Competitive Nonlinear Pricing under Adverse Selection

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A difficulty in the theory of value: 50 years after

Arrow–Debreu does not apply if a trading partner has private information about some *common* value (Akerlof 1970).

E.g., a car buyer may draw inferences about its quality from the mere fact that the seller supplies it at the market price.

But in many cases the good is divisible, which gives rise to a richer set of conditioning events, such as the quantity supplied.

Then the details of the market environment matter, and typically game theory has been used to predict market outcomes.

This revolution has deeply changed our views on how insurance, financial, and labor markets work (Riley 2001).

The focus of this talk: **Nonexclusivity**

Working definition: There is nonexclusivity when an informed agent can *privately* trade with *several* partners at the same time.

This is irrelevant in Akerlof 1970 because the traded good is indivisible and stochastic contracts are ruled out by assumption.

But when the traded good is divisible, the conditioning events under exclusivity and nonexclusivity have different meanings.

E.g., how can I "signal through quantities" if I deal with several partners who cannot control each other's trades with me ?

Why care about nonexclusivity ?

This is not only a scholastic question, because the exclusivity assumption is not satisfied in many relevant markets.

Most empirical studies assume that contracting is exclusive, leading to possible misspecification problems.

Exclusivity raises delicate modelling issues such as information sharing or contractible contracts.

Nonexclusivity is a natural benchmark for modeling competition, leaving agents as much freedom as possible.

Plan of the talk

1. Primitives

2. Inactive markets: The adverse-selection death spiral

3. Active markets: Contracting under nonexclusivity

4. A nonstrategic approach: Entry-proofness

5. Strategic approaches I: Competitive screening games

6. Strategic approaches II: Discriminatory ascending auctions

7. Empirical perspectives

Primitives

Preferences and costs

There are I types of buyers, with distribution $\boldsymbol{m}$. Type i derives continuous and quasiconcave utility u_i from the quantity q of a divisible good she consumes and the transfer t she pays in return.

SC *The u_i's satisfy single-crossing*:

For all $i < j$ and $q < q'$, $u_i(q,t) < u_i(q',t') \Rightarrow u_j(q,t) < u_j(q',t')$.

$\tau_i(q,t) \equiv$ type i's willingness-to-pay at (q,t).

Each seller has constant returns to scale and his unit cost of serving type i is c_i.

CV *Higher types are on average more costly to serve*:

$\overline{c}_i \equiv \mathbf{E}[c_j \,|\, j \geq i]$ *is nondecreasing in* $i \Leftrightarrow$ *For all* $j \leq i$, $c_j \leq \overline{c}_i$.

The complete-information benchmark (Malinvaud 1972)

Under complete information, it is efficient to open I markets for differentiated commodities indexed by buyers' types.

In the resulting competitive equilibrium, every type i purchases her demand $D_i(c_i)$ on market i at the fair price c_i.

When information becomes private, buyers are only willing to trade on the markets with the lowest price $p \equiv \min_i c_i$.

But then aggregate profits $\mathbf{E}[(p - c_i)D_i(p)]$ would be negative, except in the limiting private-value case where $c_i = c_j$ for all i, j.

From now on, we assume that each buyer privately knows her type. Then SC + CV $\Leftrightarrow$ *Weak* adverse selection.

Examples

This general description encompasses many specifications that have been considered in the literature:

Insurance markets: Rothschild and Stiglitz 1976, Prescott and Townsend 1984, Crocker and Snow 1985, Hendren 2013.

Corporate finance: Leland and Pyle 1977, DeMarzo and Duffie 1999, Biais and Mariotti 2005.

Market microstructure: Glosten 1994, Biais, Martimort, and Rochet 2000, Back and Baruch 2013.

Labor markets: Spence 1973, Miyazaki 1977.

Inactive markets: The adverse-selection death spiral

A generalization of Akerlof (1970) and Hendren (2013)

We want to understand why a market may stay idle despite there being first-best gains from trade, i.e., $\tau_i(0,0) > c_i$ for some i.

The problem is adverse selection: A contract (q,t) that attracts some type i also attracts the on average more costly types $j > i$.

As a result, the relevant costs under adverse selection are not the costs c_i's, but their upper-tail conditional expectations $\overline{c}_i$'s.

Theorem 1 *Suppose that $\tau_i(q,0) \leq \tau_i(0,0)$ for all i and $q > 0$. Then a no-trade equilibrium exists if and only if*

For each i, $\tau_i(0,0) \leq \overline{c}_i$.

Stronger version holds under strict quasiconcavity/single-crossing.

A bound on profits

Necessity only requires single-contract offers.

Sufficiency requires considering menus $(q_i, t_i)_{i=1}^{I}$, with $q_j \geq q_i$ for all $j > i$ by SC (no sorting).

Following Wilson (1993), profits may be rewritten as

$$\sum_i m_i(t_i - c_i q_i) = \sum_i \left(\sum_{j \geq i} m_j\right)[t_i - t_{i-1} - \overline{c}_i(q_i - q_{i-1})]$$

$$\leq \sum_i \left(\sum_{j \geq i} m_j\right)[\tau_i(q_{i-1}, t_{i-1}) - \overline{c}_i](q_i - q_{i-1}).$$

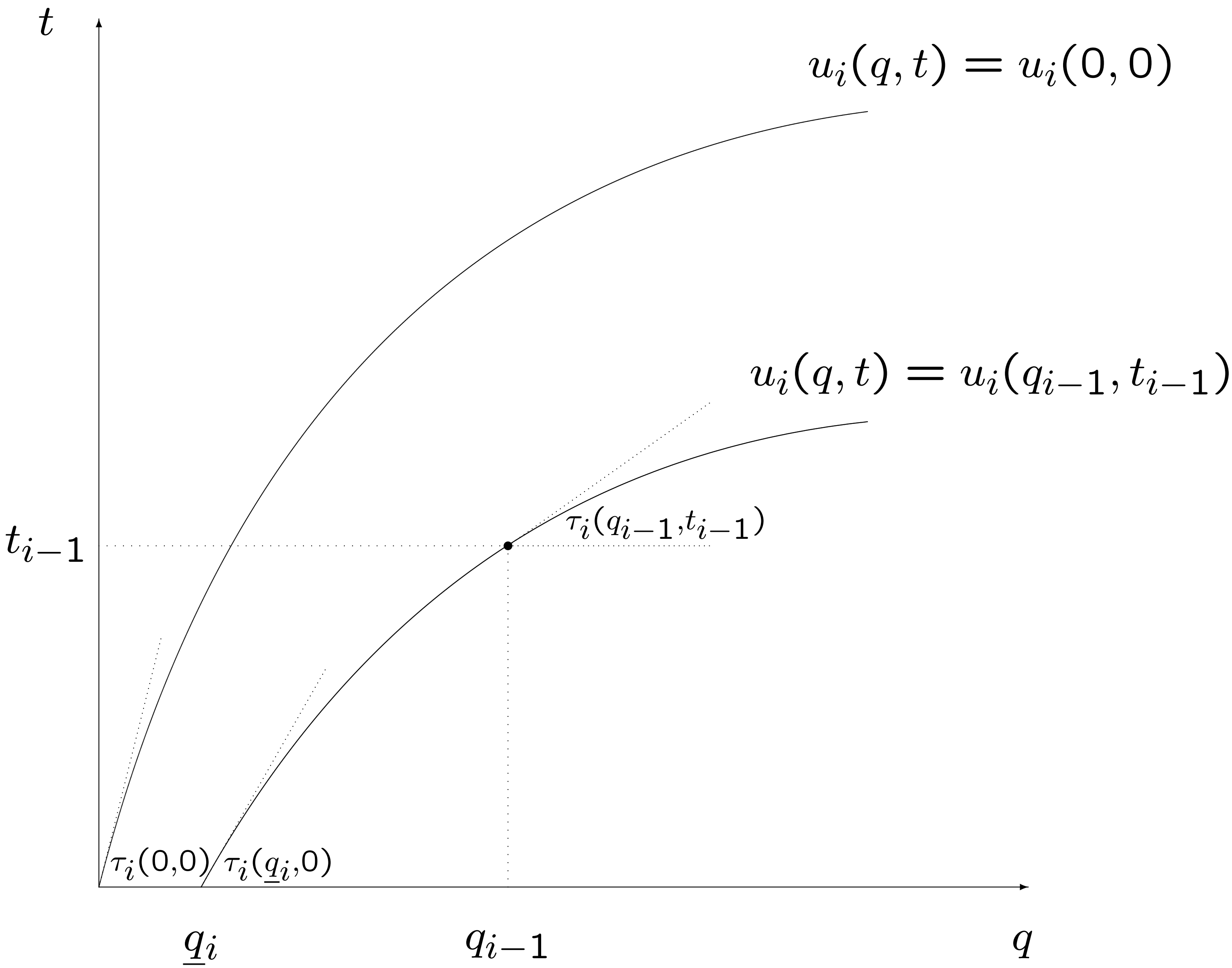

Figure 1. A graphical proof that $\tau_i(q_{i-1}, t_{i-1}) \leq \overline{c}_i$.

Active markets: Contracting under nonexclusivity

Nonexclusivity

We depart from the literature from Rothschild and Stiglitz 1976 to Azevedo and Gottlieb 2017 by focusing on nonexclusivity.

This means that no seller can monitor the trades a given buyer makes with his competitors, allowing for multiple contracting.

We assume that contracting is bilateral. Thus a contract offered by seller k is a (possibly null) quantity-transfer pair (q^k, t^k).

In particular, each seller can recognize each buyer. Thus nonlinear tariffs are feasible and we can focus on the one-buyer case.

After privately learning her type, the buyer chooses from the set of offered contracts and overall trades $(\sum_k q^k, \sum_k t^k)$.

Differences with the side-trading literature

Side trades are typically assumed to take place on Walrasian markets (Allen 1985, Hammond 1987, Jacklin 1987, Cole and Kocherlakota 2001, Golosov and Tsyvinski 2007).

This comes from an intuition forged with anonymous markets, in which a buyer can make several purchases from a seller without being recognized, which pushes towards linear prices.

But nonexclusivity in our sense does not entail anonymity: Though no seller can monitor the trades a buyer makes with the other sellers, he can monitor the trades she makes with him.

Applications

Our modeling choices are relevant for several important markets:

Insurance markets: Annuities, life, long-term care.

Corporate finance: Security issuances, securitization.

Market microstructure: Limit-order book.

Labor markets: Professionals, freelance workers.

A nonstrategic approach: Entry-proofness

The linear-pricing candidate (Pauly 1974)

A linear market tariff is entry-proof if an entrant cannot propose a linear tariff at a lower price that would make a profit.

The unique entry-proof linear market tariff is associated to the lowest price p that solves

$$p = \mathbf{E}\left[c_i \, \frac{D_i(p)}{\mathbf{E}[D_i(p)]}\right].$$

This formula is widely used in the annuity literature (Sheshinski 2008, Hosseini 2015, Rothschild 2015).

Justification: Buyers can linearize any nonlinear tariff by trading many small contracts with different sellers (Chiappori 2000).

The limit-order entry

If different types purchase different amounts at price p, offering a slightly lower price along with a ceiling on quantities is profitable.

Offering such a limit-order is actually the best an entrant can do when facing a linear market tariff (AMS 2019).

If an entrant can control prices *and* quantities, a linear market tariff is not entry-proof unless demands are 0/1 (Akerlof 1970).

Entry-proofness under nonexclusivity

Definition 1 *The market tariff* T^M *is* entry-proof *if, for any entrant's tariff* T^E, *there exists for each* i *a solution* (q_i^M, q_i^E) *to*

$$\max_{(q^M,q^E)} u_i(q^M + q^E, T^M(q^M) + T^E(q^E))$$

such that the entrant makes at most zero profit:

$$\mathbf{E}[T^E(q_i^E) - c_i q_i^E] \leq 0.$$

The entrant can offer an arbitrary menu of contracts which the buyer is free to *combine* with trades along the market tariff.

Our goal is to characterize the budget-feasible allocations that are implemented by an entry-proof market tariff.

Existence and uniqueness: The two-type case

Theorem (AMS 2020) *The* JHG allocation *(after Jaynes 1978*, *Hellwig 1988*, *Glosten 1994) defined by*

$$q_1^* \equiv \arg\max_q u_1(q, \overline{c}_1 q),$$

$$t_1^* \equiv \overline{c}_1 q_1^*,$$

$$q_2^* \equiv q_1^* + \arg\max_q u_2(q_1^* + q, t_1^* + \overline{c}_2 q),$$

$$t_2^* \equiv t_1^* + \overline{c}_2 (q_2^* - q_1^*),$$

is the only budget-feasible allocation implemented by an entry-proof market tariff. One such tariff is convex and piecewise linear, *with slopes* $\overline{c}_1$ *over* $[0, q_1^*]$ *and* $\overline{c}_2$ *over* $[q_1^*, q_2^*]$*.*

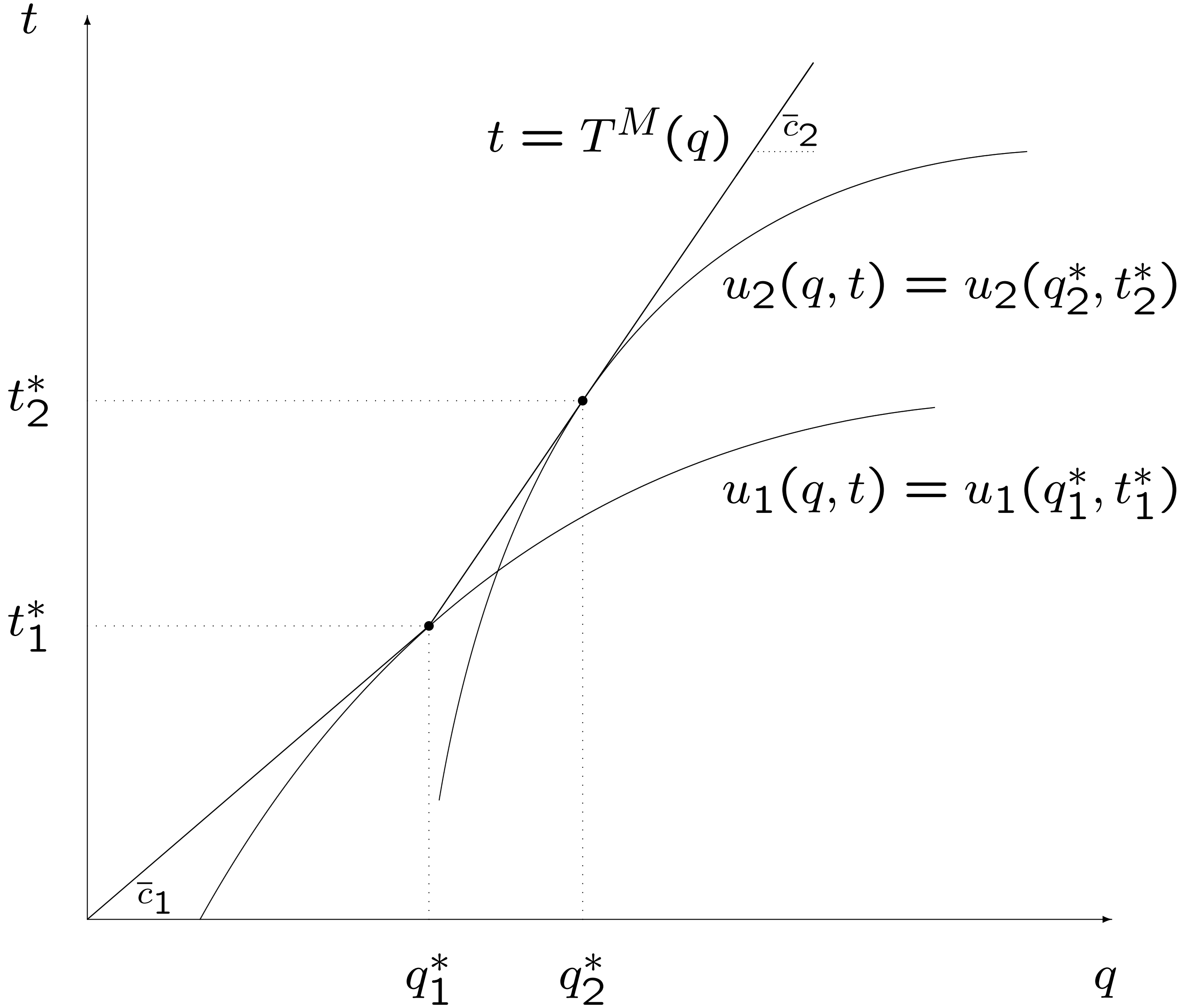

Figure 2. The JHG allocation when $I = 2$.

The uniqueness argument

If $(q_i, t_i)_{i=1}^{2}$ is implemented by an entry-proof tariff, then

$$u_1(q_1, t_1) \geq \max_q u_1(q, \overline{c}_1 q),$$

$$u_2(q_2, t_2) \geq \max_q u_2(q_1 + q, t_1 + \overline{c}_2 q).$$

This implies

$$t_1 \leq \overline{c}_1 q_1 \quad \text{and} \quad t_2 \leq t_1 + \overline{c}_2(q_2 - q_1).$$

Hence budget-feasibility (rewritten à la Wilson 1993)

$$t_1 - \overline{c}_1 q_1 + m_2[t_2 - t_1 - \overline{c}_2(q_2 - q_1)] \geq 0$$

holds if and only if these are all equalities, which yields JHG.

Differences with exclusivity (Rothschild and Stiglitz 1976)

The JHG allocation features partial pooling $\Rightarrow$ Two *layers* at prices $\overline{c}_i$ versus two *markets* at prices c_i.

The IC constraints are not binding under nonexclusivity $\Rightarrow$ The JHG allocation is not second-best unless values are private.

Under nonexclusivity, budget-feasibility and entry-proofness are never incompatible requirements $\Rightarrow$ No existence problem.

This is because the buyer can combine any entrant's contract with the market tariff $\Rightarrow$ New instruments to deter entry.

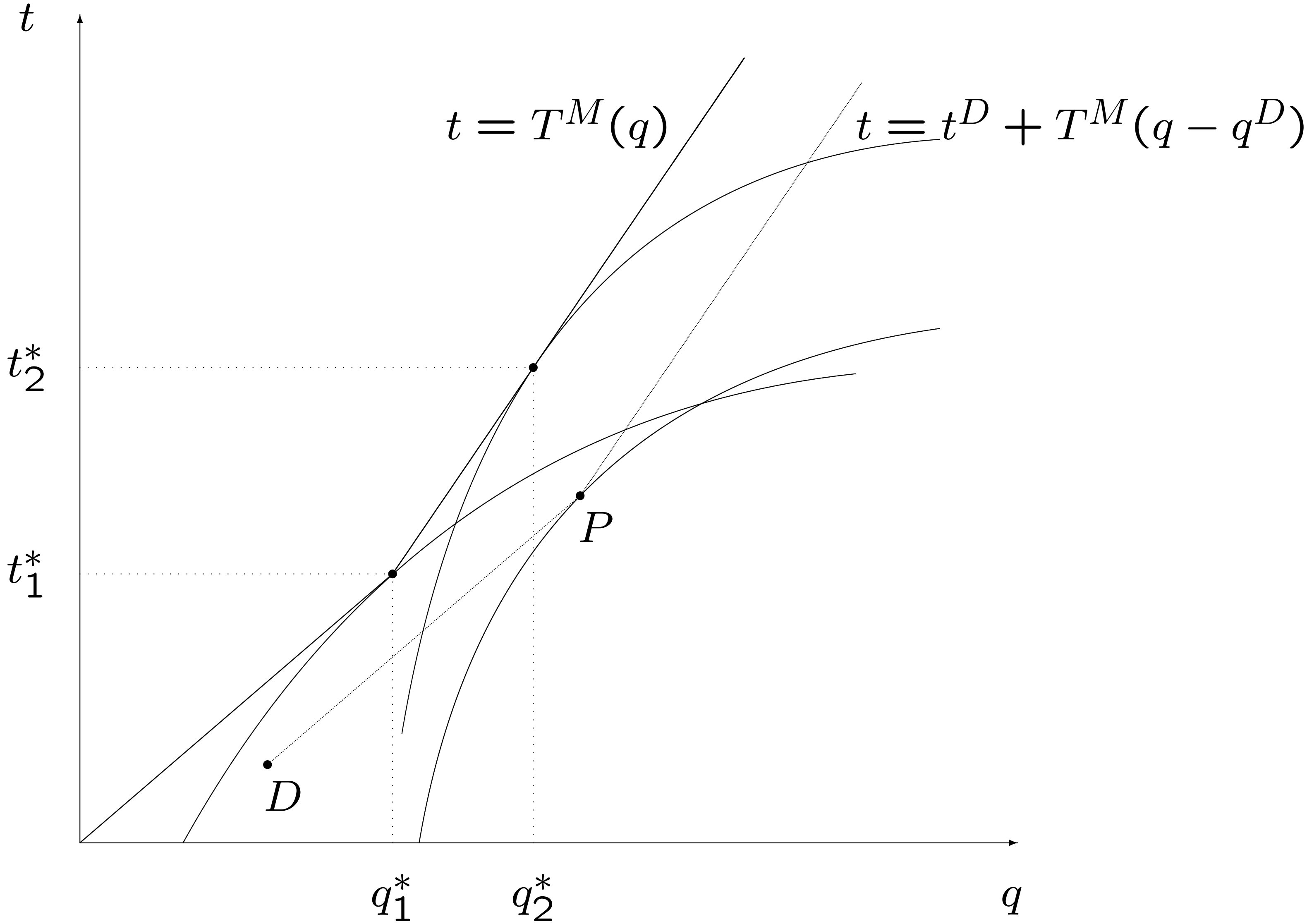

Figure 3. Blocking cream-skimming deviations.

Allocations, tariffs, and latent contracts

When trades can be perfectly monitored, the revelation principle ensures there is no need to distinguish tariffs from allocations.

Under nonexclusivity, *latent* contracts that are not traded by the buyer but are only meant to block entry need to be issued.

While such contracts are necessary, one must make sure that, by offering them, he does not create further entry opportunities.

The convex market tariff T^M strikes an optimal balance between these two requirements.

The convexity of T^M ensures that the entrant bears the full cost of adverse selection, unlike under exclusivity.

Existence and uniqueness: **The I-type case**

Generalizing the entry-proofness conditions in the two-type case, it is natural to conjecture that

For each i, $u_i(q_i, T^M(q_i)) \geq \max_{(q,q')} u_i(q + q', T^M(q) + \bar{c}_i q')$

is a necessary condition for the market tariff T^M implementing $(q_i, T^M(q_i))_{i=1}^I$ to be entry-proof. This yields

For each i, $T^M(q_i) \leq T^M(q_{i-1}) + \bar{c}_i(q_i - q_{i-1})$,

so that budget-feasibility (rewritten à la Wilson 1993) holds if and only if these are all equalities, which yields JHG.

Justifying the conjecture

We need to ensure that an interval of type is attracted by a deviation, a form of single-crossing on the indirect utilities

$$u_i^{T^M}(q', t') \equiv \max_q \, u_i(q + q', T^M(q) + t')$$

that allows us to extend the logic of entry-proofness from inactive markets to active markets, i.e., to show that

For each i, $\tau_i^{T^M}(0, 0) \leq \bar{c}_i$

is a necessary and sufficient condition for T^M to be entry-proof. One restriction on T^M that works is convexity.

Existence and uniqueness: **The I-type case**

Theorem 2 *The JHG allocation defined by* $(q_0^*, t_0^*) \equiv (0, 0)$ *and*

$$q_i^* \equiv \arg\max_q \, u_i(q_{i-1}^* + q, t_{i-1}^* + \overline{c}_i q),$$

$$t_i^* \equiv t_{i-1}^* + \overline{c}_i (q_i^* - q_{i-1}^*),$$

is the only budget-feasible allocation implemented by an entry-proof convex *market tariff. This tariff is piecewise linear*, *with slope* $\overline{c}_i$ *over the interval* $[q_{i-1}^*, q_i^*]$.

Convexity can be significantly relaxed, but the general case is an open question.

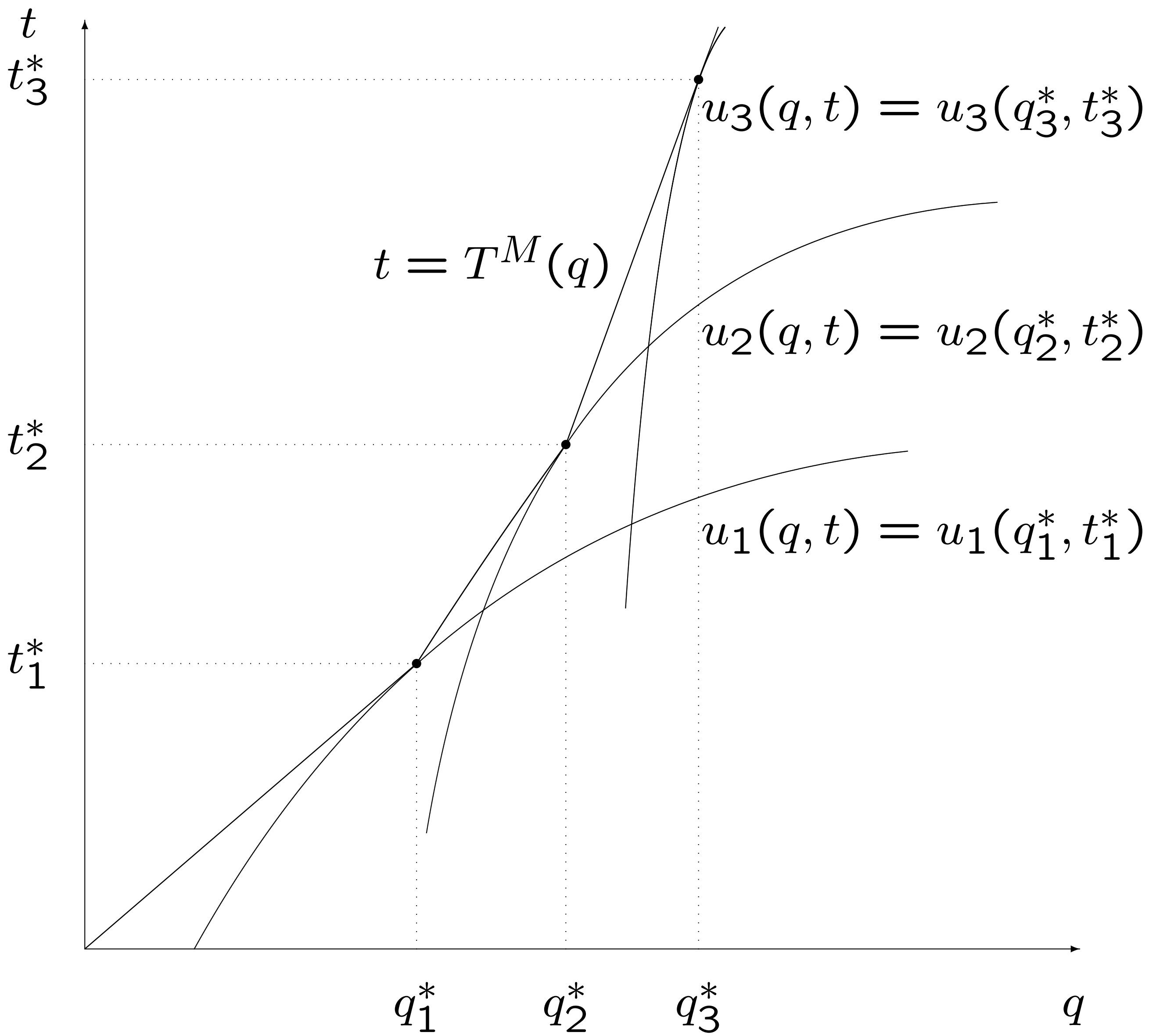

Figure 4. The JHG allocation when $I = 3$.

The limit-order-book interpretation (Glosten 1994)

Each layer corresponds to limit order with maximum quantities $q_i^* - q_{i-1}^*$ and unit price $\overline{c}_i$.

Each marginal quantity is priced at the expected cost of serving the types who purchase it.

On each layer, what sellers are ready to supply exactly matches the residual demand of the marginal type.

This generalizes Akerlof (1970), which arises when preferences are linear, subject to a capacity constraint (AMS 2011).

Strategic approaches I: Competitive screening games

The decentralization problem

The JHG allocation is the natural candidate for a competitive equilibrium under nonexclusivity and adverse selection.

But we do not know how the JHG tariff may come into existence in a decentralized market with strategic sellers.

Under exclusivity, once we have an entry-proof tariff, it is easy to turn it into an equilibrium of a competitive screening game.

This is not so under nonexclusivity, because buyers have the opportunity to combine contracts issued by different sellers.

Competitive screening games

Under simultaneous bilateral contracting, the menu theorems of Peters 2001, Martimort and Stole 2002, Page and Monteiro 2003 allow us to focus on games in which sellers compete by posting menus or tariffs:

1. Each seller k offers a compact menu of contracts C^k that contains at least the null trade $(0, 0)$.

2. After privately learning her type, the buyer selects a contract from each of the menus C^k.

Focus on pure-strategy SPNE (∞-dimensional strategy spaces).

A positive result in the linear case

Theorem (AMS 2011) *Suppose* $u_i(q,t) = v_i q - t$ *for* $q \in [0,1]$. *Then, generically, any equilibrium of the competitive screening game implements the JHG allocation, and there exists a linear-price equilibrium with price* $\overline{c}_{i^*}$, *where* i^* *is the first type* i *such that* $v_i > \overline{c}_i$.

This holds for any continuous distribution.

No seller is indispensable for anyone to reach her aggregate trade.

The equilibrium relies on linearly priced latent contracts.

A positive result in the continuous convex case (Biais, Martimort, and Rochet 2000)

In this case, a Cournot-convergence result holds:

Theorem (BMR 2000) *Suppose that* $u_i(q,t) = v_i q - \frac{\alpha\sigma^2}{2} q^2 - t$ *and* m *is continuous. Then, under regularity conditions, there exists a symmetric equilibrium in strictly convex tariffs in which the sellers earn positive profits. The aggregate tariff converges to the JHG tariff as the number of sellers grows large.*

The equilibrium is not entry-proof for finitely many sellers.

The equilibrium does not rely on ties or latent contracts.

"Un champ de ruines"

Theorem (AMS 2014) *Suppose that $I = 2$ and the u_i's are strictly quasiconcave. Then any equilibrium implements the JHG allocation, but an equilibrium exists if and only if $q_1^* = 0$ in that allocation.*

Trade can take place in equilibrium only if type 1 is left out of the market.

A necessary and sufficient condition for equilibrium existence is $\tau_1(0,0) \leq \overline{c}_1$.

Intuition: **“Lemon dropping” and “cherry picking”**

1. If $q_1^* > 0$, type 1 subsidizes type 2 at (q_1^*, t_1^*).

2. No seller is indispensable to provide (q_1^*, t_1^*).

3. Double deviation that exploits pivoting:

   - Same contract as that traded by type 1 on path.

   - Second layer $q_2^* - q_1^*$ at a unit price slightly less than $\bar{c}_2$.

The deviator neutralizes his loss with type 2 and secures a profit with type 1.

Cross-subsidies between contracts are the essence of the double deviation.

De profundis: **The convex-tariff game**

The convex-tariff game has less deviations. But still not works...

Theorem (AMS 2019) *Suppose that the u_i's are quasilinear and strictly quasiconcave. Then any equilibrium of the convex-tariff game implements the JHG allocation*, *but an equilibrium exists if and only if $q_i^* = 0$ for all $i < I$ in that allocation.*

Trade can take place in equilibrium only if all types but one are left out of the market.

A necessary and sufficient condition for equilibrium existence is $\tau_i(0,0) \leq \overline{c}_i$ for all $i < I$.

The BMR equilibrium is not a limit of equilibria of even finely discretized models.

Intuition: **Linear pricing strikes back**

1. If there is nonlinear pricing, then one must have JHG and hence sellers should earn zero profit in equilibrium.

2. But then each seller offering trades at a kink is indispensable for the types trading at this kink to reach their equilibrium utility.

3. As one can hardly be indispensable and make zero profit, this means that there is no kink, and thus pricing is linear.

Ways out

A quantum of solace is that the JHG allocation is an $O(1/K^2)$-equilibrium outcome when the number K of sellers grows large. But this is only a limit result, with no claim to uniqueness.

Mixed-strategy equilibria exist under quite general conditions (Page and Monteiro 2003, Carmona and Fajardo 2009). But they are hard to characterize and do not implement JHG.

Strategic approaches II: Discriminatory ascending auctions

Motivation

A noticeable feature of the JHG allocation is its recursive structure:

- Each layer is priced competitively.
- Supplying less on any layer would inefficiently ration demand.
- Supplying more would entail losses on the excess quantity.

Idea (Beaudry and Poitevin 1995): *Sequential* competition.

Here focus on discriminatory *ascending* auctions (no signaling).

The extensive form

Discrete price grid $\{0, \Delta, 2\Delta, \ldots\}$ + Quasilinear preferences.

Auctioning phase: When a price p is quoted, sellers publicly and simultaneously state the maximum quantities $s^k(p)$ they stand ready to trade at price p. Then move to $p + \Delta$, and so on.

$\Rightarrow$ Sellers commit, unlike in a *tatônnement* process.

$\Rightarrow$ Aggregate supply function S^M, convex market tariff T^M.

Buying phase: The buyer learns her type, and decides which quantities to purchase, at what prices, from whom. Overall, she purchases a quantity Q in exchange for a payment $T^M(Q)$.

$\Rightarrow$ Ties may occur only at the marginal price $\partial^- T^M(Q)$.

A simple equilibrium

States: Current price p and aggregate supply Q^- at prices $p' < p$.

$[D_j(p) - Q^-]^+ =$ Residual demand of type j in state (p, Q^-).

Profitable residual demand $\Rightarrow$ Type i such that $\overline{c}_i < p \leq \overline{c}_{i+1}$.

Theorem 3 *There exists a SPNE in which, in any state* (p, Q^-): *(i) If* $p \leq \overline{c}_1$, *each seller supplies* 0; *(ii) If* $\overline{c}_1 < p \leq \overline{c}_I$, *each seller supplies an equal share of the profitable residual demand*; *(iii) If* $p > \overline{c}_I$, *each seller supplies* ∞.

Aggregate equilibrium allocation $\rightarrow$ JHG allocation as $\Delta \rightarrow 0$.

The mechanics of the equilibrium

Key idea: At each price, each seller can condition his behavior on his and his competitors' past supply decisions.

No upward deviation is profitable as it leads to current losses and jeopardizes future profits.

No downward deviation is profitable as the increase in residual demand at the next price is shared with the other sellers.

The main thrust of punishments is borne by the sellers, leaving for the buyer only the task of breaking ties.

This explains the superiority of an ascending auction relative to a simultaneous one.

Convergence of equilibrium allocations

A SPNE is *robust to irrelevant offers* if no buyer type punishes a seller for deviating at a price at which she is not willing to trade.

Theorem 4 *For each n, fix any equilibrium robust to irrelevant offers of the ascending auction with tick size $\Delta/2^n$. Then the resulting sequence of aggregate equilibrium allocations converges to the JHG allocation, and the sequence of equilibrium market tariffs converges to the JHG tariff.*

A dynamic Bertrand argument

By Helly's selection theorem, S_n^M weakly converges to S_∞^M.

Let p be the last price at which profits can be earned given S_∞^M.

Each seller can attempt to reap these profits at $p' < p$.

For n large, this can be done almost without losing priority.

Thus no profits can be earned given S_∞^M, which yields JHG.

Empirical perspectives

Testing for adverse selection under exclusivity

A prediction of Chiappori and Salanié 2000 is that there should be a positive correlation between the aggregate coverage bought by a consumer and this consumer's risk.

Empirically, this may be tested by surveying consumers to get data on their total coverage and total insurance premium, or by gathering information on the contracts offered by firms.

Under exclusivity, these two approaches make no difference, as the aggregate demand of a consumer must be supplied by a single contract offered by a single firm.

An alternative empirical strategy

Under nonexclusivity, these approaches are not equivalent: the positive correlation property holds at the consumer level and consumers pay quantity premia, but the contracts offered by firms may exhibit negative correlation and quantity discounts.

An alternative approach would be to exploit price and cost data to compare the price of successive layers of insurance to their average cost, as measured by the empirical loss frequency of the consumers who trade them.

This approach would extend Einav, Finkelstein, and Cullen 2010 to environments where consumers can combine different levels of coverage from different firms, and Hendren 2013 to the case of inactive markets.