

# Credit lines, bank deposits or CBDC?

## Competition & efficiency in modern payment systems\*

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### Abstract

This paper studies the welfare effects of introducing a Central Bank Digital Currency (CBDC). Its premise is that CBDC is a new product in the market for liquidity where it competes with both commercial bank deposits and credit lines used for payments. If the central bank offers CBDC but not credit lines, then it interferes with the complementarity between credit lines and deposits built into modern payment systems. As a result, increasing CBDC may reduce welfare even if the central bank can provide deposits more cheaply than commercial banks.

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# 1 Introduction

Should central banks provide reserve accounts to everyone? A number of concrete proposals for central bank digital currency (CBDC) are now being discussed by policy makers as well as the general public. For example, the governor of the Swedish Riskbank has put the probability of issuing an "e-krona" within the next decade at greater than 50%.<sup>1</sup> Moreover, in the June 2018 Vollgeld referendum, Swiss voters assessed (and, for now, turned down), a proposal to introduce non-interest paying CBDC. Along with this general interest, an emerging literature is weighing the pros and cons of CBDC.

This paper is a theoretical study of the welfare effects of CBDC. Our approach is to view CBDC as a new product in the market for liquidity, broadly defined: CBDC competes not only with deposits, but also with credit lines offered by commercial banks for liquidity purposes. To illustrate the importance of credit lines for payments, Figure 1 plots deposits at all US commercial banks together with those banks' outstanding credit card limits. Since credit card limits provide only a lower bound on credit lines used for payments – for example, the figure leaves out credit lines provided to asset management firms by their custodian banks – the message is that credit lines matter.

Our focus on the broad market for liquidity leads to a skeptical assessment of CBDC. In particular, introducing CBDC would interfere with current payments technology that exploits complementarities between deposit taking and credit lines. Most CBDC proposals imply the emergence of a system with two types of banks: commercial banks that offer both deposits and credit lines, and a central bank that offers only the former. Our model shows that such hybrid systems entail costs that are not present in the current system when banks jointly provide credit lines and deposits. As a result, introducing CBDC may lower welfare even if the central bank can offer deposits at lower cost than commercial banks.

Our conclusions are derived from three assumptions. First, banks face leverage constraints: to be credible as payment instruments, deposits require backing by bank assets. Second, holding assets inside banks, or firms more generally, is costly: delegation of asset holdings gives rise to asset management costs. Finally, the provision of payment instruments – either credit lines or deposits – requires banks themselves to have access to liquidity to execute customers' payment instructions. A good payment system minimizes costly asset holdings that are required as collateral to back promises or as liquid assets to meet liquidity needs.

Banks that jointly offer credit lines and deposits economize on both collateral and liquid assets. Indeed, when a customer makes a payment by drawing down a credit line, the bank-

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<sup>1</sup><https://news.bitcoin.com/central-bank-digital-currencies-take-center-stage-at-imf-spring-meetings/>

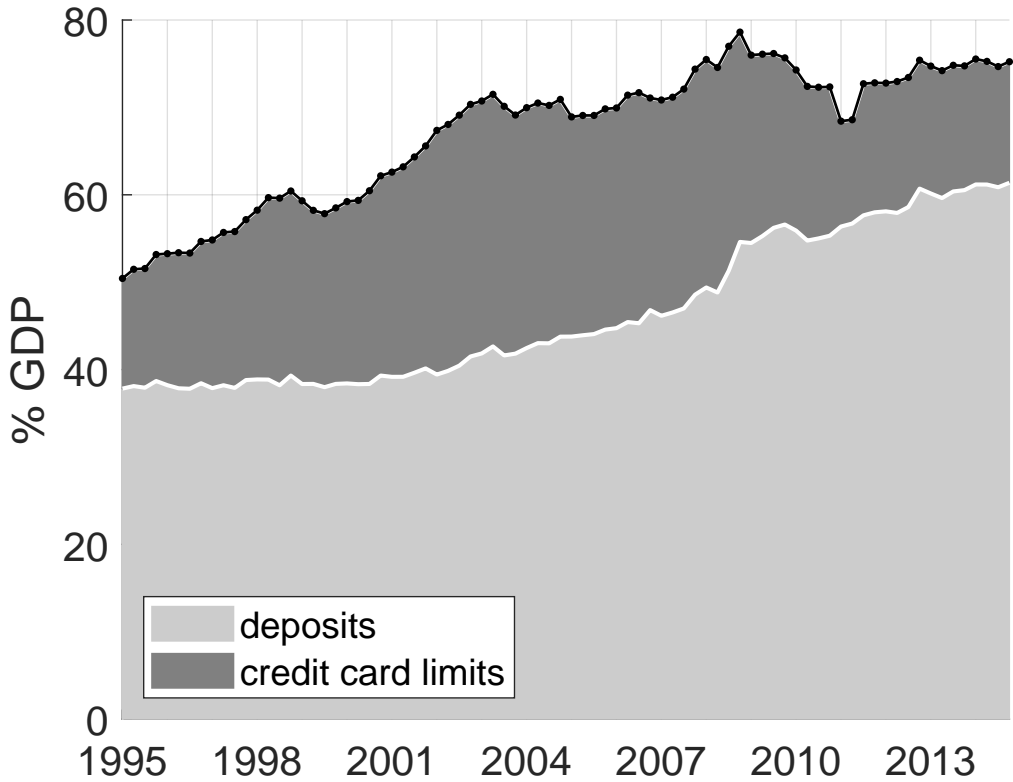


Figure 1: Deposits and credit card limits at US commercial banks

ing system creates a matching deposit account. The loan serves as collateral for these new deposits. At the same time, no liquid assets are needed to handle the payment instruction: the bank creates liquidity on its books. If instead deposits and credit lines are provided by separate banks, then more assets are needed to facilitate payments: loans have to be funded and deposits have to be backed. Moreover, banks that provide credit lines need to hold liquid assets to manage deposit outflows that result from customer payments to banks that provide only deposits.

Our model builds on the standard neoclassical growth model. We add frictions that require payment instruments supplied by banks as well as collateral constraints and balance-sheet costs. We show that a model with those frictions is equivalent to a model with a less efficient production technology. A switch to the wrong payment system thus works like a negative technology shock: it has real effects on consumption, investment and the allocation of labor that lower welfare. We also add idiosyncratic shocks to preferences and production that make the liquidity needs of an individual agent not perfectly predictable.

We then compare three types of payment system. We first show that a system in which

banks jointly provide credit lines and deposits is superior to one where they only offer deposits. The gain from credit lines is especially large if many agents have liquidity needs that are difficult to predict. The advantage of credit lines is that assets on bank-balance sheets and the associated costs only reflect actual transactions. Deposits held by agents with unpredictable liquidity needs instead have a precautionary component that requires more assets and hence costs. The gain from credit lines is also larger if there is greater liquidity demand from agents who are not natural savers, for example, firms that themselves incur balance sheet costs.

To assess CBDC, we consider a hybrid system where the central bank offers deposits, and can do so more cheaply than commercial banks. We show that such a system is superior only if the cost advantage of the central bank is sufficiently large. As commercial banks compete with the central bank for funding, they reduce their supply of deposits. As a result, credit lines become more expensive. They nevertheless continue to be used since they are still cheaper than deposits for some bank customers. If the benefit from cheaper deposits is small, it is outweighed by the higher cost of credit lines and welfare declines. This force is stronger if liquidity needs are harder to predict as well as when there is more liquidity demand from agents who are not natural savers.

Our model is motivated by a liquidity-centric view of banking. In fact, the only way in which banks add value in our model is by providing liquidity. In particular, they do not have a special ability to lend, except by extending on-demand loans when credit lines are drawn. The role of other bank assets in the model is only to provide backing for deposits. A liquidity-centric view fits well with evidence on bank portfolio composition. Indeed, in most countries banks hold not only loans, for which they might have a special ability to lend, but also securities. Moreover, a sizeable share of loans tends to be mortgages that are easily securitized. In our model, it makes sense for banks to hold securities even though they are worse at holding them than households, because securities help back deposits and thus deliver payment services.

We restrict technology and preferences so that banks' required collateral is small relative to the capital stock of the economy. In other words, it is never important to accumulate capital in order to provide enough collateral to back payment instruments. Instead, society determines what share of capital is optimally held inside banks and holds the rest outside. This approach also fits well with data on sectoral wealth in modern economies. Indeed, banks typically hold only a share of fixed income securities outstanding in the economy, with another sizeable share of both government and private debt held either directly by households or indirectly through investment intermediaries such as pension funds. Moreover, in most countries, business and housing equity are held almost entirely outside banks.

We emphasize that our assumption that bank lending is small relative to capital does not mean that banking is irrelevant for investment. In our model, capital accumulation depends on the cost of payments because capital-goods producers require liquidity in order to produce. Our assumption on balance-sheet costs imply that capital-goods producing firms find it particularly burdensome to hold deposits and prefer credit lines. Interpreting capital broadly as physical plus intangible capital, we view this feature as a stand-in for the demand for payment services from nonbank financial institutions that typically favors credit lines over deposits. The implication is that introducing CBDC can have a stronger negative effect on investment relative to consumption since it distorts effective prices faced by firms more than those faced by households.

A premise of our analysis is that it is beneficial for society to minimize the amount of assets held inside banks and firms. Here we build on a large literature that has discussed the costs of delegated portfolio management. At the same time, delegated asset management may also have benefits, for example cheaper diversification or savings of transactions costs. Our approach assumes that such benefits can be achieved more easily through investment intermediaries such as mutual funds that are funded with equity. They do not require asset holdings inside banks that also issue debt or firms that also engage in production. Moreover, the delegated monitoring problems that arise in leveraged banks and producing firms – which are arguably more complicated than those of investment intermediaries – induce costs that outweigh any benefits that can be realized through investment intermediaries. It is thus optimal to minimize assets inside banks and firms, and to think of the household sector as consolidated with investment intermediaries.

To zero in on the key interaction of credit lines and deposits, our model abstracts from a number of other interesting considerations on CBDC. First, we define CBDC narrowly as a deposit contract and do not consider the option of anonymity that would make CBDC closer to physical currency. In terms of the "money flower" taxonomy of monies introduced by Bech and Garratt (2017), we study CBDC that is widely accessible (as opposed to restricted) as well as account-based (as opposed to token-based). In fact, we abstract from physical currency altogether and require that all payments are made with deposits or credit lines. As a result, we do not engage in the discussion of how CBDC might alter a potential lower bound on interest rates.

Second, we study an entirely real model and do not consider the determination of the price

level or how the transmission of monetary policy might change if a CBDC is introduced.<sup>2</sup> This approach is guided by our focus on long-run welfare from the design of the payment system. We thus formulate policy as the elastic supply of real balances at a certain spread between the interest rate on CBDC and a safe claim that is not liquid. In practice, one would expect the central bank to fix both the price and quantity of *nominal* CBDC. In the long run, the price level would then adjust to deliver the quantity of real balances desired by the economy.

Third, we work with frictionless capital and insurance markets. In particular, households have access to a complete set of contingent claims to insure against preference shocks and banks can issue equity at no cost at all times. In addition to making the model analytically tractable, these assumptions also clarify that our mechanism does not rely on net-worth constraints in banks. Moreover, banks in our model are not needed to facilitate consumption smoothing when assets are illiquid, as in the literature that studies banks as a mechanism for liquidity insurance. Their purpose instead is to provide immediacy of payments in goods market transactions that financial markets cannot provide. Future work might explore the interaction of our mechanism with other financial frictions.

Finally, we do not explicitly model credit risk and aggregate shocks. All lending is deterministic and there is no default. While this simplification precludes us to talk about some interesting features of the data such as risk premia or credit spreads, the loss for our study of balance-sheet positions is smaller. In particular, one interpretation of balance-sheet costs incurred by banks is bankruptcy costs in lending. Suppose for example, that banks hold claims to capital not by owning capital directly, but by making loans that carry idiosyncratic risk and deadweight costs of default. On average, banks then expect to lose a certain share of the return on capital. Households who hold capital directly or through investment intermediaries do not incur the same costs since they hold equity claims.

Our approach builds on the theoretical and empirical literature on liquidity provision. Strahan (2010) provides an overview and discusses the trend towards greater use of credit lines. Berger and Udell (2014) argue that including off-balance sheet measures such as loan commitments is important to measure the role of banks in an economy and provide cross-country evidence. Sufi (2007) shows that credit lines help firms avoid costs of holding cash. Consistent with this idea, our model builds in the assumption that firms have a stronger preference for credit lines over deposits, since they are not natural savers.

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<sup>2</sup>Piazzesi, Rogers and Schneider (2019) study the transmission of CBDC in a New Keynesian model. They argue that the transmission of interest-rate policy works is similar a floor system as currently implemented by many central banks, but different from traditional corridor systems because of a lower elasticity of broad money supply. Barrdear and Kumhof (2016) also study CBDC in a New Keynesian setup and derive welfare gains from a reduction in transaction costs.

Holmström and Tirole (1998) show how credit lines can help allocate liquidity when individual agents' needs for liquidity are not perfectly predictable, an important theme in our analysis. Kashyap, Rajan and Stein (2002) provide theory and evidence on complementarity between credit lines and bank deposits. They show that, at the level of the individual bank, liquidity management is cheaper when liquidity needs – that is, outflows of funds – implied by the two products are imperfectly correlated. They provide supporting cross-sectional evidence on banks' selection of products to offer (see also Gatev, Schuermann and Strahan (2009)).

In our model, complementarity is due to both collateral and liquidity constraints. In fact, in the version of the model without a central bank, liquidity management plays no role – we assume that bank-level shocks due to customer-payment instructions net out across banks so they can be managed with a negligible amount of reserves. As a result, banks incur no liquidity-management costs regardless of whether they offer deposits or credit lines. Complementarity is nevertheless present because loans due to drawn down credit lines serve as collateral for deposits created when payments occur. This is why fewer collateral assets are needed when banks offer both products.

Liquidity management does matter once we discuss the role of CBDC. Here we use the fact that, for the aggregate banking system, the correlation of liquidity needs is perfectly negative: every drawn credit line (an outflows of funds) gives rise to a new deposit account (an inflow of funds). The impact of CBDC is then that some funds obtained via credit lines flow to the central bank that offers only deposits. Commercial bank liquidity needs thus become less negatively correlated. Again, this weakening of complementarity not only affects liquidity management but also the cost of collateral for the economy as a whole.

A growing literature discusses various forms that CBDC might take, assesses their pros and cons and compares them to other forms of electronic money. Bech and Garratt (2017) provide a taxonomy of monies together with historical examples. Chapman and Wilkins (2019) point out the joint trends of innovation in cryptoassets and decline of cash and discuss challenges for central banks once banks coexist with (public or private) providers of digital currencies. Engert and Fung (2017) survey motivations for CBDC that have been brought up in the literature.

The small theoretical literature on CBDC has focused on competition with bank deposits as opposed to credit lines. Andolfatto (2018) studies a model of monopoly banking where the introduction of CBDC reduces rents in the deposit market and increases financial inclusion. He also emphasizes that there need not be a detrimental effect on investment. While our model abstracts from bank market power, we agree that assessing its role for current bank spreads may matter for judging the impact of CBDC. In our context, it would be interesting to explore the role of concentration and customer capital in both the credit line and deposit

markets, as well as possible bundling of goods by noncompetitive financial institutions.

Faure and Gersbach (2018) and Brunnermeier and Niepelt (2019) are interested in when fractional reserve banking and a system with CBDC are equivalent in terms of allocations. We share their approach of writing down environments with monetary exchange and comparing alternative payment systems. However, a key condition that is violated in our setup is that expanding balance sheets of banks (or central banks) is costless. Indeed, the equivalence results use the idea that when CBDC attracts depositors away from commercial banks, then the central bank can undo those positions by lending to commercial banks. In our model, balance sheet costs penalize the total length of balance sheets so that the allocation would not be equivalent to one without CBDC. Our focus on balance sheet costs also leads us to assign the government a technology for borrowing and asset management that is different from that of commercial banks.

Keister and Sanches (2019) also emphasize complementarity between deposits and lending. In their model, however, bank lending is important for funding investment and banks face net worth constraints. Offering deposits at low interest rates relaxes net worth constraints and increases investment, a benefit that is weakened by CBDC. In our model, in contrast, complementarity comes from liquidity provision; our banks are not important for funding investment and do not face net worth constraints. Our approach targets economies with large banks and highly developed securities markets.

The rest of the paper is structured as follows. Section 2 uses simple balance-sheet diagrams to illustrate the main arguments. The diagrams capture the positions taken by agents in the model, but do not get into endogenous prices and choices. Section 3 then lays out the full model and Section 4 provides uses it to compare alternative payment systems.

## 2 Transactions and payment instruments

Consider a buyer and a seller who are about to transact an amount of goods  $T$ . The buyer needs liquidity in order to pay the seller. One option is for the buyer to hold deposits before the transaction takes place, and then instruct his bank to transfer funds to the seller. We assume that both the buyer and the seller hold deposits for some period of time – perhaps short – around the date of the transaction.

We further assume that the buyer did not know for sure up front how many goods he wanted to consume. He thus holds deposits  $D = T/v$  where  $v \in (0, 1)$ . We say that liquidity needs are more predictable if  $v$  is larger. In the extreme case  $v = 1$ , the buyer knows exactly how much he wants to consume and holds just enough deposits to cover spending.



The banking system must provide deposits both before and after the transaction takes place. In order to do so credibly, it must hold assets worth  $D/\phi$ . The parameter  $\phi$  works like a bound on bank leverage. Banks must further pay a proportional balance-sheet cost  $\kappa$  per unit of assets held.

Table 1 illustrates balance sheets. Panel A shows that, before the transaction, the buyer holds deposits  $D$  with the bank. The bank holds  $A = D/\phi$  assets to back those deposits and finances the remaining assets with equity  $E$ . The other items in the balance sheets are not important for the argument, so we do not show them explicitly.

Panel B shows balance sheets after the transaction. The payment  $vD$  has now been subtracted from deposits and instead appears in the deposit account of the seller. Total deposits at the bank  $D$  have not changed. We note that handling transactions does not imply a net outflow from (or inflow into) the banking system. Banks thus do not require reserves in this example.<sup>3</sup>

Since banks hold  $A = D/\phi = T/\phi v$  assets both before and after the transaction, we write the total cost of liquidity provision as

$$\kappa A + \kappa A = 2\kappa T/\phi v.$$

Liquidity is cheaper if banks can lever more as well as when liquidity needs are more predictable.

In an economy where all transactions are supported by deposits, introducing CBDC is beneficial if the central bank has a lower cost of providing deposits. For example, suppose that the central bank is more efficient at managing assets and thus has a lower balance-sheet cost  $\kappa^*$ , or that the central bank can credibly back deposits with fewer assets to achieve a higher ratio  $\phi^*$ . The costs of liquidity provision with CBDC are  $2\kappa^* T/\phi^* v$ .

An environment in which all payments are made with deposits thus provides an argument in favor of the introduction of CBDC. The same argument applies to an environment in which buyers do not qualify for a credit line.

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<sup>3</sup>We have implicitly assumed here that the transaction does not create imbalances within the banking system. For example, if all sellers had accounts at one bank and all buyers at another, then transactions would require interbank transfers. If banks are similar, we would expect transfer to net out so as to require almost no reserves.

TABLE 1: DEPOSIT PAYMENT

Panel A: Before the transaction

Buyer		Seller		Bank	
Assets	Liabilities	Assets	Liabilities	Assets	Liabilities
$D$				$A$	$D$ $E$

Panel B: After the transaction

Buyer		Seller		Bank	
Assets	Liabilities	Assets	Liabilities	Assets	Liabilities
$(1 - v) D$		$v D$		$A$	$D$ $E$

## 2.1 Deposits and Credit Lines

Suppose now the buyer arranges a credit line with his bank. The credit limit  $L$  is chosen to satisfy the same overall liquidity need as before, that is,  $L = T/v$ . The buyer then draws down the credit line to pay  $T = vL$  with his credit line to the seller, who places the funds in his deposit account.

At the time that the buyer arranges the credit line, the bank does not need to hold assets to back the credit line. After the transaction, in Panel B of Table 2, the buyer has drawn  $vL$  from the credit line, which is a liability for the buyer and an asset for the bank. The seller receives a deposit which the bank backs with assets  $\bar{A}$  that satisfy

$$vL = \phi(vL + \bar{A}) \implies \bar{A} = \frac{1 - \phi}{\phi} vL.$$

The bank finances these assets with equity  $\bar{E} = \bar{A}$ .

TABLE 2: CREDIT LINE PAYMENT

Panel A: Before the transaction					
Buyer		Seller		Bank	
Assets	Liabilities	Assets	Liabilities	Assets	Liabilities

Panel B: After the transaction					
Buyer		Seller		Bank	
Assets	Liabilities	Assets	Liabilities	Assets	Liabilities
	$v L$	$v L$		$v L$	$v L$
				$\bar{A}$	$\bar{E}$

The costs of liquidity provision are

$$\kappa v L + \kappa \bar{A} = \kappa \left( 1 + \frac{(1 - \phi)}{\phi} \right) v L = \kappa \phi T.$$

A credit line economizes on balance-sheet costs for two reasons. First, deposits require asset holdings both before and after transactions take place in order to provide liquidity to the buyer and the seller, respectively. When a credit line is used, in contrast, the transaction generates an asset that in turn backs deposits. This complementarity economizes on asset holdings and cuts cost by a factor of 2. Second, only a fraction  $v < 1$  of the credit limit is drawn, which generates a smaller amount of deposits  $v D$  than the deposit amount  $D$  involved with a deposit payment and therefore lower asset costs. This property cuts costs by another factor of  $1/v$ . Credit lines thus have a greater advantage when liquidity needs are less predictable.

A credit line is also associated with lower costs if the buyer and/or the seller are firms that face costs for holding assets as do banks. If the buyer is a firm, a deposit payment leads to an additional asset cost  $\kappa D + \kappa(1 - v) D$ . If the seller is also a firm,  $\kappa v D$  are added to the cost. A credit line payment leads to a lower additional asset management cost:  $\kappa v L$  for the seller and the buyer firm.

## 2.2 Central bank digital currency

We now introduce CBDC into an economy with deposits and credit. For simplicity, we consider a scenario where the seller strictly prefers an account at the central bank over an account at a commercial bank. We also assume that the buyer continues to prefer a credit line from a

bank. This will typically be the case if liquidity needs are hard to predict and CBDC is not too cheap.

The introduction of CBDC decouples credit and deposits. The funds received by the seller become digital currency, a liability of the central bank in Panel B of Table 3. The central bank backs the digital currency with assets  $A^* = vL/\phi^*$  and needs equity  $E^* = v(1 - \phi^*)L/\phi^*$ . Since the commercial bank uses deposits to fund the loan it extends to customers who draw the credit line, it turns instead to equity  $\bar{E}'$ .

The introduction of CBDC further implies that the transaction generates an outflow of funds from commercial banks to the central bank. In order to meet this outflow, commercial banks must hold liquid funds already before the transaction. We assume that commercial banks hold deposits at the central bank. Panel A shows that the central bank then already issues deposits worth  $vL$  before the transactions, which are held by commercial banks and funded with equity.

TABLE 3: CBDC AND CREDIT LINES

Panel A: Before the transaction							
Buyer		Seller		Bank		Central Bank	
Assets	Liabilities	Assets	Liabilities	Assets	Liabilities	Assets	Liabilities
				$vL$	$E'$	$A^*$	$vL$
							$E^*$

Panel B: After the transaction							
Buyer		Seller		Bank		Central Bank	
Assets	Liabilities	Assets	Liabilities	Assets	Liabilities	Assets	Liabilities
	$vL$	$vL$		$vL$	$E'$	$A^*$	$vL$
							$E^*$

The costs of liquidity provision are

$$2\kappa vL + 2\kappa^* A^* = 2\kappa T + 2\frac{\kappa^*}{\phi^*} T.$$

If the central bank has the same technology as the commercial bank,  $\kappa^* = \kappa$  and  $\phi^* = \phi$ , these costs are lower than those with only deposit payments,  $2\kappa T/\phi v$ . However, they are clearly higher than with credit. The reason is that the introduction of CBDC decouples credit from deposits and thereby destroys the complementarity in liquidity production.

Our model below formalizes derives the same logic in a more elaborate model. The model

describes how credit lines and deposits are priced and endogenizes customers' and banks' demand and supply decisions. These margins offer some additional insights on when gains from CBDC are present. The basic message is the same: CBDC is only desirable if it is sufficiently cheap. If it is only somewhat cheaper than commercial bank deposits, then the introduction of CBDC may reduce welfare.

### 3 Setup

#### 3.1 Preferences and technology

Time is discrete and divided into integer dates  $t = 1, 2, 3, \dots$  and intermediate dates  $t = 1.5, 2.5$  and so on. At integer dates  $t$ , capital is installed for production at the next intermediate date  $t + .5$ . At intermediate dates, final goods are made from labor. Final goods can be consumed or converted into capital goods for installation at the next integer date.

The production function for final goods at intermediate date  $t$  is

$$Y_t = K_t^\alpha N_t^{1-\alpha}, \quad (1)$$

where  $K$  is capital that depreciates at rate  $\delta$  per unit of time and  $N$  is labor. There is a continuum of capital goods, which are all perfect substitutes. At intermediate dates, a fraction  $v_i$  of capital goods can be produced one-for-one from consumption goods. The productivity shocks that select the producible goods are iid across goods and time.

There is a continuum of households. At intermediate dates, a fraction  $v_c < 1$  of individual households get utility from consumption, while all households work. A preference shock  $\zeta_t \in \{0, 1\}$  selects whether a household consumes or not. The preference shock is iid across households and time. (To keep notation simple, we do not use a household-specific index  $i$ .) Felicity over consumption and labor is

$$\zeta_t \log c_t - \theta \frac{N_t^{1+1/\varepsilon}}{1+1/\varepsilon}.$$

Preferences are time separable and households discount the future with a discount factor  $\beta$  per unit of time.

Let  $C_t = E[c_t(\zeta_t)]$  and  $I_t$  denote aggregate consumption and investment, respectively. The resource constraints for final goods at an intermediate date  $t$  and for capital at the subsequent

integer date are

$$\begin{aligned} C_t + I_t &= Y_t, \\ K_{t+1} &= I_t + (1 - \delta) K_t. \end{aligned} \tag{2}$$

Here  $K_{t+1}$  is capital installed at the integer date  $t + .5$  for production at the subsequent intermediate date  $t + 1$ .

*First best allocation*

The social planner maximizes expected utility subject to the production function and the resource constraints. Since shocks are iid, it is optimal to perfectly insure households against preference shocks: the consumption of a household who consumes at date  $t$  is  $c_t = C_t/v_c$ . In terms of aggregates, the optimal allocation is exactly that of the standard neoclassical growth model. Indeed, felicity for the social planner can be written in terms of aggregate consumption with the probability  $v_c$ : up to a constant, we have

$$v_c \log C_t - \theta \frac{N_t^{1+1/\varepsilon}}{1 + 1/\varepsilon}. \tag{3}$$

The optimal allocation is independent of  $v_i$ . It reflects  $v_c$  via the relative weight of consumption versus labor: since everyone works in the intermediate periods, hours are optimally lower in economies with lower  $v_c$ .

### 3.2 Financial frictions

We now define a decentralized equilibrium with banks and firms and introduce frictions that make money important. The basic principles are that goods trading requires payment instruments, that leverage is limited, and holding assets inside firms is costly. At the same time, we allow for financial markets that are essentially frictionless, and allow easy financing of firms as well as insurance against idiosyncratic shocks.

At integer dates, households have access to a full set of contingent claims. In particular, they can trade claims to the outcomes of their own preference shocks as well as firms' productivity shocks. Production is run by two types of competitive firms. Consumption-goods producers rent capital and hire labor to make final goods at intermediate dates. Capital-goods producers buy final goods at intermediate dates to make capital goods ready for installation at the next integer date. Factor markets are competitive; we write  $w_t$  for the wage and  $r_t$  for the rental rate of capital at an intermediate date, both in units of consumption goods. Both types of firms as well as banks introduced below are owned by households.

Since markets for trade between integer dates are complete, households will share the risk due to idiosyncratic shocks. To simplify the analysis, we work directly with a large family that pools resources at integer dates. It then decides how to arrange for liquidity for the individual members. The family further owns all firms as well as banks – competitive firms introduced below – and maximizes their shareholder value. Even though idiosyncratic shocks can be insured, they will still play a role for the allocation because payment instruments, such as deposits, that are needed in transactions, are not fully contingent.

We introduce a demand for payment instruments through cash-in-advance constraints. In particular, we assume that goods-market transactions at intermediate dates require payment of money from buyers to sellers. Buyers must make sure, at the previous integer date, to have money to pay for any purchases, either by holding deposits or by arranging a credit line. Similarly, sellers must carry funds from sales to the next integer date via money. Factor market transactions are agreed at intermediate dates, but settled only at the next integer date. While all markets are Walrasian, the cash-in-advance constraints are designed to capture the fact that goods-market trade is not perfectly synchronized and centralized.

The setup creates two types of demand for liquidity. For households and capital-goods producers who face idiosyncratic shocks, the need for liquidity is not perfectly predictable. As a result, holding a noncontingent asset such as deposits always means that there is a state of the world where the agent has "too much" liquidity. In contrast, for consumption-goods producers who just need to store funds, the need for liquidity is predictable and hence deposits are perfectly adequate. Payment instruments are provided by banks, discussed in detail below. Banks face collateral constraints: to provide deposits they must hold assets as collateral.

Holding assets inside firms or banks requires asset-management services. We assume that those services are produced by an asset-management sector that rents capital and hires workers just like consumption-goods producers. If the payments technology is more costly, it therefore requires more factor inputs, leaving fewer inputs for the production of consumption goods and capital goods. We do not require the use of payment instruments in the market for asset-management services. This approach is convenient since it avoids differences in welfare due to how resource costs of liquidity provision are themselves paid for. We believe such effects represent a relatively small share of output and do not want them to drive our conclusions.

We define  $Q_t$  as the relative price of capital at an integer date  $t$  in units of consumption goods at the prior intermediate date  $t - .5$ , and write  $Q_t/R_t$  as the relative price of capital in terms of consumption goods at the following intermediate date  $t + .5$ . Relative prices of

consumption goods between adjacent periods are indeterminate, since consumption goods cannot be converted into capital at any given date. We adopt the convention that the gross interest rate between an intermediate date and the following integer date is equal to one, so that  $R_t$  is the gross interest rate, in units of consumption goods, between an integer date  $t$  and the following intermediate date  $t + .5$ .

### 3.3 Household problem

We focus first on the household problem when the only payment instrument is deposits. The interest rate on deposits between dates  $t - .5$  and  $t$  is  $R_{t-.5}^D$ . The interest rate between dates  $t$  and  $t + .5$  is  $R_t^D$ . Consider a family that wants to fund consumption at an intermediate date  $t$ . It starts at the integer date  $t - .5$  with wealth  $W_{t-.5}$  and faces budget constraints for dates  $t - .5$  through  $t + .5$

$$\begin{aligned} D_{t-.5} + A_{t-.5} &= W_{t-.5}, \\ c_t(\xi_t) &= v_t(\xi_t) R_{t-.5}^D D_{t-.5}; \quad v_t(\xi_t) \leq 1 \\ W_{t+.5} &= R_{t-.5} A_{t-.5} + w_t N_t + R_t^D (1 - E[v_t(\xi_t)]) R_{t-.5}^D D_{t-.5}, \end{aligned} \quad (4)$$

At date  $t - .5$ , family resources are split into deposits and other assets that earn the interest rate  $R_{t-.5}$ . We focus on symmetric portfolios: every member receives the same amount of deposits. At date  $t$ , preference shocks are realized and family members choose the share  $v_t(\xi_t)$  of their deposit balance that is spent on consumption. Finally, at date  $t + .5$ , the family again pools resources, namely asset payoffs, wages and leftover deposit balances with interest.

Suppose that liquidity is costly, that is, the cumulative return on deposits  $R_{t-.5}^D R_t^D$  is below the interest rate  $R_{t-.5}$ . It is then optimal for the family to make each household hold just enough deposits to fund consumption if the idiosyncratic shock is  $\xi_t = 1$ . Since other households do not consume, we have  $v_t(1) = 1$ ,  $v_t(0) = 0$  and  $E[v_t(\xi_t)] = v_c$ . We can then consolidate the family's budget constraints into

$$W_{t+.5} = R_{t-.5} W_{t-.5} + w_t N_t - C_t \left( 1 + \frac{\omega_t^c}{v_c} \right); \quad \omega_t^c := \frac{R_{t-.5}}{R_{t-.5}^D} - 1 + (1 - v_c) (1 - R_t^D), \quad (5)$$

where we define  $\omega_t^c$  as the household's cost of liquidity. In order to provide aggregate consumption  $C_t$ , every household has to hold deposits  $C_t/v_c$ . The cost of liquidity  $\omega_t^c$  sums up the deposit spread paid by the family between two integer dates. First, all members invest in deposits at date  $t - .5$  and hence pay a spread between dates  $t - .5$  and  $t$ . Moreover, the share  $1 - v_c$  households who do not spend their deposits at  $t$  further face a spread between



dates  $t$  and  $t + .5$ . In a world with deposits, liquidity is more costly if liquidity needs are less predictable in the sense that  $v_c$  is low.

Since the cost of liquidity adds to the effective price of consumption, it drives a wedge between the marginal utilities of consumption and wealth. The first-order condition for consumption is

$$\frac{v_c}{C_t} = \lambda_{t+.5} \left( 1 + \frac{\omega_t^c}{v_c} \right). \quad (6)$$

If the spread between the deposit rate and the interest rate is zero at both dates  $t - .5$  and  $t$ , then  $\omega_t^c = 0$  and the marginal utilities of wealth and consumption are equal. More generally, a higher cost of liquidity lowers consumption, other things equal. Intuitively, our cash-in-advance setup makes liquidity and consumption goods complementary.

#### *Access to credit lines*

A credit line arranged at date  $t - .5$  specifies a fee  $f_{t-.5}$  per unit of credit limit, payable at date  $t + .5$ . It allows the household to borrow an amount  $v_t(\xi_t) L_t$  between the next intermediate date  $t$  and the next integer date  $t + .5$  at the market interest rate of zero. The liquidity benefit of the credit line will thus be priced into the fee. An alternative approach might specify an interest rate on loans taken out when the credit line is drawn – this would not alter the results in important ways. In addition, we do not introduce an explicit credit limit, say the loan must be less than some proportion of assets. We will focus below on equilibria in which the amount of loans for liquidity purposes is small relative to capital held by households so this limit would not bind.

The family budget constraints with a credit line – but without deposits – are

$$\begin{aligned} A_{t-.5} &= W_{t-.5}, \\ c_t(\xi_t) &= v_t(\xi_t) L_t; \quad v_t(\xi_t) \leq 1 \\ W_{t+.5} &= R_{t-.5} A_{t-.5} + w_t N_t - (E[v_t(\xi_t)] + f_{t-.5}) L_t. \end{aligned} \quad (7)$$

Comparing (4) and (7), deposits and credit lines are functionally equivalent: the balance  $R_{t-.5}^D D_{t-.5}$  works like the credit limit  $L_t$ , and deposits  $(1 - v_t(\xi_t)) R_{t-.5}^D D_{t-.5}$  carried over to the next integer period work like the undrawn credit line  $(1 - v_t(\xi_t)) L_t$ . The difference is in when funds have to be placed in the bank, and how costs are paid. With deposits, the household has to save  $R_{t-.5}^D D_{t-.5}$  inside the bank between  $t - .5$  and  $t$ , further has to save a share  $1 - v_t(\xi_t)$  between date  $t$  and  $t + .5$  and costs are paid as spreads on both investments. In contrast, with a credit line there is no savings between either pair of dates and the cost is paid as a fee at the end only.

It is optimal for an individual household member to fully draw down the credit line if  $\zeta_t = 1$  and to not take out any loan if  $\zeta_t = 0$ . We can again consolidate budget constraints to obtain

$$W_{t+.5} = R_{t-.5}W_{t-.5} + w_tN_t - C_t \left( 1 + \frac{f_{t-.5}}{v_c} \right).$$

The functional equivalence between deposits and credit lines implies that they give rise to the same budget constraint – the only difference is the cost of liquidity. Optimal consumption thus follows again from equation (6) but with the cost of liquidity now given by  $\omega_t^c = f_{t-.5}$ .

If the family is given a choice between a credit line and deposits, the optimal payment instrument follows from comparing the cost of liquidity and the associated price of consumption. In particular, a credit line is preferable if and only if

$$f_{t-.5} < \frac{R_{t-.5}}{R_{t-.5}^D} - 1 + (1 - v_c) (1 - R_t^D). \quad (8)$$

Apart from the market prices of the two instruments, the desirability of a credit line depends on the predictability of preference shocks: holding prices fixed, a credit line is cheaper if  $v_c$  is lower – liquidity needs are rare. The reason is that managing liquidity with deposits is costly because deposits need to be held even if liquidity needs do not materialize.

### 3.4 Goods-producing firms

There are three types of firms in the economy. Here we discuss firms that make consumption goods and capital goods. Firms that provide asset-management services are described below in the section on banking.

#### *Consumption-goods producers*

Firms that produce consumption goods sell their goods at intermediate dates, receive payments, and must carry these revenues into the next integer period, when they pay wages and rent on capital. The only way for them to receive payments and carry their revenue across sub-periods is with deposits. Given our convention of a zero interest rate between the intermediate date and the subsequent integer date, consumption-goods producers maximize

$$Y_t^c (1 - \omega_t^y) - w_tN_t - r_tK_t, \quad (9)$$

where  $\omega_t^y = 1 - R_t^D$  is the liquidity cost, here the spread on deposits.

The first-order conditions for consumption-goods producers show how frictions induce

wedges, here between marginal products and factor prices:

$$\begin{aligned} \alpha \frac{Y_t^c}{K_t} (1 - \omega_t^y) &= r_t, \\ (1 - \alpha) \frac{Y_t^c}{N_t} (1 - \omega_t^y) &= w_t. \end{aligned} \tag{10}$$

Complementarity between deposits and production implies that a higher cost of liquidity makes firms plan *as if* wages and rents are higher. This effect is familiar from work on the "cost channel" of monetary policy.

### *Capital-goods producers*

Every firm that produces capital goods makes a single variety, so it can produce with probability  $v_i$ . It must arrange liquidity at date  $t - .5$  to prepare for purchasing inputs at date  $t$ , either through deposits  $R_{t-.5}^D D_{t-.5}$  or a credit line  $L_t$ . Let  $\chi_t \in \{0, 1\}$  indicate whether an individual firm can produce its capital good at the intermediate date  $t$  (again, this omits an index  $i$  for the individual firm.) Their need for liquidity is captured by the cash-in-advance constraint

$$0 \leq i_t(\chi_t) \leq R_{t-.5}^D D_{t-.5} + L_t.$$

The constraint says that investment, must be covered by prearranged payment instruments, much like households' purchases of consumption goods at the intermediate date  $t$ ,

We assume that capital-goods producers incur balance-sheet costs when they hold financial assets such as deposits. In particular, per unit of deposit balance  $R_{t-.5}^D D_{t-.5}$  arranged for payment at date  $t$ , a firm must purchase  $\kappa_i$  units of asset-management services at a price  $p_t$ . Moreover, the same per unit cost  $p_t$  accrues per unit of deposits carried over from date  $t$  into date  $t + .5$ . The assumption applies the general principle that holding assets within a firm as opposed to outside is costly. We have not applied the same logic to capital in the process of production because doing so amounts to a renormalization of productivity.

We consider first the case where  $L_t = 0$ . The firms maximize shareholder value, with discounting at the interest rate  $R_t$ :

$$-D_{t-.5} - R_{t-.5}^{-1} E \left[ \kappa_i p_t R_{t-.5}^D D_{t-.5} \right] + R_{t-.5}^{-1} E \left[ Q_{t+.5} \chi_t i_t(\chi_t) + \left( R_t^D - \kappa_i p_t \right) \left( R_{t-.5}^D D_{t-.5} - i_t(\chi_t) \right) \right].$$

At date  $t - .5$ , firms raise equity to acquire deposits. At date  $t$ , they use these deposits, including interest on deposits, to purchase  $i_t(\chi_t)$  consumption goods to invest. The remaining deposit balance cannot be paid out to shareholders right away, but must be carried over to the next integer date via deposits. As a result, date  $t + .5$  payout only records the balance-sheet

costs on deposits due then. Finally, at date  $t + .5$ , firms sell capital at a price  $Q_{t+.5}$  and receive deposits less balance-sheet costs.

Suppose that  $Q_{t+.5} > 1$  and liquidity is costly. Firms with  $\chi_t = 1$  then want to invest as much as possible and exhaust their liquidity, so that their cash-in-advance constraint binds. The ex ante choice of liquidity compares the price of capital to the resource cost of investment plus the cost of liquidity. We define the frictional cost for capital-goods producers as

$$\omega_t^i = \left\{ \frac{R_{t-.5}}{R_{t-.5}^D} - 1 + (1 - v_i) (1 - R_t^D) \right\} + \kappa_i p_t (1 + (1 - v_i)). \quad (11)$$

The first term is a cost of liquidity that is analogous to the definition for households in (5). The second term reflects the balance-sheet cost.

Substituting for investment and spreads, we write value proportional to initial deposits.<sup>4</sup> For the firm to break even, we must have

$$Q_{t+.5} = 1 + \frac{\omega_t^i}{v_i}. \quad (12)$$

The first order condition is thus analogous to the case of consumption in (6). The wedge  $\omega_t^i$  effectively increases the price of capital goods.

Consider now capital-goods producers with access to a credit line. Their shareholder value takes the simpler form

$$R_{t-.5}^{-1} (E [Q_{t+.5} \chi_t i_t (\chi_t) - i_t (\chi_t)] - f_{t-.5} L_t).$$

At date  $t - .5$ , the firm only arranges the credit line. At date  $t$ , the firm draws down the credit line in order to pay for investment: again there is no cash flow. Finally, at date  $t + .5$ , the firm sells capital and pays back the loan. The optimal strategy is again to produce only if  $\chi_t = 1$ . The breakeven price of capital is therefore given by (12) with  $\omega_t^i = f_{t-.5}$ . We note that capital-goods producers strictly prefer a credit line to deposits. The reason is that even if liquidity is free, which means that deposits pay the real interest rate at all dates, deposits involve balance-sheet costs, while credit lines do not.

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<sup>4</sup>Substituting for investment from the binding cash-in-advance constraint, shareholder value is

$$D_{t-.5} \left( -1 + \frac{R_{t-.5}^D}{R_{t-.5}} \left( -\kappa_i p_t + v_i Q_{t+.5} + (1 - v_i) (R_t^D - \kappa_i p_t) \right) \right).$$

### 3.5 Banking

Banks are competitive firms that live from one integer date to the next. They have a technology to provide liquidity to households and firms at intermediate dates, when the latter cannot access asset markets. Since liquidity is valuable to households and firms, banks earn positive revenue from providing it. However, banks have to hold assets to back deposits, which is costly, to be able to provide this liquidity credibly. Banks trade assets in competitive capital, equity and deposit markets at all dates. There is free entry into banking and banks can be recapitalized at no cost at any date.

We focus throughout on equilibria in which not all capital is held inside banks. Below we will make assumptions on preferences and technology such that some part of the capital stock is always held directly by households. Since there is no aggregate risk, returns on all assets held by households are equated: in particular, the return on capital is the same as the interest rate  $R_t$ . Since households own banks, the return on bank equity is also equal to  $R_t$ . As for capital-goods producers, we compute shareholder value using discounting at the cost of capital  $R_t$ .

Banks incur balance-sheet costs. For any assets they hold to back deposits, banks must buy a proportional amount  $\kappa$  of asset management services at a price  $p_t$ . We assume that such services are provided by a competitive asset-management sector which produces services from capital and labor using the same technology as consumption goods. Since balance-sheet costs are positive even when the bank issues no debt, it does not make sense for shareholders to keep any assets in the bank unless it issues deposits or is holding deposits for liquidity purposes.

#### *Provision of deposits*

We consider first banks that only issue deposits. Banks face two types of constraints. The first is a leverage constraint: deposits must be below a fraction  $\phi < 1$  of the value of assets. We impose the leverage constraint at both integer and intermediate dates:

$$\begin{aligned} R_t^D D_t &\leq \phi R_t K_t, \\ D_{t+.5} &\leq \phi K_{t+.5}. \end{aligned} \tag{13}$$

In both cases, the constraint relates the deposits used in transactions to the value of assets at the time the transactions are made. In particular, we require that the balance of deposits (principal plus interest) promised at date  $t$  for use in transactions at  $t + .5$  is less than  $\phi$  times the value of assets at  $t + .5$ . At the same time, deposits that reflect customer transactions in  $t + .5$  must be backed by the value of assets at  $t + .5$ .

In addition, banks face a liquidity constraint: they must meet deposit outflows at an intermediate date with holdings of interbank deposits:

$$R_t^D D_t \leq R_t^D M_t + D_{t+.5},$$

where  $M_t$  denotes interbank deposits held between dates  $t$  and  $t + .5$ . The liquidity constraint captures the idea that assets that back money cannot be costlessly transformed into money, not even by banks. In particular, banks that have issued deposits and bought capital at date  $t$  cannot sell the capital at date  $t + .5$ , obtain cash that customers use in transactions, and then continue with fewer deposits from date  $t + .5$  to  $t + 1$ . We note that banks that do not face any deposit outflows can simply “roll over” deposits, choosing  $R_t^D D_t = D_{t+.5}$ . Rolling over deposits avoids the liquidity costs associated with money  $M_t$ . Moreover, it is always possible to open a bank at date  $t + .5$  and accept deposits – offering storage of liquid funds until  $t + 1$  does not require any liquid funds at  $t + .5$ .

Our approach abstracts from liquidity shocks and precautionary holdings of interbank deposits to buffer such shocks. With many banks, one might imagine that customer transactions generate large interbank flows. Here the idea is that such flows net out according to a law of large numbers so that no precautionary holdings are necessary as long as the banking system does not face a net outflow of funds. The latter will become relevant only once there is a central bank that offers cheaper deposit contracts. Since interbank liquidity management is not essential to our argument we also abstract from introducing reserves at this point, and capital is the only asset.<sup>5</sup>

The bank maximizes shareholder value, that is, the present value of cash flows at dates  $t + 1$ ,  $t + .5$  and  $t$ :

$$\begin{aligned} & R_t^{-1} \left( K_{t+.5} (1 - \kappa p_{t+.5}) - R_{t+.5}^D D_{t+.5} \right) \\ & + R_t^{-1} \left( R_t K_t (1 - \kappa p_{t+.5}) + R_t^D M_t (1 - \kappa p_{t+.5}) - R_t^D D_t - (K_{t+.5} - D_{t+.5}) \right) \\ & - (K_t + M_t - D_t). \end{aligned}$$

Cash flow at date  $t + 1$  consists of payoff from assets less deposits. At date  $t + .5$ , the bank receives payoff from assets and cash less deposits and chooses its new position. Date  $t$  cash flow consists only of the initial equity injection  $K_t + M_t - D_t$ .

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<sup>5</sup>Piazzesi and Schneider (2018) and Piazzesi, Rogers and Schneider (2019) study models in which liquidity management requires reserves and compare spreads and bank positions in a corridor system with scarce reserves as well as a floor system with ample reserves. While adding such detail has further interesting predictions, the key role of collateral in backing money is the same as in the present paper.

We focus on the case where the real interest rate is positive ( $R_t > 1$ ) and liquidity is costly, that is,  $R_t^D < R_t$  and  $R_{t+.5}^D < 1$ . We can rearrange shareholder value to clarify the contribution of different balance sheet positions:

$$\begin{aligned} & -\kappa p_{t+.5} K_{t+.5} + (1 - R_{t+.5}^D) D_{t+.5} \\ & -\kappa p_{t+.5} R_t K_t - \left(\kappa p_{t+.5} + R_t/R_t^D - 1\right) R_t^D M_t + \left(R_t/R_t^D - 1\right) R_t^D D_t \end{aligned}$$

The bank makes money by providing liquidity: spreads between the real rate and the deposit rate are positive. At the same time, the bank is worse at holding assets than shareholders, because it pays a balance-sheet cost. The bank therefore wants to maximize deposit issuance and minimize asset holdings.

As long as it is profitable for banks to issue deposits at intermediate dates, it is always best to "roll over" the deposits and not hold money. To see this, consider the bank's activity at date  $t + .5$ . The leverage constraint at date  $t + .5$  must always bind since capital incurs balance-sheet costs. Substituting into cash flow, deposits are profitable if the net benefit of issuing a unit of deposits is nonnegative, that is,  $1 - R_{t+.5}^D - \kappa p_{t+.5}/\phi \geq 0$ . It is then never optimal for the bank to hold money at date  $t$ . Indeed, for any plan with  $M_t > 0$ , consider an alternative plan that simply increases deposits  $D_{t+.5}$  as well as collateral  $K_{t+.5}$  so ensure that the liquidity constraint holds. The new plan satisfies all constraints by construction and cannot lower shareholder value.

In the absence of money holdings, the bank's activities at dates  $t$  and  $t + .5$  are independent. We have already seen that date  $t + .5$  deposit supply is perfectly elastic at the interest-rate spread  $1 - R_{t+.5}^D = \kappa p_{t+.5}/\phi$ . Similarly, we have that date  $t$  deposit supply is perfectly elastic at the spread

$$R_t/R_t^D - 1 = \frac{\kappa p_{t+.5}}{\phi}.$$

### *Provision of credit lines*

We now consider a bank that provides credit lines. The bank sells the commitment to provide liquidity  $L_t$  at date  $t$ . A fraction  $v_t^L$  of customers draw their credit lines. For those customers, the bank can provide liquidity either by issuing deposits, or by holding money. The leverage and liquidity constraints are

$$\begin{aligned} D_t & \leq \phi \left( v_t^L L_t + K_t \right), \\ v_t^L L_t & \leq R_t^D M_t + D_{t+.5}. \end{aligned}$$

The leverage constraint is analogous to (13), but takes into account that the drawn credit line is a collateral asset. The liquidity constraint says that if the bank wants to issue fewer deposits than the credit lines it provides, then it must cover the balance with money. One interpretation is that the bank opens a deposit account for every customer who draws down the line, and then pays money to those customers who withdraw.

Bank shareholder value is again the present value of cash flows at dates  $t$ ,  $t + .5$  and  $t + 1$ :

$$R_t^{-1} \left( f_t L_{t+.5} + v_{t+.5}^L L_{t+.5} (1 - \kappa p_{t+.5}) + K_{t+.5} (1 - \kappa p_{t+.5}) - R_{t+.5}^D D_{t+.5} \right) \\ + R_t^{-1} \left( R_t^D M_t (1 - \kappa p_{t+.5}) - \left( v_{t+.5}^L L_{t+.5} + K_{t+.5} - D_{t+.5} \right) \right) - M_t.$$

Cash flow at date  $t + 1$  consists of payoff from credit lines and capital less payoff to deposits. At date  $t + .5$ , the bank receives payoff from money and chooses its new position. Date  $t$  cash flow consists only of the initial equity injection  $M_t$ .

As for a bank that issues deposits, capital inside the bank is costly and therefore only useful if it serves as collateral. In other words, the leverage constraint at date  $t + .5$  must bind. We can use this fact to rearrange shareholder value as

$$f_t L_{t+.5} - \left( \kappa p_{t+.5} - \phi \left( 1 - R_{t+.5}^D \right) \right) \left( v_{t+.5}^L L_{t+.5} + K_{t+.5} \right) - \left( \kappa p_{t+.5} + R_t / R_t^D - 1 \right) R_t^D M_t \quad (14)$$

We thus decompose value into revenue from credit lines – the fee income  $f_t L_{t+.5}$  – and two cost terms. The cost of issuing deposits to fund loans is proportional to the collateral that backs those deposits: it reflects the cost of holding the collateral less the levered spread earned on deposits. The cost of holding money to pay out depositors reflects the cost of holding money plus the spread of paid on money.

Suppose first that the bank breaks even on deposits at date  $t + .5$ , that is  $1 - R_{t+.5}^D - \kappa p_{t+.5} / \phi = 0$ . It is then optimal for the bank to not hold any money and issue at least as many deposits as loans. Moreover, the marginal cost of providing credit lines is zero, that is, the bank breaks even for  $f_t = 0$ . Indeed, for any plan with  $M_t > 0$ , we can find an alternative plan that increases  $D_{t+.5}$  and  $K_{t+.5}$  to satisfy the constraints; if the bank breaks even on deposits, this plan strictly raises shareholder value. If deposits are not costly to issue, the bank can simply issue more rather than hold money and pay the cost of carrying money from date  $t$  to date  $t + .5$  as well as the spread between money and the real interest rate.

Now consider instead a bank that is not competitive in deposit provision at date  $t + .5$ , that is,  $1 - R_{t+.5}^D - \kappa p_{t+.5} / \phi < 0$ . In other words, the spread charged on deposits in the market is lower than the bank's marginal cost. In this case, both the leverage and liquidity constraint must bind: the bank wants to economize on all assets other than loans. In principle, whether



money or capital is better for the bank depends on relative costs of deposits at dates  $t$  and  $t + .5$ . We focus here on the case where spreads are the same at both dates, or  $1 - R_{t+.5}^D = R_t/R_t^D - 1$ . It is then optimal for the bank to not hold any capital and hold money to pay for a deposit outflow of  $(1 - \phi) v_{t+.5}^L L_{t+.5}$ . Indeed, for any plan with  $K_{t+.5} > 0$ , we can reduce capital and increase money to increase shareholder value. Intuitively, a liquidity-constrained bank can choose between money and date  $t + .5$  deposits, which requires collateral. It takes only  $\phi$  units of money (at date  $t + .5$ ) to substitute for one unit of capital whereas the balance-sheet costs are the same.

Substituting back into (14), we find the price of the credit line at which the bank breaks even as

$$f_t = v_{t+.5}^L \left( \kappa p_{t+.5} - \phi \left( 1 - R_{t+.5}^D \right) + (1 - \phi) \left( \kappa p_{t+.5} + R_t/R_t^D - 1 \right) \right) > 0 \quad (15)$$

The bank charges credit line customers a positive fee so as to make up for its high cost of deposits. This cross-subsidizations allows it to compete in the deposit market even though its marginal cost of deposits is higher than the prevailing market rate. At the same time, as long as deposits have a liquidity benefit, it is still sensible to issue deposits, as opposed to, say, funding loans with equity, because of their lower funding cost.

#### *Asset management and the pricing of liquidity*

The asset-management sector maximizes profits  $p_t Y_t^a - w_t N_t - r_t K_t$ . In equilibrium, both asset-management services and consumption goods must be produced. Since the production function of asset managers and consumption-goods producers is identical, the relative price of asset management services at an intermediate date  $t$  must be  $p_t = R_t^D$ . At this price, asset-manager profits are the same as consumption-goods producer profits in (9), so both types of firms can break even. Using the pricing of deposits at intermediate dates by banks, we thus have the equilibrium prices

$$p_t = R_t^D = \frac{1}{1 + \kappa/\phi}; \frac{R_{t-.5}}{R_{t-.5}^D} - 1 = \frac{\kappa/\phi}{1 + \kappa/\phi}$$

Since the price of asset-management services is constant, all costs of liquidity faced by non-banks are constant in equilibrium as well.

## **4 Comparing payment instruments**

We now proceed to compare equilibria with different payment technologies. In particular, we are interested in the properties of a system in which one type of bank issues deposits and

provides credit lines, like a modern commercial bank, and a second type of bank only issues deposits, like the central bank in a number of CBDC proposals. To build towards this case, we take two preliminary steps. We start first with an economy where all payments require deposits. We then clarify the complementarity between credit lines and deposits in a system where both coexist.

Throughout this section we assume that the probabilities of trading for households and capital-goods producers are the same, that is,  $v_c = v_i = v$ . In addition, we maintain two restrictions on parameters. First, we impose a sufficient condition so that the steady state capital stock is always larger than the maximal amount of assets banks may have to hold in order to support all transactions. In other words, capital accumulation is not needed in order to produce assets to back payments. Second, we assume that capital goods producers' balance-sheet cost is sufficiently high that they always prefer to arrange liquidity via credit lines rather than hold deposits. We write  $\rho = \beta^{-1} - 1$  for the rate of time preference. We then assume

*Assumption A1. (liquidity provision is not a motive for capital accumulation in steady state)*

$$\rho + \delta < \frac{\alpha\phi v}{1 + 2\kappa/\phi v + \kappa_i(2/v - 1)}.$$

*Assumption A2. (capital-goods producers always prefer credit lines)*

$$\kappa_i > \kappa(2 - \phi).$$

Assumption A1 requires sufficiently low rates of time preference and depreciation relative to the probability of trading  $v$ . The two rates capture households' distaste for capital accumulation, which must be weak relative to their need for liquidity captured by  $v$ . The condition is also easier to satisfy when the capital share in production is higher (higher  $\alpha$ ) so production requires more assets, as well as when banks can leverage more (higher  $\phi$ ), so banks need fewer assets. Finally, the denominator captures the fact that balance-sheet costs of both banks and capital-goods producers discourage capital accumulation. Assumption A2 simply requires that the balance-sheet cost of capital-goods producers is high enough.

## 4.1 Banks provide only deposits

Consider an economy in which all liquidity is provided by deposits. Combining first-order conditions of banks and the asset-management service sector, the equilibrium liquidity costs faced by households as well as capital-goods and consumption-good producers are constant

at

$$\begin{aligned}
\omega^c &= \frac{\kappa}{\phi} (2 - v) \frac{1}{1 + \kappa/\phi}, \\
\omega^i &= \left( \frac{\kappa}{\phi} + \kappa_i \right) (2 - v) \frac{1}{1 + \kappa/\phi}, \\
\omega^y &= \frac{\kappa}{\phi} \frac{1}{1 + \kappa/\phi}.
\end{aligned} \tag{16}$$

In a world with deposits, households have higher liquidity costs than consumption-goods producers because their liquidity demand is less predictable: for  $v < 1$ , they always hold some deposits that are not used for consumption. Capital-goods producers share this feature, but in addition they have higher liquidity costs because holding deposits on the balance sheet is costly, as captured by  $\kappa_i$ .

We can combine results on bank, household, and firm optimization to compute the total demand for asset-management services. Households who hold deposits in order to finance aggregate consumption  $C_t$  need a deposit balance of  $C_t/v$  at date  $t$  and invest  $(1 - v)C_t/v$  in deposits at that date. Similarly, capital-goods producers need a balance  $I_t/v$  at date  $t$  and invest  $(1 - v)I_t/v$ . Finally, consumption-goods producers require  $Y_t^c$  deposits at date  $t + .5$ . Per unit of deposits, banks demand  $\kappa/\phi$  units of asset-management services. Moreover, capital-goods producers demand  $\kappa_i$  units of asset-management services per unit of deposits held. Equilibrium in the market for asset-management services thus requires

$$\frac{C_t}{v} \frac{\kappa}{\phi} (2 - v) + \frac{I_t}{v} \left( \frac{\kappa}{\phi} + \kappa_i \right) (2 - v) + Y_t^c \frac{\kappa}{\phi} = Y_t^a.$$

An *equilibrium* consists of consumption  $C_t$ , capital  $K_t$ , labor  $N_t$  sectoral output  $Y_t^a$  and  $Y_t^c$  as well as factor allocations to each sector, together with prices  $R_t, R_t^D, Q_t$  for every integer date as well as  $p_t$  and  $R_t^D$  for every intermediate date, such that households, firms and banks optimize and markets clear. Market clearing for rental capital and labor means that factors used in the two sectors add up to total labor and capital. Market clearing for deposits means that households' and firms deposit holdings are consistent with banks' demand for asset-management services. Market clearing for consumption goods is  $C_t + I_t = Y_t^c$ .

We define total output as  $Y_t = Y_t^a + Y_t^c$ . Adding up market clearing conditions and rearranging, we have that in any equilibrium

$$C_t \left( 1 + \frac{\omega^c}{v} \right) + I_t \left( 1 + \frac{\omega_i}{v} \right) = Y_t (1 - \omega^y). \tag{17}$$

The costs of liquidity faced by households and firms  $\omega^c, \omega^i$  and  $\omega^y$  thus exactly reflect the proportional resource costs that must be incurred to support transactions. Resource costs needed to support consumption and investment can be different because of differences in liquidity risk and balance-sheet costs between the two sectors.

#### *Equivalent social planner problem and characterization of equilibrium*

Consider a planner who chooses consumption, investment, capital and output to maximize expected utility subject to the capital accumulation equation and the resource constraint (17). An allocation  $(C_t, I_t, K_t, Y_t)$  is part of an equilibrium if and only if it is a solution to the social planner problem. Indeed, eliminating prices from firms' and household marginal conditions or eliminating multipliers from the social planner's first order conditions both deliver

$$\begin{aligned} \frac{\theta}{v} C_t N_t^{\frac{1}{\varepsilon}} &= \frac{1 - \omega^y}{1 + \frac{\omega^c}{v}} (1 - \alpha) \frac{Y_t}{N_t}, \\ \beta \frac{C_t}{C_{t+1}} \left( 1 - \delta + \alpha \frac{Y_{t+1}}{K_{t+1}} \frac{1 - \omega^y}{1 + \frac{\omega^i}{v}} \right) &= 1. \end{aligned} \quad (18)$$

Both an equilibrium allocation and a solution to the social planner problem satisfy (18), the resource constraint (17) as well as the production function (1) and the capital accumulation equation in (2).

To think about the dynamics of the model, we can further exploit the fact that it is isomorphic to the standard neoclassical growth model. Indeed, dividing by the wedge on consumption, we can rewrite (17) as

$$C_t + \frac{I_t}{\gamma} = ZY_t; \quad \gamma := \frac{1 + \omega^c/v}{1 + \omega^i/v}, \quad Z := \frac{1 - \omega^y}{1 + \frac{\omega^c}{v}}.$$

The technology for liquidity provision captured by asset-management costs and banks' ability to pledge collateral therefore effectively alters the total factor productivity  $Z$  as well as the investment-specific technology parameter  $\gamma$ . For example, a decrease in  $\phi$  that tightens banks' leverage constraint works like a drop in TFP. Moreover,  $\gamma$  decreases and investment becomes relatively more expensive than consumption because it involves higher balance-sheet costs.

#### *Steady state and welfare*

To compare economies with different liquidity provision arrangements, we compute steady state welfare. In steady state, output, labor, consumption, and investment are all constant. The

capital-labor ratio  $K/N$  follows from (18) as

$$\frac{K}{N} = \left( \frac{\alpha}{\rho + \delta} \frac{1 - \omega^y}{1 + \frac{\omega^i}{v}} \right)^{\frac{1}{1-\alpha}}.$$

The effect of the payment system on capital accumulation depends on the cost of liquidity of firms. An increase in either  $\omega^y$  or  $\omega^i$  works like a bad technology shift and lowers the amount of capital accumulated relative to labor.

To verify that the capital stock is higher than assets held by banks, we observe that the capital-output ratio is  $(K/N)^{1-\alpha}$ . Substituting for  $\omega^y$  and  $\omega^i$  from 16, Assumption A.1 says that the capital stock larger than  $Y/v\phi$ . The latter is an upper bound on assets required by banks. Indeed, banks need to hold less than  $Y/\phi$  assets between an integer date and the next intermediate date, since it is enough to back deposits worth the sum of consumption and investment. Between an intermediate date and the next integer date, banks hold the same amount of assets: the only accounting change is that some deposits are transferred from buyers' accounts to sellers' accounts.

Since we have assumed balanced growth preferences and changes in liquidity costs work like technology shifts, steady state labor hours are at the first best level  $\bar{N}$ , say, independently of the  $\omega$ s. The welfare effect of liquidity derives instead from the level of consumption

$$C = \bar{C} \left( \frac{1 - \omega^y}{1 + \frac{\omega^c}{v}} \right)^{\frac{1}{1-\alpha}} \left( \frac{1 + \omega^c/v}{1 + \omega^i/v} \right)^{\frac{\alpha}{1-\alpha}}, \quad (19)$$

where  $\bar{C}$  is the first best level of consumption. All three costs of liquidity lower welfare. Naturally, the impact of capital-goods producers' cost of liquidity is more important if the economy is more capital intensive, that is,  $\alpha$  is higher. In contrast, consumption-goods producers, who hire workers, matter more if the economy is more labor intensive. The impact of households' liquidity cost on welfare does not depend on capital intensity.

## 4.2 Banks provide deposits and credit lines

Suppose now that banks offer both credit lines and deposits and that both households and capital-goods producers have the choice between the two products. Since there is free entry into the deposit market, the spreads on deposits are the same as in an economy without credit lines, namely  $1 - R_t^D = \kappa p_t/\phi$  at intermediate dates and  $R_t/R_t^D = \kappa p_{t+.5}/\phi$  at integer dates. The price of asset management services  $p_t$  also remains the same. Since banks can use loans as

collateral, credit lines are priced at their marginal cost  $f_t = 0$ . Households and capital-goods producers therefore prefer credit lines. The equilibrium costs of liquidity are now

$$\omega^c = \omega^i = 0, \quad \omega^y = \frac{\kappa}{\phi} \frac{1}{1 + \kappa/\phi}. \quad (20)$$

Consider the demand for asset-management services. As before, consumption-goods producers require  $Y_t^c$  deposits at date  $t + .5$ , so banks demand  $\kappa Y_t^c / \phi$  units of asset-management services. The key feature of credit lines that no additional services are needed: among the assets that back deposits of consumption-goods producers are credit lines drawn by households and capital-goods producers. As a result, there is no need to hold additional assets in order to provide liquidity to those sectors. It follows that, for an economy with credit lines, (17) holds with the modified costs of liquidity (20) and we can still characterize equilibrium using a planner who works with technology shifters.

Using the formula (19) and the solutions for  $\omega$  in the deposit and credit line cases, we can compare consumption – and hence welfare – between a credit-line economy and one with only deposits. As a share of first best consumption, the difference in consumption between the credit-line economy and the one with only deposits is

$$\left( \frac{1}{1 + \kappa/\phi} \right)^{\frac{1}{1-\alpha}} - \left( \frac{1}{1 + 2\kappa/\phi v} \right)^{\frac{1}{1-\alpha}} \left( \frac{1 + 2\kappa/\phi v}{1 + 2\kappa/\phi v + \kappa_i(2/v - 1)} \right)^{\frac{\alpha}{1-\alpha}} > 0.$$

In an economy with credit lines, all three costs terms are smaller, so welfare is higher. Moreover, the relative cost of liquidity used for consumption and investment is the same, that is,  $\gamma = 1$ . The only effect of frictions is that higher  $\kappa/\phi$  works like lower TFP. In contrast, deposits imply lower TFP and further alter the relative cost of consumption and investment.

### 4.3 Different banks for deposits and credit lines

What happens in the economy of the previous subsection – where banks offer credit lines and deposits – when there is a second type of bank, called the central bank, that offers only deposits? In particular, we allow for the possibility that the central bank has a better technology to manage assets or to back deposits. We thus introduce parameters  $\kappa^* \leq \kappa$  and  $\phi^* \geq \phi$  that capture the technology of the central bank. Given those parameters, the central bank has to purchase asset-management services just like commercial banks. We assume that the central bank elastically supplies deposits at marginal cost. It thus works like a competitive banking sector with technology  $(\kappa^*, \phi^*)$  that is not allowed to provide credit lines.

We are interested in the provision of deposits and credit lines in this hybrid system and its

welfare consequence. Consider first how different sectors choose their payment instruments. Consumption-goods producers always want deposits. Since the central bank provides deposits at least as cheaply as commercial banks, its deposit spreads reflect its lower marginal cost. We thus have  $1 - R_t^D = \kappa^* p_t / \phi^*$  at intermediate dates and  $R_t / R_t^D = \kappa^* p_{t+.5} / \phi^*$  at integer dates. The price of asset-management services is therefore  $p_t = 1 / (1 + \kappa^* / \phi^*)$ . We note that these equilibrium prices are independent of who in the economy holds credit lines or deposits, and how prices of credit lines are determined.

The choice of payment instruments for households and capital-goods producers depends on the relative cost of deposits and credit lines. In the economy of the previous section, the credit line was free and therefore always cheaper than deposits. We have derived in (15) that banks charge a positive fee on credit lines when they face deposit spreads that are higher than their marginal cost. Expressing deposit spreads in terms of bank costs, we have the equilibrium price of the credit line

$$f = \frac{v}{1 + \kappa^* / \phi^*} \left( \kappa - \phi \frac{\kappa^*}{\phi^*} + (1 - \phi) \left( \kappa + \frac{\kappa^*}{\phi^*} \right) \right) > 0.$$

Competition from cheap deposits makes credit-line provision more expensive for two reasons. First, deposits are now a more expensive funding source. In fact, it is no longer profitable for banks to hold other assets such as capital in order to back deposits: banks' only assets are now drawn credit lines, which are funded with deposits and equity. The first term in the bracket reflects the balance-sheet cost of the drawn credit line: per unit of drawn credit line, the bank incurs a cost  $\kappa$  which is partly offset by the deposit spread on a share  $\phi$  of assets.

The second reason comes from the liquidity constraint. Banks anticipate that at least some of the payments by their customers will involve sending funds to the central bank. To prepare for this outflow – a share  $1 - \phi$  of all drawn credit lines – at an intermediate date, they have to hold money already at the previous integer date. The last term in the bracket reflects the overall cost of this liquidity: it consists both of a direct balance-sheet cost for the commercial bank and the spread that reflects the indirect balance-sheet costs incurred by the central bank.

Households and capital-goods producers compare the relative cost of deposits and credit lines to choose the optimal payment instrument to use. Capital goods producers prefer a credit line if

$$v \left( (2 - \phi) \kappa + (1 - 2\phi) \frac{\kappa^*}{\phi^*} \right) \leq \left( \frac{\kappa^*}{\phi^*} + \kappa_i \right) (2 - v). \quad (21)$$

Assumption A.2 ensures that (21) always holds with strict inequality: the balance-sheet cost  $\kappa_i$  is high enough so that it is better for capital-goods producers to use credit lines even if  $v = 1$  and  $\kappa^* = 0$ . In other words, holding deposits is too costly for firms even if there is no

spread and the need for liquidity is entirely predictable. It follows that in any equilibrium both payment instruments coexist, since each has its set of firms that uses it for sure.

We distinguish two types of equilibria by whether or not households pay with deposits or a credit line. Their choice depends on the technology (in particular, balance-sheet costs and leverage constraints) that determine the cost of liquidity. – but also on how predictable their liquidity needs are, as captured by the preference parameter  $v$ . In particular, households prefer a credit line if

$$v \left( (2 - \phi) \kappa + (1 - 2\phi) \frac{\kappa^*}{\phi^*} \right) \leq \frac{\kappa^*}{\phi^*} (2 - v). \quad (22)$$

For any positive cost parameters, a credit line is preferred if  $v$  is sufficiently close to zero.

Households may prefer deposits if the deposit rate offered by the central bank is sufficiently high. Indeed, if  $\kappa^*/\phi^* < \kappa/\phi - \kappa/2$ , then deposits are optimal if  $v$  is sufficiently close to one. In order to relate the cost of deposits and the predictability of liquidity needs, we define  $\bar{v}(\kappa^*/\phi^*)$  as the minimum of one and the value of velocity  $v$  that satisfies (22) with equality. The function  $\bar{v}$  satisfies  $\bar{v}(0) = 0$  and is strictly increasing, concave, and bounded above by one. It captures the intuition that households continue to use credit lines if and only if their liquidity needs are not too predictable or the deposit rate offered by the central bank is not too high.

#### *Equilibrium when households use credit lines*

We study first equilibria with  $v < \bar{v}(\kappa^*/\phi^*)$ , that is, households use credit lines along with capital-goods producers. We can summarize the costs of liquidity for the three agents as

$$\begin{aligned} \omega^c = \omega^i &= \frac{v}{1 + \kappa^*/\phi^*} \left( \kappa - \phi \frac{\kappa^*}{\phi^*} + (1 - \phi) \left( \kappa + \frac{\kappa^*}{\phi^*} \right) \right), \\ \omega^y &= \frac{\kappa^*}{\phi^*} \frac{1}{1 + \kappa^*/\phi^*}. \end{aligned} \quad (23)$$

Demand for asset-management services now comes from two banks. Consumption-goods producers require  $Y_t^c$  deposits at date  $t + .5$ . A share  $1 - \phi$  is provided by central bank which thus demands  $(1 - \phi) \kappa^* Y_t^c / \phi^*$  units of asset-management services. The remaining share  $\phi$  is provided by commercial banks. It is collateralized by drawn credit lines  $vL_t = C_t + I_t$  that support consumption and investment by households and capital-goods producers, respectively. Credit lines cost  $\kappa(C_t + I_t)$  to hold. In addition, commercial banks hold deposits at the central bank between dates  $t$  and  $t + .5$  so as to have enough money to meet the deposit outflow  $(1 - \phi)(C_t + I_t)$  at date  $t + .5$ . This generates additional balance-sheet costs at both commercial banks and the central bank.



Summing up, market clearing for asset-management services is

$$(C_t + I_t) \left( \kappa + (1 - \phi) \left( \kappa + \frac{\kappa^*}{\phi^*} \right) \right) + Y_t^c (1 - \phi) \frac{\kappa^*}{\phi^*} = Y_t^a.$$

Rearranging, we have that (17) again holds, now at the liquidity costs (23). We can therefore proceed to characterize equilibrium using an equivalent social planner problem for our modified neoclassical growth model, as before. Since households and capital-goods producers face the same liquidity costs, the investment specific technology parameter  $\gamma$  is equal to one. The entry of the central bank shows up only as a change in TFP.

Are households better off in an economy where the central bank offers deposits with a technology  $(\kappa^*, \phi^*)$ ? For the economy with commercial banks from the previous section, TFP was  $Z = (1 + \kappa/\phi)^{-1}$ . In contrast, with CBDC we have  $Z = (1 + (2 - \phi)\kappa + 2(1 - \phi)\frac{\kappa^*}{\phi^*})^{-1}$ . Since welfare is monotonic in TFP, we can assess it by simply comparing the two terms. The ranking is not obvious since deposits become cheaper, while credit lines become more expensive. We have that CBDC is beneficial if and only if

$$\frac{\kappa^*}{\phi^*} < \frac{1 - \phi}{2} \frac{\kappa}{\phi}. \quad (24)$$

Naturally, central bank deposits are beneficial if they are sufficiently cheap to produce. At the same time, if the central bank offers only a small improvement in cost, that is,  $\kappa^*/\phi^*$  is smaller than but close to  $\kappa/\phi$ , then offering CBDC unambiguously reduces welfare. The reason is that one group of commercial bank customers continues to prefer credit lines, and those credit lines now become more expensive to produce. To be beneficial, the cost advantage of the central bank has to be sufficiently large. The ratio  $(\kappa/\phi)/(\kappa^*/\phi^*)$  is a measure of this cost advantage. The ratio has to be high relative to banks' ability to lever: the condition is harder to satisfy as  $\phi$  is closer to one. Intuitively,  $\phi$  measures the complementarity between deposits and credit lines at commercial banks. If this complementarity is stronger, then credit lines are better at backing deposits.

#### *Equilibrium when households use deposits*

Consider now the case where  $\kappa^*/\phi^* < \kappa/\phi - \kappa/2$  and  $v > \bar{v}(\kappa^*/\phi^*)$ , that is, central bank deposits are sufficiently attractive that households no longer use credit lines. The costs of liquidity for capital-goods producers and consumption-goods producers are unchanged. However, the cost of liquidity for households takes the same form as in (16), except that

spreads reflect the cost of the central bank:

$$\omega^c = \frac{\kappa^*}{\phi^*} (2 - v) \frac{1}{1 + \kappa^*/\phi^*}.$$

The same calculations as above establish equivalence with a social planner problem. The new feature now is that the costs of liquidity of households and capital-goods producers differ, so that the investment specific technology parameter is different from one.

We evaluate welfare by substituting into (19). Central bank deposits are beneficial if and only if

$$\frac{1}{1 + \kappa/\phi} < \frac{1}{1 + 2\kappa^*/\phi^*v} \left( \frac{1 + 2\kappa^*/\phi^*v}{1 + \frac{\kappa^*}{\phi^*} + \kappa - \phi \frac{\kappa^*}{\phi^*} + (1 - \phi) \left( \kappa + \frac{\kappa^*}{\phi^*} \right)} \right)^\alpha. \quad (25)$$

The left hand side is TFP  $Z$  in the economy without central bank deposits. The right hand side is the product of TFP and  $\gamma^\alpha$  in the economy with central bank deposits. TFP can be higher or lower depending on costs and preferences. Since we have assumed that (22) does not hold, we always have  $\gamma < 1$ . In other words, in the parameter region where household liquidity needs are more predictable, entry of the central bank works like a shift in TFP together with an adverse hit in investment-specific technology.

It is still the case that central bank deposits are beneficial if they are sufficiently cheap and liquidity needs are sufficiently predictable. We define  $\tilde{v}(\kappa^*/\phi^*)$  as the value of velocity  $v$  that satisfies (25) with equality. The function  $\tilde{v}$  is increasing, it intersects the function  $\bar{v}$  at the point  $\kappa^*/\phi^* = \frac{1-\phi}{2}\kappa/\phi$ , and is higher than  $\bar{v}$  for any higher  $\kappa^*/\phi^*$ .<sup>6</sup> Central bank deposits are beneficial if  $v > \tilde{v}(\kappa^*/\phi)$ . It follows that there is a parameter region where central bank deposits are not too cheap, but liquidity needs are quite predictable, such that households switch to using deposits, so central bank deposits are widely adopted, but nevertheless welfare declines. This region is larger the higher is the capital share of the economy.

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<sup>6</sup>By construction, along the schedule  $\bar{v}(\kappa^*/\phi^*)$  we have  $\gamma = 1$ . Welfare there is the same as in the case studied earlier when households choose credit lines. It follows from (24) that the reverse of (25) holds along the  $\bar{v}$  schedule in the range  $\kappa^*/\phi^* > \frac{1-\phi}{2}\kappa/\phi$ . For (25) to instead hold with equality,  $v$  must increase, holding fixed the cost, so we must have  $\tilde{v}(\kappa^*/\phi^*) > \bar{v}(\kappa^*/\phi^*)$  for  $\kappa^*/\phi^* > \frac{1-\phi}{2}\kappa/\phi$ .

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