \prod_{i=1}^{n_t} g(b_i^0|n_t, x_t, \varepsilon_t, \boldsymbol{\theta})\pi_{\boldsymbol{\varepsilon},\boldsymbol{\theta}}(\boldsymbol{\varepsilon}, \boldsymbol{\theta}|\boldsymbol{b}, \boldsymbol{n}, \boldsymbol{x})d(\boldsymbol{\varepsilon}, \boldsymbol{\theta}). \tag{21}$$

Note that (21) is an example of (20) with $c(\boldsymbol{\varepsilon}, \boldsymbol{\theta}) = \prod_{i=1}^{n_t} g(b_i^0|n_t, x_t, \varepsilon_t, \boldsymbol{\theta})$. Figure 3 shows the predictive premium density for each industry in the top panels, indicating the observed premiums in the data by the vertical dashed lines. The densities all predict the observed premiums well. Table 7 shows that, for example, for the consumer industry, the predictive premium (1.388) approximates the observed (1.326) with a small standard deviation.

The posterior distribution quantifies the uncertainty for every element in the model, allowing us to decompose the uncertainty about the premium into the three elements of the observed bid, i.e., $b_{it}^0 = \exp(x_t'\gamma) \times \exp(\varepsilon_t) \times (1 + b_{it})$. The middle panel of Figure 3 shows the predictive densities of $\exp(\varepsilon_t)$ by a solid line, $\exp(x_t'\gamma)$ by a dashed line, and b_{it}

Figure 2: Posterior Predictive Analysis for $f_v(\cdot|n)$

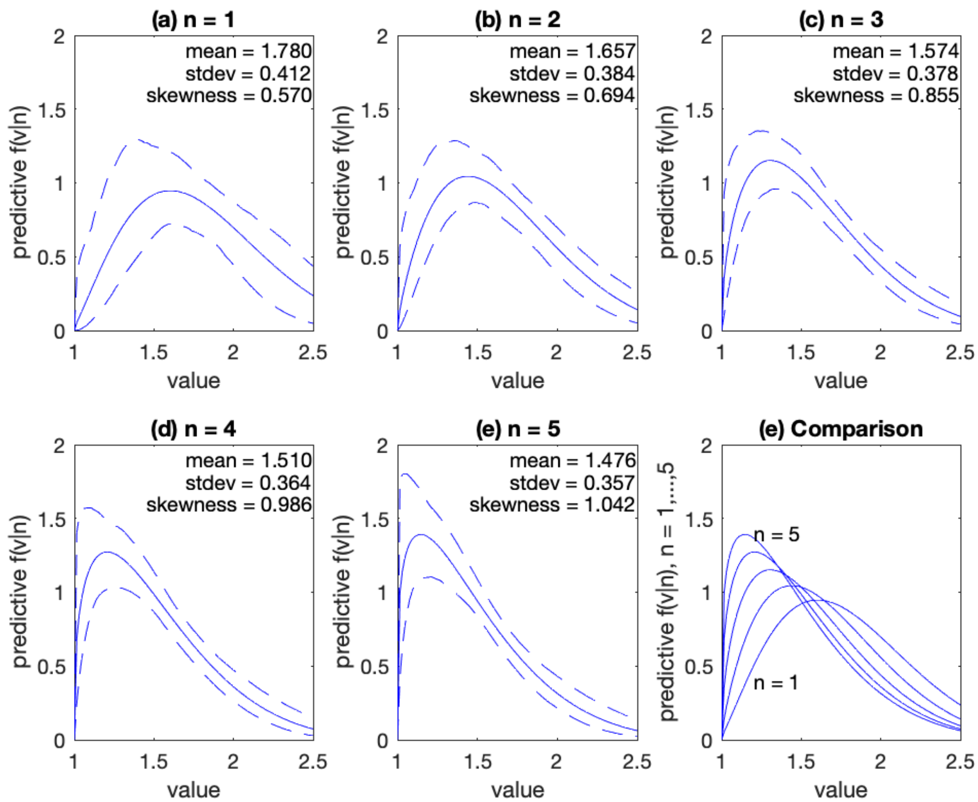


Figure 1 visualizes the implications of λ by plotting the predictive densities (solid) for v_{it} across different numbers of actual bidders, the point-wise 2.5^{th} and 97.5^{th} percentiles (dashed), i.e., a 95 percent credible band, and reporting the predictive mean, variance, and skewness.

by a dash-dotted line, where the densities are centered on their respective means. For these median companies, the predictive densities of latent components, $\exp(\varepsilon_t)$, are more diffuse than the explained components, $\exp(x_t'\gamma)$. To be specific, the small table below Figure 3 shows that the variation of $\exp(\varepsilon_t)$ takes up 31.9 percent out of the total variation of the premium, whereas the uncertainty associated with $\exp(x_t'\gamma)$ is 4.2 percent for the median auction in the consumer industry. This finding suggests that the unobserved factors play a critical role in the bidders' valuation process.

Table 6: Posterior Distribution of $\delta(n;\xi_j)$ in %

	Posterior Mean	Standard Deviation	2.5%	5%	50%	95%	97.5%
			Posterior Percentiles				
Consumer							
$n=2$	0.740	0.047	0.659	0.669	0.739	0.822	0.836
3	0.550	0.071	0.434	0.447	0.546	0.676	0.699
4	0.411	0.080	0.286	0.299	0.403	0.556	0.585
Manufacture							
$n=2$	0.417	0.051	0.342	0.349	0.401	0.508	0.524
3	0.176	0.044	0.117	0.122	0.161	0.258	0.274
4	0.076	0.029	0.040	0.043	0.064	0.131	0.144
HighTech							
$n=2$	0.890	0.038	0.810	0.824	0.893	0.949	0.957
3	0.794	0.067	0.657	0.678	0.798	0.900	0.916
4	0.709	0.089	0.532	0.559	0.713	0.854	0.877
Health							
$n=2$	0.489	0.068	0.375	0.382	0.475	0.613	0.641
3	0.243	0.069	0.140	0.146	0.225	0.375	0.410
4	0.124	0.054	0.053	0.056	0.107	0.230	0.263
Others							
$n=2$	0.965	0.023	0.913	0.921	0.969	0.995	0.997
3	0.932	0.043	0.833	0.848	0.939	0.991	0.995
4	0.901	0.062	0.760	0.781	0.910	0.986	0.992

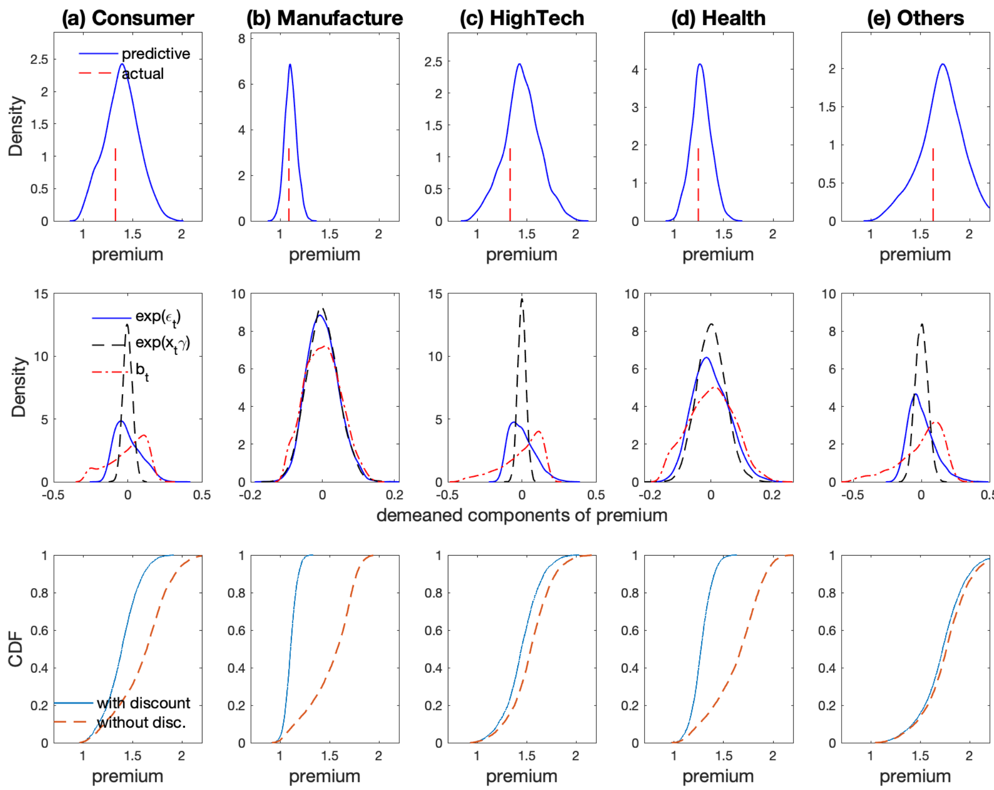
Table 6 shows the posterior predictive measure of the discount rate, $\delta(n)$, for $n \in \{2,3,4\}$ for each industry, along with the posterior standard deviation and the percentiles.

5.3 Counterfactual analysis

For the five industry-median companies, we consider hypothetical values of structural parameters. We first consider the case without value discount to measure information costs and then the case where the seller considers a different number of bidders for the *given* signal distribution to learn operating costs. We then study the premium implications of alternative mechanisms.

Information cost We measure the information costs by the difference between the predictive premium with value discount and the predictive premium without discount, i.e., by restricting $\delta(n)=1$ for all n. Table 7 summarizes the predictive premiums by the posterior mean, standard deviation, and several percentiles, showing (n_t, N_t, $\texttt{size}_t$), for the median companies. For the median company in the consumer industry, the predictive premium without discount is 1.607, which is 16 percent higher than the predictive premium of 1.388. By multiplying the premium difference by the book value, we obtain a loss of 40 million dollars. We consider this premium loss as the information cost because it arises from information disclosure. We conduct the same exercise for the other median companies. For example,

Figure 3: Predictive Analysis for Industry-Median Target Companies



	Relative Variation of Each Component (fraction)				
	Consumer	Manufacture	HighTech	Health	Others
$\exp(\varepsilon_t)$	0.319	0.303	0.271	0.317	0.251
$\exp(x_t\gamma)$	0.042	0.306	0.028	0.197	0.057
b_w	0.640	0.391	0.701	0.486	0.692

The top panel shows the posterior predictive densities of the premium (winning bid) for the industry-median target companies, indicating the observed premiums by the vertical dashed lines. The middle panel shows the posterior predictive densities of three elements, $\exp(\varepsilon_t)$, $\exp(x_t'\gamma)$, and b_{it} for the selected auctions. The densities are centered around the mean. The table below the diagrams presents the proportion as a percentage of the variation for each element. The bottom panel plots the CDF of the premium with the discount factor by a solid line and the CDF of the premium without the discount factor, i.e., $\delta(n) = 1$, by a dashed line.

we find 11 and 229 million dollars of information costs for the median companies in the manufacturing and health industries, respectively. Finally, the lower panels of Figure 3 show that the premiums with value discount are dominated by the premiums without discount for all the median companies, and the differences are economically substantial (except for the other industries) but with a large degree of heterogeneity.

Table 7: Predictive Premiums with and without Value-Discount for Median Targets

Industry	Observed	Predictive	Standard	Posterior Percentiles				
(n_t, N_t, $\texttt{size}_t$)	Premium	Premium	Deviation	2.5	5	50	95	97.5
Consumer	1.326	1.388	0.172	1.050	1.097	1.391	1.676	1.732
(1, 4, \$183.97M)	If no discount,	1.607	0.264	1.065	1.125	1.644	2.010	2.075
Manufacture	1.087	1.106	0.061	0.990	1.010	1.103	1.211	1.229
(1, 6, \$27.21M)	If no discount,	1.520	0.232	1.020	1.062	1.577	1.813	1.850
HighTech	1.331	1.447	0.181	1.078	1.133	1.444	1.755	1.815
(2, 3, \$222.11M)	If no discount,	1.536	0.207	1.094	1.167	1.540	1.874	1.947
Health	1.245	1.275	0.101	1.077	1.107	1.272	1.447	1.478
(1, 3, \$639.59M)	If no discount,	1.633	0.248	1.112	1.164	1.676	1.983	2.027
Others	1.629	1.720	0.225	1.239	1.315	1.726	2.083	2.156
(2, 3, \$380.55M)	If no discount,	1.755	0.235	1.248	1.326	1.764	2.134	2.214

Information Disclosure Discount

	Consumer	Manufacture	HighTech	Health	Others
IDD (\$M)	40.32	11.27	19.66	229.19	13.39

The first column shows the number of actual bidders, the number of potential bidders, and the book value for the industry-median target companies. The second column shows the observed premium and the third shows the predictive premiums with the discount in the first row and without in the second row. The fourth column presents the standard deviations, and the others show several percentiles of predictive premiums. The small tabular below the main table documents the dollar value of information costs.

Operating cost The operating costs include not only the accounting fees for retaining legal and financial advisors, but also other opportunity costs for running an auction associated with planning, carrying out, and concluding a takeover process. For example, if the target company's staff members assisted bidders to conduct due diligence, the operating costs arise because the staff members could have engaged in revenue-generating business activities.

To obtain a lower bound of the operating cost, we assume that the seller could have invited one more bidder with a signal following the *same* distribution, $F_v(\cdot|n_t)$. Then, the predictive premium with $\tilde{n} = n_t + 1$ bidders (Table 8, column (C)) would exceed the premium with n_t bidders (Table 8, column (B)). However, the fact that the seller invites n_t actual bidders suggests that inviting one more bidder costs more than the premium increase. Therefore, the operating costs must be larger than the premium differences, i.e., (C)-(B). The lower bounds vary between 2.5 percent to 7.3 percent of the predictive premiums across the industry median companies, which are considerably larger than the average accounting cost of 0.84 percent of the deal values documented in Hunter and Jagtiani (2003).

Moreover, for auctions with $n_t \geq 2$, we can obtain the upper bound. The seller could

Table 8: Operating Cost

	Predictive Premium (StDev)			operating Cost, %		operating Cost, $M	
	$\tilde{n} = n_t - 1$	n_t	$\tilde{n} = n_t + 1$	LB%	UB%	LB	UB
	(A)	(B)	(C)	(C)-(B)	(B)-(A)	(LB%, UB%)×$\texttt{size}_t$	
Consumer		1.388 (0.172)	1.461 (0.145)	0.073		13.38	
Manufacture		1.106 (0.061)	1.131 (0.054)	0.025		0.68	
HighTech	1.344 (0.211)	1.447 (0.181)	1.493 (0.162)	0.046	0.103	10.26	22.91
Health		1.275 (0.101)	1.318 (0.089)	0.043		27.44	
Others	1.585 (0.272)	1.720 (0.225)	1.781 (0.197)	0.062	0.134	23.42	51.17

Columns (A), (B), and (C) show the predictive premiums for the cases with $n_t - 1$ if $n_t > 1$ and the observed n_t and $n_t + 1$ actual bidders. The next columns take the difference between the premiums to compute the lower and upper bounds to compute the operating costs in terms of the percentage points and the dollar values. Note that the median target auctions in the high-tech and the other industries have $n_t = 2$ actual bidders and the other industry-median auctions have $n_t = 1$ actual bidder.

have invited $\tilde{n} = n_t - 1$, suggesting that inviting the last bidder increases premiums more than the operating cost for that bidder, i.e., the premium increase is an upper bound of the operating cost (Table 8, column (A)). Among those median companies, we obtain the upper bound for the ones in the high-tech and other industries, as they have $n_t \geq 2$, and we find that the upper bounds are roughly twice larger than the lower bounds.

Ascending (English) Auction The median company in the consumer industry has $n_t = 1$ bidder, for which the ascending auction would result in a 30.2 percentage-point (55.5 million dollars) premium decrease relative to the current sealed-bid auction; see Table 9. For the given number, $n_t = 1$, of bidders, the sole bidder would behave as if her signal is zero ($v_{it} = 0$) in the ascending auction. To make the comparison more meaningful, we assume that in the ascending auction, the seller can invite one more bidder who draws her signal from the same distribution, $F_v(\cdot|n_t)$. In that case, the ascending auction would result in a 15.1 percentage-point (27.9 million dollars) premium increase relative to the sealed-bid auction with $n_t = 1$ bidders. The premium increase exceeds the lower bound ($13.4M) of the operating cost in Table 8, but since the upper bound is unknown, the comparison cannot be decisive. The other median companies with $n_t = 1$ bidder exhibit similar patterns.

For the median companies in the high-tech and other industries with $n_t = 2$, the ascending auction with the same n_t bidders would result in 5.56 and 35.80 million dollars of price decreases, respectively; thus, the seller should prefer the current mechanism. Moreover, even if the seller can invite one more bidder to the ascending auction, the premium increments would not exceed the upper bound of the operating costs; see Table 8.

Table 9: First Price Auction vs Ascending Auction

	First-Price Auction		Ascending Auction		Ascending vs FPA		Premium
	Predictive Premium	Standard Deviation	Predictive Premium	Standard Deviation	Premium Difference	Standard Error	Difference $\times$Size$_t$, \$M
Consumer							
$n_t = 1$	1.388	0.172	1.086	0.086	-0.302	0.001	-55.51
$n_t + 1$			1.540	0.271	0.151	0.004	27.85
Manufacture							
$n_t = 1$	1.106	0.061	1.005	0.039	-0.101	0.001	-2.75
$n_t + 1$			1.238	0.130	0.132	0.002	3.60
HighTech							
$n_t = 2$	1.447	0.181	1.422	0.263	-0.025	0.004	-5.56
$n_t + 1$			1.522	0.248	0.075	0.004	16.69
Health							
$n_t = 1$	1.275	0.101	1.106	0.062	-0.168	0.001	-107.76
$n_t + 1$			1.403	0.175	0.128	0.002	81.92
Others							
$n_t = 2$	1.720	0.225	1.625	0.319	-0.094	0.005	-35.80
$n_t + 1$			1.806	0.321	0.086	0.005	32.75

This table presents the predictive premium and standard deviation under the first-price auction and ascending auction for the industry-median target companies, but replacing the value densities to the ones associated with $n \in \{1, 2, 3\}$ bidders, where (†) indicates the actual ns. The table represents the premium difference in percentage points and dollar values, which are all significant at the one percent level.

Negotiation We derive the negotiation premium upper bounds and compare them with the current premiums, instead of predicting precise negotiation outcomes under additional assumptions on bargaining behavior. We assume, just like in the seal-bid auction, that the seller identifies $F_v(\cdot|n_t)$ and bidders who would draw signals from it after stage 3. We assume that the seller initiates bargaining with one of those bidders, the seller contacts the next bidder if the deal does not go through, and each i^{th} bidder knows that $(i-1)$ bidders have already declined the deal and her signal follows $F_v^{(i)}(\cdot)$. Note that $F_v^{(1)}(\cdot) = F_v(\cdot|n_t)$ and $\mu_{v,(1)} = E[v|n_t]$. We assume that the mean $\mu_{v,(i)} = \int_0^{\bar{v}} v F_v^{(i)}(v)dv$ weakly decreases in i, which all bidders know.

The first bidder would decline any premium higher than what the seller expects from the second bidder minus the additional operating cost. Consider the case with $N_t = 2$. If the first bidder declines the deal, the seller's expected payoff from the second bidder would be $\delta(2)\mu_{v,(2)} - oc_t$ if the second bidder has no bargaining power, as relevant to the premium *upper* bound.[26] If the first bidder knew $(\mu_{v,(2)}, oc_t)$, she would not accept any offer above $\delta(2)\mu_{v,(2)} - oc_t$. But, she does not know oc_t and might not $\mu_{v,(2)}$, either. However,

[26] The seller's expected payoff from the second bidder with no bargaining power would be $\delta(2)\mu_{v,(2)} - 2oc_t$, but we subtract out the operating cost for the first bidder for all comparisons here, as it must incur.

$\delta(2)E[v|n_t] - \underline{oc}_t \geq \delta(2)\mu_{v,(2)} - oc_t$, where $\underline{oc}_t$ is the lower bound of the operating cost, and the first bidder knows $E[v|n_t]$ and can infer $\underline{oc}_t$ as this paper does. Thus, she would not accept any offer above $\delta(2)E[v|n_t] - \underline{oc}_t$, which is a premium (upper) bound.

If $N_t = 3$, the *second* bidder, when contacted, would also decline any premium higher than the expected premium with the third bidder minus additional operating costs – the second bidder's upper bound is $\delta(3)\mu_{v,(2)} - \underline{oc}_t \geq \delta(3)\mu_{v,(3)} - oc_t$. Knowing this, the first bidder would not pay more than $\delta(3)E[v|n_t] - 2\underline{oc}_t$. Applying the reasoning recursively for a general N_t, the first bidder would decline any offer above $\delta(N_t)E[v|n_t] - (N_t - 1)\underline{oc}_t$, the negotiation premium bound.

We find that for the median targets with $n_t = 1$ in the consumer, manufacturing, and health industries, the negotiation premium upper bounds are 1.214, 0.890, and 1.230, respectively, all below the current premiums. For the median targets with $n_t = 2$ in the high-tech and other industries, the negotiation premium upper bounds are 1.452 and 1.724, respectively, higher than their current premiums minus operating costs, $1.401 (= 1.447 - 0.046)$ and $1.658 (= 1.720 - 0.062)$. However, since the lower bounds are unknown, we do not know if the negotiation would generate higher premiums than the current mechanism.

The negotiation premium bound, $\delta(N_t)E[v|n_t] - (N_t - 1)\underline{oc}_t$, remains unchanged if $E[v|n_t] \geq \mu_{v,(i)}$ for all i, relaxing the monotonicity of $\mu_{v,(i)}$ in i. If the lower bound of operating cost is not proportional to the number of bidders, we can use a more conservative premium upper bound, $\delta(N_t)E[v|n_t] - \underline{oc}_t$. We find that the latter does not qualitatively change our premium comparisons, i.e., for the ones with $n_t = 1$, the current mechanism generates higher premiums than the negotiation, but the comparison is undecisive for those with $n_t = 2$. Finally, if bidders do not know $F_v(\cdot|n_t)$, the expression for the negotiation premium bounds has $\sum_{n=1}^{N_t} p(n|x_t, v_{(1)t})E[v|n]$ in place of $E[v|n_t]$, where $v_{(1)t}$ denotes the first bidder's signal. The perceived beliefs lower the premium bounds but do not alter our results qualitatively.

6 Concluding Remarks

This study sheds light on the substantial economic impact of information and operating costs borne by sellers in takeover auctions, providing insight into the widespread practice of restricting bidder participation. Our findings carry several policy implications for regulatory and judicial authorities. The considerable information costs underscore a potential social welfare loss due to informational externalities and the need for reinforced regulations governing the use of information obtained during the takeover process.

Furthermore, our quantitative approach can be used for addressing takeover-related lawsuits. The seller's decision to limit potential bidders can be contested in court by shareholders who argue that this practice stifles competition and harms their interests. However, it is essential to note that accounting costs alone cannot justify this decision. Our methodology quantifies the information costs and offers an estimated range for the operating cost, which can serve as a benchmark for court judgments, helping assess whether the implied operating costs fall within reasonable bounds.

The substantial economic burden on the seller also explains why the seller might enter an exclusivity agreement with a single bidder. Such an agreement signals to the chosen bidder that confidential information will not be exposed to other competitors, thereby preventing the bidder from undervaluing the target. Finally, although our analysis focuses on successful takeovers, the impact of information costs on failed transactions is straightforward: when a deal fails to materialize, the market value of the target company is expected to decline, as the eventual owner, the target firm, is left to bear the information costs.

A Proofs

A.1 Proposition 1

We use $\beta(v) = \beta_N(v; x, \delta)$. The second derivative of $U(\beta(u), v; x)$ with respect to u is

$$\frac{\partial^2}{\partial u^2} U(\beta(u), v; x) = \frac{\partial}{\partial y} \sum_{n=1}^{N} p(n|x, v) \left\{ f_{v,n}(u) \left[\delta(n)v - \beta(u) \right] - F_{v,n}(u)\beta'(u) \right\}$$

$$= \sum_{n=1}^{N} p(n|x, v) \left\{ f'_{v,n}(u) \left[\delta(n)v - \beta(u) \right] - 2f_{v,n}(u)\beta'(u) - F_{v,n}(u)\beta''(u) \right\}, \tag{22}$$

where the first equality holds due to (4). Moreover, (4) is zero for $u = v$, giving

$$\beta'(v) = \frac{\sum_{n=1}^{N} p(n|x, v) f_{v,n}(v) \left[\delta(n)v - \beta(v) \right]}{\sum_{n=1}^{N} p(n|x, v) F_{v,n}(v)}.$$

Then, the second derivative of the bidding strategy is

$$\begin{aligned}
\beta''(v) = & \frac{1}{\left[\sum_{n=1}^{N} p(n|x, v) F_{v,n}(v) \right]^2} \left(\sum_{n=1}^{N} \{ p_v(n|v) f_{v,n}(v) \left[\delta(n)v - \beta(v) \right] \right. \\
& + p(n|x, v) f'_{v,n}(v) \left[\delta(n)v - \beta(v) \right] - p(n|x, v) f_{v,n}(v)\beta'(v) \} \sum_{n=1}^{N} p(n|x, v) F_{v,n}(v) \\
& \left. - \sum_{n=1}^{N} p(n|x, v) f_{v,n}(v) \left[\delta(n)v - \beta(v) \right] \sum_{n=1}^{N} \left[p_v(n|v) F_{v,n}(v) + p(n|x, v) f_{v,n}(v) \right] \right).
\end{aligned} \tag{23}$$

Evaluate (22) at $u = v$ and plug (23) in (22), we obtain

$$\begin{aligned}
& \sum_{n=1}^{N} p(n|x, v) f'_{v,n}(v) \left[\delta(n)v - \beta(v) \right] - 2 \sum_{n=1}^{N} p(n|x, v) f_{v,n}(v)\beta'(v) \\
& - \sum_{n=1}^{N} p(n|x, v) F_{v,n}(v) \times \frac{1}{\left[\sum_{n=1}^{N} p(n|x, v) F_{v,n}(v) \right]^2} \left(\sum_{n=1}^{N} \{ p_v(n|v) f_{v,n}(v) \left[\delta(n)v - \beta(v) \right] \right. \\
& + p(n|x, v) f'_{v,n}(v) \left[\delta(n)v - \beta(v) \right] - p(n|x, v) f_{v,n}(v)\beta'(v) \} \sum_{n=1}^{N} p(n|x, v) F_{v,n}(v) \\
& \left. - \sum_{n=1}^{N} p(n|x, v) f_{v,n}(v) \left[\delta(n)v - \beta(v) \right] \sum_{n=1}^{N} \left[p_v(n|v) F_{v,n}(v) + p(n|x, v) f_{v,n}(v) \right] \right)
\end{aligned}$$

which is equivalent to

$$
\begin{aligned}
&\sum_{n=1}^{N} p(n|x,v) f'_{v,n}(v)\left[\delta(n)v-\beta(v)\right] - 2\sum_{n=1}^{N} p(n|x,v) f_{v,n}(v)\beta'(v) \\
&- \sum_{n=1}^{N} p_v(n|v) f_{v,n}(v)\left[\delta(n)v-\beta(v)\right] - \sum_{n=1}^{N} p(n|x,v) f'_{v,n}(v)\left[\delta(n)v-\beta(v)\right] + \sum_{n=1}^{N} p(n|x,v) f_{v,n}(v)\beta'(v) \\
&+ \underbrace{\frac{\sum_{n=1}^{N} p(n|x,v) f_{v,n}(v)\left[\delta(n)v-\beta(v)\right]}{\sum_{n=1}^{N} p(n|x,v) F_{v,n}(v)}}_{=\beta'(v)} \times \sum_{n=1}^{N}\left[p_v(n|v)F_{v,n}(v) + p(n|x,v) f_{v,n}(v)\right],
\end{aligned}
$$

which simplifies to

$$
\begin{aligned}
&-2\sum_{n=1}^{N} p(n|x,v) f_{v,n}(v)\beta'(v) - \sum_{n=1}^{N} p_v(n|x,v) f_{v,n}(v)\left[\delta(n)v-\beta(v)\right] \\
&+\sum_{n=1}^{N} p(n|x,v) f_{v,n}(v)\beta'(v) + \beta'(v)\sum_{n=1}^{N} p(n|x,v) f_{v,n}(v) + \beta'(v)\sum_{n=1}^{N} p_v(n|x,v)F_{v,n}(v) \\
=&-\sum_{n=1}^{N} p_v(n|x,v)\left\{f_{v,n}(v)\left[\delta(n)v-\beta(v)\right] - \beta'(v)F_{v,n}(v)\right\}, \qquad (24)
\end{aligned}
$$

where $p_v(n|x,v) = \frac{d}{dv}p(n|x,v) = \frac{p(n|x)f'(v|n)}{\sum_{m=1}^{N} p(m|x)f(v|m)} - \frac{p(n|x)f(v|n)\sum_{m=1}^{N} p(m|x)f'(v|m)}{\left[\sum_{m=1}^{N} p(m|x)f(v|m)\right]^2}$. Then, we multiply (24) by (-1) to flip the sign of the second-order condition, which becomes

$$
\begin{aligned}
&\sum_{n=1}^{N}\left(\frac{p(n|x)f'(v|n)}{\sum_{m=1}^{N} p(m|x)f(v|m)}\right)\left\{f_{v,n}(v)\left[\delta(n)v-\beta(v)\right] - \beta'(v)F_{v,n}(v)\right\} \\
&-\underbrace{\sum_{n=1}^{N} p(n|x)f(v|n)\left\{f_{v,n}(v)\left[\delta(n)v-\beta(v)\right] - \beta'(v)F_{v,n}(v)\right\}}_{=0 \text{ by FOC}} \times \frac{\sum_{m=1}^{N} p(m|x)f'(v|m)}{\left[\sum_{m=1}^{N} p(m|x)f(v|m)\right]^2} \\
=&\sum_{n=1}^{N}\left(\frac{p(n|x)f(v|n)}{\sum_{m=1}^{N} p(m|x)f(v|m)}\right)\left(\frac{f'(v|n)}{f(v|n)}\right)\left\{f_{v,n}(v)\left[\delta(n)v-\beta(v)\right] - \beta'(v)F_{v,n}(v)\right\} \\
=&\sum_{n=1}^{N} p(n|x,v)s(v|n)\left\{f_{v,n}(v)\left[\delta(n)v-\beta(v)\right] - \beta'(v)F_{v,n}(v)\right\} \\
=&C_{p(n|x,v)}\left\{s(v|n), \Psi(n,v,v)\right\} \geq 0
\end{aligned}
$$

where the last inequality holds due to the optimality of $\beta(\cdot)$ (and the flipped sign).

A.2 Proposition 2

Since $p(n|x,v) = p(n|x)$, $R_N(v;x) = \exp\left(\int_0^v \frac{\sum_{n=1}^N p(n|x) f_{v,n}(t)}{\sum_{n=1}^N p(n|x) F_{v,n}(t)} dt\right) = \sum_{n=1}^N p(n|x) F_{v,n}(v)$. Hence, (5) becomes

$$
\begin{aligned}
&\frac{1}{\sum_{m=1}^N p(m|x) F_{v,m}(v)} \int_0^v \alpha \sum_{n=1}^N p(n|x) F_{v,n}(\alpha) \frac{\sum_{n=1}^N p(n|x) f_{v,n}(\alpha)\delta(n)}{\sum_{n=1}^N p(n|x) F_{v,n}(\alpha)} d\alpha \\
&= \sum_{n=1}^N \left(\frac{p(n|x)}{\sum_{m=1}^N p(m|x) F_{v,m}(v)}\right) \delta(n) \int_0^v \alpha f_{v,n}(\alpha) d\alpha \\
&= \sum_{n=1}^N \left(\frac{p(n|x)}{\sum_{m=1}^N p(m|x) F_{v,m}(v)}\right) \delta(n) \left[v F_{v,n}(v) - \int_0^v F_{v,n}(\alpha) d\alpha\right] \\
&= \sum_{n=1}^N \left(\frac{p(n|x) F_{v,n}(v)}{\sum_{m=1}^N p(m|x) F_{v,m}(v)}\right) \delta(n) \left[v - \int_0^v \frac{F_{v,n}(\alpha)}{F_{v,n}(v)} d\alpha\right] = \sum_{n=1}^N \omega_n(v;x)\delta(n)\beta_n(v),
\end{aligned}
$$

which is (7). The rest of the proof extends Proposition 2.2 in (Krishna, 2002). Suppose that bidder i with v bids $\beta(u) = \beta_N(u;x,\delta)$ in (7) with $u \neq v$. Then, her expected utility is

$$
\begin{aligned}
U(\beta(y),v;x) &= \sum_{n=1}^N p(n|x) f_{v,n}(u)(\delta(n)v - \beta(u)) \\
&= \sum_{n=1}^N p(n|x) f_{v,n}(u)\delta(n)v - \sum_{n=1}^N p(n|x) f_{v,n}(u) \times \underbrace{\frac{\sum_{n=1}^N p(n|x) f_{v,n}(u)\delta(n)\beta_n(y)}{\sum_{n=1}^N p(n|x) f_{v,n}(u)}}_{=\beta(u)} \\
&= \sum_{n=1}^N p(n|x) f_{v,n}(u)\delta(n)v - \sum_{n=1}^N p(n|x) f_{v,n}(u)\delta(n)\beta_n(y) \\
&= \sum_{n=1}^N p(n|x)\delta(n) \left(f_{v,n}(u)(v-y) + \int_0^y F_{v,n}(\alpha) d\alpha\right).
\end{aligned}
$$

Hence, $\pi(\beta(v),v;x) - \pi(\beta(u),v;x)$ equals

$$
\begin{aligned}
&\sum_{n=1}^N p(n|x)\delta(n) \left(\int_0^v F_{v,n}(\alpha) d\alpha - f_{v,n}(u)(v-y) - \int_0^y F_{v,n}(\alpha) d\alpha\right) \\
&= \sum_{n=1}^N p(n|x)\delta(n) \left(\int_y^v F_{v,n}(\alpha) d\alpha - f_{v,n}(u)(v-y)\right) \geq 0
\end{aligned}
$$

regardless of whether $y \geq v$ or $y \leq v$. We have shown that if all other bidders are following (7), a bidder with an arbitrary value of $v \in [0, \overline{v}]$ cannot benefit by deviating from it.

A.3 Proposition 3

The derivative of (7) with respect to v is

$$\frac{d}{dv}\beta(v) = \frac{d}{dv}\left\{\frac{1}{\sum_{n=1}^{N} p(n|x)F_{v,n}(v)} \times \sum_{n=1}^{N} p(n|x)F_{v,n}(v)\delta(n)\left[v - \int_0^v \frac{F_{v,n}(\alpha)}{F_{v,n}(v)}d\alpha\right]\right\}$$
$$= \frac{d}{dv}\frac{\sum_{n=1}^{N} p(n|x)F_{v,n}(v)\delta(n)v}{\sum_{n=1}^{N} p(n|x)F_{v,n}(v)} - \frac{d}{dv}\frac{\sum_{n=1}^{N} p(n|x)\delta(n)\int_0^v F_{v,n}(\alpha)d\alpha}{\sum_{n=1}^{N} p(n|x)F_{v,n}(v)}. \tag{25}$$

Consider the first term in (25).

$$\left(\sum_{n=1}^{N} p(n|x)F_{v,n}(v)\right)^2 \times \frac{d}{dv}\frac{\sum_{n=1}^{N} p(n|x)\delta(n)F_{v,n}(v)v}{\sum_{n=1}^{N} p(n|x)F_{v,n}(v)}$$
$$= \sum_{n=1}^{N} p(n|x)\delta(n)[f_{v,n}(v)v + F_{v,n}(v)]\sum_{n=1}^{N} p(n|x)F_{v,n}(v) - \sum_{n=1}^{N} p(n|x)\delta(n)F_{v,n}(v)v\sum_{n=1}^{N} p(n|x)f_{v,n}(v)$$
$$= \sum_{n=1}^{N} p(n|x)\delta(n)f_{v,n}(v)v\sum_{m=1}^{N} p(m|x)F_{v,m}(v) + \sum_{n=1}^{N} p(n|x)\delta(n)F_{v,n}(v)\sum_{m=1}^{N} p(m|x)F_{v,m}(v)$$
$$- \sum_{n=1}^{N} p(n|x)\delta(n)F_{v,n}(v)v\sum_{m=1}^{N} p(m|x)f_{v,m}(v)$$

Consider the second term in (25).

$$\left(\sum_{n=1}^{N} p(n|x)F_{v,n}(v)\right)^2 \times \frac{d}{dv}\frac{\sum_{n=1}^{N} p(n|x)\delta(n)\int_0^v F_{v,n}(\alpha)d\alpha}{\sum_{n=1}^{N} p(n|x)F_{v,n}(v)}$$
$$= \sum_{n=1}^{N} p(n|x)\delta(n)F_{v,n}(v)\sum_{m=1}^{N} p(m|x)F_{v,m}(v) - \sum_{n=1}^{N} p(n|x)\delta(n)\int_0^v F_{v,n}(\alpha)d\alpha\sum_{m=1}^{N} p(m|x)f_{v,m}(v)$$

Combining the two terms, $\left(\sum_{n=1}^{N} p(n|x)F_{v,n}(v)\right)^2 \times \frac{d}{dv}\beta(v)$ equals

$$\left(\sum_{n=1}^{N} p(n|x)F_{v,n}(v)\right)^2 \times \left[\frac{d}{dv}\frac{\sum_{n=1}^{N} p(n|x)\delta(n)F_{v,n}(v)v}{\sum_{n=1}^{N} p(n|x)F_{v,n}(v)} - \frac{d}{dv}\frac{\sum_{n=1}^{N} p(n|x)\delta(n)\int_0^v F_{v,n}(\alpha)d\alpha}{\sum_{n=1}^{N} p(n|x)F_{v,n}(v)}\right]$$
$$= v\sum_{n=1}^{N} p(n|x)\delta(n)f_{v,n}(v)\sum_{m=1}^{N} p(m|x)F_{v,m}(v) - v\sum_{n=1}^{N} p(n|x)\delta(n)F_{v,n}(v)\sum_{m=1}^{N} p(m|x)f_{v,m}(v)$$
$$+ \sum_{n=1}^{N} p(n|x)\delta(n)\int_0^v F_{v,n}(\alpha)d\alpha\sum_{m=1}^{N} p(m|x)f_{v,m}(v)$$

Since the third term is positive, we focus on the first two terms;

$$\begin{aligned}
&v\sum_{n=1}^{N}\sum_{m=1}^{N} p(n|x)p(m|x)\delta(n)[f_{v,n}(v)F_{v,m}(v) - F_{v,n}(v)f_{v,m}(v)] \\
=&v\sum_{n=1}^{N}\sum_{m\neq n}^{N} \underbrace{p(n|x)p(m|x)F_{v,n}(v)F_{v,m}(v)}_{\text{common for all terms}}\delta(n)\underbrace{\left[\frac{f_{v,n}(v)}{F_{v,n}(v)} - \frac{f_{v,m}(v)}{F_{v,m}(v)}\right]}_{\text{appears twice}} \\
=&v\sum_{n=1}^{N}\sum_{m>n}^{N} p(n|x)p(m|x)F_{v,n}(v)F_{v,m}(v)(\delta(n)-\delta(m))\left[\frac{f_{v,n}(v)}{F_{v,n}(v)} - \frac{f_{v,m}(v)}{F_{v,m}(v)}\right] > 0 \qquad (26)
\end{aligned}$$

because $\delta(n) > \delta(m)$ due to Assumption 1 and $\frac{f_{v,n}(v)}{F_{v,n}(v)} > \frac{f_{v,m}(v)}{F_{v,m}(v)}$ for $n < m$ under the stochastic dominance hypothesis of the proposition.

A.3. Proposition 4

Let $G(b|x,n)$ and $g(b|x,n)$ be the CDF and density of $\mathcal{P}(b|x,n)$. We suppress the dependence of $G(b|x,n)$ and $g(b|x,n)$ and $p(n|x)$ on x other than N. By the change of variables for $b = \beta(v)$, we rewrite the first-order condition, (4), as $\sum_{n=1}^{N} p(n|x)(n-1)G(b|n,N)^{n-2}g(b|n,N)^2(\delta(n)v - b) = \sum_{n=1}^{N} p(n|x)G(b|n,N)^{n-1}g(b|n,N)$, which gives the inverse bidding function,

$$v = \frac{b\sum_{n=1}^{N} p(n|x)(n-1)G(b|n,N)^{n-2}g(b|n,N)^2 + \sum_{n=1}^{N} p(n|x)G(b|n,N)^{n-1}g(b|n,N)}{\sum_{n=1}^{N} p(n|x)(n-1)G(b|n,N)^{n-2}g(b|n,N)^2\delta(n)}. \qquad (27)$$

For $\alpha \in (0,1]$, let $v_n(\alpha) := F_v^{-1}(\alpha|n)$ and $b_{n|N}(\alpha) := G^{-1}(\alpha|p,n,N)$. Let also $A_{n,N}(\alpha) := p(n|x)\alpha^{n-1}g(b_{n|N}(\alpha)|n,N)$ and $B_{n,N}(\alpha) := p(n|x)(n-1)\alpha^{n-2}g(b_{n|N}(\alpha)|n,N)^2$. From (27),

$$v_n(\alpha) = \frac{b_{n|N}(\alpha)\sum_{n=1}^{N} B_{n,N}(\alpha) + \sum_{n=1}^{N} A_{n,N}(\alpha)}{\sum_{n=1}^{N} B_{n,N}(\alpha)\delta(n)} \qquad (28)$$

$v_n(\alpha)$ does not depend on N under Assumption 3. For $(n,N) \in \{(2,2),(2,3)\}$, (28) gives

$$\frac{B_{2,2}(\alpha)\delta(2)}{b_{2|2}(\alpha)B_{2,2}(\alpha) + \sum_{n=1}^{2} A_{n,2}(\alpha)} = \frac{B_{2,3}(\alpha)\delta(2) + B_{3,3}(\alpha)\delta(3)}{b_{2|3}(\alpha)\sum_{n=2}^{3} B_{n,3}(\alpha) + \sum_{n=1}^{3} A_{n,3}(\alpha)} \qquad (29)$$

where the only unknowns are $\delta(2)$ and $\delta(3)$. By Assumption 6, we can write $\delta(2)$ and $\delta(3)$ in the form of polynomials of order d_ζ. Consider $d_\zeta = 1$. Then, we have

$$\delta(n) = 1 - (n-1)\zeta_1 \ \text{ for } n \in \{2,3\} \qquad (30)$$

with unknown $\zeta_1 \in [0, 1/2)$, so that $\delta(n) \in (0, 1]$. Under (30), then, (29) becomes

$$\frac{B_{2,2}(\alpha) - B_{2,2}(\alpha)\zeta_1}{b_{2|2}(\alpha)B_{2,2}(\alpha) + \sum_{n=1}^{2} A_{n,2}(\alpha)} = \frac{(B_{2,3}(\alpha) + B_{3,3}(\alpha)) - (B_{2,3}(\alpha) + 2B_{3,3}(\alpha))\zeta_1}{b_{2|3}(\alpha)\sum_{n=2}^{3} B_{n,3}(\alpha) + \sum_{n=1}^{3} A_{n,3}(\alpha)}, \tag{31}$$

where the only unknown is ζ_1 and, therefore, it is identified. For a larger d_ζ, the identification strategy for $\delta(2)$ and $\delta(3)$ remains the same. For example, consider $d_\zeta = 2$. Then,

$$\delta(n) = 1 - (n-1)\zeta_1 - (n-1)^2\zeta_2 \text{ for } n \in \{2, 3\} \tag{32}$$

with parameters (ζ_1, ζ_2) to identify. Under (32), (29) becomes

$$\begin{aligned} &\frac{B_{2,2}(\alpha) - B_{2,2}(\alpha)\zeta_1 - B_{2,2}(\alpha)\zeta_2}{b_{2|2}(\alpha)B_{2,2}(\alpha) + \sum_{n=1}^{2} A_{n,2}(\alpha)} \\ &= \frac{(B_{2,3}(\alpha) + B_{3,3}(\alpha)) - (B_{2,3}(\alpha) + 2B_{3,3}(\alpha))\zeta_1 - (B_{2,3}(\alpha) + 4B_{3,3}(\alpha))\zeta_2}{b_{2|3}(\alpha)\sum_{n=2}^{3} B_{n,3}(\alpha) + \sum_{n=1}^{3} A_{n,3}(\alpha)}. \end{aligned} \tag{33}$$

We can construct another equation like (33) at a different quantile $\widetilde{\alpha}$ to have a system of two linear equations with two unknowns (ζ_1, ζ_2). The system is not homogeneous, and by solving it for (ζ_1, ζ_2), we identify (ζ_1, ζ_2). By the same idea, we can identify $(\zeta_1, \ldots, \zeta_{d_\zeta})$ for a larger d_ζ. For a larger d_ζ, we need to construct a system of d_ζ linear equations, which is not homogeneous under Assumption 6. By solving the system for $(\zeta_1, \ldots, \zeta_{d_\zeta})$, we identify a flexible $\delta(n)$ for $n \in \{2, 3\}$. Once $\delta(k-1)$ and $\delta(k)$ are identified, we recursively identify $\delta(k+1)$ without a functional form assumption for $k \geq 3$. Using $(n, N) \in \{(k, k), (k, k+1)\}$, we equate the two quantile-by-quantile inverse bidding functions,

$$\frac{\sum_{n=1}^{k} B_{n,k}(\alpha)\delta(n)}{b_{k|k}(\alpha)\sum_{n=1}^{k} B_{n,k}(\alpha) + \sum_{n=1}^{k} A_{n,k}(\alpha)} = \frac{\sum_{n=1}^{k} B_{n,k+1}(\alpha)\delta(n) + B_{k+1,k+1}(\alpha)\delta(k+1)}{b_{k|k+1}(\alpha)\sum_{n=1}^{k+1} B_{n,k+1}(\alpha) + \sum_{n=1}^{k+1} A_{n,k+1}(\alpha)},$$

where only $\delta(k+1)$ is unknown, implying the identification of $\delta(k+1)$. Finally, since $\{\delta(n)\}$ are all identified, we identify $F_v(\cdot|n)$ for all $n \in \{1, \ldots, \max\{N\}\}$ by (28).

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