Hard-to-Interpret Signals*

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Abstract

Decisions under uncertainty are often made with information that is (ambiguous or) difficult to interpret because multiple interpretations are possible. For example, during the COVID-19 pandemic, policy-makers based their decisions on the fatality rate among tested and confirmed individuals, which is only an ambiguous signal of the overall fatality rate among the infected in the population – a key unknown. Individuals may perceive and handle uncertainty about interpretation differently and in ways that are not directly observable to a modeler. This paper identifies and experimentally examines behavior that can be interpreted as reflecting an individual's attitude towards such uncertainty.

Keywords: ambiguity, information, fatality-rate, COVID-19

JEL: C91, D81, D83, D91

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1 Introduction

1.1 Objective

The two cornerstones of Bayesian theory are the subjective prior and Bayesian updating. Ellsberg (1961) demonstrates the behavioral limitations of the assumption of a prior through his celebrated thought experiments, which have the clear intuition that a probability measure does not permit a role for limited information and confidence underlying beliefs. In this paper, we present a parallel critique of the updating component consisting of both a thought experiment and a laboratory experiment that provides supporting empirical evidence.

The importance attached to Ellsberg's experiments is due to the presumption that in many instances of decision-making under uncertainty in the field, information may be lacking to justify sharp beliefs. Clearly, this presumption does not require that in all, or even most, such cases there is no information at all. Our motivation begins with the presumption that often there is a great deal of information, but the difficulty for the decision-maker is that its interpretation may not be clear in the sense that the inferences to be drawn from them are uncertain. In such cases we refer to *hard-to-interpret, or ambiguous, signals*. Fed policy communication and qualitative corporate news are two typical examples. They also illustrate the broader class of situations where complete information is complex and thus where information often comes in the form of summaries, open to multiple interpretations (and inferences), rather than in the form of detailed reports.

A timely example is provided by the COVID-19 pandemic. The policy-maker[1] had to choose an action to combat the potential new pandemic due to the novel virus. A critical unknown factor is the probability that an infected individual in the population will suffer serious health consequences (for example, require hospitalization or, at the extreme, die). The policy-maker has access to some information about the effects of the virus in other jurisdictions: how many patients required hospitalization, and most importantly, the case-fatality rate.[2] However, it is not clear what inferences to make about

[1] Vivalt and Coville (2020) document that trained policy-makers deviate from Bayesian updating just like regular subjects, even in the context within which they have expertise.

[2] The case-fatality rate (CFR) is the number of deaths of individuals who tested positive divided by the number of individuals who were confirmed as infected. Importantly, CFR is to be distinguished from the infection-fatality rate (IFR), which is the proportion of all

the risk of suffering serious health outcomes - a point that is well understood by epidemiologists and received recently attention in the media.[3]. For example, even if the policy-maker knows how many people were tested for the virus and how many of them tested positive, she plausibly does not know how many people were infected but were not tested (because they had only mild symptoms or because they died without having been tested). Consequently, an observed high rate of serious health outcomes may indicate a high infection rate and that an infected individual is likely to experience serious health outcomes, or alternatively, it may reflect that only individuals who were symptomatic and severely ill were tested. Similarly, an observed low rate of serious health outcomes may reflect that even individuals with mild (or no) symptoms were tested, or that patients who died and suffered from pre-existing conditions were not tested, or that the likelihood of the virus causing serious health outcomes is simply low. How would such hard-to-interpret information affect the policy-maker's choice of action?

We identify choices in an Ellsberg-style setting that can be understood as revealing that uncertainty about how to interpret signals matters for behavior (Section 2); and later (Appendix B), we provide behavioral definitions of the attitude (aversion, affinity or indifference) to signal ambiguity in a much more general setting. Sensitivity to signal ambiguity, which is our focal hypothesis, is conceptually distinct from sensitivity to prior ambiguity (for example, about the composition of an urn or the hazard of serious health outcomes in an evolving pandemic). Accordingly, while Ellsberg pointed to the limitation of modeling prior beliefs by an additive probability measure, our analysis points to the limitation of modeling updating in such a way that updating conforms to the *martingale property* of beliefs (that is, prior beliefs is an average of the set of posteriors). At the functional form level, additivity of prior beliefs and the martingale property are the two distinct fundamental properties of the Bayesian model. Importantly, the martingale property is extended here to a property of preference (rather than probabilistic beliefs) which need satisfy only mild nonparametric restrictions.

Studying the behavioral meaning of hard-to-interpret signals is motivated

infections that result in death.

[3] Harmon, Amy. 2020. "Why we don't know the true death rate for COVID-19." *The New York Times*, April 17. https://nyti.ms/2VgGQKj

Mooney, Chris, Juliet Eilperin and Joel Achenback. 2020. "As U.S. coronavirus fatality rate rises to 5 percent, experts are still trying to understand how deadly this virus is." *The Washington Post*, April 17. link to article in part by the inherent interest in such a fundamental notion; see below for references to papers where ambiguity in signals plays a role. We believe that although its importance is evident from applications such as the example above, it has not been investigated systematically in decision theory where prior ambiguity plays the predominant role. Moreover, experience during the COVID-19 pandemic suggests that policy-makers' attitude to ambiguous signals may have large economic and social consequences. In addition, the way in which hard-to-interpret signals are treated by decision-makers reflects on the potential importance of the existing literature on ambiguity. Specifically, if such signals themselves reduce confidence in beliefs then there is reason to believe that, at least in some circumstances, ambiguity might persist rather than being only a short-run phenomenon.

We present a thought experiment that demonstrates our proposed definition of sensitivity to ambiguous signals and we substantiate its relevance to observed behavior in a controlled experiment. We elicit probability equivalents to an event, both unconditionally and conditional on two complementary signals. The experimental design is guided by the fact that even when available information takes the form of noisy (or risky) signals, individuals often fail to update their beliefs as specified by Bayes rule. We therefore employ a between-subject design that compares deviations from Bayesian updating when signals are noisy (the control) to when they are ambiguous. We find that ambiguous signals significantly increase deviations from updating that is consistent with Bayes rule. In addition, we find a significant association between the attitude to prior ambiguity (measured indirectly through compound risk) and a non-neutral attitude to signal ambiguity.

The paper proceeds as follows. The rest of this introduction considers related literature. In Section 2, we present the thought experiment, then define and discuss the behavior that is the focus of the current paper (Appendices A and B extend and formalize this discussion). The experimental implementation is described Section 3, and Section 4 describes the implications for some existing models of preference.

1.2 Related literature

Two very recent experimental studies investigate updating when information is (in some sense) ambiguous. Neither includes behavioral definitions for different attitudes to signal ambiguity, or highlight the relevance of the martingale property. In a contemporaneous project, Liang (2019) elicits certainty equivalents (not probability equivalents) for many bets (including uncertain) and information structures (including uncertain). In the absence of behavioral definitions, it is not clear how to identify attitudes. In addition, though there is overlap in motivation, there is no overlap in design as Liang (2019) does not include our main treatment with uncertain prior and ambiguous information. Therefore, he cannot compare the effects of risky and ambiguous information when the prior is uncertain, which is the focus of our investigation. Shishkin and Ortoleva (2020) study how ambiguous information affects the valuation of risky and ambiguous bets.[4] Their main finding is that when the prior is known but the information is ambiguous, ambiguity averse subjects do not change their valuation of the bet systematically – contrary to common models of ambiguity that suggest that the set of priors will increase (dilate), and hence that valuation will fall.

Ambiguous signals are considered, implicitly or explicitly, in a number of applied studies. Ambiguous communication is shown to arise endogenously from maximizing behavior in a range of strategic settings (e.g., Bose and Renou 2014; Blume and Board 2014; Kellner and Le Quement 2017, 2018; Beauchene, Li and Li 2019; and Kellner Le Quement Riener 2019 for an experimental counterpart). Levy and Razin (2016) study settings with group communication in which communication, the signal in their model, creates ambiguity. They consider several applications including to jury deliberations and common-value auctions. Fabrizi, Lippert, Pan and Ryan (2019) study theoretically and experimentally collective decision making under the unanimity rule and show that ambiguity may sustain informative voting. Ambiguous signals have been studied also in macro/finance models (e.g., Epstein and Schneider 2010; Ilut 2012; Ilut, Kehrig and Schneider 2018; Yoo, 2019). The distinction in Daniel and Titman (2006) between tangible and intangible information is suggestive of the distinction between noisy and ambiguous signals. In all of these studies, preferences and/or the form taken by updating are assumed known to the modeler and interpretations of the model are based on functional form appearance or what seems "natural." This paper is complementary in that it takes behavior alone to be observable and asks, for example, "what behavior would reveal an aversion to ambiguity in signals?"

Fryer, Harms and Jackson (2019) study the relation between signals that are open to interpretation and polarization. They posit a particular updating

[4] Their paper was written with knowledge of our project, including experimental design and results.

rule and study its implications for polarization. In contrast, we ask what can be learned about updating from behavior with an objective of using identified behavior to distinguish between alternative models of updating. Their online experiment is designed to study polarization, while our experiment is designed to examine whether uncertainty about signal interpretation is revealed by behavior.

Updating under ambiguity has been studied in axiomatic decision theory, (see, for example, Gilboa and Schmeidler 1993; Pires 2002; Epstein and Seo 2010; Gul and Pesendorfer 2018). For beliefs represented by probability measures, the martingale property delivered by Bayesian updating is studied axiomatically by Shmaya and Yariv (2007) and Cripps (2018), and is at the heart of the test of Bayesian updating proposed in Augenblick and Rabin (2018). Gajdos et al (2008) and Hayashi and Wada (2010) incorporate imprecise information into models of preference. They assume that information comes in the form of an objective (observable) set of probability measures over the state space. None of this literature has addressed the specific questions studied here.

Epstein and Schneider (2007, 8, 10) pay explicit attention to the behavioral meaning of functional form specifications and they introduce and discuss the notion of ambiguous signals. In particular, they distinguish between noisy and ambiguous signals, and correspondingly point to a new dimension of information quality, distinct from the usual notion of the precision of a noisy signal, that pertains to the ease/difficulty of its interpretation. They also describe a thought experiment which we build upon here. An important difference is that our thought experiment "leads to" and illustrates a general model (Appendix B), while such a general analysis is not apparent in the previous work. Moreover, the current paper is the first to document empirically (in an experimental setting) the behavioral relevance of these theoretical distinctions.

2 A thought experiment

This section builds on Ellsberg's two-urn experiment, and on Epstein and Schneider (2007,8), and suggests a thought experiment to give behavioral meaning to sensitivity to hard-to-interpret signals. A more general and formal treatment is provided in Appendices A and B.

2.1 The choice problems

Consider bets on the color, red or black, of one ball to be drawn from an urn, which we call the "payoff urn". Bets pay either \$100 or \$0. The decision-maker (DM) is told further that the payoff urn contains 10 balls, with at least one of each color. Elicit *probability equivalents* in two *ordered* scenarios.

1. *Unconditional choice*: Let f_R and f_B denote bets on red and black, respectively, being drawn from the payoff urn. Elicit *unconditional* probability equivalents $p_{0,R}$ and $p_{0,B}$, where

$$f_R \sim_0 (100, p_{0,R}; 0, 1 - p_{0,R}) \text{ and } f_B \sim_0 (100, p_{0,B}; 0, 1 - p_{0,B}).$$

   where $(100, 0; 0, 1 - p)$ is a bet that pays \$100 with probability p, and \$0 with probability $1 - p$, and the relation $\sim_0$ denotes indifference at the unconditional stage. The intuitive behavior highlighted by Ellsberg in his two-urn experiment corresponds to $p_{0,R}, p_{0,B} < \frac{1}{2}$, but this is not necessary for what follows.

2. *Conditional choice*: The DM is now told about a second "signal urn" that is constructed by adding an equal number (N) of red and black balls to the payoff urn. The total number ($2N$) of balls added is not specified. Then a ball is drawn from the *signal urn* and its color is revealed: $\sigma \in \mathbf{\Sigma} = \{\sigma_R, \sigma_B\}$, where σ denotes the color of the ball drawn from the signal urn. Once again, consider bets on the color to be drawn from the *payoff urn* and elicit *conditional* probability equivalents $p_{\sigma,R}$ and $p_{\sigma,B}$ for each signal $\sigma \in \{\sigma_R, \sigma_B\}$:[5]

$$\begin{array}{llll} f_R & \sim_\sigma & P_{\sigma,R} & = & (100, p_{\sigma,R}; 0, 1 - p_{\sigma,R}) \\ f_B & \sim_\sigma & P_{\sigma,B} & = & (100, p_{\sigma,B}; 0, 1 - p_{\sigma,B}). \end{array} \tag{2.1}$$

   where $\sim_\sigma$ denotes indifference at the conditional stage (after a signal is observed).

Below we assume

$$(p_{\sigma_R,R} - p_{\sigma_B,R}) \cdot (p_{\sigma_R,B} - p_{\sigma_B,B}) < 0, \tag{2.2}$$

a property that we call *signal diversity.* Essentially, it excludes the case where the same signal is viewed as being weakly better for both bets. For

[5] Throughout, upper case Ps denote lotteries and lower case ps denote probabilities.

example, a special case that is natural for the present setting is that the DM views σ_R *as a better signal for the bet on red than is* σ_B, *and the reverse for the bet on black*, that is,

$$p_{\sigma_R,R} > p_{\sigma_B,R} \text{ and } p_{\sigma_R,B} < p_{\sigma_B,B}. \tag{2.3}$$

(See Lemma 1 in Appendix A and Appendix B for the role played by signal diversity.)

The specification of the two scenarios as ordered (or sequential) is adopted in order to isolate the processing of signals from prior ambiguity. We compare conditional choices, made after realization and processing of a signal, with (prior, ex-ante, or) unconditional choices, made in the absence of any realized *or even anticipated signals*. To clarify the relevance of "anticipated signals," suppose that a choice between prospects is made ex-ante, before realization of a signal but with the expectation that before the state of the world is realized, a signal about the state will be forthcoming. Though choice cannot be made contingent on the signal, its anticipation can still affect the ex-ante evaluation of prospects, for example, if DM backward inducts from anticipated conditional rankings. In that case, unconditional choices would be "contaminated" by the signal structure, which would leave unclear how to isolate the behavioral implications of the signal structure. Thus, to be perfectly clear, the behavior described below should not be seen as describing dynamic choice, but rather as choice in two different (with and without signals) but related settings (the payoff urn is common). Alternatively, if the environment is dynamic but the decision maker is myopic or cannot anticipate possible signals, similar behavior may arise. Since it may be challenging to identify myopia, we concentrate (both theoretically and experimentally) on the setting of two ordered scenarios.

We assume throughout that lotteries are ranked according to expected utility theory (vNM). Though restrictive on descriptive grounds, it is almost universal in the literature on ambiguity-sensitive preferences, starting with Schmeidler (1989) and Gilboa and Schmeidler (1989).[6] In addition, risk preferences are assumed to be strictly monotone in the sense of FOSD and unaffected by signal realizations.

[6] An exception is Dean and Ortoleva (2017).

2.2 Behavior: the symmetric case

It is convenient to adopt the following notation: the payoff-relevant state space is $S = \{R, B\}$, the set of prizes is $X = \{100, 0\}$, and the signal space is $\mathbf{\Sigma} = \{\sigma_R, \sigma_B\}$. Conditional and unconditional preferences are defined on bets and lotteries, that is, on $\{f_R, f_B\} \cup \Delta(X)$, where $\{f_R, f_B\}$ are bets on red and black from the payoff urn and $\Delta(X)$ are objective lotteries over X.

Because information about both payoff and signal urns is color-symmetric, one would expect a "rational" individual to satisfy also the following *symmetry* condition:

$$p_{0,R} = p_{0,B},\ p_{\sigma_R,R} = p_{\sigma_B,B},\ p_{\sigma_R,B} = p_{\sigma_B,R}. \tag{2.4}$$

We assume (2.4) throughout this section, in the ensuing discussion of models and also in the laboratory experiment. See Appendix B for the more general case where symmetry is not imposed, and for other generalizations of the above choice problems whereby $S, \mathbf{\Sigma}$ and X can be any nonbinary finite sets, and bets on colors can be replaced by arbitrary Savage acts from S into X.

Assuming (2.4), our focal behavior corresponding to *(strict) aversion to signal ambiguity* is:

$$p_{0,R} > \tfrac{1}{2}p_{\sigma_R,R} + \tfrac{1}{2}p_{\sigma_B,R}. \tag{2.5}$$

In the rest of this section, we describe some intuition for (2.5). Weak aversion, strict and weak affinity, and indifference or neutrality are defined by the obvious modifications of (2.5) and can be motivated similarly. (All these inequalities refer explicitly only to bets on red, corresponding inequalities for bets on black follow immediately from symmetry.) Though all forms of nonindifference (inequality in (2.5)) are of equal interest, as is common in the literature our discussion focuses on strict aversion.

The intuition we suggest for (2.5) centers on uncertainty about the number of balls added to the signal urn and hence about how to interpret a signal. To explain, note first that the unconditional probability equivalent p_0 reflects the attitude towards the uncertain composition of the payoff urn, as in Ellsberg's experiment, but is not affected by uncertainty about signal interpretation because even the possibility of signals is presumably unknown at the unconditional stage. However, uncertainty about signal interpretation is relevant for conditional probability equivalents. For example, a red draw (σ_R) is a strong signal in favor of a bet on red (and against a bet on black) if only a small number of balls were added in constructing the signal urn, but it is only a weak signal for red (and against black) if a large number of balls were added. A conservative decision-maker facing this uncertainty might interpret σ_R as a weak positive signal when evaluating a bet on red, (corresponding to large N), hence leading to a small probability equivalent $p_{\sigma_R,R}$ for betting on a red ball drawn from the payoff urn. But similar uncertainty applies when interpreting the implication of observing a black ball drawn from the signal urn, and a conservative attitude would lead to viewing it as a strong negative signal for a bet on red (corresponding to N small), and hence to a small probability equivalent $p_{\sigma_B,R}$. This suggests how aversion to signal ambiguity might explain (2.5).

There is a parallel with the Ellsberg-based approach to prior ambiguity. Given symmetry (2.4), and hence $p_{\sigma_B,R} = p_{\sigma_R,B}$, then (2.5) can be described as saying that a given signal (σ_R) is interpreted as providing weak support for both an event (drawing red) and its complement (drawing black). This is a counterpart of the essence of Ellsberg's two-urn experiment, namely that both an event and its complement are deemed unlikely.

To illustrate (2.5), consider a numerical example in which the DM has the following additional information regarding the payoff and signal urns: All 8 unknown balls of the payoff urn are red (black) if a fair coin toss gives heads (tails); and the signal urn is constructed by adding N balls of each color, where $N = 0$ or 45. Thus, if DM calculates objective probabilities correctly (satisfies the Reduction of Compound Lotteries axiom, ROCL), presumably $p_{0,R} = \frac{1}{2}$ as there is no prior ambiguity.[7] The posterior probabilities satisfy

$$\Pr(R \mid \sigma_R) \in \{.53,\ .82\} \text{ and } \Pr(R \mid \sigma_B) \in \{.18,\ .47\}.$$

Then aversion to uncertainty about signal interpretation (the signal is strong if $N = 0$ but weak if $N = 45$) plausibly leads to probability equivalents $p_{\sigma_R,R} < .67$ and $p_{\sigma_B,R} < .33$, and hence to (2.5).

Our thought experiment can be seen as an idealized version of the situation faced by the policy-maker deliberating costly actions to combat a novel virus, as described in the introduction. In a simplified version, ambiguity stems from the uncertain incidence of the virus in the population. Epidemiologists have access to the rate of patients with serious health outcomes only among tested and confirmed individuals, but they are uncertain how many other individuals were infected and were not severely ill, and also how many

[7] Extensive research has documented that ROCL is not a good behavioral assumption in this case, and we will use this fact in our experimental design (see Section 3.1).

others suffered serious health outcomes that were not attributed to the virus. Obviously, ambiguity concerning the signal may have other dimensions: various risk factors and immunization regimes may make a signal more uncertain (both risky and ambiguous), as may limited understanding about the network connections of infected individuals and the status of the health system. Some of these factors may be thought of as noisy (risky), but others are reasonably viewed as ambiguous. The question arises whether sensitivity to ambiguity of signals affects behavior, and if so, how. This paper identifies such behavior in an abstract setting (and is the first in the literature to do so).

The virus example demonstrates another important feature of our thought experiment: the policy-maker may have made also unconditional choices, very possibly without anticipating the later arrival of information. This "lack of foresight" concerning the information structure provides external validity to the specific sequencing of choices we employ in the thought experiment and in the experimental implementation to follow.

Readers who find the above intuition and motivation for (2.5) convincing can proceed directly to the laboratory experiment (Section 3). For the benefit of other readers, Appendix A elaborates on and deepens the above intuition.[8]

3 A laboratory experiment

In the section we report the design and results of a lab experiment whose goal is to evaluate the empirical applicability of the signal-sensitive behavior proposed in (2.5). There are a few major practical challenges that a lab experiment must overcome. First, subjects may not be Bayesian even when the accuracy of the signal is known (see Grether 1980, Ambuehl and Li 2018, for example). Second, they might not reduce compound objective lotteries, a behavior that has been shown to be empirically associated with sensitivity to ambiguity. Third, they may not satisfy expected utility even when dealing with objective probabilities. Fourth, even if all the above are non-issues, and subjects are sensitive to signal ambiguity as we suggest, they may use the elicitation system to hedge such ambiguity. In the following subsection we detail how we dealt with these challenges and provide the details of the experimental design.

[8]One possible concern is that the equal weighting of $p_{\sigma,R}$ for the two signals $\sigma \in \{\sigma_R, \sigma_B\}$ may seem arbitrary. However, Lemma 1 should resolve this concern.

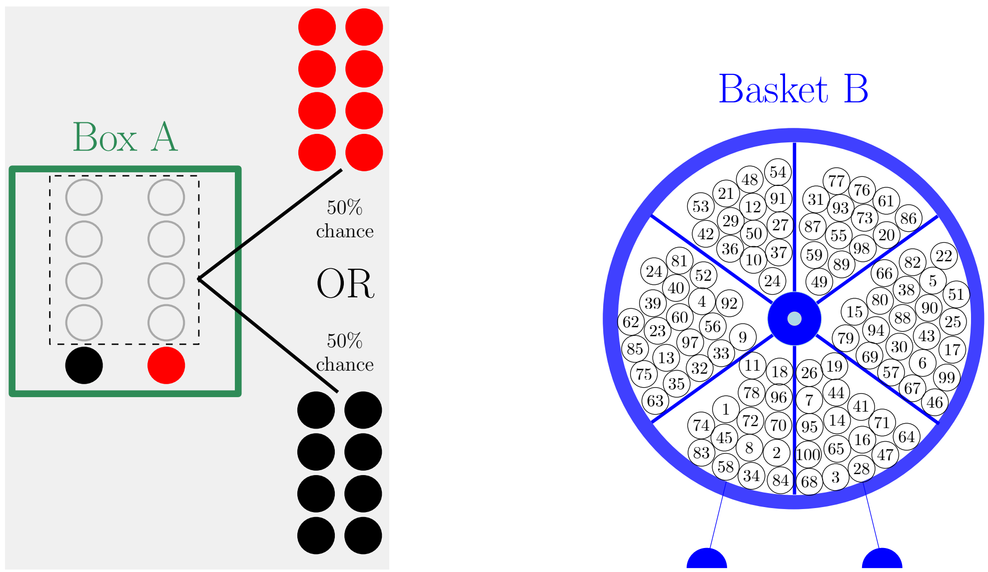


Figure 3.1: The payoff urn (left) and the basket used to elicit probability equivalents (right)

3.1 Experimental Design

We adopted a between-subject design, where the control group received a known risky-signal, while the treatment group received an ambiguous signal. Our focus is on the differential effect of signal ambiguity on updating.

The environment is similar to the numerical example presented in Section 2.2. The payoff urn (for both groups) consisted of 10 balls, that had either 9 red balls (and 1 black ball) or 1 red ball (and 9 black balls), each with probability .5, as on the left panel of Figure 3.1. Hence, the probability of drawing a red ball is either .1 or .9, each with probability .5. Two considerations motivated us to eliminate prior ambiguity from the payoff urn. First, symmetry between a bet on red and a bet on black is universal in such a case, while if the compositions were only symmetric but included prior ambiguity, then some subjects may have had non-symmetric belief. We deal with this possibility in Appendix B, but the elicitation is more complex in that case. Second, there exists now strong empirical evidence that many subjects do not distinguish between symmetric ambiguous environments and similar environments with compound risk (Halevy 2007; Dean and Ortoleva, 2019; Gillen, Snowberg and Yariv, 2019; and especially Chew, Miao and Zhong

2017). The payoff urn is a special case of Chew et al's two-point compound-risk, which they show is similar to two-point ambiguity. We therefore expect many (ambiguity averse) subjects to prefer a one-stage lottery with a winning probability of .5 to a bet on either color from the payoff urn. We eliminated hedging opportunities in the unconditional choice[9] by asking the subject to choose a color to bet on (red or black) from the payoff urn.

The elicitation of probability equivalents was implemented as in Freeman, Halevy and Kneeland (2019). Subjects were presented with a basket containing 100 balls numbered from 1 to 100 (as on the right panel of Figure 3.1), and on each line of a choice list they were asked to choose between their bet on the payoff urn and a bet that the ball drawn from the basket has a number that is smaller or equal to the line number, so the latter increases when the subject moves down the list. In the initial choice list the step was 10 percent, and then subjects were presented with a zoom-in list were the resolution was 1 percent.[10] One may worry that if subjects have non-expected utility preferences, the elicitation of probability equivalents is not incentive compatible. As demonstrated in Freeman et al (2019) and again in Freeman and Mayraz (2019), this poses a challenge only when the constant alternative in the choice list is certain, while in our case it is uncertain.

The control group facing a risky signal was then introduced to a signal urn constructed by adding 5 red and 5 black balls to the payoff urn (left panel of Figure 3.2). The signal urn therefore included 20 balls, which were equally likely to be 14 red (and 6 black) or 6 red (and 14 black). Conditional probability equivalents were elicited, that is – the probability equivalent of the chosen bet on the payoff urn conditional on each color being drawn from the signal urn. By Bayes rule, if the prior probability of drawing red from the payoff urn is p, and if the signal urn contains N additional balls of each color, then the probability of drawing red from the payoff urn conditional on a red ball being drawn from the signal urn is:

[9] Strictly speaking, there should not be a concern for hedging here, as the payoff urn is compound risk and not ambiguous. However, if a subject identifies the two, and reacts to ambiguity using hedging, she may hedge here as well. As a result, she may report the probability equivalent .5 for bets on both red and black from the payoff urn, even if her true probability equivalent is smaller than .5 (Baillon, Halevy and Li 2019).

[10] If subjects switched more than once in a choice list, a pop-up explained to them the logic of monotonic preferences. However, if they wished to switch multiple times - they were allowed to do so. In other words, we did not impose a single crossing, but tried to make sure subjects understood their choices. This technique was first used in Freeman et al (2019).

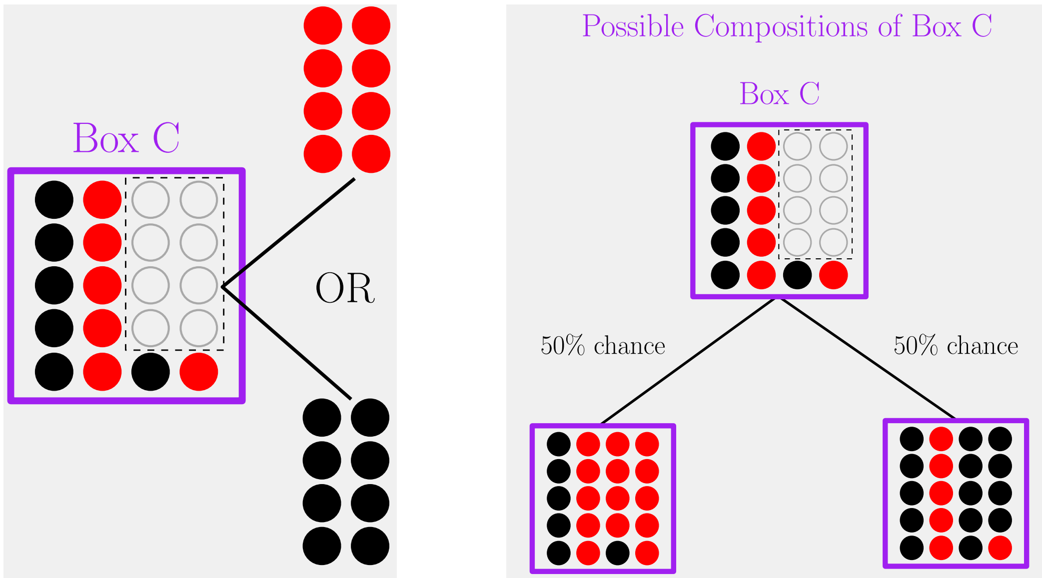


Figure 3.2: The signal urn: the risk control

$$P\left(R|\sigma_R, N\right) = \frac{\left(\frac{9+N}{10+N}\right)\frac{9}{10}\left(1-p\right) + \left(\frac{1+N}{10+N}\right)\frac{1}{10}p}{\left(\frac{9+N}{10+N}\right)\left(1-p\right) + \left(\frac{1+N}{10+N}\right)p} \tag{3.1}$$

Applied to the risky signal urn ($N = 5$), the Bayesian updates of $p = .5$ are .66 and .34 for a favorable and unfavorable signal, respectively. We did not expect subjects to calculate Bayes rule exactly. In order to facilitate a reasonable approximation to Bayes rule, and inspired by Gigerenzer and Hoffrage (1995), we presented to subjects the two possible signal urn compositions (right panel of Figure 3.2), which suggest that the probabilities of drawing the chosen color from the *signal* urn are .7 or .3 depending on the composition of the *payoff* urn.

The treatment group faced an ambiguous signal urn, constructed by adding N balls of each color to the payoff urn, where N was either 0 or 45 (left panel of Figure 3.3). That is, the signal urn contained either 10 balls (with a composition of 9R1B or 1R9B) or 100 balls (with a composition of 54R46B or 46R54B). If $N = 0$ the signal is much more informative than if $N = 45$; accordingly, if $p = .5$, then $P\left(R|\sigma_R, N=0\right) = .82$ while $P\left(R|\sigma_R, N=45\right) = .532$. As done for the control group, we presented subjects with images of the possible compositions of the signal urn (right panel

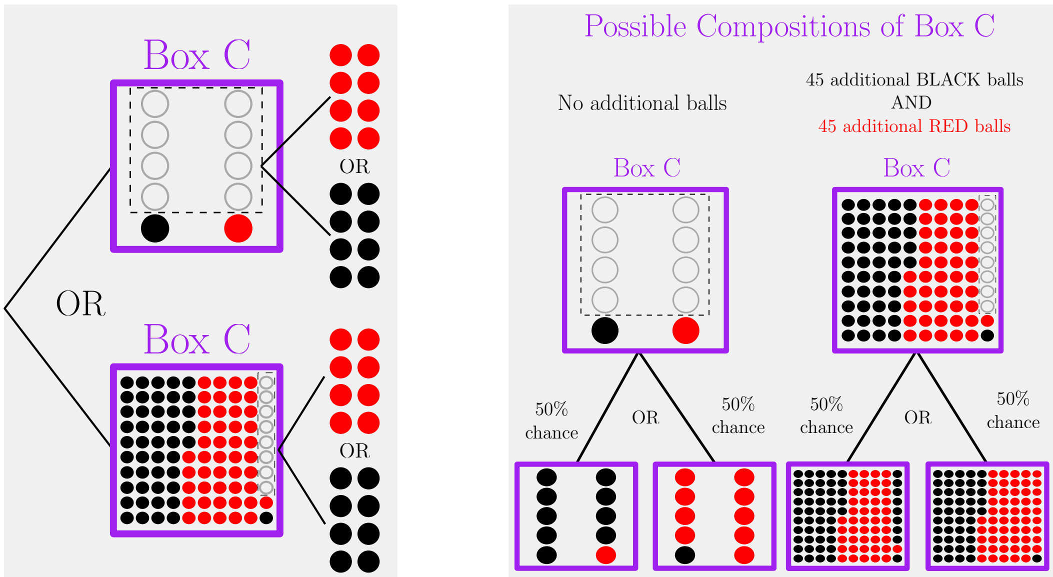


Figure 3.3: The signal urn: the ambiguous treatment

of Figure 3.3) in order to facilitate their intuitive reasoning when eliciting conditional probability equivalents.

We chose the risky and ambiguous signal urns such that if $p = .5$ then the Bayesian posterior of the risky signal is approximately equal to the average of the two possible Bayesian posteriors of the ambiguous signals.

Skeptics may wonder whether the fact that we elicited two (conditional) probability equivalents generates more noise compared to the prior probability equivalent. We believe this is not the case as random noise would tend to cancel out, and the average conditional probability equivalents would be close to the prior. Hence, the noise hypothesis would work against finding non-neutrality to signal ambiguity. More crucially, we did the same in the control (risky) group, and we will concentrate on the differential effect of ambiguous signals relative to this control.

As we elicited probability equivalents conditional on both red and black balls being drawn from the signal urn and paid only one choice, we expose the experimental design in the conditional stage to the theoretical possibility of hedging. That is, if a subject is ambiguity averse, she may use the incentive system to hedge part of the ambiguity concerning the inferences made based on the signal urn. Although this is a theoretical possibility, we find it highly improbable that subjects will be sophisticated enough to hedge the ambiguity in this way. In any case, the resulting bias would be that ambiguity averse subjects who do not have probabilistic beliefs about the structure of the signal will behave as if they are Bayesian, which is the null hypothesis in the current investigation.

To sum up, we elicited for each subject three probability equivalents (assuming the subject chose to bet on red): $p_{0,R}$ – the unconditional probability equivalent; $p_{\sigma_R,R}$ and $p_{\sigma_B,R}$ – the probability equivalents conditional on drawing a favorable (red) and unfavorable (black) signal from the signal urn. Since symmetry is built into the payoff urn using the compound lottery, Bayesian updating implies that the unconditional probability equivalent is the (equally weighted) average of the conditionals. Therefore, our interest is in measuring the difference

$$(0.5p_{\sigma_R,R} + 0.5p_{\sigma_B,R}) - p_{0,R}. \tag{3.2}$$

However, since we do not expect subjects to be exactly Bayesian even in the case of a risky signal, we compare this measure for hard-to-interpret signals with the corresponding measure for risky signals.

Before moving to results, one may wonder what would be the effect of replacing the ambiguous signals with two equally likely possible signals. As argued above, there is now considerable empirical evidence that many DMs identify ambiguous environments with compound but risky environments in which they do not reduce compound lotteries. This evidence suggests that modeling ambiguity as a compound object has sound behavioral support, and contributes to the understanding of new dimensions of ambiguity.

The experiment was conducted at the Toronto Experimental Economics Laboratory in March 2018. Subjects had to answer 12 comprehension questions, and were incentivized by \$0.25/question to answer each correctly on their first attempt (they had to answer it correctly before moving to the next question/stage). The experiment was programmed in zTree (Fischbacher 2007). The potential prize in the experiment was \$20 plus a show-up payment of \$7 and a maximum of \$3 as payment for answering the comprehension question correctly. We recruited 153 subjects: 67 for the risk control and 86 for the ambiguous signal treatment. The instructions as well as the experimental interface are included in Appendix E.

3.2 Results

In this section, we first report results for all subjects. Later, we omit subjects whose behavior is inconsistent with any model of updating based on the objective information provided in the experiment, as we believe that it reflects confusion rather than deliberate choice. We report mainly probability equivalents (PE) that were elicited using choice lists. Option A was always held constant and represented a bet on the payoff urn (unconditional and conditional), while option B was an objective one-stage bet on the number of the ball drawn from a basket containing balls numbered from 1 to 100.[11]

44.81% of subjects (69 out of 154) have unconditional PE of approximately .5, close to 46.75% (72 subjects) have unconditional PE lower than .475 and the remainder 8.44% (13 subjects) higher than .525. These are standard results for 2-point ambiguity and compound risk attitude (Halevy 2007; Chew et al 2017), and justify our behavioral approach of using compound lotteries to mirror two point ambiguity.[12] As expected, there is no treatment effect when measuring unconditional PE.

Although not the main focus of our study, as it does not take into account the unconditional PE, it is interesting to note how subjects respond to favorable and unfavorable signals when the signal is risky and when it is ambiguous, assuming an unconditional prior of .5. In the risky-signal control, only 13.24% of subjects (9 out of 68) had favorable PE that was approximately the Bayesian update of .5 (.66 +/- .025), while almost two-thirds of the others had PE that was below this. In the ambiguous signal treatment, and assuming equally likely signals, only a single subject (out of 86) had PE that was approximately the Bayesian update of .5 (.676 +/- .025), while more than three quarters of the others had PE that was below the Bayesian benchmark of .5. For the unfavorable signal, the difference between

[11] We calculated the probability equivalent as the average of the last line in which Option A was chosen and the first line in which Option B was chosen. For subjects whose choices are consistent with monotonic and transitive preferences, these lines will be consecutive (single switching point, e.g. if a subject switched to B at .5 then the PE would be .495). If a subject switches multiple times between A and B, this is the midpoint in the range of switching (so if the last time A was chosen is .6 and the first time B was chosen was .5, the reported PE would be .55). Throughout this section we allow subjects 2.5% margin to standard behavior: reduce compound lotteries and Bayesian updating. We term this "approximate".

[12] 10 subjects had multiple switching, only 1 of them had unconditional PE of approximately .5 (6 had PE$<$.475 and 3 had PE$>$.525).

the treatments is even starker. 19.12% of subjects in the risk control (13 of 68) are close to the Bayesian update of .5 (.34 +/- .025), and the remainder are equally split between those whose unfavorable PE is higher and lower than the benchmark. In the ambiguous signal treatment, only 3 out of 86 are approximately Bayesian (.324 +/- .025), assuming equally likely signals and a prior of .5), and almost 94% (78 subjects) of the others have PE that is higher than the Bayesian benchmark.

Three comments are in order regarding these observations. First, as noted above, most subjects do not start from an unconditional prior of .5 (it is typically lower). Second, even if the average behavior in the risky signal treatment is not too far from the Bayesian update of .5, there is huge heterogeneity at the individual level. Third, the tendency to underreact to an unfavorable ambiguous signal can be rationalized by the belief that the signal is more likely to be less informative (it is more likely that the signal urn contains 100 balls rather than 10 balls). Indeed, this is consistent with the tendency to underreact to a favorable signal as well. This demonstrates the necessity to measure *individual* behavior using (3.2), as it ties together the conditional (favorable and unfavorable) PEs and unconditional PE, allows for non-neutral attitude to prior ambiguity (compound-risk), and answers the crucial question if there exists a prior over the possible signals that can rationalize the unconditional/conditional PEs as a result of Bayesian updating.

3.2.1 Bayesian updating

In the control risky-signal treatment 32 out of 68 subjects (47.06%) are approximately Bayesian in the sense that their unconditional PE is approximately the average of their conditional PE, while in the ambiguous-signal treatment the proportion falls to 27 out of 86 (31.4%). The increase in the incidence of non-Bayesian behavior as a response to hard-to-interpret signals is significant at the 5% level (p-values of one-sided proportion test and one sided Fisher exact test are .0162 and .024, respectively).

One might suspect that the noted difference is due to subjects making more mistakes in the cognitively more challenging ambiguous-signal treatment. We therefore omit from both treatments subjects who made various errors. We consider two types of "errors" that are incompatible with standard models of preference and updating. The first is multiple switching between options A and B in one or more of the choice lists (and here there is no signif-

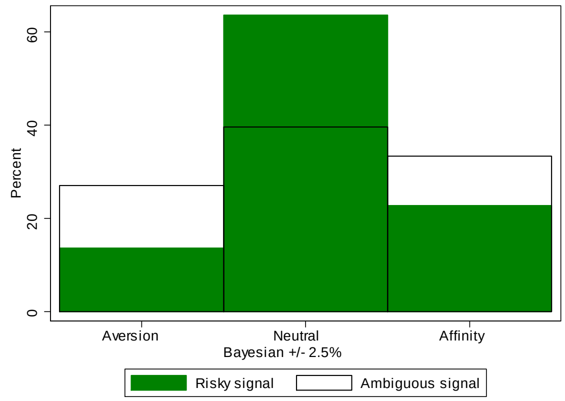


Figure 3.4: Bayesian behavior and attitude to signal ambiguity - regular subjects

icant difference between the treatments).[13] The second mistake is updating in the wrong direction between the unconditional and conditional PEs.[14] In the ambiguous-signal treatment 36 out of 86 subjects (41.86%) updated in the wrong direction, and in the risky-signal treatment 21 out of 68 subjects (30.88%) updated in the wrong direction. The difference is marginally significant (p-values of one-sided proportion test and one-sided Fisher exact test are .0806 and .109, respectively). In order to test if the difference in behavior that is incompatible with Bayesian updating is due to error, we omit the subjects who committed one or more of the above mistakes. This leaves us with 44 (out of 68) "regular" subjects in the risky-signal treatment and 48 (out of 86) "regular" subjects in the ambiguous-signal treatment. These "regular" subjects switched exactly once between options A and B in all the choice lists, and always updated in the correct direction.

Figure 3.4 demonstrates that the proportion of "regular" subjects who are approximately Bayesian falls significantly from 63.64% in the risky-signal treatment to 39.58% in the ambiguous-signal treatment (p-values of one-sided

[13]In the risky-signal control treatment 4 out of 68 subjects switched more than once between A and B, while in the ambiguous-signal treatment the number was 6 out of 86 (and only 1 of them in one of the conditional PEs).

[14]That is, if the signal is favorable to the chosen bet on the payoff urn - the conditional PE should not decrease, and if the signal is unfavorable - the conditional PE should not increase.

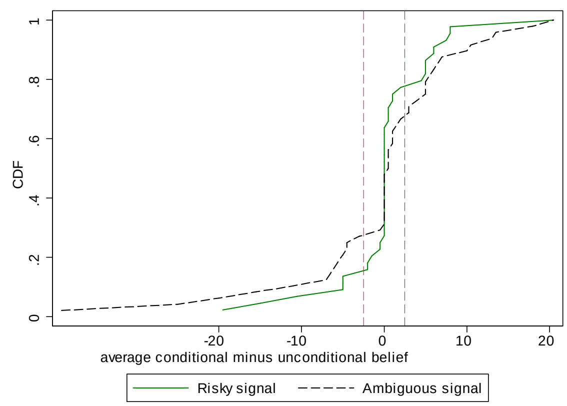


Figure 3.5: CDF of Bayesian updating for regular subjects

proportion test and one sided Fisher exact test are .0106 and .018, respectively). Because these subjects are "mistake-free", we attribute the greater departure from Bayesian updating under ambiguous signals as being due to a deliberate response to uncertainty concerning the signals. It is interesting to note that the marginal effect of signal ambiguity is almost equally split between an increase in the proportion of subjects who are averse to signal ambiguity - the average of the conditionals is lower than the unconditional PE (from 13.64% under risk to 27.08% with ambiguous signals), and those who exhibit affinity to signal ambiguity - the average of the conditionals is higher than the unconditional PE (from 22.73% to 33.33%).

Figure 3.5 plots the difference in the distributions of our proposed measure for Bayesian behavior (3.2) between the risky-signal treatment (green full line) and the ambiguous-signal (dashed line). A value of "0" on the horizontal axis implies that the unconditional PE equals the average of the two conditional PEs, that is – the subject is *exactly Bayesian*. The interval between the two vertical dashed lines is the 5% interval: $[-2.5\%, +2.5\%]$ in which we classify subjects as *approximate Bayesian*. Subjects that are to the left (right) of the interval have PEs that indicate that they are averse (seeking) to signal ambiguity. As can be easily seen from the figure, the distribution of (3.2) in the ambiguous-signal treatment has a higher variance than in the risky-signal control (p-value of Levene's test for equality of variances is .0425).

Finally, it is of interest to understand the relation between the attitudes towards prior-ambiguity and towards signal-ambiguity. As discussed above, we have only an indirect measure of the former, since using unconditional PE measures attitude to 2-point compound-risk that has been shown to be strongly associated with attitude to two-point ambiguity (Halevy, 2007; Chew et al 2017). We find that among "regular" subjects the association between approximately reducing compound lotteries (and indirectly – being neutral to prior ambiguity), and being approximately Bayesian in the risky-signal control is insignificant at the 5% level (p-value of Fisher's exact test is .065). In contrast, in the ambiguous-signal treatment the association is very strong (p-value of Fisher's exact test is .001).[15]

4 Models

The previous section showed that sensitivity to signal ambiguity is common, but not universal. Decision-makers vary in their attitude to hard-to-interpret signals: some are close to the Bayesian benchmark, others are averse, while the remainder like signal ambiguity. Moreover, we found that non-neutrality to signal ambiguity is associated with aversion to prior ambiguity (as captured by compound risk). The goal of this section (and Appendix C) is to demonstrate (in a non-exhaustive way) how some popular models accommodate the various patterns of behavior and the associations among behaviors documented in the experiment.

As emphasized, we explore choice given two different information structures – no signals, and then a particular signal structure as defined above. Existing static models of ambiguity-sensitive preference restrict attention to one fixed (implicit) information structure (Gajdos et al (2008) is the only exception of which we are aware) and thus do not apply directly. Put another way, one could apply any of these models separately to model $\succeq_0$ and each conditional order $\succeq_\sigma$. Applied in this way, received theories would not address updating in that they would not restrict how unconditional and conditional preferences are related, rendering (2.5), as well as many other

[15] If we consider *all* subjects, the associations are strong in both treatments (p-values of Fisher's exact tests are .007 in the risky-signal control and $<$.001 in the ambiguous-signal treatment), as subjects who make mistakes tend not to reduce compound lotteries and not to be Bayesian. However, we find this observation less interesting for understanding signal-ambiguity in a rational framework.

patterns of unconditional and conditional choices, rationalizable. We view this approach as conceding that received theories are orthogonal to the issues considered here. We proceed instead by examining whether extensions of these models that include plausible and/or commonly used updating rules can accommodate signal ambiguity. Another point to emphasize is that our treatment of models is intended to be illustrative rather than exhaustive. After examining the benchmark Bayesian model, we focus on the maxmin model (Gilboa and Schmeidler 1989) with two alternative updating rules. (See also Appendix D for an examination of the smooth ambiguity model.)

For all models, preferences are defined on a set $\mathcal{F}$ of Savage acts over the state space S with outcomes in X. As above, we assume for the most part that $S = \{R, B\}$, $X = \{100, 0\}$, $\mathcal{F} = \{f_R, f_B\}$, and that the signal space is $\mathbf{\Sigma} = \{\sigma_R, \sigma_B\}$, but arguments extend readily to the general setup treated in Appendix B. Risk preferences are expected utility with vNM index u normalized by

$$u(100) = 1,\ u(0) = 0.$$

Utility functions on $\mathcal{F}$, denoted $V_\sigma(\cdot)$, for $\sigma \in \{0\} \cup \mathbf{\Sigma}$, are defined by probability equivalents:

$$V_\sigma(f_R) = p_{\sigma,R} \text{ and } V_\sigma(f_B) = p_{\sigma,B}.$$

Symmetry (2.4) and signal diversity (2.2) are assumed throughout. We examine the capacity of models to accommodate (2.5) and its ambiguity loving counterpart where the inequality is reversed.

4.1 Models with "Bayesian updating"

In the Bayesian model, unconditional utility has the subjective expected utility (SEU) form with respect to prior belief m_0,

$$V_0(f) = \int_S u(f)\, dm_0(s),\ \ f \in \mathcal{F}.$$

Conditional utility $V(\cdot \mid \sigma)$ is given by SEU with the posterior $m(\cdot \mid \sigma)$ which is derived by Bayesian updating using a likelihood function $\ell(\sigma \mid s)$. Exclude the degenerate case where signals are uninformative and assume that

$$\ell(\sigma_R \mid R) \neq \ell(\sigma_R \mid B); \tag{4.1}$$

this implies signal diversity. The well-known implication of this model is that

$$\begin{aligned} m_0(\cdot) &= \sum_\sigma L(\sigma)\, m(\cdot \mid \sigma) \text{ and} \\ V_0(f) &= \sum_\sigma L(\sigma)\, V_\sigma(f),\ f \in \mathcal{F}, \end{aligned} \tag{4.2}$$

where

$$L(\sigma) \equiv \int_S \ell(\sigma \mid s)\, dm_0(s).$$

Therefore, by Lemma 1, indifference to signal ambiguity (that is, equality in (2.5)) is implied.

Note that the preceding applies to *any likelihood function* ℓ (and more generally, for any L consistent with (4.2)), just as the Ellsberg paradox is robust to which prior is assumed. In particular, it applies to two variants of the above Bayesian model that have been explored in the literature. To capture uncertainty about interpretation of signals, and hence about the true likelihood function, Acemoglu, Chernozhukov and Yildiz (2016) assume that updating of m_0 is done using an average likelihood $\overline{\ell}$ of the form

$$\overline{\ell}(\sigma \mid s) = \int \ell(\sigma \mid s)\, d\lambda_s(\ell),$$

where, for each s, $\lambda_s \in \Delta(\Delta(\{\sigma_R, \sigma_B\}))$ is a subjective distribution over likelihoods. Conclude that this specification does not model hard-to-interpret signals in the sense of the behavior we have identified. In another variant, it is assumed that the Bayesian agent uses the "wrong" likelihood function, specifically, one in which signals are taken to be more precise than they really are. Such agents are often called "overconfident" (Daniel, Hirshleifer and Subrahmanyam 1998). We see that such overconfidence is behaviorally distinguishable from an affinity to signal ambiguity.

Indifference to signal ambiguity is implied also in models that can rationalize (unconditional) Ellsbergian ambiguity aversion if a suitable "Bayesian-like" updating rule is added. See Epstein and Seo (2015) for one such model.

4.2 Maxmin utility

Following Gilboa and Schmeidler (1989), ambiguity about S is represented by a subjective set $\mathcal{M}_0 \subset \Delta(S)$, and unconditional utility is given by

$$V_0(f) = \min_{m \in \mathcal{M}_0} \int u(f)\, dm.$$

In the alternative scenario, the individual is informed that a signal will be realized. Thus she contemplates uncertainty about $\{\sigma_R, \sigma_B\} \times S$, which is modeled by a subset $\mathcal{M}$ of $\Delta(\{\sigma_R, \sigma_B\} \times S)$. We assume that $\mathcal{M}$ is consistent with $\mathcal{M}_0$ in the sense that $\mathcal{M}_0$ equals the set of all S-marginals of measures in $\mathcal{M}$, that is,

$$\mathcal{M}_0 = \{mrg_S\ m : m \in \mathcal{M}\}. \tag{4.3}$$

After realization of the signal σ, the individual updates her set of priors to $\mathcal{M}_\sigma \subset \Delta(S)$ and she evaluates acts using the conditional maxmin utility function

$$V_\sigma(f) = \min_{m \in \mathcal{M}_\sigma} \int u(f)\, dm.$$

It remains to describe $\mathcal{M}$ and the sets $\mathcal{M}_\sigma$ in greater detail. We consider two widely used update rules: prior-by-prior Bayesian updating (also known as generalized Bayes' rule (GBR)), and maximum likelihood updating (ML), whereby only those priors that maximize the likelihood of the realized signal are retained and updated by Bayes' rule.[16] We further divide the discussion into two cases that highlight the main message regarding how to model signal ambiguity within the framework of maxmin utility. (See Appendix C for supporting details.)

Single-likelihood: For each s, let $\ell(\cdot \mid s) \in \Delta(\{\sigma_R, \sigma_B\})$ describe the distribution of signals conditional on the true state satisfying (4.1). The critical feature of this special case is that this conditional distribution, or likelihood function, is unique as in Bayesian modeling. To incorporate this sharp view of likelihoods let $\mathcal{M}$ consist of all measures m on $\{\sigma_R, \sigma_B\} \times S$ for which the S-marginal lies in $\mathcal{M}_0$ and the S-conditional is ℓ. Then, *for both of the noted updating rules, affinity to signal ambiguity* is implied (and the affinity is strict if $\mathcal{M}_0$ is not a singleton).

Multiple-likelihoods: To sharpen the contrast with the preceding case, suppose that unconditional beliefs about S are represented by the single (full support) prior m_0, that is, $\mathcal{M}_0 = \{m_0\}$. Multiplicity arises at the level of conditional distributions or likelihoods: let $\mathcal{L}$ denote a subjective set of possible likelihood functions ℓ, where $\ell(\cdot \mid s) \in \Delta(\{\sigma_R, \sigma_B\})$ for every s. Define

[16] Pires (2002) provides axiomatic foundations for GBR; Gilboa and Schmeidler (1993) axiomatize ML in the special case where the maxmin model of preference with set $\mathcal{M}$ also conforms with Choquet expected utility (Schmeidler 1989).

$\mathcal{M}$ to be the set of all measures m on $\{\sigma_R, \sigma_B\} \times S$ for which the S-marginal is m_0 and, for which the S-conditional is an element of $\mathcal{L}$. Then *signal ambiguity aversion is implied for both updating rules.* The intuition is apparent at the functional form level: the multiplicity of likelihoods captures uncertainty about how to interpret a given signal and permits the adopted interpretation to vary with the bet being evaluated. For example, when evaluating the bet on red (black), a signal σ is interpreted conservatively in the way least (most) favorable to red being drawn. This acts to reduce conditional utility levels for each bet and each signal, consistent with (2.5).

Note that unconditional ambiguity aversion (satisfied by single-likelihood but not by multiple-likelihood as defined above) and signal ambiguity aversion (satisfied by multiple-likelihood but not by single-likelihood) are independent properties. Simultaneous aversion to both kinds of ambiguity can be achieved by perturbing initial beliefs in the multiple-likelihoods model slightly and taking $\mathcal{M}_0$ to be a small neighborhood of m_0.

5 Conclusion

We provide a counterpart to Ellsberg's experiments, which focus on prior ambiguity, by considering the response to information in environments where information is available but is compatible with different interpretations and hence inferences. We suggest that such decision environments are common, including, for example, the recent COVID-19 pandemic. After identifying revealed sensitivity to hard-to-interpret information with failure of the martingale property of belief, we document experimentally that many subjects (especially those who are averse to prior ambiguity) respond in such a way to ambiguous signals.

A Appendix: Discussion

Here we provide additional theoretical motivation for (2.5). We do so by adapting the usual practice in the literature on unconditional ambiguity-sensitive preferences where behavior in the "ambiguous" domain (bets on Ellsberg's unknown urn) is compared with behavior in the risk domain (bets on Ellsberg's known urn). Behavior in the risk domain is assumed to be "standard" (expected utility), and differences in behavior across the domains are attributed to non-indifference to ambiguity. Because our focus is on updating behavior, we go further in these respects. In our case, the comparison risk domain includes also risky ("noisy") signals and in addition, updating in the risk domain is assumed to conform to Bayes' rule.[17] Then, for each color in turn, we compare bets on that color from the payoff urn versus from a risky urn (constructed below) *both unconditionally, and then also conditionally* after realization of both ambiguous and noisy signals.

To elaborate, fix two possible noisy signals, denoted also σ_R and σ_B, and their probabilities given by $\alpha \in \Delta(\mathbf{\Sigma})$. For each color $s \in \{R, B\}$, the following table describes two trees, a subjective one corresponding to the choice problems in Section 2.1 (left column), and an objective tree (right column). In the objective tree, if the α_σ-signal is realized, Bayesian updating leads to the posterior lottery $P_{\sigma,s}$. In the subjective tree, the signal σ leads to the conditional preference order $\succeq_\sigma$ according to which f_s is indifferent to $P_{\sigma,s}$ (recall (2.1)).

$$\begin{array}{ccc}
f_s & \succ_0 & \sum_\sigma \alpha_\sigma P_{\sigma,s} \\
\downarrow & & \downarrow \\
\text{signal } \sigma & \& & \alpha_\sigma\text{-event for risk} \\
\Downarrow & & \Downarrow \\
\text{subjective updating} & & \text{Bayes' rule} \\
f_s & \sim_\sigma & P_{\sigma,s}
\end{array} \tag{A.1}$$

Moreover, this is true for both colors s and both signals σ. Yet, for each s, f_s is strictly preferable to $\sum_\sigma \alpha_\sigma P_{\sigma,s}$ unconditionally. We attribute this to

[17] This discussion applies to the thought experiment. In the experiment itself, however, we will not make such an assumption, but compare the updating behavior under risk and ambiguity (see Section 3.1.)

the higher cognitive cost of updating based on hard-to-interpret signals that is reflected in the conditional lottery equivalents $P_{\sigma,s}$, but is absent at the unconditional stage and also in the objective tree. (An alternative explanation is that the objective mixture is ranked lower because risk preferences exhibit a dislike for randomization – utility over lotteries is quasiconvex. This interpretation is excluded only by the assumption of vNM risk preferences.)

The preceding suggests the following definition of (strict) aversion to signal ambiguity:[18] There exists $\alpha \in \Delta(\mathbf{\Sigma})$ such that, for each $s = R, B$,

$$\begin{array}{lll} f_s & \succ_0 & \alpha_{\sigma_R} P_{\sigma_R,s} + \alpha_{\sigma_B} P_{\sigma_B,s} \\ f_s & \sim_{\sigma_R} & P_{\sigma_R,s} \\ f_s & \sim_{\sigma_B} & P_{\sigma_B,s}, \end{array} \tag{A.2}$$

or equivalently, there exists $\alpha \in \Delta(\mathbf{\Sigma})$ such that,

$$\begin{array}{lll} p_{0,R} & > & \alpha_{\sigma_R} p_{\sigma_R,R} + \alpha_{\sigma_B} p_{\sigma_B,R} \text{ and} \\ p_{0,B} & > & \alpha_{\sigma_R} p_{\sigma_R,B} + \alpha_{\sigma_B} p_{\sigma_B,B}. \end{array} \tag{A.3}$$

Though it might seem less demanding than (2.5), where each α_σ is fixed at $\frac{1}{2}$, in fact, the two conditions are equivalent under present assumptions.

Lemma 1 *Assuming signal diversity, the conditions (2.5) and (A.3) are equivalent.*

Proof. Given symmetry (2.4), condition (2.5) implies (A.3) with $\alpha_\sigma = \frac{1}{2}$ for all σ. Conversely, suppose there exists α as indicated but that $p_{0,R} \leq \frac{1}{2} p_{\sigma_R,R} + \frac{1}{2} p_{\sigma_B,R}$. Then, for $s = R, B$,

$$\begin{array}{rcl} \Sigma_\sigma \alpha_\sigma p_{\sigma,s} & < & p_{0,s} \leq \frac{1}{2} p_{\sigma_R,s} + \frac{1}{2} p_{\sigma_B,s} \Longrightarrow \\ 0 & < & \left(\frac{1}{2} - \alpha\right) \left(p_{\sigma_R,s} - p_{\sigma_B,s}\right). \end{array}$$

But this is impossible given signal diversity (2.2). ■

The intuition leading to (A.2) thus serves to explain and motivate (2.5). Next we add perspective by examining the existential quantifier "there exists α" in (A.2). An alternative that may have occurred to some readers is to require that the behavior indicated in (A.2) be exhibited for all signal

[18] An equivalent condition would have unconditional indifference and conditional strict preference for f_s for each signal.

structures, that is, "for all α." This condition is equivalent (given symmetry) to requiring that

$$p_{0,R} > p_{\sigma_R,R} \text{ and } p_{0,R} > p_{\sigma_B,R},$$

or, more explicitly in terms of preferences,

$$f_R \succ_0 P_{\sigma,R} \text{ for } \sigma \in \{\sigma_R, \sigma_B\}\,. \tag{A.4}$$

This says roughly that, for each σ, $\succeq_\sigma$ is *more ambiguity averse than* $\succeq_0$ in the sense of the comparative notion widely adopted in the decision theory literature and built on the following intuition: given that $f_R \sim_\sigma P_{\sigma,R}$, that $P_{\sigma,R}$ is ambiguity-free, and that $\succeq_0$ is less averse to ambiguity, then $f_R \succ_0 P_{\sigma,R}$ follows. Thinking of ambiguous signals as increasing ambiguity aversion, one might take (A.4) as the behavioral meaning of signal ambiguity aversion.

Clearly, (A.4) is strictly more demanding than our condition in (A.2), or equivalently (2.5).[19] Moreover, it is too strong in our view. A signal σ may reduce utility because of uncertainty about its interpretation, but, in general, a signal also contains information about the state space that could render the bet under consideration more attractive, thus raising utility. Think of two dimensions of a signal – its "mean informational content" and "uncertainty about that content" – that may affect utility in opposite directions. The condition (A.4) identifies as ambiguous only signals for which the uncertainty effect dominates for all signal realizations. In contrast, the behavior that we propose can identify uncertainty about a signal's interpretation even if, for some realizations, its mean informational content dominates and results in an overall increase in utility. To see how, suppose that contrary to (A.4), $P_{\sigma_R,R} \succ_0 f_R$, and that there is nevertheless uncertainty about the interpretation of σ_R. The indicated strict ranking reveals that σ_R is a very strong positive signal in the mean dimension for drawing red, strong enough to more than offset difficulties with interpretation. But then it is also a very unfavorable signal for drawing black. Thus σ_R makes the bet on black unattractive because of *both* uncertainty about interpretation *and* because of its negative mean informational content. This can lead to its lottery equivalent $P_{\sigma_R,B}$ being sufficiently unattractive that, assuming symmetry (2.4), $f_R \succ_0 \frac{1}{2}P_{\sigma_R,R} + \frac{1}{2}P_{\sigma_B,R}$ is satisfied. (This possibility is illustrated in the numerical example in Section 2.2 when, for instance, $p_{\sigma_R,R} > \frac{1}{2}$.)

[19] By the independence axiom, $P_{\sigma,R} \succeq \frac{1}{2}P_{\sigma_R,R} + \frac{1}{2}P_{\sigma_B,R}$ for at least one $\sigma \in \{\sigma_R, \sigma_B\}$. Thus, by transitivity, (A.4) implies (A.2). It is easy to see that the converse is false.

B Online Appendix: A more general analysis

B.1 Primitives

- S: finite (payoff relevant) state space
- $\mathbf{\Sigma}$: finite set of signals
- X: set of real-valued outcomes with largest and smallest elements (say 100 and 0)
- Acts f map S into X; $\mathcal{F}$ is a *(fixed) finite subset* of acts
- $\Delta(X)$: the set of all (simple) lotteries P
- Preferences $\succeq_0$ and $\{\succeq_\sigma\}_{\sigma\in\mathbf{\Sigma}}$ on $\mathcal{F}\cup\Delta(X)$

Adopt the following basic assumptions on preferences:

Pref0 All preferences are complete and transitive.

Pref1 All conditional preferences agree with $\succeq_0$ in the ranking of lotteries.

Pref2 $\succeq_0$ restricted to lotteries conforms to expected utility theory.

Pref3 $\succeq_0$ is strictly FOSD-monotone on lotteries

Pref4 For each $\sigma\in\{0\}\cup\mathbf{\Sigma}$, and act f, $\exists$ *probability-equivalent* $p_{\sigma,f}$, such that $P_{\sigma,f}=(100,p_{\sigma,f};0,1-p_{\sigma,f})\sim_\sigma f$

Pref1 expresses the assumption that signals are unrelated to the objective prospects (lotteries). Pref2 is almost universal in the decision theory literature focussing on ambiguity. Pref3 and Pref4 are self-explanatory and common. These assumptions permit construction of utility functions $V_\sigma(\cdot)$ for $\succeq_\sigma$, $\sigma\in\{0\}\cup\mathbf{\Sigma}$, where,

$$V_\sigma(f)=p_{\sigma,f},\ \text{ for all } f\in\mathcal{F},$$

and, for all $P\in\Delta(X)$,

$$V_\sigma(P)=p,\text{ where } P\sim_0(100,p;0,1-p).$$

These utility functions render meaningful the comparison of utility levels unconditionally and across different signals. In particular, the inequality

$$V_{\sigma'}(f) > V_{\sigma}(f), \text{ for given } \sigma' \neq \sigma \in \mathbf{\Sigma},$$

is equivalent to the preference statement

$$[f \sim_{\sigma'} P' \text{ and } f \sim_{\sigma} P] \implies P' \succ_0 P,$$

It is interpreted to mean that σ' *is a better signal for* f *than is* σ.

Remark 2 *Finiteness of the set of acts $\mathcal{F}$ is not typical in axiomatic studies but is entirely appropriate in underpinnings for experiments where one elicits risk equivalents of only finitely many acts. The attitudes towards signal ambiguity defined below depend on the empirically relevant set $\mathcal{F}$.*

Refer to *signal diversity* (relative to $\mathcal{F}$) if: For every disjoint subsets $\Sigma_I, \Sigma_{II} \subset \Sigma \backslash \{\sigma_1\}$, at least one nonempty, $\exists f \in \mathcal{F}$ s.t.

$$\begin{aligned} p_{\sigma,f} &> p_{\sigma_1,f} \text{ if } \sigma \in \Sigma_I \\ p_{\sigma,f} &< p_{\sigma_1,f} \text{ if } \sigma \in \Sigma_{II}, \end{aligned}$$

that is, σ is better (worse) than σ_1 for f if $\sigma \in \Sigma_I$ (Σ_{II}). If $\mathbf{\Sigma} = \{\sigma_1, \sigma_2\}$ is binary, then reduces to: $\exists g, h \in \mathcal{F}$ s.t.

$$(p_{\sigma_2,g} - p_{\sigma_1,g})(p_{\sigma_2,h} - p_{\sigma_1,h}) < 0$$

that is, σ_1 is better for one act and σ_2 is better for the other, as in (2.2).

B.2 Attitudes: definitions and characterizations

Define attitudes to signal ambiguity as follows (strict notions can be defined in the obvious way).

Definition 3 *Weak aversion: There exists $\alpha \in \Delta(\mathbf{\Sigma})$ s.t.*

$$V_0(f) \geq \sum_{\sigma} \alpha_\sigma V_\sigma(f) \text{ for all } f \in \mathcal{F}. \tag{B.1}$$

Weak affinity: There exists $\alpha \in \Delta(\mathbf{\Sigma})$ s.t.

$$V_0(f) \leq \sum_{\sigma} \alpha_\sigma V_\sigma(f) \text{ for all } f \in \mathcal{F}. \tag{B.2}$$

Indifference: There exists $\alpha \in \Delta(\mathbf{\Sigma})$ *such that*

$$V_0(f) = \sum_{\sigma} \alpha_\sigma V_\sigma(f) \text{ for all } f \in \mathcal{F}. \tag{B.3}$$

Intuition for these definitions is similar to that derived from (A.1). In the SEU framework, when updating conforms to Bayes' rule, (B.3) reduces to the familiar martingale condition relating prior and posterior beliefs. Our intention here is to identify it as a meaningful condition more generally where preferences over acts are not necessarily SEU and beliefs are not necessarily representable by probability measures.

The presence of the existential quantifiers $\exists\alpha$ raises two questions about these definitions. First, is indifference equivalent to the conjunction of weak aversion and weak affinity? Second, and more practically, can the defining conditions be verified? The next theorem addresses these concerns.

Theorem 4 *(i) There is weak aversion indifference to signal ambiguity iff*

$$\min_{\sigma\in\mathbf{\Sigma}} \left(\int V_\sigma(f)\, d\beta \right) \leq \int V_0(f)\, d\beta \text{ for all } \beta \in \Delta(\mathcal{F}). \tag{B.4}$$

(ii) There is weak affinity to signal ambiguity iff

$$\int V_0(f)\, d\beta \leq \max_{\sigma\in\mathbf{\Sigma}} \left(\int V_\sigma(f)\, d\beta \right) \text{ for all } \beta \in \Delta(\mathcal{F}). \tag{B.5}$$

(iii) There is weak indifference to signal ambiguity iff $\forall\beta \in ba(\mathcal{F})$,[20]

$$\min_{\sigma\in\mathbf{\Sigma}} \left(\int V_\sigma(f)\, d\beta \right) \leq \int V_0(f)\, d\beta \leq \max_{\sigma\in\mathbf{\Sigma}} \left(\int V_\sigma(f)\, d\beta \right). \tag{B.6}$$

Assuming signal diversity, then: (a) weak indifference is also equivalent to the conjunction of weak aversion and affinity; and (b) $\alpha = (\alpha_\sigma)$ *in the martingale condition is unique.*

In each case, the corresponding equivalent statement replaces the existential quantifiers for α with more customary and preferable universal quantifier (see Section B.3 for how the reformulation aids verifiability). The condition

[20] $ba(\mathcal{F})$ is the set of signed measures on $\mathcal{F}$. Given finiteness of $\mathcal{F}$, it is isomorphic to $\mathbb{R}^{|\mathcal{F}|}$.

(B.6) can be simplified since the left-hand inequality is redundant given that β is not restricted in sign. However, it is strictly stronger than the act-by-act condition

$$\min_{\sigma \in \mathbf{\Sigma}} V_\sigma(f) \leq V_0(f) \leq \max_{\sigma \in \mathbf{\Sigma}} V_\sigma(f) \text{ for all } f \in \mathcal{F},$$

which would be sufficient if in (B.3) we allowed α to vary with f. The conjunction of (i) and (ii) is weaker than (B.3), because the former asserts only existence of two measures, one for (B.1) and another, generally distinct, measure for (B.2), while (B.3) asserts that there is a single measure satisfying both inequalities. Accordingly, the characterization (B.6) is stronger than the conjunction of (B.4) and (B.5) because the βs are not restricted to be probability measures. However, under signal diversity, the conjunction of weak aversion and weak affinity is equivalent to indifference.[21]

Signal diversity also guarantees other desirable properties. For example, define strict attitudes by the obvious strict inequality counterparts of (B.1) and (B.2). Then, for example, strict aversion (affinity) and weak affinity (aversion) are disjoint if signal diversity is satisfied.

Part (iii) of the theorem can be interpreted as providing an *axiomatization* for the property (B.3) of updating and doing so for a very broad class of preferences.[22] There is arguably a rough parallel with Machina and Schmeidler (1992). They generalize SEU and axiomatize probabilistically sophisticated preferences – those for which underlying beliefs can be represented by a probability measure; and they do so without unduly restricting other aspects of preference. We generalize the other main component of the Bayesian model, namely Bayesian updating, and we axiomatize those collections $\{V_\sigma\}_{\sigma \in \{0\} \cup \mathbf{\Sigma}}$ of preferences that satisfy the key martingale property of Bayesian updating; and we do so without assuming maxmin or any other parametric class of preferences, and without specifying a particular updating rule beyond what is implicit in (B.3) or (B.6). Another parallel is that just as probabilistic sophistication defines a benchmark for modeling sensitivity to unconditional ambiguity of the sort highlighted by Ellsberg, we propose (B.3) as a benchmark for modeling sensitivity to signal ambiguity.

---

[21] The proof is elementary. For example, assume that (B.1) and (B.2) are satisfied with α and α' respectively. Then $\sum_{\sigma \neq \sigma_1} (\alpha_\sigma - \alpha'_\sigma)(V_\sigma(f) - V_{\sigma_1}(f)) \leq 0$ for all f, which contradicts signal diversity unless $\alpha = \alpha'$ (take $\Sigma_I = \{\sigma \neq \sigma_1 : \alpha_\sigma > \alpha'_\sigma\}$ and $\Sigma_{II} = \{\sigma \neq \sigma_1 : \alpha_\sigma < \alpha'_\sigma\}$).

[22] Condition (B.6) is a full-fledged axiom because the utility values $V_\sigma(f)$ are probability equivalents and hence observable. Its interpretation is not clear however.

B.3 Verifiability

Here we show that the alternative characterizations (B.4) and (B.5) in Theorem 4 provide a tractable way to check whether a given data set is consistent with weak aversion or weak affinity. By "data," we mean probability equivalents elicited along the lines of our thought (and laboratory) experiments. Utility values for each act are equal to the corresponding probability equivalents–hence, it merits emphasis that the utility values appearing in the theorem are observable. When a similar procedure is applied to check for strict aversion, (using the obvious strict counterpart of the theorem), one obtains a generalization of the inequality (2.5) which is the focus of the text. That presumed a binary environment and the symmetry expressed by (2.4), while these restrictions are not needed in Theorem 4.

Consider the practical value of the characterization (B.4) for verifying (B.1): The former can be written as

$$\max_{\beta \in \Delta(\mathcal{F})} \Phi(\beta) \leq 0,$$

where $\Phi(\beta) = \min_{\sigma \in \mathbf{\Sigma}} \left(\int V_\sigma(f)\, d\beta \right) - \int V_0(f)\, d\beta$. Finding a maximizer is a matter of linear programming because Φ is piecewise linear.

To illustrate, consider the thought experiment and revert to earlier notation. Then

$$\begin{aligned} \Phi(\beta) \quad = \quad & \min \left\{ \beta p_{\sigma_R,R} + (1-\beta)\, p_{\sigma_R,B}, \beta p_{\sigma_B,R} + (1-\beta)\, p_{\sigma_B,B} \right\} \\ & -(\beta p_{0,R} + (1-\beta)\, p_{0,B}) \end{aligned} \tag{B.7}$$

The maximum is achieved at $\beta^* = 0, 1$, or β^c,

$$\beta^c = \frac{1}{1 + \frac{p_{\sigma_R,R} - p_{\sigma_B,R}}{p_{\sigma_B,B} - p_{\sigma_R,B}}}.$$

β^c is that weight β for which the two terms inside the minimization in (B.7) are equal:

$$\beta^c p_{\sigma_R,R} + (1-\beta^c)\, p_{\sigma_R,B} = \beta^c p_{\sigma_B,R} + (1-\beta^c)\, p_{\sigma_B,B}. \tag{B.8}$$

Thus weak aversion is *equivalent* to $\Phi(\beta) \leq 0$ at these three values of β and hence (by brute calculation) to:

$$\begin{aligned} & \beta^* p_{0,R} + (1-\beta^*)\, p_{0,B} \\ \geq \quad & \min \left\{ \beta^* p_{\sigma_R,R} + (1-\beta^*)\, p_{\sigma_R,B}, \beta^* p_{\sigma_B,R} + (1-\beta^*)\, p_{\sigma_B,B} \right\}, \end{aligned} \tag{B.9}$$

where

$$\beta^* = \begin{cases} 0 & p_{0,R} - p_{0,B} > p_{\sigma_R,R} - p_{\sigma_R,B} \\ 1 & p_{0,R} - p_{0,B} < p_{\sigma_B,R} - p_{\sigma_B,B} \\ \frac{1}{1+\frac{p_{\sigma_R,R}-p_{\sigma_B,R}}{p_{\sigma_B,B}-p_{\sigma_R,B}}} & p_{\sigma_B,R} - p_{\sigma_B,B} \le p_{0,R} - p_{0,B} \\ & \le p_{\sigma_R,R} - p_{\sigma_R,B} \end{cases} \tag{B.10}$$

Under the intuitive assumption

$$p_{\sigma_B,R} - p_{\sigma_B,B} < p_{0,R} - p_{0,B} = 0 < p_{\sigma_R,R} - p_{\sigma_R,B}, \tag{B.11}$$

(B.9)-(B.10) are equivalent to the single inequality

$$p_{0,R} \ge \frac{p_{\sigma_B,B}-p_{\sigma_R,B}}{(p_{\sigma_B,B}-p_{\sigma_R,B})+(p_{\sigma_R,R}-p_{\sigma_B,R})} p_{\sigma_R,R} + \frac{p_{\sigma_R,R}-p_{\sigma_B,R}}{(p_{\sigma_B,B}-p_{\sigma_R,B})+(p_{\sigma_R,R}-p_{\sigma_B,R})} p_{\sigma_R,B}.$$

If (B.11) is strengthened to symmetry (2.4), then $\beta^* = \frac{1}{2}$ and one obtains the weak inequality form of (2.5).

B.4 Proof of Theorem 4

For vector inequalities, adopt the notation

$$\begin{aligned} x &\gg y\text{: } x_i > y_i \text{ all } i \\ x &> y\text{: } x_i \ge y_i \text{ all } i \text{ and } x \ne y \\ x &\ge 0\text{: } x_i \ge y_i \text{ all } i \end{aligned}$$

All vectors are column vectors unless transposed by superscript $^\intercal$.

We use Tucker's Theorem of the Alternative (Mangasarian 1969): Exactly one of the following systems of inequalities has a solution:

$$\begin{aligned} (1)\ Bx &> 0,\ Cx \ge 0,\ Dx = 0 \quad (B \text{ nonvacuous}) \\ (2)\quad 0 &= B^\intercal y_2 + C^\intercal y_3 + D^\intercal y_4,\ y_2 \gg 0,\ y_3 \ge 0. \end{aligned}$$

Purely for notational simplicity, let $\mathbf{\Sigma} = \{\sigma_1, \sigma_2\}$ and $\mathcal{F} = \{g, h\}$ be binary; the reader will see that the argument is perfectly general.

Proof of (iii): If we denote by x the vector $(1, \alpha_{\sigma_1}, \alpha_{\sigma_2})^\intercal$, or as any positive scalar multiple thereof, then existence of solution α satisfying (B.3) can be restated as: $\exists x \in \mathbb{R}^3$ solving

$$Cx = 0,\ x > 0$$

where[23]

$$d^\intercal = \left[\begin{array}{ccc} 1 & -1 & -1 \end{array}\right], C = \left[\begin{array}{c} A \\ d^\intercal \end{array}\right], \text{ and}$$

$$A = \left[\begin{array}{c} A_g^\intercal \\ A_h^\intercal \end{array}\right], A_f^\intercal = \left[\begin{array}{ccc} V_0(f) & -V_{\sigma_1}(f) & -V_{\sigma_2}(f) \end{array}\right], \ f = g, h.$$

By Tucker's Theorem, the alternative is: $\exists y = (y^2, y^4)$ such that

$$y^2 + C^\intercal y^4 = 0, \, y^2 \gg 0,$$

or equivalently $C^\intercal y^4 << 0$, or equivalently (let $y^4 = (\beta_g, \beta_h, \beta_0)$):

$$\left[\begin{array}{cc} A^\intercal & d \end{array}\right] y^4 \ll 0,$$

$$\left[\begin{array}{ccc} V_0(g) & V_0(h) & 1 \\ -V_{\sigma_1}(g) & -V_{\sigma_1}(h) & -1 \\ -V_{\sigma_2}(g) & -V_{\sigma_2}(h) & -1 \end{array}\right]\left[\begin{array}{c} \beta_g \\ \beta_h \\ \beta_0 \end{array}\right] << 0,$$

or equivalently: $\exists \left(\beta_g, \beta_h, \beta_0\right)$ s.t.

$$\begin{array}{rcl} \Sigma_f \beta_f V_0(f) + \beta_0 & < & 0 \quad \text{and} \\ \Sigma_f \beta_f V_\sigma(f) + \beta_0 & > & 0 \text{ for all } \sigma \end{array}$$

which is true iff

$$\Sigma_f \beta_f V_0(f) < -\beta_0 < \Sigma_f \beta_f V_\sigma(f) \ \text{ for all } \sigma.$$

Conclude that the alternative is: $\exists \left(\beta_g, \beta_h\right)$ s.t.

$$\Sigma_f \beta_f V_0(f) < \Sigma_f \beta_f V_\sigma(f) \ \text{ for all } \sigma.$$

Therefore, (B.3) obtains iff: $\forall \left(\beta_g, \beta_h\right)$

$$\Sigma_f \beta_f V_0(f) \geq \Sigma_f \beta_f V_\sigma(f) \ \text{ for some } \sigma.$$

But taking $-\beta$, obtain also that: $\forall \left(\beta_g, \beta_h\right)$

$$\Sigma_f \beta_f V_0(f) \leq \Sigma_f \beta_f V_\sigma(f) \ \text{ for some } \sigma.$$

[23] Note that $x > 0$ and $d^\intercal x = 0$ imply that $x_1 > 0$. Below keep in mind also that C is $3 \times (\Sigma + 1)$.

Combine to obtain: $\forall\beta$,

$$\min_{\sigma} \Sigma_f \beta_f V_\sigma(f) \leq \Sigma_f \beta_f V_0(f) \leq \max_{\sigma} \Sigma_f \beta_f V_\sigma(f).$$

Consider (iii.a). Assume that (B.1) and (B.2) are satisfied with α and α' respectively, $\alpha \neq \alpha'$. Then $\sum_{\sigma\neq\sigma_1} (\alpha_\sigma - \alpha'_\sigma)(V_\sigma(f) - V_{\sigma_1}(f)) \leq 0$ for all $f \in \mathcal{F}$. Obtain a contradiction by taking $\Sigma_I = \{\sigma : \alpha_\sigma - \alpha'_\sigma > 0\}$ and $\Sigma_{II} = \{\sigma : \alpha_\sigma - \alpha'_\sigma < 0\}$ in the definition of signal diversity,

Uniqueness follows similarly.

Proof of (i): Use notation from the preceding proof. x denotes the vector $(1, \alpha_{\sigma_1}, \alpha_{\sigma_2})^\intercal$, or any positive scalar multiple thereof. We want a solution to

$$x > 0,\ Ax \geq 0,\ d^\intercal x = 0.$$

By Tucker's Theorem, the alternative is:

$$\begin{aligned} y^2 + A^\intercal y^3 + dy^4 &= 0, \\ y^2 &\gg 0, y^3 \geq 0 \end{aligned}$$

or, letting $y^3 = \left(\beta_g, \beta_h\right)^\intercal \geq 0$,

$$\begin{bmatrix} V_0(g) & V_0(h) \\ -V_{\sigma_1}(g) & -V_{\sigma_1}(h) \\ -V_{\sigma_2}(g) & -V_{\sigma_2}(h) \end{bmatrix} \begin{bmatrix} \beta_g \\ \beta_h \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} y^4 \ll 0.$$

Thus (adding 1st and 2nd components, then 1st and 3rd) the *alternative* to (B.1) is that $\exists \left(\beta_g, \beta_h\right)^\intercal \geq 0$ s.t.

$$\Sigma_{f=g,h} \beta_f V_0(f) < \Sigma_{f=g,h} \beta_f V_\sigma(f) \text{ for each } \sigma.$$

Conclude that (B.1) is equivalent to: $\forall \left(\beta_g, \beta_h\right)^\intercal \geq 0$,

$$\Sigma_{f=g,h} \beta_f V_0(f) \geq \min_{\sigma} \Sigma_{f=g,h} \beta_f V_\sigma(f).$$

The proof for (ii) is similar. ■

C Online Appendix: Details for the maxmin model

We provide some supporting details for the maxmin model defined in Section 4.2. Accordingly, $S = \{R, B\}$ and $\mathbf{\Sigma} = \{\sigma_R, \sigma_B\}$. Both symmetry (2.4) and signal diversity (2.2) are assumed.

C.1 Maxmin with single-likelihood

We are given that $\ell(\cdot \mid s) \in \Delta(\mathbf{\Sigma})$ for each $s = R, B$, satisfying (4.1). Without loss of generality, renaming signals if necessary, suppose that

$$\ell(\sigma_R \mid R) > \ell(\sigma_R \mid B). \tag{C.1}$$

Let $\mathcal{M}_0 \subset \Delta(S)$ be compact and a non-singleton, and let $\mathcal{M} \subset \Delta(\mathbf{\Sigma} \times S)$ be constructed as in Section 4.2. Unconditional utilities are

$$\begin{aligned} V_0(f_B) &= \min_{m \in \mathcal{M}} m(B) = m^*(B), \text{ and} \\ V_0(f_R) &= \min_{m \in \mathcal{M}} m(R) = m^{**}(R). \end{aligned}$$

By symmetry, the probability interval $[m^{**}(R), 1 - m^*(B)]$ for red is symmetric about $\frac{1}{2}$ and

$$V_0(f_R) = m^{**}(R) < \tfrac{1}{2}.$$

For any given m in $\mathcal{M}$, its Bayesian update is

$$\begin{aligned} m(s \mid \sigma) &= [m(s)\,\ell(\sigma \mid s)] / L_m(\sigma), \text{ where} \\ L_m(\cdot) &\equiv \int \ell(\cdot \mid s')\, dm(s'). \end{aligned}$$

Prior-by-prior updating (GBR): Conditional utilities are given by, for each $\sigma = \sigma_B, \sigma_R$,

$$\begin{aligned} V_\sigma(f_B) &= \min_{m \in \mathcal{M}} \frac{m(B)\,\ell(\sigma \mid B)}{L_m(\sigma)} \\ &= \frac{m^*(B)\,\ell(\sigma \mid B)}{L_{m^*}(\sigma)}. \end{aligned}$$

Therefore,

$$\begin{aligned} V_0(f_B) &= m_0^*(B) = m_0^*(B)\,\ell(\sigma_B \mid B) + m_0^*(B)\,\ell(\sigma_R \mid B) \Longrightarrow \\ V_0(f_B) &= L_{m^*}(\sigma_B)\, V_{\sigma_B}(f_B) + L_{m^*}(\sigma_R)\, V_{\sigma_R}(f_B). \end{aligned} \tag{C.2}$$

Similarly for R,

$$V_0(f_R) = L_{m^{**}}(\sigma_B)\, V_{\sigma_B}(f_R) + L_{m^{**}}(\sigma_R)\, V_{\sigma_R}(f_R). \tag{C.3}$$

In addition, because $\max m(B) = m^{**}(B) > m^{*}(B) = \min m(B)$,

$$\begin{aligned} L_{m^*}(\sigma_B) &= m_0^*(B)\,\ell(\sigma_B \mid B) + m_0^*(R)\,\ell(\sigma_B \mid R) \\ &< m_0^{**}(B)\,\ell(\sigma_B \mid B) + m_0^{**}(R)\,\ell(\sigma_B \mid R) = L_{m^{**}}(\sigma_B). \end{aligned}$$

By (C.1),

$$V_{\sigma_B}(f_B) > V_{\sigma_R}(f_B) \text{ and } V_{\sigma_R}(f_R) > V_{\sigma_B}(f_R). \tag{C.4}$$

Therefore, from (C.2), (C.3), and (C.4),

$$\begin{aligned} V_0(f_R) &< L_{m^*}(\sigma_B)\,V_{\sigma_B}(f_R) + L_{m^*}(\sigma_R)\,V_{\sigma_R}(f_R), \text{ and} \\ V_0(f_B) &< L_{m^{**}}(\sigma_B)\,V_{\sigma_B}(f_B) + L_{m^{**}}(\sigma_R)\,V_{\sigma_R}(f_B). \end{aligned}$$

Combine these with the equalities (C.2) and (C.3) to obtain

$$V_0(f_s) < \sum_{\sigma} \alpha_\sigma V_\sigma(f_s), \; s = R, B,$$

where $\alpha_\sigma = \frac{1}{2}L_{m^*}(\sigma) + \frac{1}{2}L_{m^{**}}(\sigma)$. This proves strict signal ambiguity affinity.

Maximum likelihood updating (ML): Conditional on each realized signal σ, one retains only those measures in $\mathcal{M}$ that maximize the probability of σ. Each is updated by Bayes' rule and the minimum conditional probability of s, $s = R, B$, defines the conditional utilities $V_\sigma^{ML}(f_s)$. Since the minimum is taken over a smaller set than under GBR, it is immediate that, for each σ and s,

$$V_\sigma^{ML}(f_s) \geq V_\sigma(f_s). \tag{C.5}$$

Unconditional utilities are identical for the two updating rules. It follows that there is signal ambiguity loving also under ML.

C.2 Maxmin with multiple-likelihoods

We have $\mathcal{M}_0 = \{m_0\}$. By symmetry for unconditional utility,

$$m_0(R) = m_0(B) = \tfrac{1}{2}.$$

Let the nonsingleton set $\mathcal{L}$ be such that $\ell(\sigma \mid s) > 0$ for every σ, s, and $\ell \in \mathcal{L}$. Each likelihood is determined by a pair $(\ell(\sigma_R \mid R), \ell(\sigma_R \mid B)) \in [0,1]^2$. Thus $\mathcal{L}$ can be identified with a subset of the unit square (it is assumed compact).

Prior-by-prior updating (GBR): Aversion to signal ambiguity follows from (C.5) and the result below for ML.

Maximum likelihood updating (ML): The priors maximizing the likelihood of σ are obtained by combining m_0 with every ℓ in $\mathcal{L}_\sigma$,

$$\mathcal{L}_\sigma = \arg\max_{\ell \in \mathcal{L}} L_\ell(\sigma), \; L_\ell(\sigma) \equiv \Sigma_s \ell(\sigma \mid s) m_0(s).$$

Utilities are given by

$$\begin{aligned} V_\sigma(f_s) &= \tfrac{1}{2} \frac{\min_{\ell \in \mathcal{L}_\sigma} \ell(\sigma \mid s)}{L^*(\sigma)}, \quad s = R, B, \\ L^*(\sigma) &= \max_{\ell \in \mathcal{L}} L_\ell(\sigma). \end{aligned}$$

Therefore, using symmetry (2.4),

$$\begin{aligned} V_{\sigma_R}(f_R) + V_{\sigma_B}(f_R) &= V_{\sigma_R}(f_R) + V_{\sigma_R}(f_B) \\ &= \frac{1}{2L^*(\sigma_R)} \left[\min_{\ell \in \mathcal{L}_{\sigma_R}} \ell(\sigma_R \mid R) + \min_{\ell \in \mathcal{L}_{\sigma_R}} \ell(\sigma_R \mid B) \right] \\ &\leq \frac{1}{2L^*(\sigma_R)} \min_{\ell \in \mathcal{L}_{\sigma_R}} [\ell(\sigma_R \mid R) + \ell(\sigma_R \mid B)] \\ &= \frac{\min_{\ell \in \mathcal{L}_{\sigma_R}} [\ell(\sigma_R \mid R) + \ell(\sigma_R \mid B)]}{\max_{\ell \in \mathcal{L}} [\ell(\sigma_R \mid R) + \ell(\sigma_R \mid B)]} \leq 1, \end{aligned}$$

which implies the weak-inequality counterpart of (2.5). ■

D Online Appendix: Smooth ambiguity

Consider the smooth model (Klibanoff, Marinacci and Mukerji 2005) adapted as follows. For concreteness and simplicity only, adopt the setting in the experiment. Accordingly, take $S = \{R, B\}$ and $\mathbf{\Sigma} = \{\sigma_R, \sigma_B\}$. Denote by $n = 1$ or 9 the possible number of red balls in the payoff urn and by $m_n \in \Delta(S)$ the corresponding probability distribution for the color drawn from the payoff urn ($m_n(R) = n/10$). At the unconditional stage, before becoming aware of the signal structure, uncertainty about n is represented by $\mu_0 \in \Delta(\{1, 9\})$; since equal probabilities are announced to subjects, take $\mu_0(1) = \frac{1}{2}$, though any subjective prior would do equally. Her unconditional utility function is V_0 given by

$$\phi \circ V_0(f) = \phi(p_{0,f}) = \int_{\{1,9\}} \phi\left(\int_S u(f)\, dm_n\right) d\mu_0(n), \tag{D.1}$$

for $f = f_R, f_B$, where $\phi(\cdot)$ is (strictly) increasing. Unconditional ambiguity aversion is modeled by taking ϕ concave.

In the alternative scenario, the individual is informed about the signal structure, (where $N = 0$ or 45 balls of each color are added when constructing the signal urn), and that a signal, either σ_R or σ_B, has been realized. Inferences about the payoff urn composition depend on beliefs about both n and N represented by the measure $\mu \in \Delta(\{1,9\} \times \{0,45\})$. The only restriction on μ is that the marginal probability $\mu(n)$ satisfy

$$\mu(n) = \mu_0(n), \;\; n = 1, 9. \tag{D.2}$$

The likelihood of each signal σ given any pair (n, N) is well-defined (e.g. $L(\sigma_R \mid n = 9, N = 45) = L(\sigma_B \mid n = 1, N = 45) = 54/100$), which permits Bayesian updating to $\mu(\cdot \mid \sigma)$. Conditional utility is defined by

$$\phi \circ V_\sigma(f) = \phi(p_{\sigma,f}) = \int_{\{1,9\}\times\{0,45\}} \phi\left(\int_S u(f(s))\, dm_n(s)\right) d\mu(n, N \mid \sigma). \tag{D.3}$$

Remark 5 *Following Klibanoff, Marinacci and Mukerji (2009), one might replace $m_n(s)$ above by $m_n(s \mid \sigma, N)$. However, the natural assumption is that draws from the payoff and signal urns are independent conditional on (n, N). Therefore, $m_n(s \mid \sigma, N) = m_n(s \mid N) = m_n(s)$, and we are back to (D.3).*

The utility functions in (D.1) and (D.3), plus (D.2), constitute a version of the smooth model for our setting. Define $L^*(\sigma)$ by

$$L^*(\sigma) = \Sigma_{n,N} L(\sigma \mid n, N)\, \mu(n, N).$$

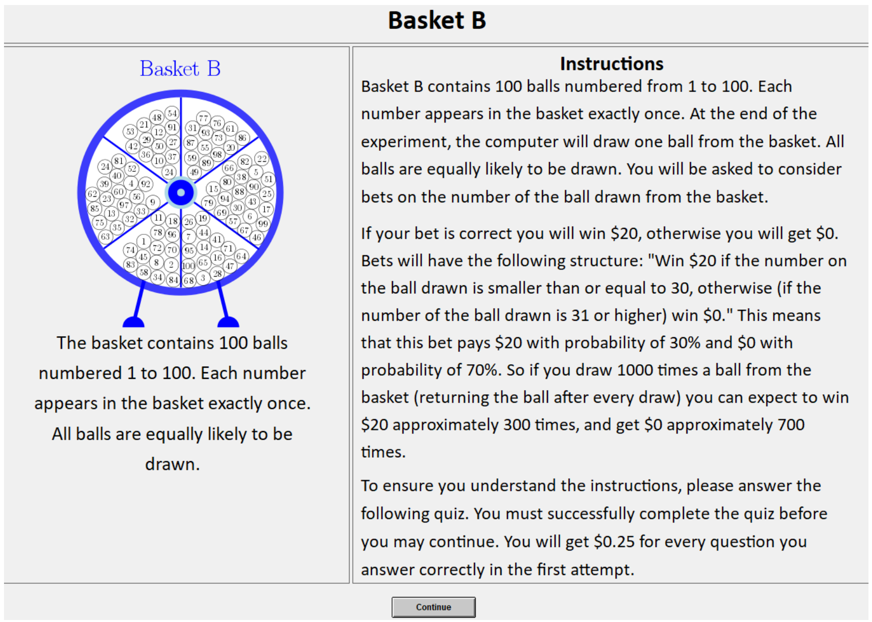
Then it follows from the martingale property of Bayesian updating that

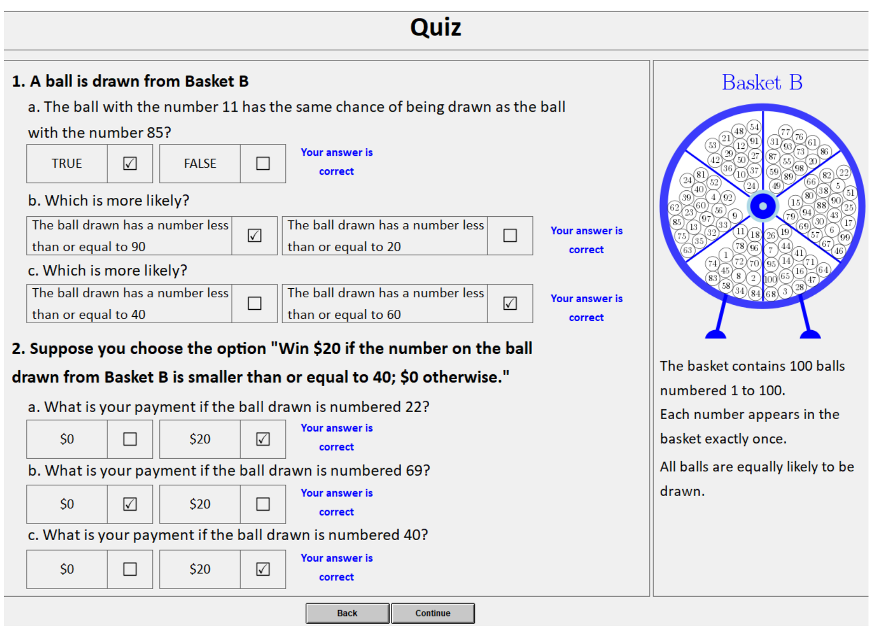
$$\begin{aligned} \phi(p_{0,f}) &= L^*(\sigma_R)\, \phi(p_{\sigma_R,f}) + L^*(\sigma_B)\, \phi(p_{\sigma_B,f}) \\ &\leq \phi\left(L^*(\sigma_R)\, p_{\sigma_R,f} + L^*(\sigma_B)\, p_{\sigma_B,f}\right) \Longrightarrow \\ p_{0,f} &\leq L^*(\sigma_R)\, p_{\sigma_R,f} + L^*(\sigma_B)\, p_{\sigma_B,f}, \end{aligned}$$

which, by Lemma 1 (and its obvious extension to weak inequalities), is equivalent to (weak) signal ambiguity loving. Conclude that unconditional (Ellsberg) ambiguity aversion implies signal ambiguity loving.

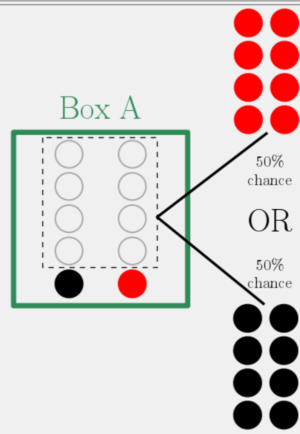
E Online Appendix: Experimental interface

E.1 Risk control





Box A



The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).

Instructions

Box A contains 10 red or black balls. There are two possible compositions - each has a chance of 50%:

-- the box contains 9 RED and 1 BLACK balls.

OR

-- the box contains 1 RED and 9 BLACK balls.

Remember that the chance of each composition is 50%.

The computer will draw a ball at random from Box A. You are asked to bet on the colour (RED or BLACK) of the ball drawn without knowing the composition of the box.

-- If you bet on RED: if the ball drawn is RED - you will win $20, and if the ball drawn is BLACK - you will get $0.

-- If you bet on BLACK: if the ball drawn is BLACK - you will win $20, and if the ball drawn is RED - you will get $0.

To ensure you understand the instructions, please answer the following quiz. You must successfully complete the quiz before you may continue. You will get $0.25 for every question you answer correctly in the first attempt.

Continue

Quiz

1. If there are more RED balls than BLACK balls in Box A, the chances of drawing a RED ball from Box A are:

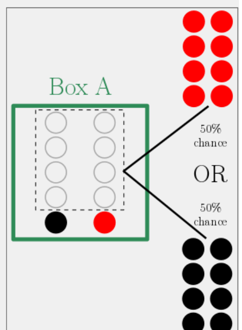
- ☐ 0%
- ☐ 10%
- ☐ 50%
- ☑ 90%
- ☐ 100%

Your answer is correct

2. If there are more BLACK balls than RED balls in Box A, the chances of drawing a RED ball from Box A are:

- ☐ 0%
- ☑ 10%
- ☐ 50%
- ☐ 90%
- ☐ 100%

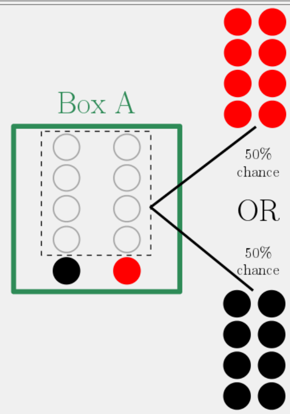
Your answer is correct



The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).

Back Continue

Box A



The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).

Instructions

Box A contains 10 red or black balls. There are two possible compositions - each has a chance of 50%:

-- the box contains 9 RED and 1 BLACK balls.

OR

-- the box contains 1 RED and 9 BLACK balls.

Remember that the chance of each composition is 50%.

The computer will draw a ball at random from Box A. You are asked to bet on the colour (RED or BLACK) of the ball drawn without knowing the composition of the box.

-- If you bet on RED: if the ball drawn is RED - you will win $20, and if the ball drawn is BLACK - you will get $0.

-- If you bet on BLACK: if the ball drawn is BLACK - you will win $20, and if the ball drawn is RED - you will get $0.

Please select the colour you would like to bet on. The rest of the experiment will invoke bets on the colour you choose.

BLACK ☐ **RED** ☑

Submit your Bet

Instructions

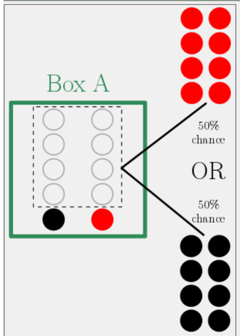
Following these instructions, you will be asked to make some choices. There are no correct choices. Your choices depend on your preferences and beliefs, so different participants will usually make different choices. You will be paid according to your choices, so read these instructions carefully and think before you decide.

One of the choice problems will be selected at random, and your chosen option in that choice problem will determine your payment. This protocol of determining payments suggests that you should choose in each choice problem as if it is the only choice problem that determines your payment.

Next

Instructions

Option A
Bet on RED from Box A



The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).

Examples of Choice Problems

In this part of the experiment, you will face a sequence of choice problems in which you will be asked to choose between two uncertain options. Specifically, on every line (see the example below), you will be asked to choose between:

-- Option A. A bet that pays $20 if the ball drawn from Box A is RED.

-- Option B. A bet that pays $20 if the ball drawn from Basket B is smaller than or equal to a certain number.

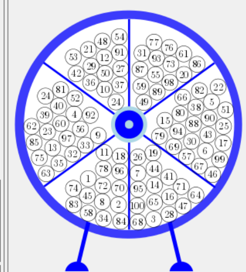
Example

Choice No.	Option A Win $20 if a RED ball is drawn from Box A Win $0 if a BLACK ball is drawn from Box A	A	B	Option B Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line. Win $0 otherwise
1	Bet on RED from Box A	☐	☐	10
2	Bet on RED from Box A	☐	☐	20
3	Bet on RED from Box A	☐	☐	30

Note that as you go down the list, the number in Option B increases, so the chances of being paid if you choose option B increase.

Option B
Play Lottery of Basket B

Basket B



The basket contains 100 balls numbered 1 to 100.

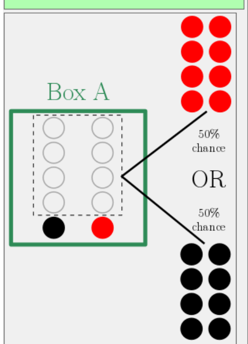
Each number appears in the basket exactly once.

All balls are equally likely to be drawn.

Back Next

Instructions

Option A
Bet on RED from Box A



The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).

Switching Between Options

In the initial list of choice problems, the increase in number (in Option B) as you move from one line to the consecutive line is 10 (or 10 percent).

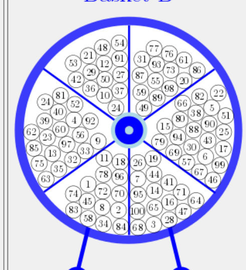
Note that the first (#0) and the last (#10) lines are pre-filled for you, and that once you switch from choosing Option A to Option B, it makes sense that you will continue to choose Option B in all consecutive choice problems (lower lines), since the chances of being paid according to Option B increase while Option A does not change. So it makes sense to switch only once on the list from Option A to Option B.

Once you finish making your choices in the initial list, you will face a "zoom-in" list in which the increase in the number in Option B is only 1 (1 percent).

If a choice problem (either from the initial list or the zoom-in list) is selected for payment, you will be paid according to your choice in that problem.

Option B
Play Lottery of Basket B

Basket B

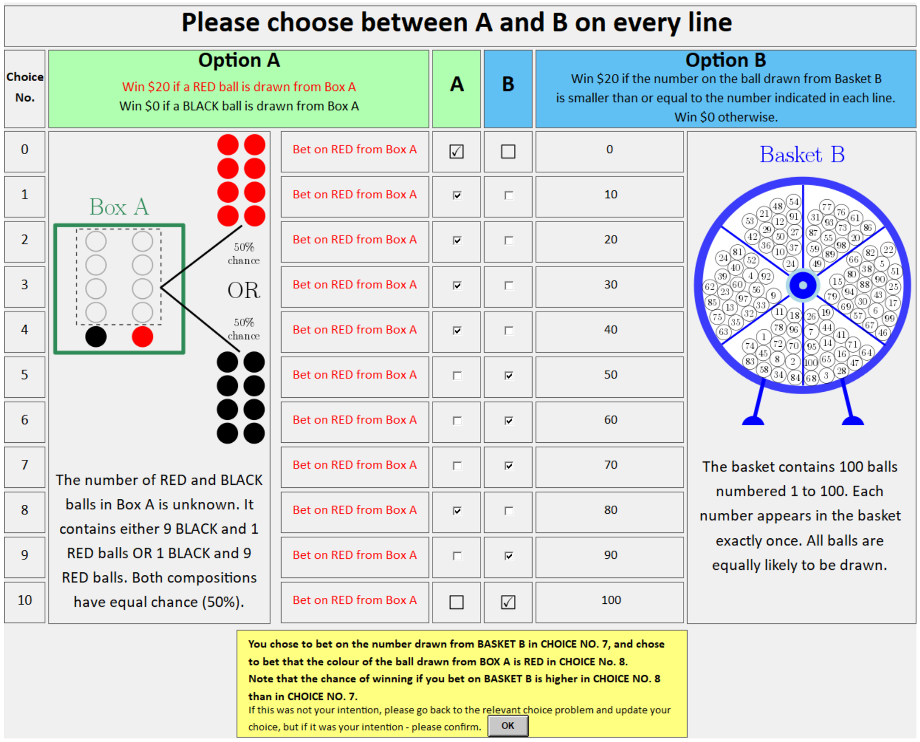


The basket contains 100 balls numbered 1 to 100.

Each number appears in the basket exactly once.

All balls are equally likely to be drawn.

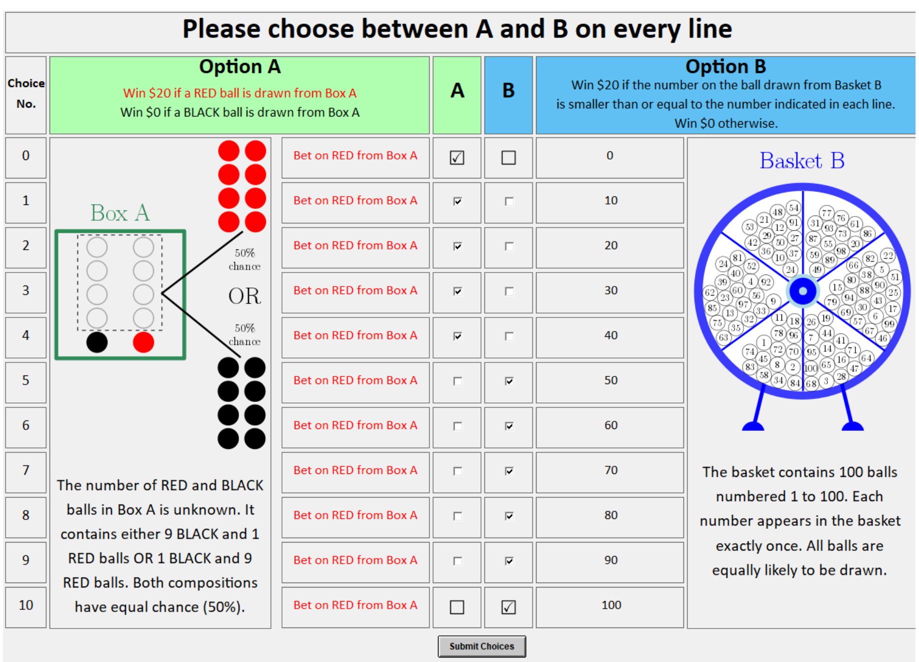
Back Next



Please choose between A and B on every line

Choice No.	Option A: Win $20 if a RED ball is drawn from Box A. Win $0 if a BLACK ball is drawn from Box A	A	B	Option B: Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line. Win $0 otherwise.
0	Bet on RED from Box A	✓		0
1	Bet on RED from Box A	✓		10
2	Bet on RED from Box A	✓		20
3	Bet on RED from Box A	✓		30
4	Bet on RED from Box A	✓		40
5	Bet on RED from Box A		✓	50
6	Bet on RED from Box A		✓	60
7	Bet on RED from Box A		✓	70
8	Bet on RED from Box A	✓		80
9	Bet on RED from Box A		✓	90
10	Bet on RED from Box A		✓	100

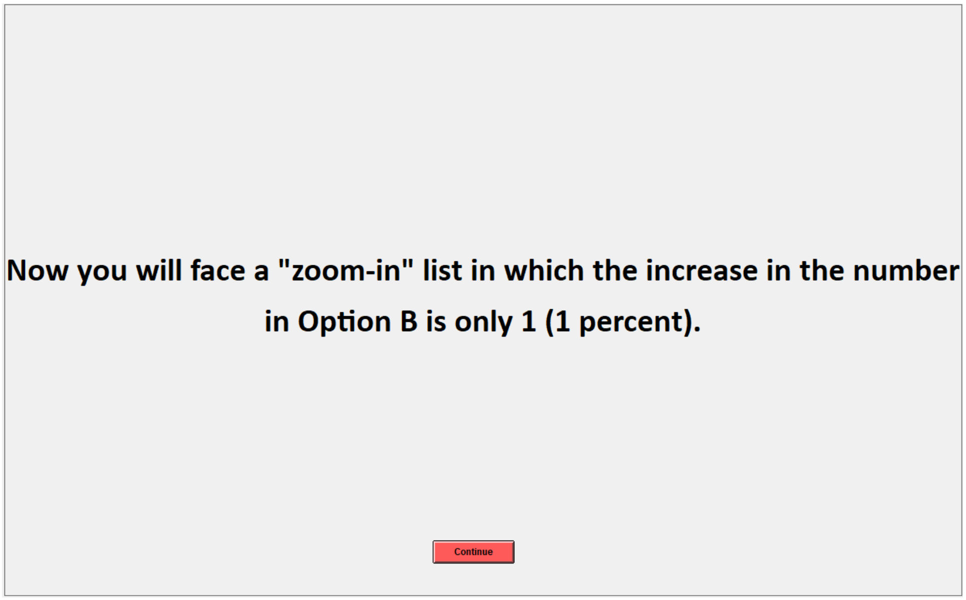
Box A
50% chance
OR
50% chance
The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).
Basket B
The basket contains 100 balls numbered 1 to 100. Each number appears in the basket exactly once. All balls are equally likely to be drawn.
You chose to bet on the number drawn from BASKET B in CHOICE NO. 7, and chose to bet that the colour of the ball drawn from BOX A is RED in CHOICE No. 8. Note that the chance of winning if you bet on BASKET B is higher in CHOICE NO. 8 than in CHOICE NO. 7.
If this was not your intention, please go back to the relevant choice problem and update your choice, but if it was your intention - please confirm.
OK

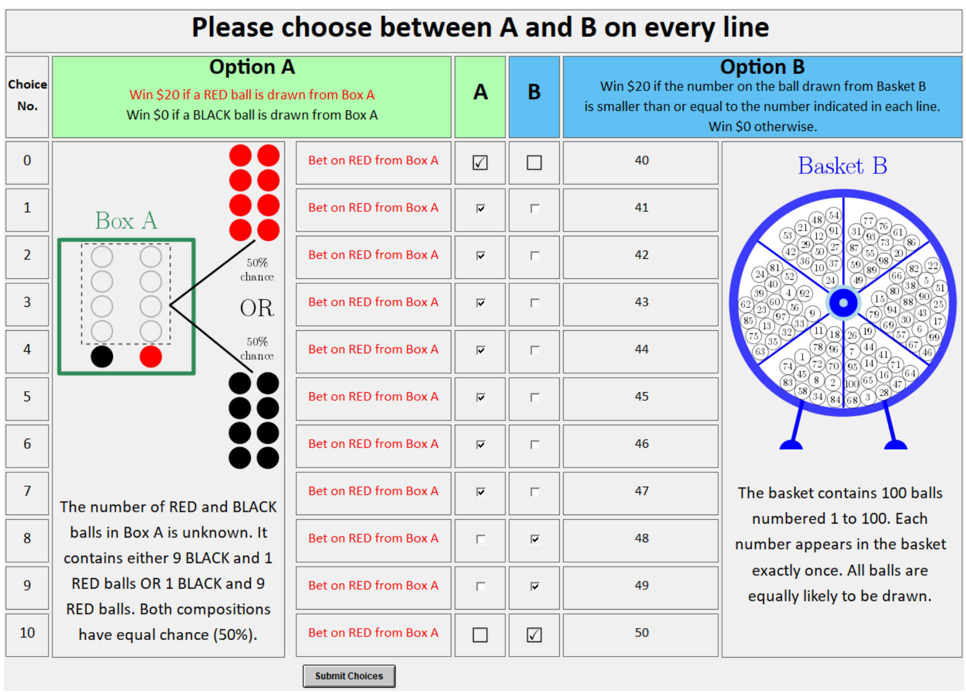


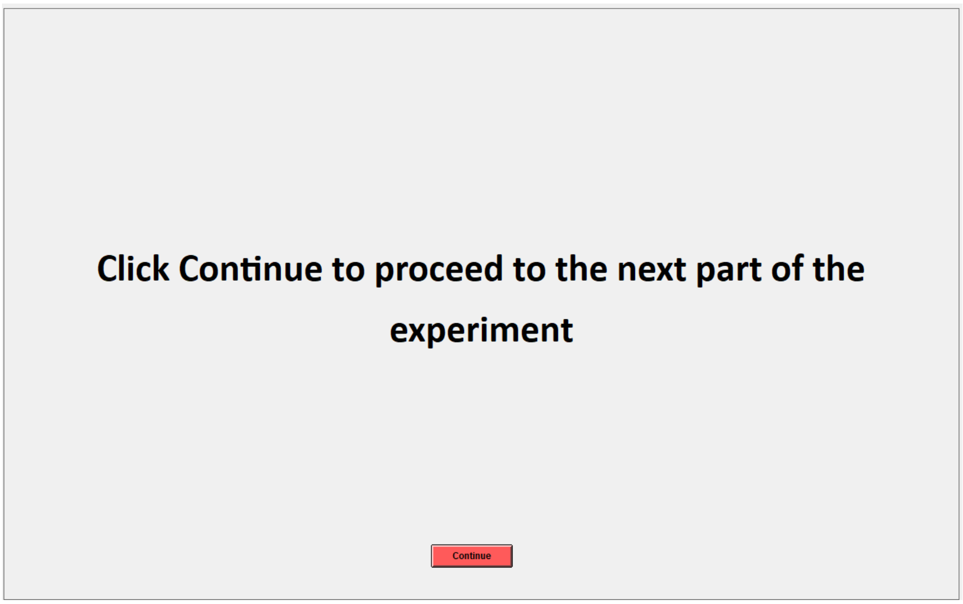
Please choose between A and B on every line

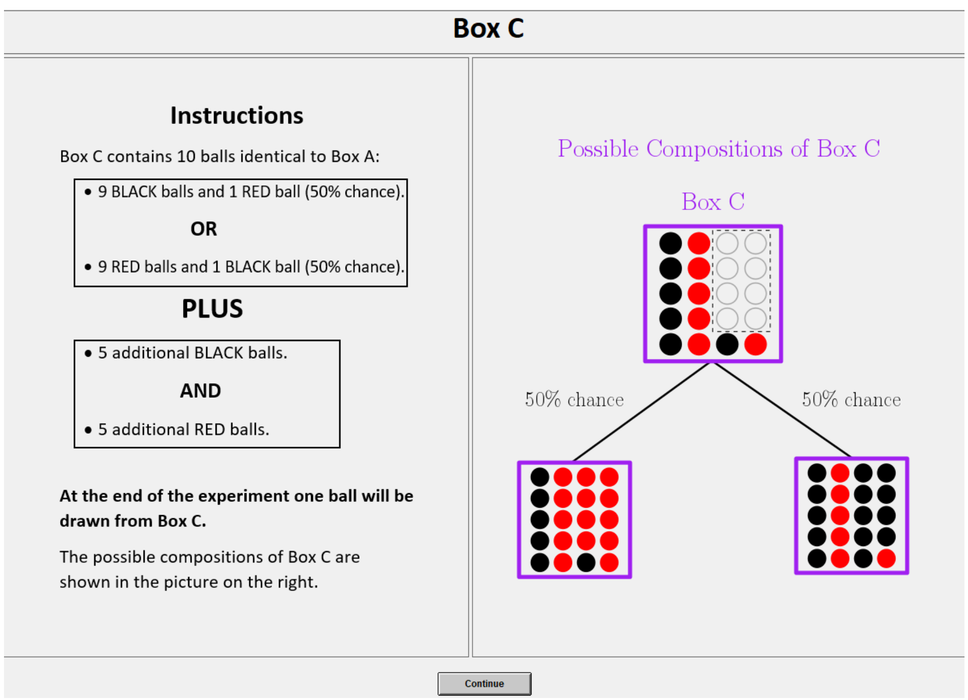
Choice No.	Option A: Win $20 if a RED ball is drawn from Box A. Win $0 if a BLACK ball is drawn from Box A	A	B	Option B: Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line. Win $0 otherwise.
0	Bet on RED from Box A	✓		0
1	Bet on RED from Box A	✓		10
2	Bet on RED from Box A	✓		20
3	Bet on RED from Box A	✓		30
4	Bet on RED from Box A	✓		40
5	Bet on RED from Box A		✓	50
6	Bet on RED from Box A		✓	60
7	Bet on RED from Box A		✓	70
8	Bet on RED from Box A		✓	80
9	Bet on RED from Box A		✓	90
10	Bet on RED from Box A		✓	100

Box A
50% chance
OR
50% chance
The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).
Basket B
The basket contains 100 balls numbered 1 to 100. Each number appears in the basket exactly once. All balls are equally likely to be drawn.
Submit Choices

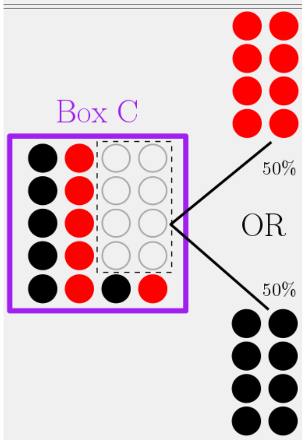

Now you will face a "zoom-in" list in which the increase in the number in Option B is only 1 (1 percent).
Continue


Please choose between A and B on every line
Choice No.
Option A
Win $20 if a RED ball is drawn from Box A
Win $0 if a BLACK ball is drawn from Box A
A
B
Option B
Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line.
Win $0 otherwise.
0 Bet on RED from Box A 40
1 Bet on RED from Box A 41
2 Bet on RED from Box A 42
3 Bet on RED from Box A 43
4 Bet on RED from Box A 44
5 Bet on RED from Box A 45
6 Bet on RED from Box A 46
7 Bet on RED from Box A 47
8 Bet on RED from Box A 48
9 Bet on RED from Box A 49
10 Bet on RED from Box A 50
Box A
50% chance
OR
50% chance
The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).
Basket B
The basket contains 100 balls numbered 1 to 100. Each number appears in the basket exactly once. All balls are equally likely to be drawn.
Submit Choices


Click Continue to proceed to the next part of the experiment
Continue


Box C
Instructions
Box C contains 10 balls identical to Box A:
• 9 BLACK balls and 1 RED ball (50% chance).
OR
• 9 RED balls and 1 BLACK ball (50% chance).
PLUS
• 5 additional BLACK balls.
AND
• 5 additional RED balls.
At the end of the experiment one ball will be drawn from Box C.
The possible compositions of Box C are shown in the picture on the right.
Possible Compositions of Box C
Box C
50% chance
50% chance
Continue

Box C



Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS 5 additional RED balls AND 5 additional BLACK balls.

Instructions

In the next part of the experiment, you will be asked to choose between bets on Basket B (as before) and bets on the colour of a ball drawn from Box A, if you observe that a RED ball is drawn from Box C. Similarly, you will be asked to choose between bets on Basket B and bets on the colour of a ball drawn from Box A, if you observe that a BLACK ball is drawn from Box C. You will be asked to make your choices before you know the actual colour of the ball drawn from Box C.

If a choice problem from this part of the experiment is selected for payment, a ball will be drawn from Box C and your choice for that case (BLACK from Box C or RED from Box C) will determine your payment.

To ensure you understand the instructions, please answer the following quiz. You must successfully complete the quiz before you may continue. You will get $0.25 for every question you answer correctly in the first attempt.

Back | Continue

Quiz

1. If you bet on RED from Box A and a BLACK ball is drawn from Box C. What would be your payment?

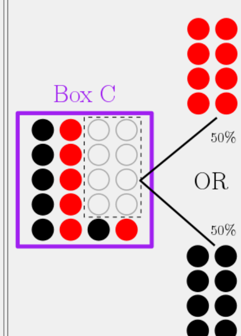
☐	$0
☐	$20
☑	It depends on the colour of the ball drawn from Box A.

Your answer is correct

2. If there are more BLACK balls than RED balls in Box A:

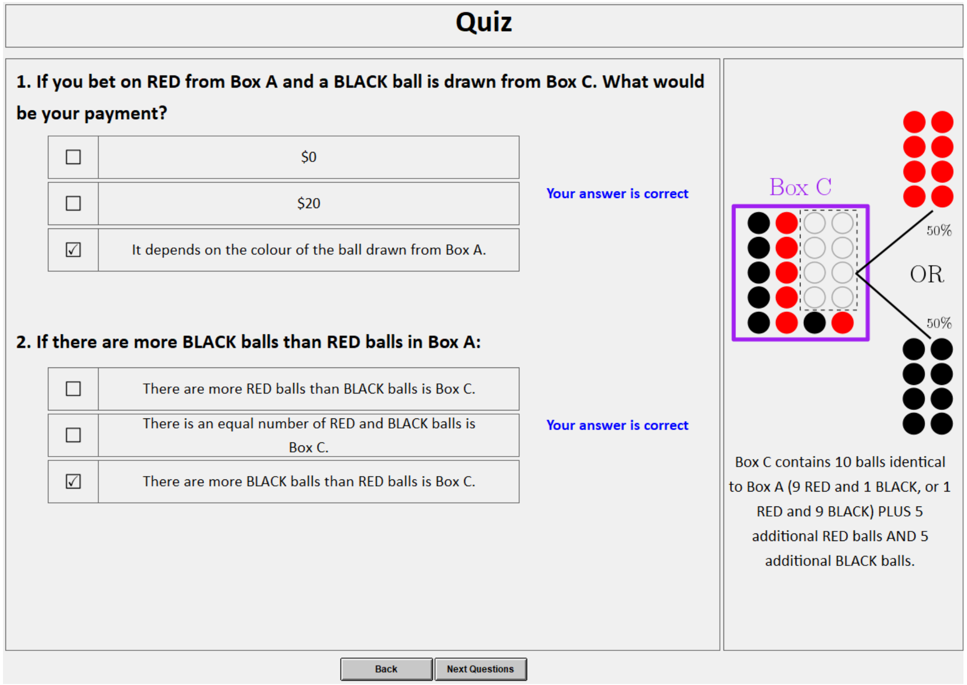
☐	There are more RED balls than BLACK balls is Box C.
☐	There is an equal number of RED and BLACK balls is Box C.
☑	There are more BLACK balls than RED balls is Box C.

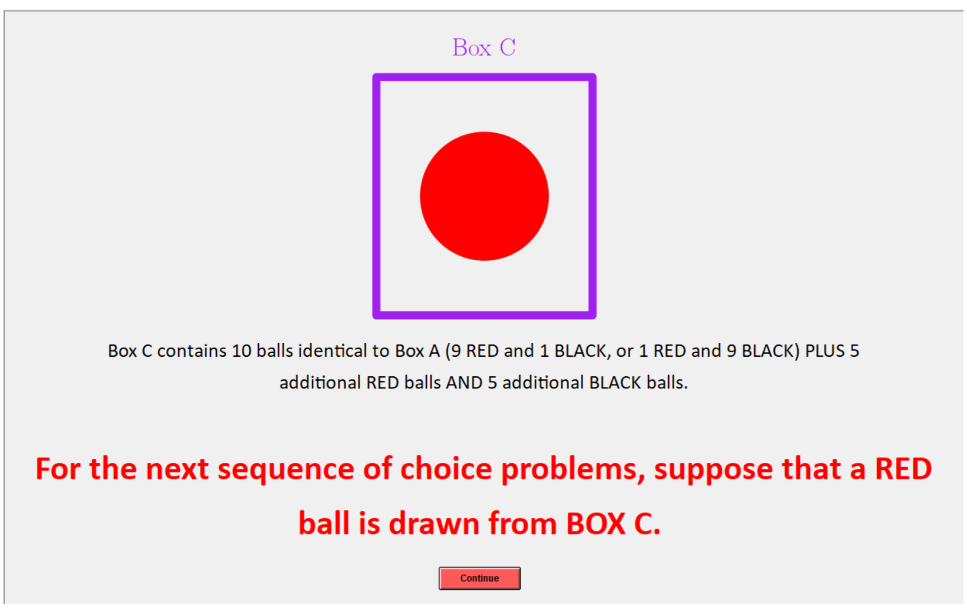
Your answer is correct

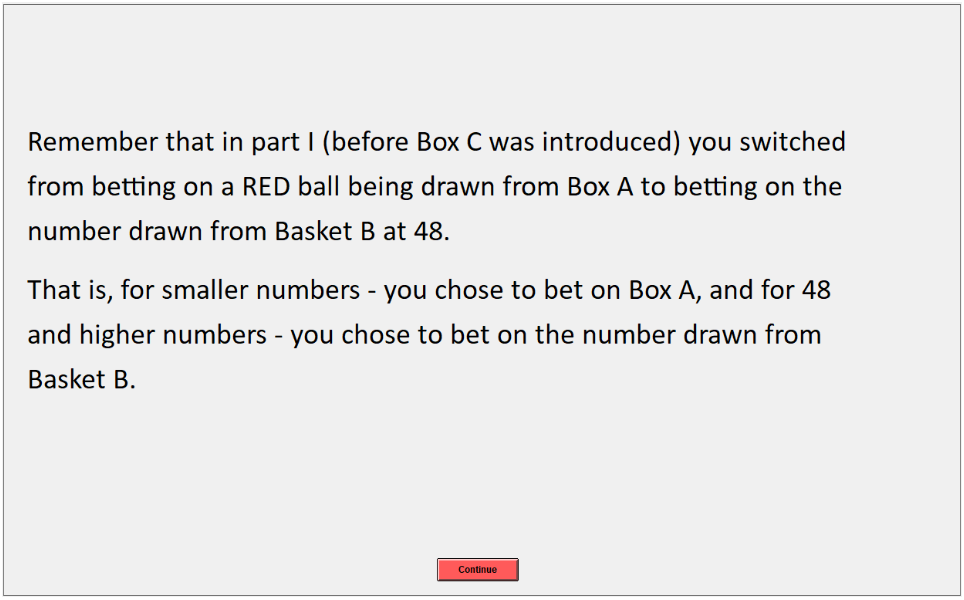


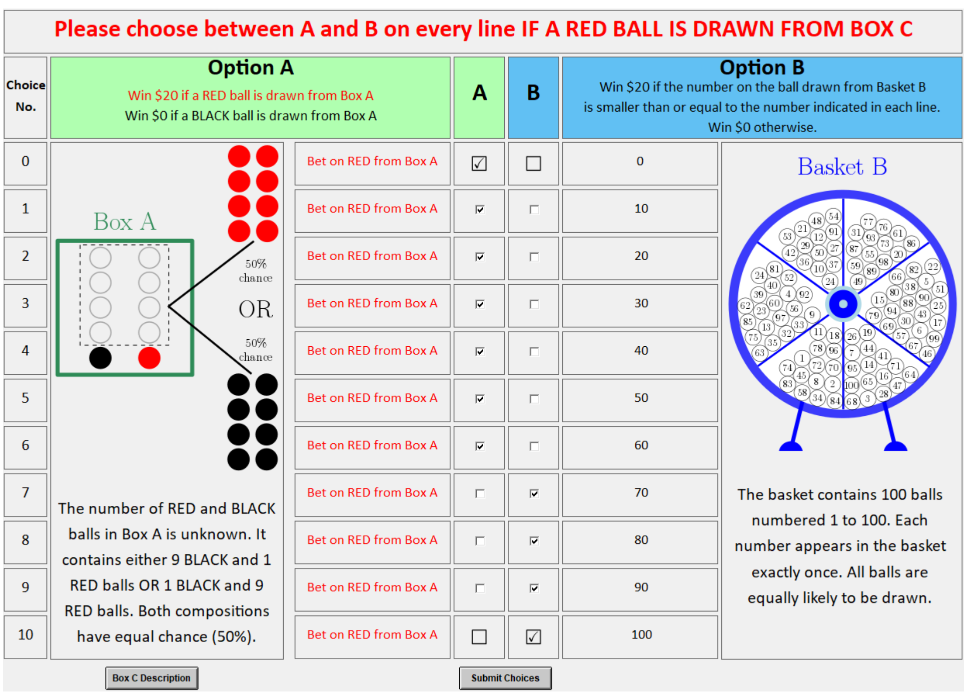
Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS 5 additional RED balls AND 5 additional BLACK balls.

Back | Next Questions


Quiz
1. If you bet on RED from Box A and a BLACK ball is drawn from Box C. What would be your payment?
$0
$20
It depends on the colour of the ball drawn from Box A.
Your answer is correct
2. If there are more BLACK balls than RED balls in Box A:
There are more RED balls than BLACK balls is Box C.
There is an equal number of RED and BLACK balls is Box C.
There are more BLACK balls than RED balls is Box C.
Your answer is correct
Box C
50%
OR
50%
Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS 5 additional RED balls AND 5 additional BLACK balls.
Back
Next Questions


Box C
Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS 5 additional RED balls AND 5 additional BLACK balls.
For the next sequence of choice problems, suppose that a RED ball is drawn from BOX C.
Continue


Remember that in part I (before Box C was introduced) you switched from betting on a RED ball being drawn from Box A to betting on the number drawn from Basket B at 48.
That is, for smaller numbers - you chose to bet on Box A, and for 48 and higher numbers - you chose to bet on the number drawn from Basket B.
Continue


Please choose between A and B on every line IF A RED BALL IS DRAWN FROM BOX C
Choice No.
Option A
Win $20 if a RED ball is drawn from Box A
Win $0 if a BLACK ball is drawn from Box A
A
B
Option B
Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line. Win $0 otherwise.
0 Bet on RED from Box A 0
1 Bet on RED from Box A 10
2 Bet on RED from Box A 20
3 Bet on RED from Box A 30
4 Bet on RED from Box A 40
5 Bet on RED from Box A 50
6 Bet on RED from Box A 60
7 Bet on RED from Box A 70
8 Bet on RED from Box A 80
9 Bet on RED from Box A 90
10 Bet on RED from Box A 100
Box A
50% chance
OR
50% chance
The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).
Basket B
The basket contains 100 balls numbered 1 to 100. Each number appears in the basket exactly once. All balls are equally likely to be drawn.
Box C Description
Submit Choices

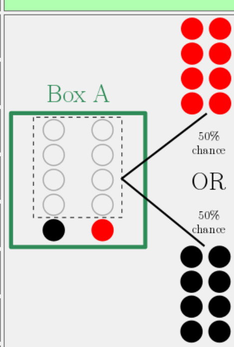
Remember that in part I (before Box C was introduced) you switched from betting on a RED ball being drawn from Box A to betting on the number drawn from Basket B at 48.

That is, for smaller numbers - you chose to bet on Box A, and for 48 and higher numbers - you chose to bet on the number drawn from Basket B.

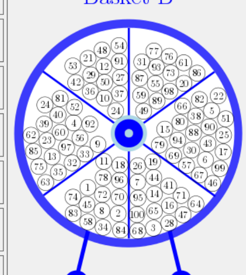
Continue

Please choose between A and B on every line IF A RED BALL IS DRAWN FROM BOX C

Choice No.	Option A: Win $20 if a RED ball is drawn from Box A. Win $0 if a BLACK ball is drawn from Box A		A	B	Option B: Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line. Win $0 otherwise.
0		Bet on RED from Box A	☑	☐	0
1		Bet on RED from Box A	☑	☐	10
2		Bet on RED from Box A	☑	☐	20
3		Bet on RED from Box A	☑	☐	30
4		Bet on RED from Box A	☑	☐	40
5		Bet on RED from Box A	☑	☐	50
6		Bet on RED from Box A	☑	☐	60
7		Bet on RED from Box A	☐	☑	70
8		Bet on RED from Box A	☐	☑	80
9		Bet on RED from Box A	☐	☑	90
10		Bet on RED from Box A	☐	☑	100



The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).



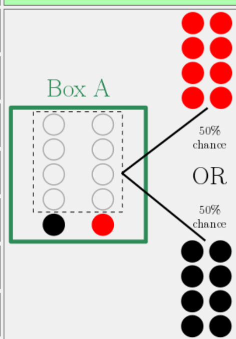
The basket contains 100 balls numbered 1 to 100. Each number appears in the basket exactly once. All balls are equally likely to be drawn.

Box C Description

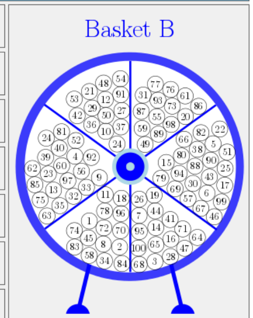
Submit Choices

Please choose between A and B on every line IF A RED BALL IS DRAWN FROM BOX C

Choice No.	Option A Win $20 if a RED ball is drawn from Box A Win $0 if a BLACK ball is drawn from Box A		A	B	Option B Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line. Win $0 otherwise.
0		Bet on RED from Box A	☑	☐	60
1		Bet on RED from Box A	☑	☐	61
2		Bet on RED from Box A	☑	☐	62
3		Bet on RED from Box A	☑	☐	63
4		Bet on RED from Box A	☑	☐	64
5		Bet on RED from Box A	☐	☑	65
6		Bet on RED from Box A	☐	☑	66
7		Bet on RED from Box A	☐	☑	67
8		Bet on RED from Box A	☐	☑	68
9		Bet on RED from Box A	☐	☑	69
10		Bet on RED from Box A	☐	☑	70

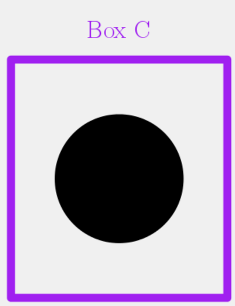


The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).



The basket contains 100 balls numbered 1 to 100. Each number appears in the basket exactly once. All balls are equally likely to be drawn.

Box C Description | Submit Choices



Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS 5 additional RED balls AND 5 additional BLACK balls.

For the next sequence of choice problems, suppose that a BLACK ball is drawn from BOX C.

Continue

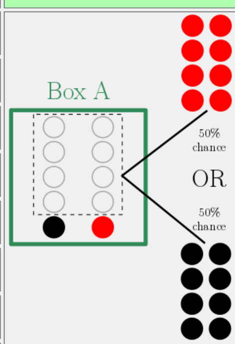
Remember that in part I (before Box C was introduced) you switched from betting on a RED ball being drawn from Box A to betting on the number drawn from Basket B at 48.

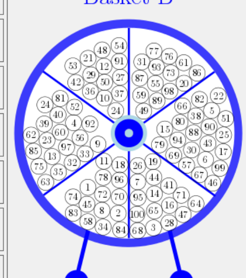
That is, for smaller numbers - you chose to bet on Box A, and for 48 and higher numbers - you chose to bet on the number drawn from Basket B.

Continue

Please choose between A and B on every line IF A BLACK BALL IS DRAWN FROM BOX C

Choice No.	**Option A** Win $20 if a RED ball is drawn from Box A Win $0 if a BLACK ball is drawn from Box A		A	B	**Option B** Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line. Win $0 otherwise.	
0	Box A The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).	Bet on RED from Box A	☑	☐	0	Basket B The basket contains 100 balls numbered 1 to 100. Each number appears in the basket exactly once. All balls are equally likely to be drawn.
1		Bet on RED from Box A	☑	☐	10	
2		Bet on RED from Box A	☑	☐	20	
3		Bet on RED from Box A	☑	☐	30	
4		Bet on RED from Box A	☐	☑	40	
5		Bet on RED from Box A	☐	☑	50	
6		Bet on RED from Box A	☐	☑	60	
7		Bet on RED from Box A	☐	☑	70	
8		Bet on RED from Box A	☐	☑	80	
9		Bet on RED from Box A	☐	☑	90	
10		Bet on RED from Box A	☐	☑	100	



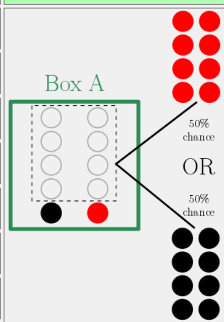


Box C Description

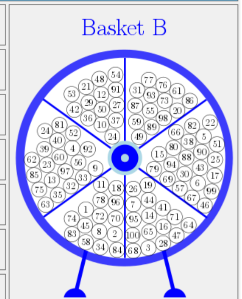
Submit Choices

Please choose between A and B on every line IF A BLACK BALL IS DRAWN FROM BOX C

Choice No.	Option A Win $20 if a RED ball is drawn from Box A Win $0 if a BLACK ball is drawn from Box A		A	B	Option B Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line. Win $0 otherwise.
0		Bet on RED from Box A	☑	☐	30
1		Bet on RED from Box A	☑	☐	31
2		Bet on RED from Box A	☑	☐	32
3		Bet on RED from Box A	☑	☐	33
4		Bet on RED from Box A	☐	☑	34
5		Bet on RED from Box A	☐	☑	35
6		Bet on RED from Box A	☐	☑	36
7		Bet on RED from Box A	☐	☑	37
8		Bet on RED from Box A	☐	☑	38
9		Bet on RED from Box A	☐	☑	39
10		Bet on RED from Box A	☐	☑	40



The number of RED and BLACK balls in Box A is unknown. It contains either 9 BLACK and 1 RED balls OR 1 BLACK and 9 RED balls. Both compositions have equal chance (50%).



The basket contains 100 balls numbered 1 to 100. Each number appears in the basket exactly once. All balls are equally likely to be drawn.

Box C Description

Submit Choices

End of the Experiment

The experiment is over. Take a look at your performance.

Part 2 was randomly selected for payment. The ball drawn from Box C is RED.

The table below displays the choice problem that was selected for payment.

Please choose between A and B IF A RED BALL IS DRAWN FROM BOX C

Choice No.	Option A Win $20 if a RED ball is drawn from Box A Win $0 if a BLACK ball is drawn from Box A	A	B	Option B Win $20 if the number on the ball drawn from Basket B is smaller than or equal to the number indicated in each line. Win $0 otherwise.
4	Bet on RED from Box A	☑	☐	40

The ball drawn from Box A is RED.

The breakdown of your payment is:

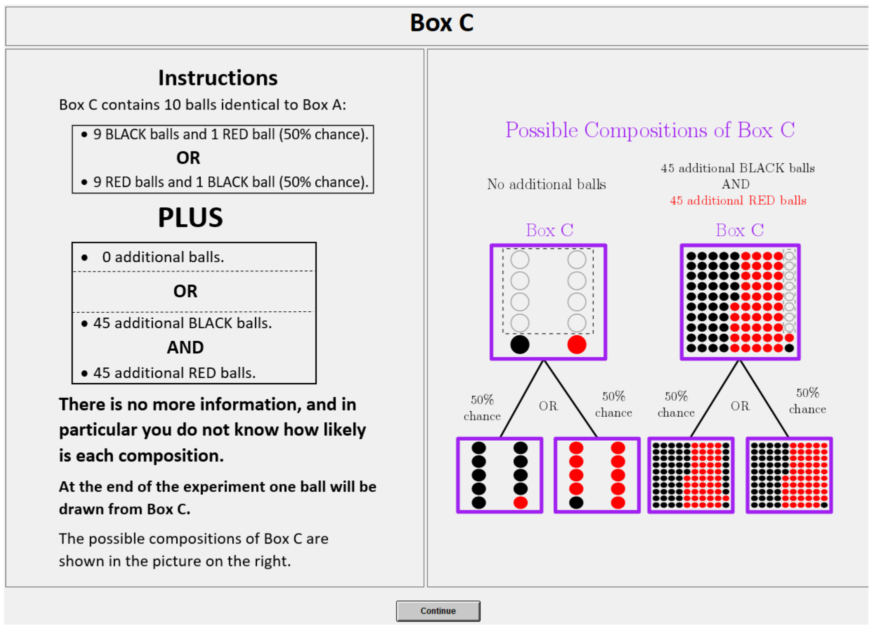
Show-up fee:	**$7.00**
Quiz (12 X $0.25):	**$3.00**
Payment from the Experiment:	**$20.00**
Your Total Payment is:	**$30.00**

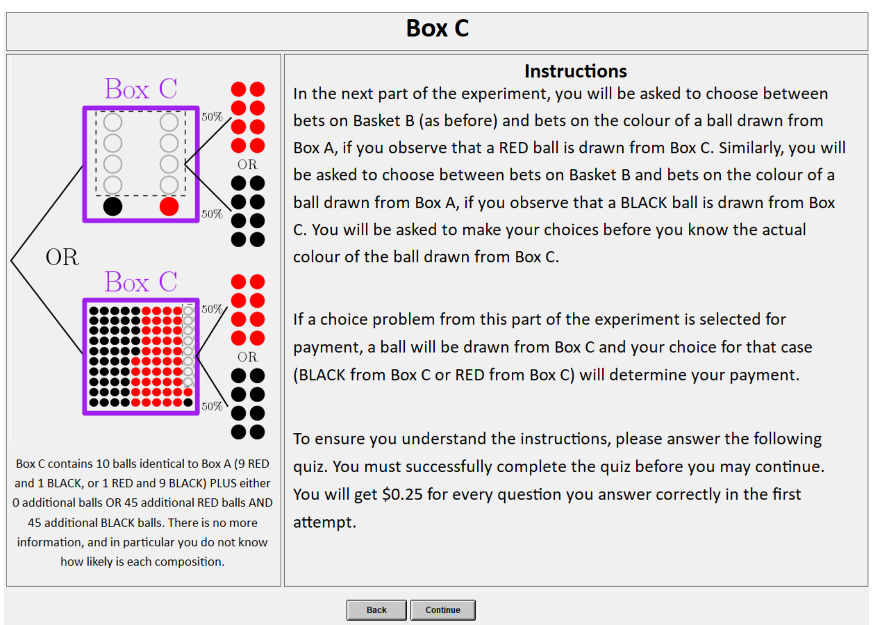
Please record your payment in your receipt.

OK

E.2 Ambiguous-signals treatment

Below are only the screens that are different in the ambiguous signal treatment





Quiz

1. If you bet on RED from Box A and a BLACK ball is drawn from Box C. What would be your payment?

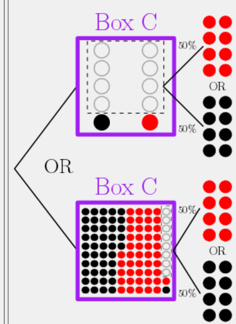
- ☐ $0
- ☐ $20
- ☑ It depends on the colour of the ball drawn from Box A.

Your answer is correct

2. If there are more BLACK balls than RED balls in Box A:

- ☐ There are more RED balls than BLACK balls is Box C.
- ☐ There is an equal number of RED and BLACK balls is Box C.
- ☑ There are more BLACK balls than RED balls is Box C.

Your answer is correct



Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS either 0 additional balls OR 45 additional RED balls AND 45 additional BLACK balls. There is no more information, and in particular you do not know how likely is each composition.

Back | Next Questions

Quiz

1. If you bet on RED from Box A and a BLACK ball is drawn from Box C. What would be your payment?

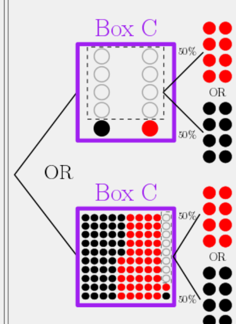
- ☐ $0
- ☐ $20
- ☑ It depends on the colour of the ball drawn from Box A.

Your answer is correct

2. If there are more BLACK balls than RED balls in Box A:

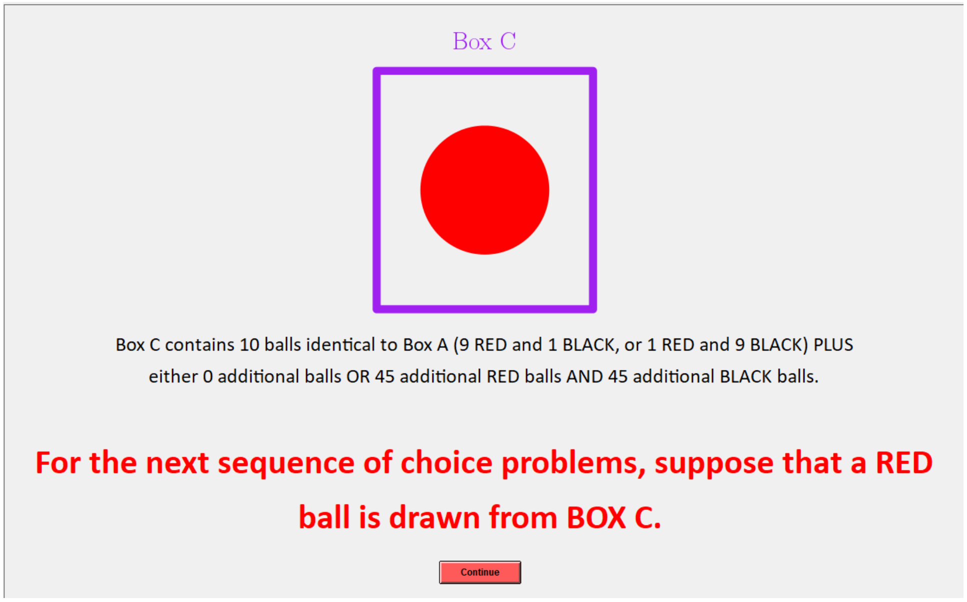
- ☐ There are more RED balls than BLACK balls is Box C.
- ☐ There is an equal number of RED and BLACK balls is Box C.
- ☑ There are more BLACK balls than RED balls is Box C.

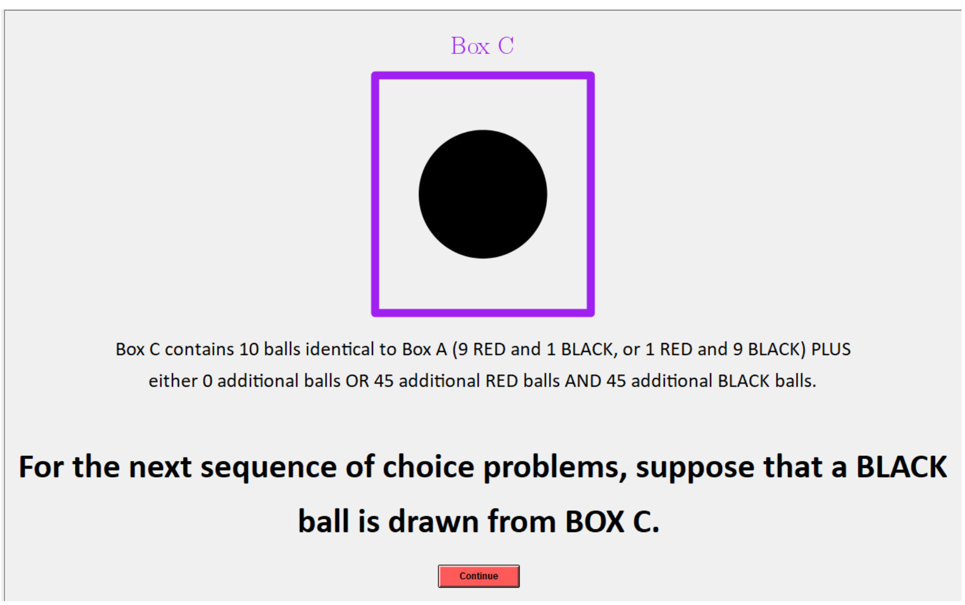
Your answer is correct



Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS either 0 additional balls OR 45 additional RED balls AND 45 additional BLACK balls. There is no more information, and in particular you do not know how likely is each composition.

Back | Next Questions


Box C
Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS
either 0 additional balls OR 45 additional RED balls AND 45 additional BLACK balls.
For the next sequence of choice problems, suppose that a RED ball is drawn from BOX C.
Continue


Box C
Box C contains 10 balls identical to Box A (9 RED and 1 BLACK, or 1 RED and 9 BLACK) PLUS
either 0 additional balls OR 45 additional RED balls AND 45 additional BLACK balls.
For the next sequence of choice problems, suppose that a BLACK ball is drawn from BOX C.
Continue

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