Buyer-Optimal Platform Design*

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Abstract

A platform matches a unit-mass of sellers, each owning a single product of heterogeneous quality, to a unit-mass of buyers with differing valuations for unit-quality. After matching, sellers make take-it-or-leave-it price-offers to buyers. Initially, valuations of buyers are only known to them and the platform, but sellers make inferences from the matching algorithm. The efficient matching is positive-assortative, but buyer-optimal matchings are, often, stochastically negative-assortative (i.e., compared to lower-quality sellers, high-quality ones are matched to buyers with lower expected valuation). Albeit everyone trades, generating rents for the side lacking bargaining power results in inefficient matching.

KEYWORDS: Two-sided markets, matching, asymmetric information, platforms.

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1 Introduction

Outcomes in two-sided markets are defined by (i) who transacts with whom and (ii) how the surplus from each transaction is divided. In digital marketplaces, a *matchmaker* typically controls (i), but not always (ii). Often, especially if transaction costs for agreeing the terms of the exchange are significant, matched parties bargain bilaterally, with limited or no access to alternatives within the platform. For example, Google search exposes consumers to a subset of service providers chosen from a vast pool of potential candidates.[1] Similarly, Airbnb provides only a tailored sample of all available rental options for popular destinations and dating services restrict the pool of suitors for their users.

In this paper, we model the *two-sided matching design* problem of a platform that has superior information about its users but does *not* control their bargaining. Our approach raises the following issue: Since platforms can use information they have on their users to optimize matching, agents are able to make inferences from the matching algorithm that may affect post-match bargaining. For instance, Google, which can likely infer the wealth of its users with great accuracy, may display more prominently high-quality opportunities to consumers known to have higher willingness-to-pay.[2] As a result, it is conceivable that sellers advertising on Google base their prices not just on their product quality and prior-information about consumers, but also on the sample of buyers that they usually interact with. When this feedback effect is present, platforms can not treat the post-match bargaining outcome of any pair as exogenous, which makes optimal matching design more complex.[3]

In our model, a platform matches buyers one-to-one with quality-differentiated sellers, each owning a single good. A buyer's willingness to pay is known to the platform but not to sellers. The quality of a seller is publicly observed. After being matched, sellers make take-it-or-leave-it price offers to buyers. The matching indirectly leaks information, either because it is public or as a result of equilibrium reasoning, allowing sellers to tailor their subsequent price offers and price-discriminate. The main insight we obtain follows from characterising both the welfare optimal matching and the buyer-optimal one. We find that a platform can use its control of the matching to garble the information of sellers in a way that offsets their bargaining power and generates information rents for buyers. However, this comes at the cost of creating *sorting* inefficiencies compared to both the first-best and, sometimes, even the fully-random matching, which a platform implements if it does not use any information.[4]

[1] Varian (2006) writes: "First, what does Google do? The answer, I claim is that Google is a yenta - a traditional Yiddish word for matchmaker. [...] From an economics perspective, Google runs a two sided matching mechanism."

[2] Businesses bid for impressions based on information provided by Google and high-quality business likely bid more for high-value consumers. A documented case is that of Orbitz, an online travel agent that used to show to Mac users more expensive hotels than those it showed to Windows users. See Dana Mattioli, "On Orbitz, Mac Users Steered to Pricier Hotels", WSJ (2012).

[3] This differentiates us from most, if not all, of the literature on two-sided matching. Consider the paradigmatic example of the National Residency Matching Program (NRMP) studied in Roth and Peranson (1999). Since salaries are set in advance there is little scope for the platform to reveal information on the preferences of both sides that would affect the outcome post-match.

[4] The sorting inefficiency in the buyer-optimal matching would persist even if the platform could directly com-

Before we discuss our key assumptions, let us elaborate briefly on our main result. The welfare optimal matching is positive-assortative in value and quality. To see this, note two things. First, any deterministic matching induces complete information and results in efficient trade for all buyer-seller pairs, albeit sellers extract all the surplus from the transactions. Second, a deterministic positive-assortative matching maximises efficiency of sorting, as the total surplus generated by each match reflects a complementarity between value and quality. In contrast, the *buyer-optimal* matching is distorted and, sometimes, (stochastically) negative-assortative. That is, the expected valuation of buyers matched to lower quality sellers is higher than that of buyers matched to higher quality ones. Intuitively, by concealing information about valuations and thereby manipulating sellers'beliefs, such a random matching keeps prices lower and generates rents for buyers. A matching platform wishing to maximise buyer-surplus therefore faces a trade-off between the sorting efficiency of the matching and control of prices. This trade-off often resolves in favour of heavily distorting the matching away from positive assortative. While sorting inefficiencies are unavoidable, all matched agents trade despite post-match information is asymmetric.[5]

By assuming that the matching is one-to-one we abstract away from the platform potentially exploiting price-competition.[6] We believe this is a good approximation, especially when firms cannot commit to prices before the match. First, even if consumers are exposed to multiple firms, the top-ranked enjoys substantial market power, as many consumers may be reluctant to search. It is known that consumers are, *ceteris paribus*, heavily biased toward most prominently located opportunities, such as the Buy-box placement in Amazon or a top place in Google's ranking (see Narayanan and Kalyanam (2015)), placements for which firms are willing to pay higher prices. Second, suppose the platform could minimise such prominence by creating a ranking of all firms and consumers could continue their search beyond their first match at a small cost, as it is sometimes the case. Search might still be limited in equilibrium and the first firm could enjoy market power, as illustrated by Diamond (1971).

Our focus on buyer-optimal matching has, primarily, a normative motivation. We present a framework where some of the welfare trade-offs of regulating a "matching algorithm" to benefit the side lacking bargaining power are exposed and can be evaluated. Nonetheless, there are several plausible scenarios where maximization of buyer-surplus is a good proxy for the incentives of a two-sided platform. For example, two platforms might be in direct competition to attract buyers who join the platform ex-ante, because sellers face switching costs or have already sunk investments that lock them in one or the other. Also, a monopolist platform may be constrained regarding its ability to charge one side, which may lead to maximisation of the ex-ante surplus

municate with sellers (e.g., by suggesting prices), because there is no additional information that the platform could credibly offer to sellers.

[5]As we briefly illustrate in Section 6, the absence of trading inefficiencies is a result of the platform being fully informed. Trading may not be always efficient if the platform is not able to perfectly tell high value buyers from low-value ones.

[6]Relatedly, the assumption that each firm has a single product for sale is not crucial because we can treat a seller with multiple units as multiple identical sellers. Finite supply, however, is important to create a role for matching with (only) vertically differentiated sellers. We discuss in section 6 how our main insights extend to the case of horizontally differentiated firms.

of the side that can be charged; or a platform might earn from advertising to one side only and therefore attempt to maximize participation on that side.

The view that big-tech (gatekeeping) platforms, such as Google and Amazon, should be regulated is popular. Concerns have been raised about what we may call *match-discrimination*, that is, the practice of exploiting information on users to determine who they will be able to interact with.[7] Then, our results can be read as expressing caution toward tampering with the algorithm in an attempt to increase the surplus of the side with less bargaining power. Generating information rent is expensive. Instead, regulators should explore alternative interventions that increase the bargaining power of buyers post-match. Letting buyers make proposals to sellers is, indeed, an existing business model in platform markets. Such policy has been popularized by Priceline, an online travel agent, and since then has been widely adopted, quite possibly in an attempt to increase buyers' surplus (e.g., EBay now allows buyers to make offers to consenting sellers).

The key motivating assumption in our work is *post-match* bargaining. A direct implication is that the platform is unable to condition the matching on prices. We claim this is a realistic assumption in many cases of interest (e.g., Google search). Nonetheless, it is worth assessing to what extent our results are driven by it. We therefore consider a variant of the model where the platform chooses a matching *after* sellers have posted their prices. Our main conclusion is that sorting distortions are a direct consequence of the attempt of the platform to mitigate post-match bargaining power.[8] In fact, when the platform can condition the matching on prices, then there is no trade-off between buyer surplus and sorting inefficiencies. Independently of which side's surplus the platform is maximizing, the equilibrium outcome is positive assortative matching of buyers to sellers. Moreover, trade is efficient for every matched pair. Nonetheless, whether buyers benefit from firms committing to prices beforehand remains ambiguous. One the one hand, sellers lose their market power against matched buyers compared to our benchmark model. On the other hand, the platform loses the ability to directly persuade sellers to post lower prices.

We now proceed with presenting the model and our results. Formal proofs are in Appendix A. In Section 5 we relax the assumption of post-match bargaining. We outline the related literature in Section 4. The model also assumes that the platform is fully informed, sellers are only vertically differentiated, qualities are known and buyers have only two possible valuations. The concluding section and the complementary Appendix B show that our main insight is robust to all these variations.

[7]The current chair of the US FTC has expressed this opinion in her landmark paper on Amazon, Khan (2016). In Europe, the Digital Markets Act imposes specific rules of behavior on systemically important platforms, deemed "gatekeepers". Among such rules is one that forbids platforms from distorting their algorithms in favor of own products, a practice called "self-preferencing".

[8]Post-match bargaining also renders cross-subsidization ineffective. Suppose the platform plans to charge one side and subsidise the other to correct the imbalance in bargaining power. Post-match, the subsidised side will be held-up by the side with bargaining power.

2 The Model

There is a unit mass of buyers. Each buyer has either low valuation, $l\,(>0)$, or high valuation, $h\,(>l)$. The fraction of buyers with high valuation is μ. There is also a unit mass of sellers. Each seller has a single good to sell. The quality of a seller is q and distributed according to the atomless CDF F with support in $[\underline{q},\overline{q}]$. If a buyer with valuation $v\,(\in\{l,h\})$ purchases the good from a seller with quality q at price t, the buyer's payoff is $vq-t$ and the seller's payoff is t.[9] The buyers and sellers are matched by a platform. We assume that the platform and the buyer know the buyer's valuation but the seller does not. We assume that sellers'qualities are publicly observed. Once a buyer and a seller are matched, the seller makes a take-it-or-leave-it price-offer to the buyer. If the buyer accepts the seller's offer, they trade at the price set by the seller. Otherwise, each of them gets their reservation payoff of zero.

Matching.— We describe a *matching* by the probabilities that each seller q is matched with a buyer with valuations h or l. That is, a matching is given by a measurable mapping $p=(p_h,p_l)$ such that $p_h,p_l\;:\;[\underline{q},\overline{q}]\;\rightarrow\;[0,1]$, where $p_h(q)$ and $p_l(q)$ denote the probabilities that a q-seller is matched with a buyer with valuations h and l, respectively. A *feasible* matching p must satisfy the following constraints:

$$\begin{aligned} p_h(q)+p_l(q) &\leq 1, \\ \int_{\underline{q}}^{\overline{q}} p_h(q)\,dF(q) &\leq \mu, \\ \int_{\underline{q}}^{\overline{q}} p_l(q)\,dF(q) &\leq 1-\mu. \end{aligned}$$

The first constraint guarantees that the probability that a seller with type q is matched with a buyer is weakly less than one. We do not require that each buyer and seller is matched with probability one. The second and third constraints guarantee the measure of high-value (low-value) buyers who are matched with sellers does not exceed the total measure of high-value (low-value) buyers.

We say that a matching p is *positive-assortative* if $p_h(q)=1$ for $q\geq F^{-1}(1-\mu)$, $p_h(q)=0$ elsewhere and $p_l(q)=1-p_h(q)$, where F^{-1} stands for the inverse of F. We say that the matching is *stochastically negative-assortative* whenever p_h is monotonically non-increasing in quality. Conversely, the matching is *stochastically positive-assortative* when p_h is monotonically non-decreasing. A matching is *fully-random* if and only if $p_h(q)=\mu$ and $p_l(q)=1-\mu$ for all $q\in[\underline{q},\overline{q}]$.

Optimal Prices.— The matching is observed by sellers. Then, if the matching is given by $p=(p_h,p_l)$, the posterior probability of seller with quality q that she is matched with a high-value buyer is $\mu^p(q)=p_h(q)/(p_h(q)+p_l(q))$. Note that $\mu^p=p_h$ if all sellers are always

[9]Our results do not qualitatively rely on this functional form. The characterization of the welfare-optimal and buyer-optimal matching and the welfare analysis can be extended with little modification to a general $u(q,v)$ assuming u is increasing in both arguments and log-supermodular. We have retained a simpler functional form to avoid burdening the reader with further notation. Likewise, assuming that seller had an opportunity cost cq of selling a product of quality q would not affect results.

matched. So, the q-quality seller is willing to set price qh if, and only, if $\mu^p(q) \geq l/h$. Such seller is willing to set price ql if, and only if, $\mu^p(q) \leq l/h$. It is without loss of generality to assume that, when a seller is indifferent between prices, it charges the lowest.

Buyer Surplus.— Let $\chi^p(q) \in [0,1]$ denote the probability that a seller with value q charges price lq following matching p. That is

$$\chi^p(q) = \begin{cases} 1 & \text{if } \mu^p(q) \leq l/h, \\ 0 & \text{if } \mu^p(q) > l/h. \end{cases}$$

Then, for given matching p, the buyers' surplus can be expressed as

$$\int_{\underline{q}}^{\overline{q}} \chi^p(q)\, p_h(q)\, q\,(h-l)\, dF(q)\,.$$

3 Optimal Matching

The end goal of this section is to characterize the matching which maximizes buyers'surplus and to study its welfare properties. The primary benchmark against which the buyer-optimal matching will be evaluated is the matching that maximizes welfare, defined as the sum of expected buyers'surplus and sellers'profit. We therefore start with the following result, which should not be surprising in light of Becker (1975).

Proposition 1. *A matching maximizes total welfare if and only if it is a positive-assortative matching (PAM) almost-everywhere. In the PAM buyers obtain zero surplus.*

A formal proof of Proposition 1 relies on two observations. First, for any matching p that induces *complete-information*, that is $\mu^p(q) \in \{0,1\}$, trade will take place with probability one and sellers will obtain all surplus. Second, PAM induces complete information and the total welfare of a match between a seller with quality q and a buyer with value v is given by the supermodular function qv.

Because every matched pair trades and sorting is optimal, PAM achieves the first-best level of surplus. Moreover, it is an immediate consequence of Proposition 1 that PAM also maximizes profits of sellers.

Having noted that a buyer-surplus maximizing matching must generate either zero surplus or inefficiencies, we now fully characterize the buyer-optimal matching and equilibrium pricing and show that such inefficiencies can be sizable. We focus on *buyer-optimal* matchings in the *Pareto frontier*, that is, such that there is no other matching that, in equilibrium, gives higher sellers' surplus without reducing buyer-surplus below its maximum level. This has one main implication: no buyer or seller remains unmatched, even if additional matches do not increase buyer surplus.

Theorem 1. *Let $p^* = (p_h^*, p_l^*)$ be a matching defined as follows:*

$$p_h^*(q) = \begin{cases} l/h & \text{if } q \geq q^* \\ 0 & \text{if } q \leq q^* \text{and } \mu \leq l/h \\ 1 & \text{if } q \leq q^* \text{and } \mu \geq l/h \end{cases},$$

$$p_l^*(q) = 1 - p_h^*(q),$$

$$\text{where } q^* = \begin{cases} F^{-1}\left(\frac{l-\mu h}{l}\right) & \text{if } \mu \leq l/h, \\ F^{-1}\left(\frac{\mu h - l}{h-l}\right) & \text{if } \mu \geq l/h. \end{cases}$$

Any buyer-optimal matching in the Pareto frontier is equal to p^* almost everywhere.

The buyer-optimal matching (henceforth also BOM) of high-value buyers, p_h^*, is exemplified in the two panels below. Observe that, when $\mu \geq l/h$ the matching is stochastically negative assortative. Instead, when $\mu < l/h$ then the matching is stochastically positive assortative.

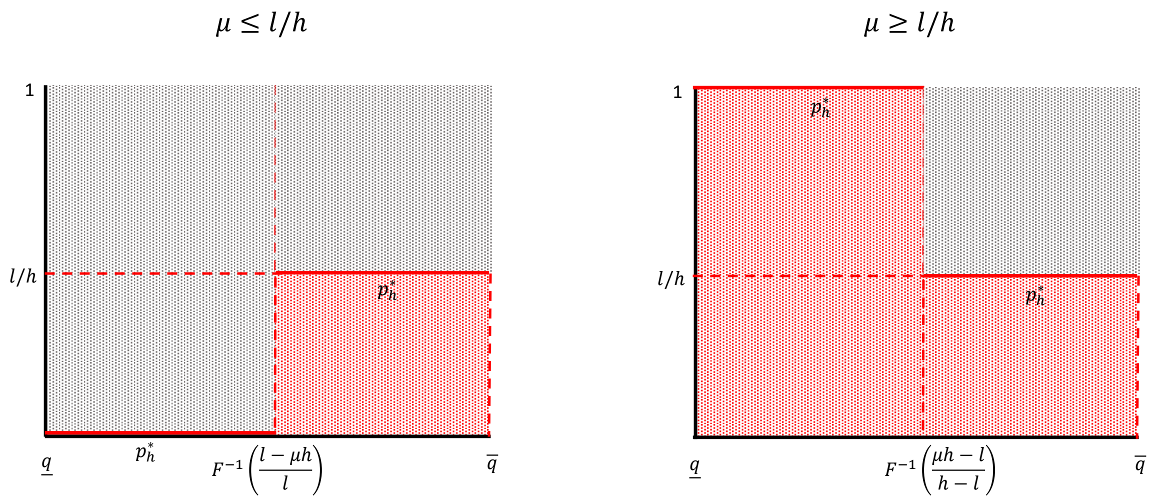

Figure 1: Sketch of Efficient Buyer-Optimal matching, p_h^*.
The red-dotted area has measure μ and the gray-dotted area has measure $1-\mu$.

Let us explain the arguments leading to this proposition. Recall that a q-seller sets price qh if the probability of being matched with a high-value buyer exceeds l/h and sets price ql otherwise. So, a buyer's payoff is positive only if his valuation is high and the seller sets the lower of these prices. The surplus of a high-value buyer who purchases a q-quality good at price ql is $q(h-l)$. Therefore, in order for a seller to generate positive consumer surplus, she must be matched with a mixture of high- and low-value buyers. Furthermore, consumer surplus is maximized at a given seller if she is matched with as many high-value buyers as possible as long as she is willing to set the lower price. That is, she is indifferent between the two prices, so the fraction of high-value buyers among her matches is exactly l/h. Since the high-value buyers' surplus, $q(h-l)$, is increasing in q, the BOM generates consumer surplus at high quality sellers.

Above a quality threshold, q^*, q-sellers are matched so that they are indifferent between the two prices, so $p_h^*(q) = 1 - p_l^*(q) = l/h$. Of course, to maximize buyer surplus, the threshold q^* should be as low as possible and it is determined by the initial distribution of the buyers, μ. If low-value buyers are abundant, $\mu \leq l/h$, then q^* is defined so that the fraction of high-value buyers who are matched with sellers with quality above q^* is exactly μ. In this case, sellers with quality below q^* are matched with the remaining low-value buyers. If there are few low-value buyers, $\mu \geq l/h$, then q^* is defined so that the fraction of low-value buyers who are matched with sellers with quality above q^* is exactly $1-\mu$. In this case, sellers below q^* are matched with the remaining high-quality buyers. The matching is stochastically negative assortative.[10]

Let's now define *sorting distortions* in any given matching as the difference between the total welfare generated by PAM and the welfare generated by that specific matching, assuming all transactions occur. In other words, sorting distortions measure the inefficiencies that cannot be directly attributed to informational frictions. Theorem 1 implies that the sorting distortions resulting from maximizing buyer surplus can be greater than those arising from a fully random matching (henceforth FRM), which would be implemented by a platform that does not use any information on buyers and sellers. Visually, when $\mu > l/h$, BOM generates a decreasing p_h, while PAM induces an increasing one and FRM a constant one.

We make a few additional observations on the nature of the optimal matching. First, the profit obtained by sellers in the BOM is not necessarily monotone in their quality. Low-quality sellers may end up with more profit than high-quality ones when BOM is (stochastically) negative assortative. Second, even if the BOM is stochastically negative assortative in value and quality, the average surplus of buyers matched to high quality sellers is larger than that of buyers matched to lower-quality ones. Third, when low-value buyers are rare, that is $\mu > l/h$, the platform could also benefit buyers by re-balancing participation of high and low values in a somewhat counterintuitive way, that is by raising the share of low-value buyers.[11]

Welfare analysis. We conclude this section by comparing BOM's payoffs with those arising under two benchmarks, the PAM and the fully random matching.

It is obvious that BOM generates strictly larger buyer surplus than PAM and FRM. It is also immediate to see that buyer surplus is strictly larger in the FRM than in PAM, as long as $\mu \leq l/h$ (i.e., all sellers set the lower price when they have no additional information on buyers). Regarding the remaining welfare variables, we have the following proposition. The main insight from this welfare comparison is that, in some cases, BOM generates, not only larger sorting distortions, but also lower welfare than if the platform ignored information altogether.

[10]An alternative buyer-optimal matching *not* in the Pareto frontier would leave high-valuation buyers unmatched. In this case, matching would not be negative assortative. Hence, readers may wonder whether negative assortativeness is an essential property of buyer-optimal matchings. We show that this is the case in Appendix B (Example 1). There, we consider the case of buyers with three possible valuations and show that any buyer-optimal matching is negative assortative in some cases.

[11]The phenomenon that the marginal value of having additional types of buyers is not only equal to the surplus they generate from trading, which is zero for low-value buyers, is studied in depth in Galperti et al. (2022).

Proposition 2. *(i) PAM generates higher welfare and higher profit than BOM; (ii) If $\mu \leq l/h$, then FRM generates the same profit as BOM and, hence, lower welfare. If $\mu > l/h$ then FRM generates higher-profit than BOM but the welfare ranking between the two matchings is, in general, ambiguous; when μ is sufficiently large, that is $\mu \geq \frac{l}{h} + \frac{h-l}{h}\frac{l}{h}$, then the welfare of BOM is larger than that of FMR.*

Part (i) is just a restatement of Proposition 1. It is also immediate to see that when $\mu \leq l/h$ profit of BOM and FRM are the same, as in both cases each seller sets price ql and trades takes place. Hence, because BOM generates higher buyer-surplus by Theorem 1, then the first part of (ii) follows. The proof in the Appendix shows that, when $\mu > l/h$, profits are lower under BOM than under FRM. Because setting the high price maximizes profit for all sellers in both the FRM (since $\mu^p = p_h = \mu > l/h$) and BOM (due to the indifference condition for $q > q^*$), this inequality in profit is the result of the negative assortative nature of BOM, which creates more sorting distortions than a FRM. Then the proof shows, by means of two non knife-edge examples, that the welfare comparison between BOM and FRM is ambiguous.

Since a platform without information on at least one of the two sides implements a FRM, these findings suggest that the overall welfare effect of the platform collecting more information about users may depend on the platform's objective and is non-obvious. As we discuss further in the concluding section, more information always reduces bargaining disagreement but may bring about a larger sorting distortion.

4 Matching on Prices

Because bargaining is *post-match*, the platform is unable to condition the matches on prices. In this section, we consider a variant of the model where firms post (observable) prices *before* the platform implements its matching. Our main conclusion is that, in this case, there is no trade-off between buyer surplus and efficient sorting. The outcome is PAM and trade always takes place. Once the platform loses incentives to steer pricing, it also loses its need to distort the matching.

There are two natural avenues to modify the model. First, we could allow the platform to *commit* to a price-dependent matching before prices are posted by sellers. This option trivializes the problem. The platform can implement PAM and any pricing that gives sellers more than their outside option of zero. It can do so by threatening sellers with the prospect of remaining unmatched. Second, we could assume that sellers post prices *before* the matching algorithm is chosen. In the remainder of the section we study this variant, which may also appear more realistic in light of the flexibility that platforms have in updating their algorithms.

In this modified game, we look for a perfect equilibrium where, first, sellers simultaneously post prices. Then, after observing prices, the platform chooses and implements a matching and, finally, the buyers decide whether to buy or not. We denote with $\tau : [\underline{q}, \overline{q}] \to \mathbb{R}_+$ the pure strategy of sellers. Since there are continuum many sellers and buyers, even when the platform cannot commit to a matching mechanism, it can punish individual deviations at no cost. This gives rise to multiple equilibria, some of which do not capture the platform's commitment problem. In what follows, we consider only equilibria which are limits of equilibria in a finite model where there are the same number of buyers and sellers.

Regarding the platform's objective, we consider three scenarios: it maximizes either buyer surplus, profits, or welfare. Our main observation is the following.

Proposition 3. *The equilibrium matching is PAM irrespective of the platform's objectives.*

Let us present only the sketch of the proof. Consider first the case of platform which maximises buyer surplus. First, note that if a q-seller is matched with an l-buyer in equilibrium, she sets price lq. Setting any price lower than that is strictly dominated by a slightly higher price. As a result, the surplus of low value buyers is zero. Second, consider the largest q for which a q-seller trades with an l-buyer, and denote it by $\widehat{q}$. Note that matching this seller with an h-buyer would generate a surplus of $(h-l)\widehat{q}$ for the buyer. Then, the following equation pins down the price of each seller trading with an h-buyer:

$$hq - \tau(q) = (h-l)\widehat{q}.$$

To see this, observe that if the price was higher, the platform would prefer to match the h-buyer with the $\widehat{q}$-seller. If the price was lower, the q-seller could profitably deviate by raising its price slightly. To conclude that the equilibrium matching is PAM, we only need to show that for all $q < \widehat{q}$, the q-seller trades with an l-buyer. Suppose, by contradiction, that there exists a $\widetilde{q}$, such that the $\widetilde{q}$-seller trades with an h-buyer. Then,

$$\tau(\widetilde{q}) = h(\widetilde{q} - \widehat{q}) + l\widehat{q} < l(\widetilde{q} - \widehat{q}) + l\widehat{q} = l\widetilde{q},$$

where the equality follows from the previous displayed equality, the strict inequality impied by $l < h$ and $q < \widehat{q}$. This inequality chain implies that the $\widetilde{q}$ seller can profitably deviate by setting price slightly below $l\widetilde{q}$ because such a price would guarantee that the platform match her with an l-buyer and trade occurs.

Consider now a welfare-maximizing platform. Again, the q-seller who is matched with an l-buyer sets price lq in equilibrium. In contrast to the previous case, the q-seller who is matched with an h-buyer sets price hq. If the price was lower, then the seller could raise the price slightly. Since that increase in price does not affect welfare, the platform will still match this seller with an h-buyer, for otherwise she would not trade. Let us now argue that the equilibrium matching is PAM. Suppose, by contradiction, that $q < q'$ and that the q-seller sells to an h-buyer and the q'-seller sells to an l-buyer. Then the q'-seller could deviate by setting price slightly below hq' instead of setting lq'. Such a deviation essentially committs the seller not to generate surplus unless she is matched with an h-buyer. After such a deviation, the platform strictly prefers matching an h-buyer with the q'-seller to matching him with the q-seller. Finally, note that the arguments we made in this paragraph are also applicable if the platform maximizes the sellers' profits. Hence, the matching will also be PAM in this case.

The seller-optimal matching when the platform can condition on prices coincides with the seller optimal one in our benchmark model. In contrast, the buyer-optimal matching differs in the two versions of the model. Despite the sorting distortions of BOM in the benchmark model, whether buyers prefer one platform type to the other is not a priori obvious. On the one hand, if sellers post prices before being matched, they lose bargaining power. On the other hand, if the platform moves after prices are posted, it loses ability to persuade sellers and control their prices. In fact, the next example shows that the comparison of the two models from the buyers' perspective is, in general, ambiguous.

Example. Let quality be uniformly distributed in $[\underline{q},\overline{q}]$. That is $F(q) = \frac{q-\underline{q}}{\overline{q}-\underline{q}}$. We can then compute buyers surplus in the PAM where the platform conditions on prices as

$$\mu(h-l)F^{-1}(1-\mu) = \mu(h-l)\left((1-\mu)(\overline{q}-\underline{q})+\underline{q}\right) \tag{1}$$

and buyer surplus in the BOM of the benchmark model as

$$\begin{cases} \mu(h-l)\mathbb{E}\left[q \mid q \geq F^{-1}\left(1-\mu\frac{h}{l}\right)\right] = \mu(h-l)\left(\frac{\underline{q}+\overline{q}}{2}+\frac{\overline{q}-\underline{q}}{2}\left(1-\mu\frac{h}{l}\right)\right) & \text{if } \mu < l/h \\ (1-\mu)l\mathbb{E}\left[q \mid q \geq F^{-1}\left(1-(1-\mu)\frac{h}{h-l}\right)\right] = l(1-\mu)\left(\frac{\underline{q}+\overline{q}}{2}+\frac{\overline{q}-\underline{q}}{2}\left(1-(1-\mu)\frac{h}{h-l}\right)\right) & \text{if } \mu > l/h. \end{cases} \tag{2}$$

To establish our conclusion that a comparison of the two models in terms of buyer-surplus is ambiguous it is sufficient to compare (1) and (2) when $\mu < l/h$. After factoring out $\mu(h-l)$ from both formulas, we can see that buyer surplus from BOM in the benchmark model is lower than buyer surplus in the PAM arising from the variant model if

$$\frac{\underline{q}+\overline{q}}{2}+\frac{\overline{q}-\underline{q}}{2}\left(1-\mu\frac{h}{l}\right) < (1-\mu)(\overline{q}-\underline{q})+\underline{q}.$$

That is if

$$(\overline{q}-\underline{q})\left(1-\mu\frac{h}{2l}\right) \quad < \quad (1-\mu)(\overline{q}-\underline{q}) \quad \text{or} \quad (\mu \leq)\ l/h < 1/2.$$

5 Literature Review

Starting with Shapley and Shubik (1971), an important strand of the literature on two-sided markets has imposed joint restrictions on who matches with whom and at what prices by requiring that no coalition of agents benefits from a different matching and sharing of output that they can implement.We depart from tradition by considering an environment where, due to the informational spillovers and bargaining under asymmetric information, the ex-post surplus-sharing of a matched couple is not exogenous, but depends on the matching. Notable exceptions, within the smaller literature that considers stability of matching under asymmetric information, are Liu et al. (2014) and Liu (2020). There, in order to evaluate potential deviations, agents form interim expectations on the value of their match which, as in our paper, depend on the putative mapping from states of the world into matchings, which is publicly known.

The design of an optimal matching, but by a revenue maximizing platform, is studied in Damiano and Li (2007), Johnson (2013), Gomes and Pavan (2016), Aoyagi and Yoo (2022) and Gomes and Pavan (2022). We share with these works the presence of a monopolistic platform that matches two-sides of the market and the emphasis placed on the distortion introduced by a matching that maximises an objective other than total welfare. In stark contrast to our model, the platform in these papers is uninformed about valuations and aims at maximizing its own profit by setting prices to agents on both sides. The approach is in the spirit of optimal mechanism design, where the outcome is now given by the matching rather than by the allocation of an object as in classic Myersonian mechanism design. These papers are complementary to ours. Aoyagi and Yoo (2022) is the closest in spirit. There, bargaining is post-match and the sorting distortions, as in our paper, result from an attempt to manipulate information rents. All the other works look at platforms that dictate the conditions at which trading takes places among matched pairs.

A number of other papers have looked at the incentives of platforms who charge per-click to distort matching in order to boost costly search (i.e., clicking). In Eliaz and Spiegler (2011) a consumer searches from a pool of firms whose boundary is determined by the platform. Inefficiently too many low-quality sellers may be allowed in the pool, to induce consumers to search more. In De Corniere (2016) a platform can match consumers to their preferred segment of firms in a more or less noisy way. Consumers search within the set of firms they are matched with. Because perfect matching traps consumers into monopoly prices, a platform may want to bias the algorithm to foster consumer participation. In a similar vein, in Hagiu and Jullien (2011) an intermediary may match consumers with stores that are worse for them in order to persuade them to search more. They also show that the intermediary gains from marginally biasing the matching away from the perfect one, if it reduce firms' prices and convinces more consumers to visit at least one store. While focusing strictly on platform's incentives, these papers suggest reasons why perfect matching may not be consumer-optimal, with De Corniere (2016) and Hagiu and Jullien (2011) also identifying a feedback effect of matching on prices. In our work, we abstract from platform's incentives, horizontal differentiation and competition. The additional simplicity allows us to fully characterise the buyer-optimal matching, thus highlighting a trade-off between matching efficiency and consumers'information rent.

In our model, if a pair is formed its participants won't be available to form other matches. Hence, the platform encounters a trade-off when forming a match. We refer to Elliott et al. (2023b) and Bergemann and Bonatti (2022) for models where sellers do not have limited supply and the role of the platform is to control which sellers compete for which buyers. In Elliott et al. (2023b) a fully informed platform manipulates the outcome by making firms uncertain about the set of buyers they can sell to and the other firms they are competing with. In contrast to our work, consumer optimal outcomes does not imply sorting distortions.[12] In Bergemann and Bonatti (2022), the consideration set of each consumer is auctioned off by the platform, which also revels the information it has about the buyer to the winning firm.

[12]In Elliott et al. (2023a), the platform does not control the consideration set of buyers but the information that sellers receive about the buyers. On the theme of a platform providing information to sellers see also Yang (2022).

In search for a buyer-optimal matching, a designer resolves a trade-off between a more efficient matching and information rent for buyers. This trade off, between efficiency and rent, recurs in other contexts that also share with us a flexible information-design-like approach.[13] In Condorelli and Szentes (2020) a buyer can choose her distribution of value for the product of a seller with bargaining power. Therefore, it faces a related trade-off between having a higher valuation and larger information rent. In Armstrong and Zhou (2021), perfectly informing consumers about which of two differentiated products is best for them relaxes competition but maximizes welfare, while the consumer-optimal information structure dampens differentiation to some extent. In a search model, Dogan and Hu (2021) show that total welfare would be maximized by giving a buyer as much information as possible to find a good match among several firms, but that would lead to too little competition within firms. In our own papers, information is a byproduct of choices (i.e., distribution of value and matching) that affect both the information structure and feasible surplus, while in Armstrong and Zhou (2021) and in Dogan and Hu (2021) the information structure is a primitive that affects agents'choices, which may end up being inefficient.

Intuitively, one would expect the above trade-off between efficient matching and information rents to be exacerbated when sellers are more heterogenous, as in that case it becomes more costly to move away from PAM. Indeed, if all sellers are the same, sorting efficiency is irrelevant and a platform that maximises buyer surplus can focus on producing information rent. In fact, it is not difficult to see that in this case our problem is isomorphic to that of identifying the segmentation of buyer's demand that maximizes buyer-surplus under a single price-discriminating monopolist. It then follows from Bergemann et al. (2015) (BBM) that there exists an efficient matching where buyers obtain all surplus minus the profit from random matching. As it turns out, in our BOM the buyer market is subdivided into two segments, one composed entirely by either high or low types, and the other by a mix of high and low type such that a monopolist facing such segment would be indifferent between setting the high or low price. Remarkably, these are the same two segments composing the segmentation that maximises consumer surplus in BBM.

6 Concluding Remarks

In this last section, we discuss the case where buyers may have more than two values and we further elaborate on the relation of our work with BBM. Then, we also discuss extending the analysis to the case of a partially informed platform, private qualities and horizontally differentiated sellers. The take-away is that sorting inefficiency remains a feature of the buyer-optimal matching with post-match bargaining.

Multiple values. Our key characterisation result for the multiple-value case is formally stated in Appendix B. Any Pareto efficient matching outcome can be obtained starting from a seg-

[13] We can map to each matching a distribution of posteriors over buyer valuations that satisfies the martingale property. However, the techniques used to solve Bayesian persuasion problems (e.g., see Kamenica and Gentzkow (2011), Bergemann and Morris (2016), and Dworczak and Martini (2019)) are not directly applicable because the objective function of the matching platform it is not linear in posterior beliefs.

mentation of the buyer market that is *extremal*, in the sense of BBM.[14] Assuming the designer maximises some weighted average of buyer and seller-surplus, there will be an extremal segmentation such that an optimal matching can be built by ordering segments in terms of the average weighted surplus they generate for *unit-quality* and then positively assortatively matching them to groups of sellers. Buyers in each segment are matched to sellers so that all sellers matched to a certain segment believe they are facing a random buyer from that segment. Trade always takes place because sellers tailor the price to the lowest value in the support of the segment to which they are matched. Any inefficiency is due to sorting.

We make three further observations regarding optimal matching, which we substantiate in Appendix B. First, PAM will continue to maximise welfare and sellers' profits even when buyers have multiple values. The relevant extremal segmentation will be one where each segment contains all and only buyers with the same value. Second, in the extremal segmentation that maximises buyer-surplus, there is at most a single segment for which buyers have only one possible value. If two such segments existed, buyer surplus could be raised by merging them. This implies that sorting will be inefficient. Third, the buyer optimal matching can still be stochastically negative assortative.

Last but not least, in light of our discussion in this and the previous section, it could be conjectured that the buyer-optimal matching problem may be solved using consumer-surplus maximising extremal segmentations characterised by BBM. Unfortunately, this is not always true and obtaining a full solution remains an open question. As we show in Appendix B with an example, the segmentation of buyers in our buyer-optimal matching with heterogenous quality and more than two values may deliver higher than BBM's monopoly profit under fixed unit-quality. The idea underlying the example is that when only a small fraction of sellers is able to generate value, the platform maximising buyer surplus may want to focus on extremal segmentations that, albeit resulting in higher than monopoly profit and, therefore, lower total consumer surplus, nonetheless produce higher consumer surplus for some small fraction of buyers, which will be those matched with the few high-quality sellers.

Private qualities. The assumption that qualities of sellers are observable is often plausible, but it is worth asking whether our characterization and main insight would survive if quality was not observable by buyers. We now argue that this is the case. However, while analogous sorting inefficiencies remain, we observe that the platform can achieve a higher payoff for buyers in some equilibria.

It is easy to see that, even with unknown quality, the platform can implement the same buyer-surplus as in the BOM. For instance, assume $\mu > l/h$ and that sellers in $(q^*, \overline{q}]$ are matched to a mix of high and low-vale buyers in such a way that each such seller is indifferent and each buyer's posterior is such that the quality of sellers they are matched with is distributed in $(q^*, \overline{q}]$ according to the truncated prior. Also assume that the remaining high-value buyers are matched

[14]Extremal markets are those where the seller is indifferent among any price equal to valuations in the support of the market and an extremal segmentation is simply a segmentation of the initial buyer market made up only of extremal markets.

one-to-one to sellers in $[\underline{q},q^*]$. Then, there is an equilibrium in which all sellers with q in $[\underline{q},q^*]$ charge qh, since buyers know the sellers they are matched with, while all sellers with q in $(q^*,\overline{q}]$ charge $\mathbb{E}[q \mid q > q^*]\,l$. In this equilibrium, sellers in $(q^*,\overline{q}]$ deviating to a different price are believed to be of quality q^*. It is immediate to see that buyer surplus in this equilibrium is equal to that achieved by BOM with observable quality. An analogous construction can be performed for $\mu < l/h$.

However, there are other equilibria of the matching above in which buyers do better. In particular, there is an equilibrium where all sellers in $[q^*,\overline{q}]$ post price q^*l sustained by the same out-of-equilibrium beliefs that a deviating seller is of type q^*. The fact that the set of equilibrium prices will depend on the beliefs of buyers, suggests that the buyer-optimal matching might might induce posterior beliefs for *sellers* that are different than those induced by our BOM with known quality. Remarkably, this is not the case. Roughly speaking, by placing sellers in a neighborhood of $\underline{q}$ in the support of the two segments of buyers we built for our Theorem 1, we can construct a matching and an equilibrium such that all sellers in $(\underline{q},q^*]$ charge $\underline{q}h$ and sellers in $(q^*,\overline{q}]$ charge $\underline{q}l$. Note that the share of sellers charging a price that results in a purchase by high-value buyers at a price acceptable also by the low-value buyers is maximized by posteriors induced by the BOM. Hence, this is the buyer-optimal equilibrium. We omit the tedious details involved in formalizing such a construction.

Partially informed platform. Suppose the platform does not know buyers'values. Instead, it receives independent binary signals about them, which are informative in the sense that a monopolist seller sets the low price following one realization and the high one following the other. It is easily seen that, on the one hand, the matching that maximizes sellers' revenues is positive assortative in the binary signal and sellers' quality. On the other hand, in the buyer-optimal matching a portion of higher quality sellers are matched with a mixture of low-signal and high-signal buyers, so that they are, as in the full information case, indifferent between charging the low and the high prices. Therefore, if $\mu > l/h$ BOM is also stochastically negative assortative. However, compared to the case of full information, sorting distortions when $\mu > l/h$ ($\mu < l/h$) are mitigated (amplified) by the lower leverage that a partially informed platform has to manipulate sellers' beliefs. A further important difference with the full information case is that PAM is not always welfare optimal. Indeed, sometimes BOM is optimal. This is because, for lower levels of signal informativeness, charging a high price results in a substantial likelihood of even the high-signal buyer refusing the offer. Hence, persuading sellers to charge a low price becomes welfare relevant and an additional trade-off arises, which either resolves in favor of BOM or PAM.

Vertically differentiated sellers. We conclude this section with an example suggesting vertical differentiation is not key to observing inefficient sorting in buyer-optimal matchings, when bargaining is post-match. Consider a model with two buyers, 1 and 2, and two sellers, A and B. Assume that, with some probability, buyer 1 has value h for the product of A and value l for the product of B, while buyer 2 has value h for the product of B and l for that A. With the remaining probability, preferences are reversed. Given perfect correlation in values, an informed platform can always match each product to the buyer that values it the most. However, such a matching fully informs both sellers that they are facing a value h buyer, thus leaving no surplus to buyers. Instead, by mismatching sufficiently often, that is matching sellers to buyers that like them least until both sellers become indifferent between asking h or l, the platform can make sure that both sellers charge price l, thus raising buyer surplus.

Appendix A: Proofs

Proof of Proposition 1. Denote with G the distribution of buyers'values. Since vq is supermodular, a classic result, the Fan Lorentz Theorem, implies that $\sup_{\pi\in\mathscr{M}(F,G)}\mathbb{E}_\pi[vq]$, where $\mathscr{M}(F,G)$ is a *coupling* of probabilities F and G, has a unique *comonotone* solution which is given by PAM (e.g., see Kleiner et al. (2021)). We have already argued the second statement follows from the fact that PAM induces complete information. □

Proof of Theorem 1. Focusing on the case $\mu \geq l/h$, we show that p^* generates strictly larger consumer surplus than p unless $p = p^*$ almost everywhere. The case $\mu \leq l/h$ is analogous and we omit the proof.

For each p, let us defined the CDF G^p as follows:

$$G^p(x) = \frac{\int_{\underline{q}}^{x} \chi^p(q)\, p_h(q)\, dF(q)}{\int_{\underline{q}}^{\overline{q}} \chi^p(q)\, p_h(q)\, dF(q)}.$$

Also, define q^p by

$$\int_{q^p}^{\overline{q}} \frac{l}{h} dF(q) = \int_{\underline{q}}^{\overline{q}} \chi^p(q)\, p_h(q)\, dF(q). \tag{3}$$

Finally, define the CDF H^p by $H^p(x) = 0$ if $x \leq q^p$ and by

$$H^p(x) = \frac{\int_{q^p}^{x} \frac{l}{h} dF(q)}{\int_{q^p}^{\overline{q}} \frac{l}{h} dF(q)}$$

if $x > q^p$.

We now show that H^p first-order stochastically dominates G^p. To see this, first note that if $x \leq q^p$ then $H^p(x) = 0 \leq G^p(x)$. Moreover, for all $x > q^p$,

$$1 - H^p(x) = \frac{\int_{x}^{\overline{q}} \frac{l}{h} dF(q)}{\int_{q^p}^{\overline{q}} \frac{l}{h} dF(q)} = \frac{\int_{x}^{\overline{q}} \frac{l}{h} dF(q)}{\int_{\underline{q}}^{\overline{q}} \chi^p(q)\, p_h(q)\, dF(q)} \geq \frac{\int_{x}^{\overline{q}} \chi^p(q)\, p_h(q)\, dF(q)}{\int_{\underline{q}}^{\overline{q}} \chi^p(q)\, p_h(q)\, dF(q)} = 1 - G^p(x),$$

where the first and last equalities are the definitions of H^p and G^p, respectively, the second equality follows from (3) and the inequality follows from $\chi^p(q) \leq 1$ and $p_h(q) \leq l/h$ whenever $\chi^p(q) = 1$. Then previous inequality chain implies that $H^p(x) \leq G^p(x)$ even when $x > q^p$.

Therefore,

$$\begin{aligned}
&\int_{\underline{q}}^{\overline{q}} \chi^p(q)\, p_h(q)\, q\,(h-l)\, dF(q) = \left[\int_{\underline{q}}^{\overline{q}} \chi^p(q)\, p_h(q)\, dF(q)\right]\left[\int_{\underline{q}}^{\overline{q}} q\,(h-l)\, dG^p(q)\right] \\
= &\left[\int_{q^p}^{\overline{q}} \frac{l}{h} dF(q)\right]\left[\int_{\underline{q}}^{\overline{q}} q\,(h-l)\, dG^p(q)\right] \leq \left[\int_{q^p}^{\overline{q}} \frac{l}{h} dF(q)\right]\left[\int_{\underline{q}}^{\overline{q}} q\,(h-l)\, dH^p(q)\right] \\
= &\int_{q^p}^{\overline{q}} \frac{l}{h} q\,(h-l)\, dF(q),
\end{aligned}$$

where the first and last equalities follows from the definitions of G^p and H^p, respectively. The second equality is implied by (3) and the inequality follows from the fact that H^p first-order stochastically dominates G^p.

It remains to show that

$$\int_{q^p}^{\overline{q}} \frac{l}{h} q(h-l)\, dF(q) \leq \int_{q^*}^{\overline{q}} \frac{l}{h} q(h-l)\, dF(q).$$

In order to do so, it is enough to argue that $q^p \geq q^*$. Observe that

$$\begin{aligned} & \int_{q^*}^{\overline{q}} \left(1-\frac{l}{h}\right) dF(q) = 1-\mu \geq \int_{\underline{q}}^{\overline{q}} \chi^p(q)\, p_l(q)\, dF(q) \geq \frac{h-l}{l} \int_{\underline{q}}^{\overline{q}} \chi^p(q)\, p_h(q)\, dF(q) \\ = & \frac{h-l}{l} \int_{q^p}^{\overline{q}} \frac{l}{h} dF(q) = \int_{q^p}^{\overline{q}} \left(1-\frac{l}{h}\right) dF(q), \end{aligned}$$

where the first equality is the explicit definition of q^* and the first inequality is a feasibility constraint for the matching p. The second inequality follows from the fact that if $\chi^p(q) = 1$ then $p_h(q)\, h \leq (p_l(q) + p_h(q))\, l$, that is, $p_l(q) \geq p_h(q)\, [(h-l)/l]$. The second equality is again implied by (3). □

Proof of Proposition 2. Profit under the FRM matching is $\mu h \mathbb{E}[q]$. If $\mu > l/h$, then the profit under BOM is smaller than that under FRM if

$$F(q^*)\mathbb{E}[q \mid q \leq q^*]\, h + (1-F(q^*))\frac{l}{h}\mathbb{E}[q \mid q > q^*]\, h < \mu h \mathbb{E}[q]$$

where we have used the fact that albeit sellers with $q \geq q^*$ charge l, they are indifferent between charging l and h. Dividing both sides by μ, simplifying h away and rewriting $\mathbb{E}[q]$ we get

$$\frac{F(q^*)}{\mu}\mathbb{E}[q \mid q \leq q^*] + \frac{1-F(q^*)}{\mu}\frac{l}{h}\mathbb{E}[q \mid q > q^*] < \mathbb{E}[q] = F(q^*)\mathbb{E}[q \mid q \leq q^*] + (1-F(q^*))\mathbb{E}[q \mid q > q^*].$$

Now focus on the inequality between the left and right side of the above. Since $q^* = F^{-1}\left(\frac{\mu h - l}{h-l}\right)$, we have

$$\frac{F(q^*)}{\mu} + \frac{1-F(q^*)}{\mu}\frac{l}{h} = 1.$$

Hence, both sides are weighted sums of the same conditional expectations. Then, to conclude the proof of the statement that profit under BOM is below profit with FRM observe that

$$\mathbb{E}[q \mid q > q^*] > \mathbb{E}[q \mid q \leq q^*]$$

and, because $\mu \geq l/h$,

$$\frac{F(q^*)}{\mu} > F(q^*) \text{ and } \frac{1-F(q^*)}{\mu}\frac{l}{h} < 1-F(q^*).$$

Again, continue to assume $\mu > l/h$. The FRM welfare is as profit, that is $\mu h \mathbb{E}[q]$. Rewrite it as

$$\mu h \mathbb{E}[q \mid q \leq q^*] F(q^*) + \mu h \mathbb{E}[q \mid q \geq q^*](1 - F(q^*))$$

The BOM welfare is

$$h\mathbb{E}[q \mid q \leq q^*] F(q^*) + \left(\frac{l}{h}h + \frac{h-l}{h}l\right) \mathbb{E}[q \mid q > q^*](1 - F(q^*)).$$

Subtracting FRM from BOM we get

$$h(1-\mu)\mathbb{E}[q \mid q \leq q^*] F(q^*) - \left[\left(\mu - \frac{l}{h}\right)h - \frac{h-l}{h}l\right] \mathbb{E}[q \mid q > q^*](1 - F(q^*)).$$

The first term is clearly positive. We now focus on when the second term of the difference. It is positive if

$$\left[\left(\mu - \frac{l}{h}\right)h - \frac{h-l}{h}l\right] > 0,$$

or

$$\mu > \frac{l}{h} + \frac{h-l}{h}\frac{l}{h}$$

It is now easy to see that when $\mu \leq \frac{l}{h} + \frac{h-l}{h}\frac{l}{h}$ then the welfare of BOM is higher than the welfare of FRM. To construct an example where the welfare of BOM is lower then FRM one assumes $\mathbb{E}[q \mid q \leq q^*]$ is sufficiently small and $\mu > \frac{l}{h} + \frac{h-l}{h}\frac{l}{h}$. □

Appendix B: Extension to multiple buyer values

In this Appendix we relax the assumption that buyers can only have two valuations. We present a partial characterisation of efficient matchings under the assumption that there are a finite number of possible values drawn from the naturally ordered set $V = \{v_1, v_2, ..., v_K\}$ according to some prior distribution with full support. We continue to assume that quality is distributed according to F.

As a first step we introduce some notation, which we borrow from BBM. We define a market (on the buyer-side) as a distribution on V and we denote the set of all markets as $X = \Delta(V)$. For each $x \in X$ let $x(v)$ the probability of $v \in V$. We denote the market defined by the prior distribution on V as $x^* \in X$. A *segmentation* σ of x^* is a subdivision of buyers in x^* into various submarkets, called segments of σ. Formally, σ is a distribution on X such that the aggregate market, that is the mixture distribution, is equal to x^*. If σ has finite support, we let $\sigma(x)$ be the probability of market $x \in supp\ \sigma$. Finally, we say that a segmentation σ is *extremal* if, for each $x \in supp\ \sigma$, a monopolist selling to it is indifferent between setting any price equal to the valuations in the support of the segment. Denote an extremal market with support $E \subseteq V$ as x^E.

We now introduce some additional pieces of notation that are needed for our purposes. First, we define a matching as the assignment of a market $x \in X$ to each quality level q. Implicit in this formulation is the restriction, which is without loss, that a matching assigns the same market to sellers with the same quality. Denoting x_q the market assigned to any seller with quality q, a matching must satisfy the feasibility condition

$$\int x_q(v)\; dF(q) = x^*(v)\, forall v \in V. \tag{4}$$

Next, we introduce the platform's objective. We will be interested in matchings that maximize a weighted average of buyer-surplus and sellers-surplus. That is, for $\lambda \in [0,1]$ our objective function is

$$\lambda \int q\; cs(x_q) dF(q) + (1-\lambda) \int q\; \pi(x_q) dF(q) = \int q \left[\lambda cs(x_q) + (1-\lambda)\pi(x_q)\right] dF(q),$$

where $cs(x)$ and $\pi(x)$ denote the consumer surplus and the monopoly profit in a standard monopoly market $x \in X$ with unit quality, respectively. Writing $u_\lambda(x) = \lambda cs(x) + (1-\lambda)\pi(x)$, the platform problem we want to solve is

$$\max_{x_q \; satisfies (4)} \int u_\lambda(x_q) q dF(q). \tag{PP-λ}$$

As it is well known a matching will be Pareto-efficient if and only if it solves (PP-λ) for some $\lambda \in [0,1]$.

Note that we can think of a matching as a segmentation of x^* made up of a continuum of segments. Conversely, to solve (PP-λ) it will be useful to construct a matching from a generic segmentation of the buyer market with finite support. To this end, we will say that a segment $x \in supp\; \sigma$ is matched *uniformly at random* with a certain mass $\sigma(x)$ of sellers if for each of those sellers $x_q = x$. Hence, each of those sellers has posterior equal to $x(v)$ for any value $v \in supp\; x$.

Of course, which segments are matched with which seller is important to our purposes. For any segmentation σ with finite support, let's order all the segments in its support as $\{x_1^\sigma, x_2^\sigma, \ldots, x_n^\sigma\}$, with the property that $u_\lambda(x_1^\sigma) \geq u_\lambda(x_1^\sigma) \geq \cdots \geq u_\lambda(x_n^\sigma)$. We then introduce the following key definition. A matching is a *u_λ-assortative matching based on a segmentation σ* (with finite support) if it is built by pairing sellers in $[\overline{q}, F^{-1}(\sigma(x_1^\sigma)))$ to consumers in x_1^σ uniformly at random , then pairing sellers in $[F^{-1}(\sigma(x_1^\sigma)), \overline{q}, F^{-1}(\sigma(x_2^\sigma)))$ to consumers in x_2^σ, and so on until all buyers and sellers are exhausted.

Our main observation is the following.

Proposition 4. *There is solution to (PP-λ) which is an u_λ-assortative matching based on an extremal segmentation of x^*.*

The proposition above indicates that the problem of finding an optimal matching can be simplified by restricting attention to the finite set of all extremal segmentations of the buyers market, further characterized in BBM. The proof follows from two observation. First, for any market $x \in X$ there exists an extremal segmentation that has the same distribution over valuations and where the price charged for all segments of such extremal segmentation is the same as that charged in x (Proposition 2 in BBM). Second, in maximizing $(PP-\lambda)$ we must have that $q' > q''$

implies $u_\lambda(x_{q'}) \geq u_\lambda(x_{q''})$. If not, then we could swap segments without affecting the feasibility condition and without lowering the objective function.

A consequence of the proposition is that, as for the case of two values, in a (Pareto-efficient) optimal matching (in the sense made clear in the statement of the proposition) every matched couple will trade. There will be no inefficiency due to private information, despite information will not be symmetric at the bargaining stage. This is because sellers set the lowest possible price, which is equal to quality times the lowest value in the support of the (extremal) segment they are matched with.

It is a simple corollary of the previous proposition, and therefore stated without proof, that PAM, defined now as u_0-assortative matching based on the perfect price discrimination segmentation of x^*, maximizes producer surplus and achieves a first-best. Formally, a perfect-price discrimination segmentation of x^* the σ composed by K extremal segments $x^{\{v_i\}}$ with $\sigma(x^{\{v_i\}}) = x^*(v_i)$ for all $i = 1, \dots, K$.

Corollary 1. *$PP-0$ is solved by PAM.*

Our final result further characterises the matching and the extremal segmentation arising in the maximisation of buyer-surplus. It suggests that the BOM will continue to be coarse, hence inefficient compared to PAM, even with multiple valuations. It extends our main qualitative insight from the previous sections to the case where buyers have multiple valuations.

Corollary 2. *PP-1 is solved by a u_λ-assortative extremal segmentations of x^* which contains at most one extremal segment that has a single valuation in its support.*

In words, the corollary restates that a buyer optimal matchings can be constructed from some extremal segmentation (but, as we shall see, not necessarily one that maximises consumer surplus in BBM) by ordering the segments in terms of consumer surplus. It also states that it is not possible that the segmentation contains two segments that both have a single value in their support.

We conclude this Appendix with two examples, 1 and 2. First, we present an example where the consumer surplus maximizing segmentation of BBM can be used to construct a buyer-optimal matching that result in stochastic negative assortative matching. Second, we show that, in some cases, the optimal extremal segmentation of the buyer-market used in the BOM is not the efficient one that maximises consumer surplus in BBM. The example also demonstrates that the problem of fully characterising a buyer-optimal matching is not trivial. The idea underlying the example is that when only a small fraction of sellers is able to generate value, the platform maximising buyer surplus may want to focus on segmentations that, albeit resulting in higher profit and therefore lower total consumer surplus, however produce higher consumer surplus for some small fraction of buyers that are those matched with the few high-quality sellers.

Example 1. Suppose the three possible valuations of buyers, (l, m, h), with $l = 1$, $m = 2$ and $h = 3$, as in BBM's leading example. The prior x^* is first-order stochastic shift from the uniform that places more mass on h. In particular $x^*(l) = 1/3, x^*(m) = 2/9, x^*(h) = 4/9$. Note that a unit-quality monopoly prices are 2 and 3. We assume the distribution of quality is F.

The efficient and consumer-surplus maximising extremal segmentation according to BBM is composed by a segment of mass $2/3$ with support $\{l,m,h\}$, that is $x^{\{l,m,h\}}$, a segment of mass $1/6$ with support $\{m,h\}$, that is , that is $x^{\{m,h\}}$, and a segment with support $\{h\}$ of mass $1/6$. In the buyer optimal matching all buyers in $x^{\{l,m,h\}}$ are matched to high quality sellers, buyers in $x^{\{l,m,h\}}$ are matched to medium quality sellers and buyers in $x^{\{h\}}$ are matched to low-quality sellers.

It is immediate to verify that the average value in $x^{\{l,m,h\}}$ is $11/6$ while the average value in $x^{\{m,h\}}$ is 8/3 and the average value in $x^{\{h\}}$ is 3. Hence the matching above is stochastically negative assortative and it remains so even if agents in x^h are left unmatched.

To verify that the above is the buyer optimal matching we proceed by using the characterisation result we obtained above and, hence, by computing average consumer surplus for unit quality in the various extremal segments. We have $cs(x^{\{l,m,h\}}) = 5/6$, $cs(x^{\{m,h\}}) = 1/3$, $cs(x^{\{l,m\}}) = 1/2$, $cs(x^{\{l,h\}}) = 2/3$, $cs(x^{\{j\}}) = 0$ for $j = l,m,h$. Then, it is a matter of algebra and brute force to show that the BBM segmentation described above first-order stochastically dominate any other extremal segmentation in terms of consumer surplus generated. Hence, it is optimal for any distribution of quality.

Example 2. Suppose the three possible valuations of buyers, (l,m,h), with $l = 1/8$, $m = 1/2$ and $h = 1$. The prior x^* is such that the medium and high value have both probability 1/5. Note that a unit-quality monopoly profit is $8/40$. If everyone buys in x^*, the unit-quality total welfare is equal to $15/40$. We assume the distribution of quality is such that the quality is either high, $q = 1$, with probability 2/5, or otherwise low, where we set $q = 0$ for simplicity.

The efficient and consumer-surplus maximising extremal segmentation according to BBM is composed by a segment of mass $4/5$ with support $\{l,m,h\}$, that is $x^{\{l,m,h\}}$, and a segment of mass $1/5$ with support $\{m,h\}$, that is $x^{\{m,h\}}$. Because it is needed for computations, we note that $x^{\{l,m,h\}}(h) = x^{\{l,m,h\}}(m) = 1/8$, while $x^{\{m,h\}}(m) = x^{\{m,h\}}(h) = 1/2$. By construction, while the unit-quality monopolist is indifferent among all prices equal to valuations in the support of each segment, price l is charged in the first segment and price m in the second. Hence, consumer surplus with a unit-quality monopolist is $7/40$. Now suppose we were to build an u_1-*assortative* matching based on it. Observe that $cs(x^{\{m,h\}}) = 1/4 > 5/32 = cs(x^{\{l,m,h\}})$. Hence, we would match segment $x^{\{m,h\}}$ first with 1/5 of the high quality sellers, while the remaining sellers will be matched to buyers in segment $x^{\{l,m,h\}}$. Of these, another 1/5 will be of high quality. We conclude that the buyer-surplus generated by the matching built using the BBM extremal segmentation, is $13/160$.

Next, we show that there is an extremal segmentation of the buyer-market that generates higher unit-quality monopoly profit than $8/40$ but allows a matching with higher buyer surplus. The idea is to maximise the size of the segment producing the highest consumer surplus among all extremal segments, in this case $x^{\{m,h\}}$. Indeed, the buyer-optimal matching in this example requires the creation of an extremal segmentation with a segment with support $\{h,m\}$ of mass 2/5 and a segment with support $\{l\}$ of mass 3/5. When the segment $x^{\{m,h\}}$ is matched uniformly at random to the high-quality sellers the buyer surplus is $1/10$, which is larger than $13/160$. Crucially, note that this segmentation generates a profit to a unit-quality monopolist equal to $11/40$, which is higher than $1/5$. Hence, it also generates lower consumer surplus overall.

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