Intermediate Goods Trade, Technology Choice and Productivity

Shin-Kun Peng

Academia Sinica and National Taiwan University

Raymond Riezman

University of Iowa, Aarhus University, and University of California, Santa Barbara

Ping Wang

Washington University in St. Louis and NBER

January 2017

Abstract: We develop a dynamic model of intermediate goods trade which focuses on the role of intermediate goods trade as a mechanism to transfer technology. We consider a developing economy whose final good production employs an endogenous array of intermediate goods, from low to high technology. We show that domestic trade liberalization reduces the range of exports, the range of domestic intermediate goods production, reduces intermediate producer markups and increases final good output. Domestic intermediate good producers also have lower technology but both aggregate and average productivities are higher.

Correspondence: Raymond Riezman, Department of Economics, University of Iowa, Iowa City, IA 52242; raymond-riezman@uiowa.edu (E-mail).

1 Introduction

Technology plays a major role in explaining cross country income differences (Caselli 2005). One way technology gets transferred from rich to poor countries is through international trade in intermediate goods. Motivated by empirical evidence discussed below, we investigate this by considering trade in intermediate goods between developed and developing countries. Specifically, we ask: what effect does trade liberalization have on the level of technology and productivity in developing countries?

Recent availability of micro data sets in many countries has enabled scholars to conclude that trade liberalization leads to productivity improvement and faster economic growth.[1] In addition, there have been many other interesting empirical findings, including three related to this paper: (i) more substantial productivity gains are found in firms using newly imported intermediate inputs (see Goldberg *et al.* 2010 for the case of India); (ii) trade liberalization results in lower mark-ups and greater competition (see Griffith, Harrison and Macartney 2006 for the case of EU), (iii) firms facing greater competition incur significantly larger productivity gains (see Amiti and Konings 2007 for the case of Indonesia).[2] Moreover, recent research has shown that R&D and technology are concentrated in just a few developed countries (Eaton and Kortum 2001; Keller 2002). We develop a unified framework to explain these empirical findings.

In addition to the overall increase in international trade, intermediate goods trade has also increased. In fact, the empirical evidence suggests that reductions in intermediate good tariffs generate larger productivity gains than final good tariff reductions.[3] [4] Keller's (2000) explanation is that *technology is transferred through intermediate goods trade.* We take Keller's empirical insight and develop a dynamic model to assess the impact of intermediate goods trade liberalization on technology levels and productivity and to explain the empirical facts mentioned above.

We consider a developing economy whose final good production uses an endogenous range of intermediate goods. For both domestic and foreign intermediate goods, the technology endogenously

---

[1]For example, after trade liberalization in the 1960s (Korea and Taiwan), 1970s (Indoesia and Chile), 1980s (Colombia) and 1990s (Brazil and India), the economic growth over the decade is mostly 2% or more higher than the previous decades. Sizable productivity gain resulting from trade liberalization is documented for the cases of Korea (Kim 2000), Indonesia (Amiti and Konings 2007), Chile (Pavcnik 2002), Colombia (Fernandes 2007), Brazil (Ferreira and Rossi 2003) and India (Topalova and Khandelwal 2011).

[2]For additional references to empirical regularities, the reader is referred to two useful survey articles by Dornbusch (1992) and Edwards (1993).

[3]As documented by Hummels, Ishii and Yi (2001), the intensity of intermediate goods trade measured by the VS index has risen from below 2% in the 1960s to over 15% in the 1990s.

[4]The larger effects of intermediate input tariffs have been found in Colombia (Fernandes 2007), Indonesia (Amiti and Konings 2007) and India (Topalova and Khandelwal 2011).

evolves over time. The developing economy is assumed to be less advanced in intermediate goods production and hence, imports intermediate goods that embody more advanced technology while exporting those having less advanced technology.

To allow for endogenous markups and endogenous ranges of exports and imports in a tractable manner, we depart from CES aggregators, and instead use a generalized quadratic production technology that extends earlier work by Peng, Thisse and Wang (2006). In addition, the existence of trade barriers means that there may be a range of intermediate goods that are nontraded. Accordingly, the ranges of imports, exports and nontraded intermediate goods, as well as the entire range of intermediate products used are all endogenously determined. We analyze the responses of these ranges, aggregate productivity and aggregate technology to domestic trade liberalization.

Consider final goods producers in the developing country. They can take advantage of advanced technology by purchasing domestically produced intermediate goods or by importing intermediate goods from a more technologically advanced country. Domestic intermediate goods producers actively invest in research and development to improve their level of technology. So, while it may be more profitable in the short run for final good producers to *import* technology, this may reduce the incentive to invest in technological improvement for domestic firms and therefore, decrease *domestic* technological advancement in the long run. This tension plays a crucial role in our results and will play an important role in assessing the steady-state effects of trade liberalization.

We establish a set of sufficient conditions for the following to occur. Domestic trade liberalization leads to domestic final goods producers using a narrower range of intermediate goods. It causes the import price schedule to decrease, the domestic producer price schedule to increase leading to a smaller range of exported intermediate goods and a smaller range of domestically produced intermediate goods, but its effect on the range of imports is indeterminate. We show using numerical methods, that these responses are larger for less developed, less technologically advanced countries.

Domestic trade liberalization affects technology through three channels. First, it encourages final producers to import rather than use domestically produced intermediate goods. Next, trade liberalization raises competitiveness and reduces markups for intermediate good producers. The third channel is a negative extensive margin effect which dominates a positive intensive margin effect. The total effect of all three channels is to lead to lower domestic R&D and technology levels.

The effect on home country productivity is less clear. There are two positive effects: a cost saving effect (cheaper intermediate goods) and an intensive margin effect. There are also two negative effects: a negative technology effect and a negative markup effects. In our numerical analysis, we find that these positive effects dominate the negative effect. Thus, domestic intermediate good producers end up having a lower level of technology but are more productive. This overall higher productivity effect is consistent with recent evidence (e.g., see Tybout 2003, Chen, Imbs and Scott,

2009, and Goldberg *et al.,* 2010).

In terms of related work, Ethier (1982) argues that the expansion of the use of intermediate goods is crucial for improving the productivity of final goods production. While Ethier (1982) determines the endogenous range of intermediate products with embodied technologies, there is no trade in intermediate goods. Yi (2002) and Peng, Thisse and Wang (2006) examine the pattern of intermediate goods trade, the range of intermediate products with exogenous embodied technology. In Flam and Helpman (1987), a North-South model of final goods trade is constructed in which the North produces an endogenous range of high quality goods and South produces an endogenous range of low quality goods. Although their methodology is similar to ours, their focus is again on final goods trade. Impullitti and Licandro (2016) have a model of final goods trade in which trade liberalization leads to higher productivity through increased firm competition, lower markups, and higher R&D investment. In contrast with all these papers, our paper determines endogenously both the pattern and the extent of intermediate goods trade with endogenous technology choice. Thus, our framework focuses on the trade-off between importing technology embodied in intermediate goods and advancing domestic technology. Furthermore, we characterize intermediate good producer markups and the productivity gains from trade liberalization on both the intensive and extensive margins.

The remainder of the paper is organized as follows. In Section 2, we construct the model with a small country conducting international trade of intermediate goods embedded with different technologies. The optimization problems facing final and intermediate producers are solved in Sections 3. We then define and characterize the steady state equilibrium in Section 4, focusing on technology choice, pattern of production and trade and the consequences of trade liberalization. Our numerical implementation of the model is in Section 5 and Section 6 concludes.

2   The Model

We consider a small country model in which the home (or domestic) country is less advanced technically than the large foreign country (Rest of the World.) Both the home and foreign country (ROW) consists of two sectors: an intermediate sector that manufactures a variety of products and a final sector that produces a single nontraded good using a basket of traded intermediate goods as inputs. All foreign (ROW) variables are labelled with the superscript *. We focus on the efficient production of the final good using either self-produced or imported intermediate goods. Whether to produce or import depends on the home country's technology choice decision and the international intermediate good markets. The final good is the numeraire whose price is normalized to one.

2.1 The Final Sector

The output of the single final good at time t is produced using a basket of intermediate goods of measure M_t. The endogenous determination of the overall length of the production line M_t plays a crucial role in assessing the "extensive margin" effects of trade liberalization on the respective ranges of export, import and domestic production.

Each variety requires ϕ units of labor and each unit of labor is paid at a market wage $w > 0$. The more varieties used in producing the final good the more labor is required to coordinate production. This follows Becker and Murphy (1992). Denoting the mass of labor for production-line coordination at time t as D_t, we have:

$$M_t = \frac{1}{\phi} D_t \tag{1}$$

In the absence of coordination cost ($\phi \to 0$), the length of the production line M_t goes to infinity. Notably, in Melitz and Ottaviano (2008), there is a choke price which sets an upper bound on M_t. In our model, higher M_t is associated with better technology and lower prices, so there is no choke price. Thus, in order to have an interior solution for M_t, we introduce a coordination cost associated with final good production.

The production technology of the final good at time t is given by:

$$Y_t = \alpha \int_0^{M_t} x_t(i)\,di - \frac{\beta - \gamma}{2} \int_0^{M_t} [x_t(i)]^2\,di - \frac{\gamma}{2} \left[\int_0^{M_t} x_t(i)\,di \right]^2 \tag{2}$$

where $x_t(i)$ measures the amount of intermediate good i that is used and $\alpha > 0$, $\beta > \gamma$. Therefore, Y_t displays strictly decreasing returns. In this expression, α measures final good productivity, whereas $\beta > \gamma$ means that the level of production is higher when the production process is more sophisticated. We thus refer to $\beta - \gamma > 0$ as the *production sophistication effect*, which measures the positive effect of the sophistication of the production process on the productivity of the final good. For a given value of β, the parameter γ measures the complementarity/substitutability between different varieties of the intermediate goods: $\gamma >$ (resp., $<$) 0 means that intermediate good inputs are Pareto substitutes (resp., complements).

It is important to note that, with the conventional Spence-Dixit-Stiglitz-Ethier setup, ex post symmetry is imposed to get closed form solutions. For our purposes it is crucial to allow different intermediate goods in the production line to have different technologies. A benefit of using this production function is that even without imposing symmetry, we can still solve the model analytically. Moreover, under this production technology, intermediate producer markups are endogenous, varying across different firms.

4

2.2 The Intermediate Sector

Each variety of intermediate goods is produced by a single intermediate firm that has local monopoly power *domestically* as long as varieties are not perfect substitutes. Consider a Ricardian technology in which production of one unit of each intermediate good $y_t(i)$ requires η units of nontraded capital (e.g., building and infrastructure) to be produced:

$$k_t(i) = \eta y_t(i) \tag{3}$$

where $i \in I$ that represents the domestic production range (to be endogenously determined).

In addition to capital inputs, each intermediate firm $i \in I$ also employs labor, both for manufacturing and for R&D purposes. Denote its production labor as $L_t(i)$ and R&D labor as $H_t(i)$. Thus, an intermediate firm i's total demand for labor is given by,

$$N_t(i) = L_t(i) + H_t(i) \tag{4}$$

With the required capital, each intermediate firm's production function is specified as:

$$y_t(i) = A_t(i) L_t(i)^\theta \tag{5}$$

where $A_t(i)$ measures the level of technology and $\theta \in (0, 1)$. By employing R&D labor, the intermediate firm can improve the production technology according to,

$$A_{t+1}(i) = (1 - \nu) A_t(i) + \psi_t(i) H_t(i)^\mu \tag{6}$$

where $\psi_t(i)$ measures the efficacy of investment in technological improvement, ν represents the technology obsolescence rate, and $\mu \in (0, 1)$. To ensure an interior solution, we impose: $\theta + \mu < 1$.

Remark 1: It should be emphasized here, that we have technology choice, not technology adoption or technology spillovers. These concepts are sometimes confused. Technology adoption permits the use of foreign technologies to produce goods domestically by paying licensing fees. Technology spillovers are uncompensated positive effects of foreign technologies on domestic technologies. What we mean by technology choice, is that domestic producers of final goods implicitly choose the level of technology they use through their choice of intermediate goods used in the production process. They can use lower technology, domestically produced intermediate goods as well as imported higher technology intermediate goods produced using foreign technologies. The trade-off these firms face is that adopting higher technology production means a larger range of intermediate goods resulting in higher coordination costs.

One may easily extend our setup to incorporate technology spillovers. In particular, consider the case in which foreign technologies embodied in imported intermediate goods also contribute to

domestic technology improvements via reverse engineering. We can modify equation (6) to allow for spillovers

$$A_{t+1}(i) = (1 - \nu) A_t(i) + [(1 - \varsigma) \psi_t(i) + \varsigma \psi_t^*(i)] H_t(i)^\mu$$

where $\psi_t^*(i)$ measures the efficacy of investment in technological improvement for the foreign country and $\varsigma \geq 0$ indicates the strength of international technology spillovers. While we will discuss the implication of this modification in Section 5 below, it is clear that such an extension would not affect our main findings so long as ς is not too large.

3 Optimization

When a particular intermediate good is produced domestically but not exported to the world market, such an intermediate producer has local monopoly power. Thus, we will first solve for the final sector's demand for intermediate goods and then each intermediate firm's supply and pricing decisions for the given demand schedule. Throughout the paper, we assume the final good sector and the intermediate good sector devoted to producing the industrial good under consideration is a small enough part of the entire economy that they take all factor prices as given.

3.1 The Final Good Sector

For now, assume that the home country produces all intermediate goods in the range $\left[0, n_t^P\right]$ and they export intermediates in the range $\left[0, n_t^E\right]$ where $n_t^E \leq n_t^P$ while intermediates in the range $\left[n_t^P, M_t\right]$ are imported (see Figure 1 for a graphical illustration). We will later verify this assertion and solve for n_t^E, n_t^P, and M_t endogenously.

The firm that produces the final good has the following first-order condition with respect to intermediate goods demand $x_t(i)$ given by,

$$\frac{dY_t}{dx_t(i)} = \alpha - (\beta - \gamma)x_t(i) - \gamma \left[\int_0^{M_t} x_t(i^{'})di^{'} \right] = p_t(i), \ \forall \ \ i \in [0, M_t] \tag{7}$$

which enables us to derive intermediate good prices $p_t(i)$ as:

$$p_t(i) = \begin{cases} PE_t(i) \equiv p_t^*(i), \ \ \forall \ \ i \in [0, n_t^E] \\ PP_t(i) \equiv \alpha - \beta x_t(i) - \gamma \widetilde{X}_t^{-i} = \alpha - (\beta - \gamma)x_t(i) - \gamma \widetilde{X}_t, \ \ \forall \ i \in [n_t^E, n_t^P] \\ PM_t(i) \equiv (1 + \tau)p_t^*(i), \ \ \forall \ \ i \in [n_t^P, M_t] \end{cases} \tag{8}$$

where $p_t^*(i)$ is the foreign intermediate goods price and τ is the domestic tariff. Given unlimited foreign demand, they would not price lower domestic than this tariff adjusted foreign price. Also,

$\widetilde{X_t} \equiv \int_0^{M_t} x_t(i') di'$ and $\widetilde{X}_{t,}^{-i} \equiv \int_{i' \neq i} x_t(i') di' = \widetilde{X_t} - x(i)$. One can think of $\widetilde{X_t}$ as a measure of aggregate intermediate good usage. Given these results we have the following Lemma.

Lemma 1: (Demand for Intermediate Goods) *Within the nontraded range $[n_t^E, n_t^P]$, the demand for intermediate good is downward sloping. If intermediate goods are Pareto substitutes ($\gamma > 0$), then larger aggregate intermediate goods (higher $\widetilde{X}_t$) imply individual intermediate good demand will be smaller.*

Manipulating (8) yields the relative inverse demands for intermediate goods and the demand for the Mth intermediate good:

$$p_t(i) - p_t(i') = \begin{cases} [p_t^*(i) - p_t^*(i')], & \forall \ i, i' \in [0, n_t^E] \\ -(\beta - \gamma)[x_t(i) - x_t(i')], & \forall \ i, i' \in [n_t^E, n_t^P] \\ (1 + \tau)[p_t^*(i) - p_t^*(i')], & \forall \ i, i' \in [n_t^P, M_t] \end{cases} \tag{9}$$

We can then derive the final good producing firm's first-order condition with respect to the length of the production line M_t (see the Appendix):[5]

$$\frac{\beta - \gamma}{2}[x_t(M_t)]^2 - [\alpha - \gamma \widetilde{X}_t - (1 + \tau)p_t^*(M_t)]x_t(M_t) + w\phi = 0 \tag{10}$$

Given $\beta > \gamma$, the solution to relative demand exists if $[\alpha - \gamma \widetilde{X}_t - (1 + \tau)p_t^*(M_t)]^2 > 2(\beta - \gamma)w\phi$.

Lemma 2: (Relative Demand for Intermediate Goods) *Within the nontraded range $[n_t^E, n_t^P]$, the relative demand for intermediate goods is downward sloping. Additionally, the stronger the production sophistication effect is (higher $\beta - \gamma$), the less elastic the relative demand will be.*

Next, we determine how the intermediate goods sector works.

3.2 The Intermediate Sector

With local monopoly power, each intermediate firm can jointly determine the quantity of intermediate good to supply and the associated price. By utilizing (3) and (4), its optimization problem is described by the following Bellman equation:

$$\begin{aligned} V(A_t(i)) &= \max_{p_t(i),\, y_t(i),\, L_t(i),\, H_t(i)} [(p_t(i) - \eta)y_t(i)] - w_t [L_t(i) + H_t(i)] + \frac{1}{1 + \rho}V(A_{t+1}(i)) \\ \forall \ i \in [n_t^E, n_t^P] & \\ &\text{s.t.} \quad (5), (6) \text{ and } (8) \end{aligned} \tag{11}$$

We solve for the value functions for both nontraded intermediate goods $i \in [n_t^E, n_t^P]$ and exported intermediate goods $i \in [0, n_t^E]$. As shown in the Appendix, the first-order conditions with respect

---

[5]It is assumed there is a very large M^* being produced in the world so that any local demand for M can be met with imports from the rest of the world.

to the two labor demand variables $L_t(i)$ and $H_t(i)$ can be derived as:

$$[p_t(i) - \eta - \beta A_t(i)L_t(i)^\theta]\theta A_t(i)L_t(i)^{\theta-1} = w_t \quad \forall \ i \in [n_t^E, n_t^P] \tag{12}$$

$$\frac{\mu}{1+\rho}V_{A_{t+1}}(i)\psi_t(i)H_t(i)^{\mu-1} = w_t \quad \forall \ i \in [n_t^E, n_t^P] \tag{13}$$

The Benveniste-Scheinkman condition with respect to $A_t(i)$ is given by,

$$V_{A_t}(i) = [p_t(i) - \eta - \beta A_t(i)L_t(i)^\theta]L_t(i)^\theta + \frac{1-\nu}{1+\rho}V_{A_{t+1}}(i) \quad \forall \ i \in [n_t^E, n_t^P] \tag{14}$$

Similarly, we have the value function for $i \in [0, n_t^E]$ as:

$$\begin{aligned} V(A_t(i)) &= \max_{p_t(i),\ y_t(i),\ L_t(i),\ H_t(i)} [p_t^*(i)\text{-}\eta]Ay_t(i)\text{-}w_t\left[L_t(i)+H_t(i)\right]+\frac{1}{1+\rho}V_{t+1}(A_{t+1}(i)) \\ \forall\ i\in[0,n_t^E] & \\ & \text{s.t.} \quad (5),\ (6)\ \text{and}\ (8) \end{aligned} \tag{15}$$

where we have used (8) for $i \in [0, n_t^E]$. We can obtain the first-order conditions with respect to $L_t(i)$ and $H_t(i)$, respectively, as follows:

$$\theta[p_t^*(i) - \eta]A_t(i)L_t(i)^{\theta-1} = w_t, \quad \forall \ i \in [0, n_t^E] \tag{16}$$

$$\frac{\mu}{1+\rho}V_{A_{t+1}}(i)\psi_t(i)H_t(i)^{\mu-1} = w_t, \quad \forall \ i \in [0, n_t^E] \tag{17}$$

By using (5), the Benveniste-Scheinkman condition is given by,

$$V_{A_t}(i) = [p_t^*(i) - \eta]L_t(i)^\theta + \frac{1-\nu}{1+\rho}V_{A_{t+1}}(i), \quad \forall \ i \in [0, n_t^E] \tag{18}$$

We now turn to solving the system for a steady state.

4 Steady-State Equilibrium

Our steady state focuses on efficient production of the final good using a basket of intermediate goods. We take the wage as given since the intermediate sector under consideration is assumed to be a small part of the larger economy.

4.1 Labor Allocation

In steady-state equilibrium, all endogenous variables are constant over time. Thus, (6) implies:

$$H(i) = \left[\frac{\nu A(i)}{\psi(i)}\right]^{\frac{1}{\mu}}, \quad i \in [0, n^P] \tag{19}$$

8

This expression implies a positive relationship between the investment in domestic technology in forms of $H(i)$. By manipulation (see the Appendix), we obtain the steady-state level of domestic technology $A(i)$ over the range $i \in [0, n^P]$:

$$A(i) = \overline{A}\psi(i)L(i)^{\mu}, \quad \forall \ i \in [0, n^P] \tag{20}$$

where

$$\overline{A} \equiv \frac{1}{\nu^{1-\mu}} \left[\frac{\mu}{\theta(\rho+\nu)} \right]^{\mu} > 0.$$

One can think of $\overline{A}$ as the technology scaling factor and $\psi(i)$ as the technology gradient that measures how quickly technology improves as i increases.

Next, we substitute (8) and (20) into (16) to eliminate $p(i)$ and $A(i)$, yielding the following expression in $L(i)$ alone:

$$\theta[p^*(i) - \eta]\overline{A}\psi(i)LE(i)^{\theta+\mu-1} = w, \quad \forall \ i \in [0, n^E] \tag{21}$$

which can be used to derive labor demand for $i \in [0, n^E]$: $LE(i) = \{\frac{\theta}{w}[p^*(i) - \eta]\overline{A}\psi(i)\}^{\frac{1}{1-\theta-\mu}}$. For $i \in [n^E, n^P]$, we have:

$$MPL(i) = \theta\overline{A}\psi(i)LP(i)^{-(1-\mu-\theta)}\left[\alpha - \eta - \gamma\widetilde{X}^{-i} - 2\beta\overline{A}\psi(i)LP(i)^{\mu+\theta}\right] = w \tag{22}$$

The marginal product of labor $MPL(i)$ is strictly decreasing in $L(i)$ with $\lim_{L(i)\longrightarrow 0} MPL(i) \longrightarrow \infty$ and $\lim_{L(i)\longrightarrow L_{\max}} MPL(i) = 0$, where

$$L_{\max} \equiv \left[\frac{\alpha - \eta - \gamma\widetilde{X}^{-i}}{2\beta\overline{A}\psi(i)} \right]^{\frac{1}{\theta+\mu}}$$

Figure 2 depicts the $MPL(i)$ locus, which intersects w to pin down labor demand in steady-state equilibrium (point E). It follows that $\frac{dL(i)}{dw} < 0$ and $\frac{dL(i)}{d\alpha} > 0$, $\frac{dL(i)}{d\eta} < 0$, $\frac{dL(i)}{d\beta} < 0$ and $\frac{dL(i)}{d\gamma} < 0$.

That is, an increase in the final good productivity (α), or a decrease in the unit capital requirement (η), the magnitude of variety bias (β), or the degree of substitutability between intermediate good varieties (γ) increases the intermediate firm's demand for labor. Note that the direct effect of improved efficiency of investment in intermediate good production technology ($\psi(i)$) is to increase the marginal product of labor and induce higher labor demand by intermediate firms. This we call the induced demand effect. However, there is also a labor saving effect. Under variable monopoly markups, a better technology enables the intermediate good firm to supply less and extract a higher markup which will save labor inputs. Thus, the overall effect is generally ambiguous. Finally, and also most interestingly, when final good production uses more sophisticated technology (larger M), it is clear that the $\widetilde{X}^{-i}$ will rise, thereby shifting the $MPL(i)$ locus downward and lowering each variety's labor demand for a given wage rate. Summarizing these results we have:

9

Lemma 3: (Labor Demand for Intermediate Goods Production) *Within the nontraded range* $[n^E, n^P]$, *labor demand is downward sloping. Moreover, an increase in final good productivity (α) or a decrease in the overall length of the final good production line (M), the unit capital requirement (η), the magnitude of variety bias (β), or the degree of substitutability between intermediate good varieties (γ) increases the intermediate firm's demand for labor in the steady state.*

Next, we can use (4), (19) and (21) to derive R&D labor demand and total labor demand by each intermediate firm as follows:

$$H(i) = \left(\nu\overline{A}\right)^{\frac{1}{\mu}} L(i), \quad \forall \ i \in [0, n^E] \tag{23}$$

$$N(i) = L(i) + H(i) = \left[1 + \left(\nu\overline{A}\right)^{\frac{1}{\mu}}\right] L(i), \quad \forall \ i \in [0, n^E] \tag{24}$$

Combining the supply of and the demand for the Mth intermediate good, (5) with $i = n^P$ and (6), we have

$$y(i) = \overline{A}\psi(i) L(i)^{\theta+\mu}, \quad i \in [0, n^P] \tag{25}$$

In equilibrium, we can re-write the supply of intermediate good i as:

$$y(i) = \begin{cases} yE(i) \equiv x(i) + z^*(i) > x(i), & i \in [0, n^E] \\ yP(i) \equiv x(i), & \text{if} \quad i \in [n^E, n^P] \\ yM(i) \equiv x(i) = z(i) > 0 & i \in [n^P, M] \end{cases} \tag{26}$$

where $z^*(i)$ is home country exports of intermediate good i and $z(i)$ is home country imports of intermediate good i. Substituting (25) into (8), we have:

$$z^*(i) = y(i) - x(i) = \overline{A}\psi(i) L(i)^{\theta+\mu} - \frac{\alpha - \gamma\widetilde{X} - p^*(i)}{\beta - \gamma}, \quad \forall i \in [0, n^E] \tag{27}$$

From (8) and (21), we can derive aggregate intermediate good usage as:

$$\widetilde{X} = \int_0^{n^P} \overline{A}\psi(i) L(i)^{\theta+\mu} \, di + \int_{n^P}^{M} z(i)di - \int_0^{n^E} z^*(i)di \tag{28}$$

The aggregate labor demand is given by,

$$\overline{N} = \phi M + \left[1 + \left(\nu\overline{A}\right)^{\frac{1}{\mu}}\right] \left[\int_0^{n^P} L(i)di\right] \tag{29}$$

We assume that labor supply in the economy is sufficiently large to ensure the demand is met.

4.2 Technology Choice and Pattern of Production and Trade

The local country's technology choice with regards to intermediate goods production depends crucially on whether local production of a particular variety is cheaper than importing it. For convenience, we arrange the varieties of intermediate goods from the lowest technology to highest technology. Consider

$$\psi(i) = \overline{\psi}(1 + \delta \cdot i), \quad \psi^*(i) = \overline{\psi}^*(1 + \delta^* \cdot i) \tag{30}$$

It is natural to assume that the advanced country has weakly better basic technology $\psi_0^* \geq \psi_0$ and strictly better advanced technologies, implying a steeper technology gradient $\delta^* > \delta$.

From (7) and (8), we have:

$$x(i) = \begin{cases} xE(i) \equiv \frac{\alpha - \gamma \widetilde{X} - p^*(i)}{\beta - \gamma} & i \in [0, n^E] \\ xP(i) \equiv \overline{A}\psi(i) L(i)^{\theta + \mu} & \text{if} \quad i \in [n^E, n^P] \\ xM(i) \equiv \frac{\alpha - \gamma \widetilde{X} - (1+\tau)p^*(i)}{\beta - \gamma} & i \in [n^P, M] \end{cases} \tag{31}$$

where $L(i)$, $i \in [n^E, n^P]$, is pinned down by (21). Thus, the value of net exports of intermediate goods is:

$$E = \int_0^{n^E} p^*(i) xE(i) di - (1+\tau) \int_{n^P}^{M} p^*(i) xM(i) di \tag{32}$$

Trade balance therefore implies that domestic final good consumption is given by,

$$C = Y + E \tag{33}$$

Notice that $p(i)$ is decreasing in $\psi(i)$, which implies that better technology corresponds to lower costs and hence lower intermediate good prices. As a result, it is expected that $\frac{dp(i)}{di} < 0$; that is, the intermediate good price function is downward-sloping in ordered varieties (i). Thus, we have the following Lemma.

Lemma 4: (Producer Price Schedule) *Within the nontraded range $[n^E, n^P]$, the steady-state intermediate good price schedule is downward sloping in ordered varieties (i).*

We can now derive an expression for aggregate intermediate goods (see the Appendix):

$$\widetilde{X} = \frac{\overline{A} \int_{n^E}^{n^P} \psi(i) L(i)^{\theta + \mu} di + \frac{\alpha}{\beta - \gamma}(M + n^E - n^P) - \frac{1}{\beta - \gamma}\left[(1+\tau) \int_{n^P}^{M} p^*(i) di + \int_0^{n^E} p^*(i) di\right]}{1 + \frac{\gamma}{\beta - \gamma}(M + n^E - n^P)} \tag{34}$$

which we call the *intermediate-good aggregation* (XX) locus. In addition, by substituting (31) into (10), we can get the boundary condition at M:

$$\alpha - \gamma \widetilde{X} - (1+\tau)p^*(M) = \sqrt{2(\beta - \gamma)w\phi} \tag{35}$$

11

which will be referred to as the *production-line trade-off* (MM) locus.

Before characterizing the relationship between M and $\widetilde{X}$, it is important to check the second-order condition with respect to the length of the production line. From (10), and (34), we can derive the second-order condition as:

$$\frac{\gamma M x(M)}{(1+\tau)p^*(M)} > -\frac{M}{p^*(M)}\frac{dp^*(M)}{dM}$$

For tractability, world price is specified by:

$$p^*(i) = \overline{p} - b \cdot i$$

The second-order condition becomes:

Condition S: (Second-Order Condition) $(1+\tau)\, b < \gamma \sqrt{\frac{2w\phi}{\beta-\gamma}}$.

Thus, it is necessary to assume that intermediate goods are Pareto substitutes in producing the final good $(\gamma > 0)$, which we shall impose throughout the remainder of the paper. This condition requires that the gradient of the tariff augmented imported intermediate goods prices be properly flat.

The next condition to check is the nonnegative profit condition for the intermediate good firms. For $i \in [n^E, n^P]$, maximum profit is $\pi(i) = \Lambda(i)wN(i)$, and the markup for the producer of intermediate good i is (see the Appendix):

$$\Lambda(i) \equiv \frac{p(i) - \eta}{\theta[1 + (\nu\overline{A})^{1/\mu}]\,[p(i) - \eta - \beta x(i)]} - 1 \tag{36}$$

For $i \in [0, n^E]$, maximum profit is $\pi(i) = \Lambda_0 wN(i)$, where the markup becomes a constant given by (see the Appendix),

$$\Lambda_0 \equiv \frac{1}{\theta[1 + (\nu\overline{A})^{1/\mu}]} - 1$$

Note that in this general quadratic setup, when price $(p(i)-\eta)$ increases, the marginal cost $(\theta[1+(\nu\overline{A})^{1/\mu}]\,[p(i) - \eta - \beta x(i)])$ increases more than proportionately, thus yielding a lower markup. This differs sharply from the constant markup CES aggregator. By using (31) and (8), markup can be expressed as,

$$\Lambda(i) = \frac{1}{\theta[1 + (\nu\overline{A})^{1/\mu}]\left[1 - \beta\frac{x(i)}{p(i)-\eta}\right]} - 1 = \frac{1}{\theta[1 + (\nu\overline{A})^{1/\mu}]\left[1 - \frac{\beta}{\frac{\alpha-\eta-\gamma\widetilde{X}_t}{\overline{A}\psi(i)L(i)^{\theta+\mu}}-(\beta-\gamma)}\right]} - 1$$

which is positively related to $L(i)$ and $\widetilde{X}_t$ for $i \in [n^E, n^P]$. It is noteworthy that while the demand for labor (for producing intermediate goods), $L(i)$, is purely an intensive margin effect, aggregate intermediate goods, $\widetilde{X}_t$, involves both an intensive and an extensive margin. When either the

12

demand for labor or aggregate intermediate good supply is higher, then the supply of the individual intermediate good i is higher and hence its price falls, which in turn increases markups because the convex cost effect dominates the linear price effect. It is clear that the intermediate good supply schedule $(xP(i) = \overline{A}\psi(i) L(i)^{\theta+\mu})$ is upward sloping, as is $\Lambda(i)$. By similar arguments, an increase in the technology scaling factor $(\overline{A})$ or the technology gradient $(\psi(i))$ reduces the marginal cost more than the price of intermediate good, thus leading to a higher markup.

To ensure nonnegative profit, we must impose $\frac{p(i)-\eta}{p(i)-\eta-\beta x(i)} > \theta[1 + (\nu\overline{A})^{\frac{1}{\mu}}]$ (i.e., $\Lambda(i) > 0$) for $i \in [n^E, n^P]$ and $\theta[1 + (\nu\overline{A})^{\frac{1}{\mu}}] < 1$ for $i \in [0, n^E]$. Since the latter condition always implies the former, we can use the definition of $\overline{A}$ to specify the following condition to ensure positive profitability:

Condition N: (Nonnegative Profit) $\frac{\mu\nu}{\rho+\nu} < 1 - \theta$.

This condition requires that the technology obsolescence rate be small enough. We then have:

Lemma 5: (Producer Markup Schedule) Under Condition N, the steady-state intermediate good markup schedule possesses the following properties:

(i) it is upward sloping in ordered varieties (i) within the nontraded range $[n^E, n^P]$, but is a constant Λ_0 over the exporting range $[0, n^E]$;

(ii) an increase in labor demand and intermediate good supply via either the intensive or extensive margin lowers the producer price schedule and raises the markup schedule;

(iii) an increase in the technology scaling factor or the technology gradient leads to a higher producer markup schedule.

We next turn to the determination of the length of the production line. This is best illustrated by the MM and XX loci as drawn in Figure 3. The MM locus (equation (35)) and the XX locus (equation (34)) are the loci that relate $\widetilde{X}$ to M and both are positively sloped. To begin, consider the MM locus. Notice that since intermediate goods are Pareto substitutes, the direct effect of an increase in aggregate intermediate goods, $\widetilde{X}$, reduces the demand for each intermediate good. As M increases, the price of the intermediate good at the boundary, $p^*(M)$, falls, as does the cost of using this intermediate good. This encourages the demand for $x(M)$ and, to restore equilibrium in (35), one must adjust $\widetilde{X}$ upward, implying that the MM locus is upward sloping. The intuition underlying the XX locus is more complicated. For illustrative purposes, let us focus on the direct effects. As indicated by (34), the direct effect of a more sophisticated production line (higher M) is to raise the productivity of manufacturing the final good as well as the cost of intermediate inputs. While the productivity effect increases aggregate demand for intermediate goods, the input cost

13

effect reduces it. On balance, it is not surprising that the positive effect dominates as long as such an operation is profitable. Nonetheless, due to the conflicting effects, the positive response of $\widetilde{X}$ to M is not too large.

Since the MM locus is the boundary condition pinning down the overall length of the production line, it is expected to be more responsive to changes in M compared to the XX locus. As a result, we claim that the XX locus is flatter than the MM locus. This slope requirement is formally specified as:

Condition C: (Correspondence Principle) $\left.\frac{d\widetilde{X}}{dM}\right|_{XX\ locus} < \left.\frac{d\widetilde{X}}{dM}\right|_{MM\ locus}$

This condition is particularly important for producing reasonable comparative statics in accordance with Samuelson's Correspondence Principle.[6] Specifically, consider an improvement in technology (higher $\overline{\psi}$ or δ, or lower ν). While the MM locus is unaffected, the XX locus will shift upward. Should the XX locus be steeper than the MM locus, better technology would cause the aggregate supply of intermediate goods ($\widetilde{X}$) to fall, which is counter-intuitive. Thus, based on Samuelson's Correspondence Principle, one may rule out this type of equilibrium. The equilibrium satisfying Samuelson's Correspondence Principle is illustrated in Figure 3 by point E. In Section 5, we will further support these arguments with numerical examples.

Defining the expression in (34) as $\widetilde{X}(M)$, we can substitute it into (10) to obtain:

$$\Gamma(M) \equiv \gamma\widetilde{X}(M) + (1+\tau)p^*(M) = \alpha - \sqrt{2(\beta-\gamma)w\phi} \tag{37}$$

By examining $\Gamma(M)$, it is seen that M has two conflicting effects: a positive effect via the aggregate intermediate goods input $\widetilde{X}(M)$ and a negative effect via the import price $p^*(M)$. Specifically, an increase in the overall length of the production line raises the aggregate intermediate goods input but lowers the import price. Since the XX locus is flatter than the MM locus as discussed above, the negative effect via the import price dominates the positive effect via the aggregate intermediate goods input. We summarize this result below.

Lemma 6: (The Length of the Production Line) *Under Conditions S, N, and C the steady-state overall length of the production line is uniquely determined by the XX and MM loci.*

4.3 Trade Liberalization

Now consider the effect of trade liberalization on the pattern of production and trade, the intermediate firms' markups, aggregate and average technology as well as overall productivity.

---

[6]Samuelson (1947) highlights the purpose of Correspondence Principle as: "to probe more deeply into its analytical character, and also to show its two-way nature: not only can the investigation of the dynamic stability of a system yield fruitful theorems in statical analysis, but also known properties of a (comparative) statical system can be utilized to derive information concerning the dynamic properties of a system."

4.3.1 Effects on Pattern of Production and Trade

We begin by determining the effect of trade liberalization on the overall length of the production line. Consider a decrease in the domestic tariff (τ). This decrease in domestic protection lowers the domestic cost of imported intermediate inputs i, $(1 + \tau)p^*(i)$ and hence increases demand. This causes the MM locus to shift up (see Figure 3a). The effect on the XX locus is, however, ambiguous. While there is a direct positive effect of domestic trade liberalization on $\widetilde{X}$, there are many indirect channels via the endogenous cutoffs, n^E and n^P. While we will return to this later, our numerical results show that the shift of the XX locus is small compared to the shift in the MM locus. Therefore, in this case one expects the net effect of domestic trade liberalization to decrease the overall length of the production line (lower M) as seen in Figure 3. On the one hand, domestic trade liberalization increases imported intermediate inputs on the *intensive margin.* However, final producers react to it by importing intermediate goods at n^P and shifting resources away from this higher type to lower type intermediate inputs $i < n^P$. Given ϕ, this implies a decrease in the overall length of the production line. This latter effect is via the *extensive margin* of import demand. Mathematically, we can differentiate (37) to obtain:

$$\frac{dM}{d\tau} = \frac{p^*}{(1 + \tau)b - \gamma(d\widetilde{X}/dM)}$$

which is positive if $(1 + \tau)b > \gamma d\widetilde{X}/dM$. Therefore we have Condition **E.**

Condition E: (Strong Extensive Margin Effect) $(1 + \tau)b > \gamma d\widetilde{X}/dM$

Thus, if Condition **E** holds, domestic trade liberalization leads to a shorter production line. This condition holds if the positive response of $\widetilde{X}$ to M is not too large, the degree of substitution between different varieties of intermediate goods is not too strong (low γ) and the price gradient is sufficiently steep (high b).

We summarize these results in Proposition 1.

Proposition 1: (The Length of the Production Line) *Under Conditions S, N, C and E the steady-state overall length of the production line is decreasing in response to domestic trade liberalization (lower τ).*

We next turn to determining the effect of domestic tariffs on the pattern of domestic production and export. From (8) and (31), we can obtain the following two key relationships that determine the cutoff values, n^E and n^P, respectively:

$$PP(n^E) = \alpha - \gamma\widetilde{X} - (\beta - \gamma)\overline{A}\psi\left(n^E\right)LP\left(n^E\right)^{\theta+\mu} = p^*(n^E) = PE(n^E) \tag{38}$$

$$PP(n^P) = \alpha - \gamma\widetilde{X} - (\beta - \gamma)\overline{A}\psi\left(n^P\right)LP\left(n^P\right)^{\theta+\mu} = (1 + \tau)p^*(n^P) = PM(n^P) \tag{39}$$

The two loci are plotted in Figure 4 along with the locus for $PM(i)$ given by equation (8). The equilibrium price locus is captured by $\widehat{ABCD}$. To see this, notice that the equilibrium price is pinned down by $PE(i)$ over $[0, n^E]$. In that range, producers of intermediate goods can sell their output for a higher price if they export than if they sell to domestic customers ($PE(i)$ is above $PP(i)$.) In the range $[n^E, n^P]$ we see that $PP(i)$ lies above $PE(i)$ indicating that producers receive a higher price by selling in the domestic market than if they export. Finally, in the range $[n^P, M]$ it is clear that $PP(i)$ is above $PM(i)$ indicating that imports are cheaper than domestically produced intermediate goods.

To better understand the comparative statics with respect to the effects of trade liberalization on the two cutoffs, we separate the conventional effects via the intensive margin from the effects via the extensive margin on the overall length of the production line. We first consider the effects on non-traded intermediate goods, i.e. those in the range $[n^E, n^P]$.

$$\frac{dPP(i)}{d\tau} = \frac{\partial PP(i)}{\partial \tau} + \frac{\partial PP(i)}{\partial LP(i)}\frac{dLP(i)}{d\tau} + \frac{\partial PP(i)}{\partial M}\frac{dM}{d\tau}$$

Since domestic trade liberalization increases imported intermediate good demand, it induces re-allocation of labor toward imported intermediates, which causes the $PP(i)$ locus to shift up. In addition, on the extensive margin, the overall length of the production line shrinks, thereby decreasing aggregate intermediate inputs and also causing the $PP(i)$ locus to shift up. Nonetheless, from (35), there is a direct positive effect of domestic trade liberalization on $\widetilde{X_t}$ via the demand for $x(M)$ on the intensive margin, which in turn shifts the $PP(i)$ locus down. When the effect via the extensive margin is strong (as is observed empirically; see an illustration in Figures 5-1a,b), trade liberalization will lead to a upward shift in the $PP(i)$ locus, i.e., $\frac{dPP(i)}{d\tau} > 0$ (see Figures 5-2a,b).

The responses of $PM(i) = (1+\tau)p^*(i)$ is clear-cut, the domestic trade liberalization rotates the $PM(i)$ locus downward. We now examine the first cutoff pinned down by (38), which determines the range of exports.

$$\frac{dn^E}{d\tau} = \frac{\partial n^E}{\partial \tau} + \frac{\partial n^E}{\partial M}\frac{dM}{d\tau}$$

From the discussion above, lower domestic tariffs yield a negative direct effect on the $PP(i)$ locus, which leads to a higher cutoff n^E and hence a larger range of exports. However, there is a general equilibrium labor reallocation effect and an extensive margin effect via the overall length of the production line, both shifting the $PP(i)$ locus upward. When the effect via the extensive margin is strong, the cutoff n^E decreases and the range of exports shrinks.

We now turn to the second cutoff n^P. Based on (39) we can determine the range of domestic production of intermediate inputs and the range of imports.

$$\frac{dn^P}{d\tau} = \frac{\partial n^P}{\partial \tau} + \frac{\partial n^P}{\partial M}\frac{dM}{d\tau}$$

16

Recall that, when the effect via the extensive margin is strong, a lower domestic tariff causes the $PP(i)$ locus to shift up. In addition, the $PM(i)$ locus rotates downward. Both result in a lower cutoff n^P and hence a smaller range of domestic production. Should the overall length M be unchanged, the range of imports would increase. But, since M shrinks, the net effect on the range of imports is generally ambiguous.

We illustrate these comparative statics results in Figures 5-1a,b and 5-2a,b and summarize the results in Proposition 2.

Proposition 2: (The Range of Exports, Domestic Production and Imports) *Under Conditions S, N, C and E, the steady-state pattern of international trade features exporting over the range $\left[0, n^E\right]$ and importing over the range $\left[n^P, M\right]$ with the range $\left[n^E, n^P\right]$ being nontraded. And in response to domestic trade liberalization (lower τ), the steady-state equilibrium possesses the following properties:*

(i) *the import price $PM(i)$ falls whereas the domestic producer price $PP(i)$ increases;*

(ii) *both the range of exports $\left[0, n^E\right]$ and the range of domestic production $\left[0, n^P\right]$ shrink;*

(iii) *the range of imports is generally ambiguous.*

Remark 2: (Exogenous Length of the Production Line) When the length of the production line M is fixed, domestic trade liberalization increases aggregate intermediate goods (as shown Appendix A1). In this case, domestic trade liberalization causes producer prices to drop, thus expanding the export range (as shown in Figure A 2). Recall that in the case with endogenous length of production line, domestic trade liberalization shortens the overall length and forces the export range to shrink, thereby leading to an ambiguous effect on the import range.

4.3.2 Markups, Productivity and Technology

We next turn to consideration of the effect of trade liberalization on markups. In the domestic exporting range $\left[0, n^E\right]$, an intermediate firm's markup is constant over i. In the nontraded range $i \in [n^E, n^P]$, we can see from (36) that markups will respond endogenously to domestic trade policy. As shown in Proposition 2, in response to a reduction in the domestic tariff τ, the domestic producer price $PP(i)$ rises when the effect via the extensive margin is strong. Moreover, there is a shift from domestic to imported intermediate inputs and hence $x(i)$ falls. Both lead to lower markups received by domestic intermediate good firms. Thus, we have:

Proposition 3: (Markups) *Under Conditions S, N, C and E, domestic intermediate firms' markups in the steady-state equilibrium is always decreasing in the home tariff, τ.*

Remark 3: It is noted that, with exogenous overall length of the production line, the markup becomes higher with domestic trade liberalization, which seems counter-intuitive.

We now turn to determining how trade liberalization affects productivity and technology. It can be seen from Proposition 2 that under domestic trade liberalization, the range of domestic production $[0, n^P]$ shrinks. Thus, some higher technology intermediate goods are now imported, which are produced in the North with lower costs, thereby resulting in unambiguous productivity gains. The effect on average technology is, however, not obvious. Define the aggregate technology used by domestic producers as $\widetilde{A} = \int_0^{n^P} A(i, M)\, di$. Utilizing (20), we can write:

$$\widetilde{A} = \overline{A} \int_0^{n^P} \psi(i)\, LP(i)^\mu\, di \tag{40}$$

Consider the benchmark case where Conditions *S, N, C and E* hold. Then, domestic trade liberalization (lower τ) will reduce the overall length of the production line, $[0, M]$, as well as the range of domestic production, $[0, n^P]$. While the latter decreases aggregate technology, the former raises individual labor demand and hence individual technology used for each intermediate good employed by the domestic final producer (recall Proposition 3). Thus, for domestic producers, domestic trade liberalization will reduce average technology $\frac{\widetilde{A}}{n^P}$. Nonetheless, average productivity measured by $\frac{Y}{X}$ will increase due to the use of more advanced imported intermediate inputs. These results are summarized in the following proposition.

Proposition 4: (Productivity) *Under Conditions S, N, C and E, domestic trade liberalization results in productivity gains for newly imported intermediate goods as well as an increase in average productivity. Both aggregate and average technology of domestic producers are lower in response to domestic trade liberalization (lower τ).*

This result is interesting because it points out that productivity and technology do not always move together. In this model, trade liberalization leads to higher productivity because input prices fall. This fall in input prices implies that it is profitable to import intermediate goods from a more technologically advanced country, rather than buying intermediate goods from domestic producers who are actively investing in improving the level of technology. As a result, it leads to a lower level of technology for domestic producers in steady-state equilibrium, as can be seen from (19). Thus, there is a tension between producing final goods with the highest level of productivity and encouraging the development of domestic technology.

5   Numerical Analysis

While we would like to use existing data to back out the intermediate good price, quantity, endogenous markup and endogenous technology schedules over various intermediate variety ranges, we do not have suitable data available to do so. Without the option for a full calibration of the model, we have to rely on simple numerical analysis based on a careful selection of parameters and subsequent sensitivity analysis. In so doing, we (i) check the validity of Conditions S, N, C and E, (ii) gain some feel for the relative magnitudes of extensive and intensive margin results, and (iii) get a feel for how trade liberalization affects productivity quantitatively.

For our baseline economy we set the time preference rate as $\rho = 5\%$, as in the literature. Given that the physical depreciation rate is usually set at 10%, the technology obsolescence rate is set at a higher rate $\nu = 25\%$. We select the intermediate sector production parameters as $\theta = 0.6$ and $\mu = 0.2$, which satisfies the requirement for decreasing returns to scale, $\theta + \mu < 1$ and leads to an overall markup of 70%, through the entire production process, over the final good producer, which is a reasonable figure. Turning now to the final sector production parameters, we set $\alpha = 10$, $\beta = 0.17$ and $\gamma = 0.1$, which satisfy the requirements $\beta - \gamma > 0$, as well as Condition C. Normalize $\eta = 1$ and set $\phi = 0.04$ so that Condition N is met. To meet Condition E, the technology and world price schedules are given by: $\psi(i) = 16\,(1 + 0.04 \cdot i)$ and $p^*(i) = 2.5 - 0.05 \cdot i$. Letting $w = 50$, this insures that Condition S is met. Finally, we choose $\tau = 7.5\%$. Under this plausible parameterization, Conditions S, N, C and E are all valid.[7] We summarize the numerical results in Table 1.

The computed ranges of exports, nontraded intermediate goods and imports turn out to be: $[0, n^E] = [0, 9.20]$, $[n^E, n^P] = [9.20, 14.24]$ and $[n^p, M] = [14.24, 20.56]$, respectively. While aggregate intermediate goods demand and production turn out to be $\widetilde{X} = 78.89$ and $Xp \equiv \int_0^{n^E} yE\,(i)\,di + \int_{n^E}^{n^P} yP\,(i)\,di = 243.91$, aggregate and average technology used by domestic producers are $\widetilde{A} = 594.88$ and $\frac{\widetilde{A}}{n^P} = 41.78$, respectively. The average markup of domestic non-exporting producers is: $\frac{\widetilde{\Lambda}}{n^P} \equiv \dfrac{n^E \Lambda_0 + \displaystyle\int_{n^E}^{n^P} \Lambda(i)di}{n^P} = 0.710$. The computed final good output is $Y = 307.51$ and the corresponding productivity measure is $\frac{Y}{\widetilde{X}} = 3.90$. In this benchmark economy, the extensive margin of import demand is sufficiently strong for the overall length of the production line to play a dominant role.

Now consider domestic trade liberalization in the form of a 10% decrease in the domestic tariff τ.

---

[7]Specifically, we have: $\gamma\sqrt{\frac{2w\phi}{\beta-\gamma}} - (1+\tau)\,b = 0.7022$; $1 - \theta - \frac{\mu\nu}{\rho+\nu} = 0.2333$; $\frac{d\widetilde{X}}{dM}\big|_{XX\ locus} = 0.3655 < \frac{d\widetilde{X}}{dM}\big|_{MM\ locus} = 0.5375$; and, $(1+\tau)b - \gamma d\widetilde{X}/dM = 0.0172$.

The overall length of the production line M shrinks from 20.56 to 19.50. Both the range of exports and the range of domestic production decrease. In particular, the computed range of exports falls to $[0, n^E] = [0, 8.39]$. The range of nontraded intermediate goods is $[n^E, n^P] = [8.39, 13.28]$ which shrinks slightly. The range of imports is now $[n^p, M] = [13.28, 19.50]$ which decreases slightly from 6.33 to 6.23 as a result of a shortened production line. Aggregate intermediate goods demand and production fall to $\widetilde{X} = 78.43$ and $Xp = 224.62$ due to the decrease in the extensive margin. Aggregate and average technology used by domestic producers fall to $\widetilde{A} = 546.85$ and $\frac{\widetilde{A}}{n^P} = 41.19$. Moreover, the average markup of domestic intermediate producers decreases to $\frac{\tilde{\Lambda}}{n^P} = 0.677$. What happens to output and productivity? Both of them increase significantly. Computed final good output increases to $Y = 357.49$ and productivity increases to $\frac{Y}{\widetilde{X}} = 4.56$. Total exports decrease to 127.16 and imports fall slightly to 47.07. These numerical results show that domestic trade liberalization leads to higher final good output and productivity by reducing the range of intermediate goods used while increasing the intensity with which each variety is used thereby saving on the coordination cost associated with final good production. In the end, *average technology used by domestic producers falls but both aggregate and average productivities are higher.*

The results concerning the effect of trade liberalization on domestic technology and productivity are surprising but intuitive. Domestic technology refers to the technology level used by domestic firms producing intermediate goods. Trade liberalization results in higher technology imports being cheaper. This discourages investment in domestic technology improvements. Moreover, trade liberalization increases competitiveness and reduces markups for intermediate good producers. Also, there is a negative extensive margin effect which dominates a positive intensive margin effect. The total effect of all three channels leads to a lower level of technology in the steady state for domestic intermediate goods producers.

Recall that productivity is measured by dividing final good output by aggregate intermediate good usage. Our results show that trade liberalization increases productivity by a significant amount. The intuition for this is that the price for more technologically advanced inputs available via import has decreased. Final good producers buy these advanced intermediate goods more intensively and this results in more output per unit input, i.e. higher productivity. Our numerical results indicate that this cost saving effect dominates the negative extensive margin effect. This finding, regarding the productivity gains from trade liberalization, is consistent with the gains from trade result in Hsieh, Li, Ossa and Yang (2016). Using very different frameworks, we both obtain a positive intensive margin effect outweighing a negative extensive margin effect.

Our numerical results lend support to the empirical findings summarized in the introduction. We show that domestic trade liberalization causes some domestically produced intermediate goods to become imported. Such a change leads to a productivity gain, as observed by Tybout (2003), Chen,

Imbs and Scott (2009) and Goldberg et al. (2010). Also, we also show that trade liberalization directly leads to lower mark-ups, which is consistent with the empirical finding by Griffith, Harrison and Macartney (2006).

Finally, we decompose the domestic trade liberalization effects on trade and intermediate goods production into extensive and intensive margin effects and report the results in Table 2. We find that the extensive margin is by far the dominant force for the effect of trade liberalization on exports and imports, while the intensive margin plays the major role in domestic production changes.

Remark 4: We have changed key parameter values $\left(\bar{\psi}, \delta, \nu, \mu, \phi, \beta - \gamma, \rho\right)$ up and down by 10% and found that all conditions are met, the unique steady-state equilibrium exists, and all of our main results concerning the effects of domestic trade liberalization are robust (see the Appendix Table).

6   Concluding Remarks

We have constructed a dynamic model of intermediate goods trade to study the effect of trade liberalization on the pattern and the extent of intermediate goods trade and the resulting effect on technology and productivity. We have established that, although domestic trade liberalization increases imported intermediate inputs on the intensive margin, final goods producers react to it by shifting imports to lower types of intermediate inputs to lower the production cost. This decreases the overall length of the production line.

Domestic trade liberalization leads to a reduction of the ranges of export and domestic production, but its effects on the range of imports are generally ambiguous. We have shown that, domestic trade liberalization leads to lower markups and greater competition and results in productivity gains. However, these productivity gains are associated with lower aggregate and average technology by domestic intermediate goods producers. We have also established numerically that domestic trade liberalization (lower domestic tariffs) can yield large benefits to final goods producers, resulting in sharp increases in both the final good output and measured productivity.

References

[1] Amiti, Mary and Jozef Konings (2007), "Trade Liberalization, Intermediate Inputs, and Productivity: Evidence from Indonesia," *American Economic Review* 97, 1611-1638.

[2] Becker, Gary S. and Kevin M. Murphy (1992), "The Division of Labor, Coordination Costs, and Knowledge," *Quarterly Journal of Economics* 107, 1137-1160.

[3] Bernard, A. B., J. B. Jensen, J. Eaton, and S. Kortum (2003), "Plants and Productivity in International Trade," *American Economic Review* 93, 1268-1290.

[4] Caselli, F. (2005), "Accounting for Cross-Country Income Differences," Handbook of Economic Growth, 1, 679-741.

[5] Chen, N., J. Imbs and A. Scott (2009) "The Dynamics of Trade and Competition," *Journal of International Economics* 77, 50-62

[6] Dornbusch, Rudiger (1992), "The Case for Trade Liberalization in Developing Countries," *Journal of Economic Perspectives* 6, 69-85.

[7] Eaton, J., and S. Kortum (2001), "Trade in Capital Goods," *European Economic Review* 45, 1195-1235.

[8] Edwards, Sebastian (1993), "Trade Liberalization, and Growth in Developing Countries," *Journal of Economic Literature* 31, 1358-1393.

[9] Ethier, Wilfred (1982), "National and International Returns to Scale in the Modern Theory of International Trade," *American Economic Review* 73, 389-405.

[10] Ferreira, Pedro Cavalcanti and José Luiz Rossi (2003), "New Evidence from Brazil on Trade Liberalization and Productivity Growth," *International Economic Review* 44, 1383-1405.

[11] Fernandes, Ana M. (2007), "Trade Policy, Trade Volumes and Plant-Level Productivity in Colombian Manufacturing Industries," *Journal of International Economics* 71, 52-71.

[12] Flam, Harry and Elhanan Helpman (1987), "Vertical Product Differentiation and North-South Trade," *American Economic Review* 77, 810-822.

[13] Goldberg, Pinelopi, Amit Khandelwal, Nina Pavcnik, and Petia Topalova (2010), "Imported Intermediate Inputs and Domestic Product Growth: Evidence from India," *Quarterly Journal of Economics* 125, 1726-1767.

[14] Griffith, R., R. Harrison and G. Macartney (2006) "Product market reforms, Labour market institutions and Unemployment," IFS Working Papers W06/06.

[15] Hsieh, Chang-Tai, Nicholas Li, Ralph Ossa, and Mu-Jeung Yang (2016), "Accounting for the New Gains from Trade Liberalization," NBER Working Paper #22069.

[16] Hummels, David, Jun Ishii, and Kei-Mu Yi (2001), "The Nature and Growth of Vertical Specialization in World Trade," *Journal of International Economics* 54, 75–96.

[17] Impullitti, G., and O. Licandro (2016): "Trade, Firm Selection, and Innovation: the Competition Channel," forthcoming in *Economic Journal.*

[18] Keller, W. (2000), "Do Trade Patterns and Technology Flows Affect Productivity Growth?" *World Bank Economic Review* 14, 17-47.

[19] Keller, W. (2002), "Trade and the Transmission of Technology," *Journal of Economic Growth* 7, 5-24.

[20] Kim Euysung (2000), "Trade Liberalization and Productivity Growth in Korean Manufacturing Industries: Price Protection, Market Power, and Scale Efficiency," *Journal of Development Economics* 62, 55-83.

[21] Melitz, Marc and I. P. Ottaviano (2008), "Market Size and Productivity," *Review of Economic Studies* 75, 295-360.

[22] Pavcnik, Nina (2002), "Trade Liberalization, Exit, and Productivity Improvements: Evidence from Chilean Plants," *Review of Economic Studies* 69, 245-276.

[23] Peng, Shin-Kun, Jacques-F. Thisse and Ping Wang (2006), "Economic Integration and Agglomeration in a Middle Product Economy," *Journal of Economic Theory* 131, 1-25.

[24] Samuelson, Paul A. (1947), *Foundations of Economic Analysis*, Harvard University Press, Cambridge, MA.

[25] Topalova, P. and A. Khandelwal (2011), "Trade Liberalization and Firm Productivity: The Case of India," *Review of Economics and Statistics* 93, 995-1009.

[26] Tybout, J. R. (2003) "Plant and Firm Level Evidence on the 'New' Trade Theories," in E.K. Choi and J. Harrigan (ed.), Handbook of International Trade, Blackwell: Malden, MA., 388-415.

[27] Yi, Kei-Mu (2003), "Can Vertical Specialization Explain the Growth of World Trade?" *Journal of Political Economy* 111, 52-102.

Table 1: Domestic Trade Liberalization Effects with Tariff Lowered by 10%

	n^E	n^P	M	$M\text{-}n^P$	$\widetilde{X}$	Xp	$\widetilde{A}$	$\frac{\widetilde{A}}{n^P}$	$\frac{\widetilde{\Lambda}}{n^P}$	Y	$\frac{Y}{\widetilde{X}}$	Ex	Im
Benchmark	9.20	14.24	20.56	6.33	78.89	243.91	594.88	41.78	0.710	307.51	3.90	137.76	47.81
$\tau \downarrow 10\%$	8.39	13.28	19.50	6.23	78.43	224.62	546.85	41.19	0.677	357.49	4.56	127.16	47.07

Table 2: Decomposition of the Domestic Trade Liberalization Effects on
Trade and Intermediate Goods Production: Extensive vs. Intensive Margin

	Export Ex		Import Im		Intermediate Goods Production Xp	
	Extensive	Intensive	Extensive	Intensive	Extensive	Intensive
$\tau \downarrow$	-11.189	0.591	-0.748	0.000	-2.575	-16.719
10%	(-8.12%)	(0.43%)	(-1.57%)	(0.00%)	(-1.06%)	(-6.85%)

Appendix

In this Appendix, we provide detailed mathematical derivations of some expressions in the main text.

Derivation of the first-order condition with respect to the production line (10): Using Leibniz's rule, the final good producing firm's first-order condition with respect to M_t can be derived as:

$$\frac{dY_t}{dM_t} = \left[\alpha - \frac{\beta - \gamma}{2}x_t(M_t) - \gamma \widetilde{X_t}\right] x_t(M_t) = w\phi + p_t(M_t)x_t(M_t)$$

which can then be combined with the last expression of (9) to yield (10).

Derivation of the first-order conditions with respect to the two labor demand (12) *and* (13): The intermediate firm's marginal revenue can be derived as:

$$\begin{aligned}
\frac{d[(p_t(i) - \eta)y_t(i)]}{dy_t(i)} &= p_t(i) - \eta + y_t(i)\frac{dp_t(i)}{dy_t(i)} \\
&= p_t(i) - \eta - \beta y_t(i) \\
&= p_t(i) - \eta - \beta A_t(i)L_t(i)^\theta
\end{aligned}$$

where $p_t(i)$ can be substituted out with (8). Using this expression, we can obtain (12) and (13).

Derivation of the steady-state level of domestic technology (20): Since $V_{A_t}(i) = V_{A_{t+1}}(i)$, we can also use (19) to rewrite (17) as:

$$V_A = \frac{(1 + \rho)wH(i)^{1-\mu}}{\mu\psi(i)}, \quad i \in [0, n^P]$$

which can then be plugged into (18) to obtain:

$$\frac{(\rho + \nu)w}{\mu\psi(i)}\left[\frac{\nu A(i)}{\psi(i)}\right]^{\frac{1-\mu}{\mu}} = [p^*(i) - \eta]L(i)^\theta, \quad \forall\ i \in [0, n^E]$$

Using (16) to simplify the above expression, we have:

$$\frac{(\rho + \nu)w}{\mu\psi(i)}\left[\frac{\nu A(i)}{\psi(i)}\right]^{\frac{1-\mu}{\mu}} = \frac{wL(i)}{\theta A(i)}$$

Manipulating this last expression gives (20).

Derivation of aggregate intermediate goods (34): Using (25)-(27) and (31), we derive:

$$
\begin{aligned}
\widetilde{X} &= \int_0^{n^P} \overline{A}\psi\left(i\right) L\left(i\right)^{\theta+\mu} di + \int_{n^P}^{M} z(i)di - \int_0^{n^E} z^*(i)di \\[2mm]
&= \int_0^{n^P} \overline{A}\psi\left(i\right) L\left(i\right)^{\theta+\mu} di + \int_{n^P}^{M} \frac{\alpha - \gamma\widetilde{X} - (1+\tau)p^*(i)}{\beta - \gamma} di \\[2mm]
&\quad - \int_0^{n^E} \left[\overline{A}\psi\left(i\right) L\left(i\right)^{\theta+\mu} - \frac{\alpha - \gamma\widetilde{X} - \frac{p^*(i)}{1+\tau^*}}{\beta - \gamma} \right] di \\[2mm]
&= \overline{A}\int_{n^E}^{n^P} \psi\left(i\right) L\left(i\right)^{\theta+\mu} di - \frac{1}{\beta - \gamma}\left[(1+\tau)\int_{n^P}^{M} p^*(i)di + \frac{1}{1+\tau^*}\int_0^{n^E} p^*(i)di \right] \\[2mm]
&\quad + \frac{\alpha - \gamma\widetilde{X}}{\beta - \gamma}(M + n^E - n^P),
\end{aligned}
$$

which can be rearranged to yield the $\widetilde{X}$ expression (34).

Derivation of the intermediate good firms' markups: By using (22) and (24), the maximized profit function for the intermediate-good firms $i \in [n^E, n^P]$ can be derived as:

$$
\pi(i) = [\alpha - \gamma\widetilde{X}^{-i} - \eta - \beta x(i)]\overline{A}\psi\left(i\right) L\left(i\right)^{\theta+\mu} - wL(i)[1 + (\nu\overline{A})^{\frac{1}{\mu}}] = \Lambda(i)wN(i)
$$

where the intermediate producer i's markup is given by (36). For $i \in [0, n^E]$, we can use (21) and (24) to obtain:

$$
\begin{aligned}
\pi(i) &= [p^*(i) - \eta]\overline{A}\psi\left(i\right) L\left(i\right)^{\theta+\mu} - wL(i)[1 + (\nu\overline{A})^{\frac{1}{\mu}}] \\[2mm]
&= \overline{A}\psi\left(i\right) L\left(i\right)^{\theta+\mu} [p^*(i) - \eta]\{1 - \theta[1 + (\nu\overline{A})^{\frac{1}{\mu}}]\} \\[2mm]
&= \Lambda_0 wN(i)
\end{aligned}
$$

Appendix Table: Sensitivity Analysis (all parameters increase or decrease by 10%)

|	n^E	n^P	M	M-n^P	$\widetilde{X}$	Xp	$\widetilde{A}$	$\frac{\widetilde{A}}{n^P}$	$\frac{\widetilde{\Lambda}}{n^P}$	Y	$\frac{Y}{\widetilde{X}}$	Ex	Im
Benchmark	9.20	14.24	20.56	6.33	78.89	243.91	594.88	41.78	0.710	307.51	3.90	137.76	47.81
$\overline{\psi}\uparrow$	9.84	14.76	20.89	6.13	79.06	409.11	739.03	50.09	0.764	276.02	3.49	296.25	46.35
$\overline{\psi}\downarrow$	8.31	13.52	20.07	6.55	78.62	136.15	462.13	34.19	0.659	347.86	4.42	39.25	49.54
$\delta\uparrow$	9.46	14.48	20.73	6.26	78.98	268.97	627.45	43.34	0.727	294.78	3.73	160.13	47.30
$\delta\downarrow$	8.91	13.97	20.37	6.39	78.78	221.13	562.71	40.28	0.694	321.20	4.08	117.94	48.34
$\nu\uparrow$	8.44	13.62	20.14	6.52	78.66	146.53	477.38	35.06	0.659	342.47	4.35	48.33	49.32
$\nu\downarrow$	9.88	14.79	20.91	6.12	79.07	423.41	749.65	50.70	0.775	274.05	3.47	310.15	46.25
$\mu\uparrow$	8.48	13.65	20.17	6.51	78.67	198.15	525.12	38.46	0.631	340.50	4.33	99.49	49.24
$\mu\downarrow$	9.82	14.74	20.88	6.14	79.06	292.09	663.72	45.02	0.801	277.03	3.50	179.45	46.40
$\phi\uparrow$	10.00	14.92	21.89	6.97	79.34	261.38	663.37	42.46	0.679	192.47	2.43	145.44	55.25
$\phi\downarrow$	8.16	13.36	18.98	5.61	78.30	220.13	545.34	40.81	0.754	399.33	5.10	125.37	40.26
$\beta-\gamma\uparrow$	7.45	12.81	19.34	6.54	77.62	202.25	509.92	40.82	0.808	428.58	5.52	121.74	44.33
$\beta-\gamma\downarrow$	10.16	15.02	20.84	5.82	79.72	265.77	643.10	42.83	0.655	78.90	0.99	130.41	50.59
$\rho\uparrow$	9.17	14.22	20.55	6.33	78.88	239.54	590.33	41.52	0.714	308.68	3.91	133.65	47.87
$\rho\downarrow$	9.22	14.26	20.58	6.32	78.89	248.43	599.54	42.05	0.705	306.33	3.88	142.03	47.76

Figure 1. Determination of Intermediate Goods Allocation

produced

import

export

n_t^E

n_t^P

M_t

0
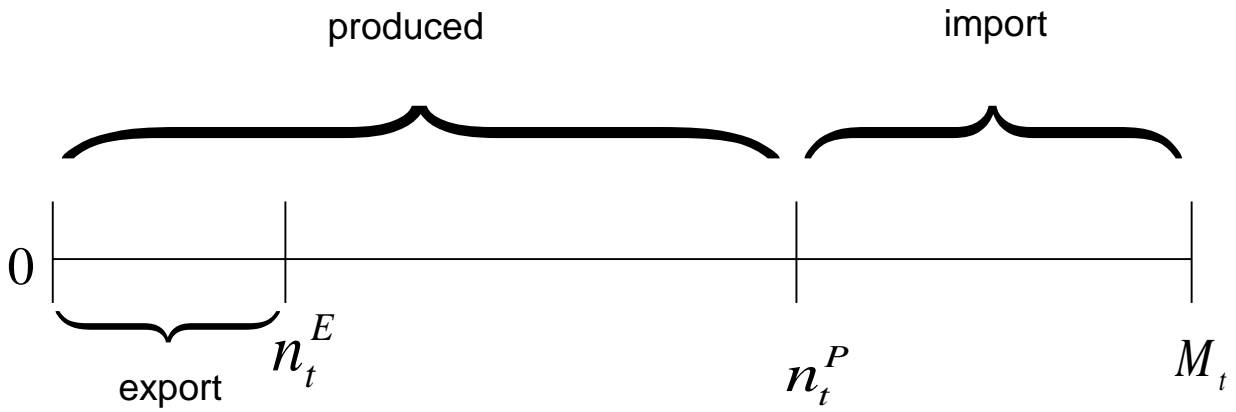
Figure 2. Labor Allocation

$MPL(\mathrm{i})$

$M\downarrow, \tau\downarrow$

E

E'

w

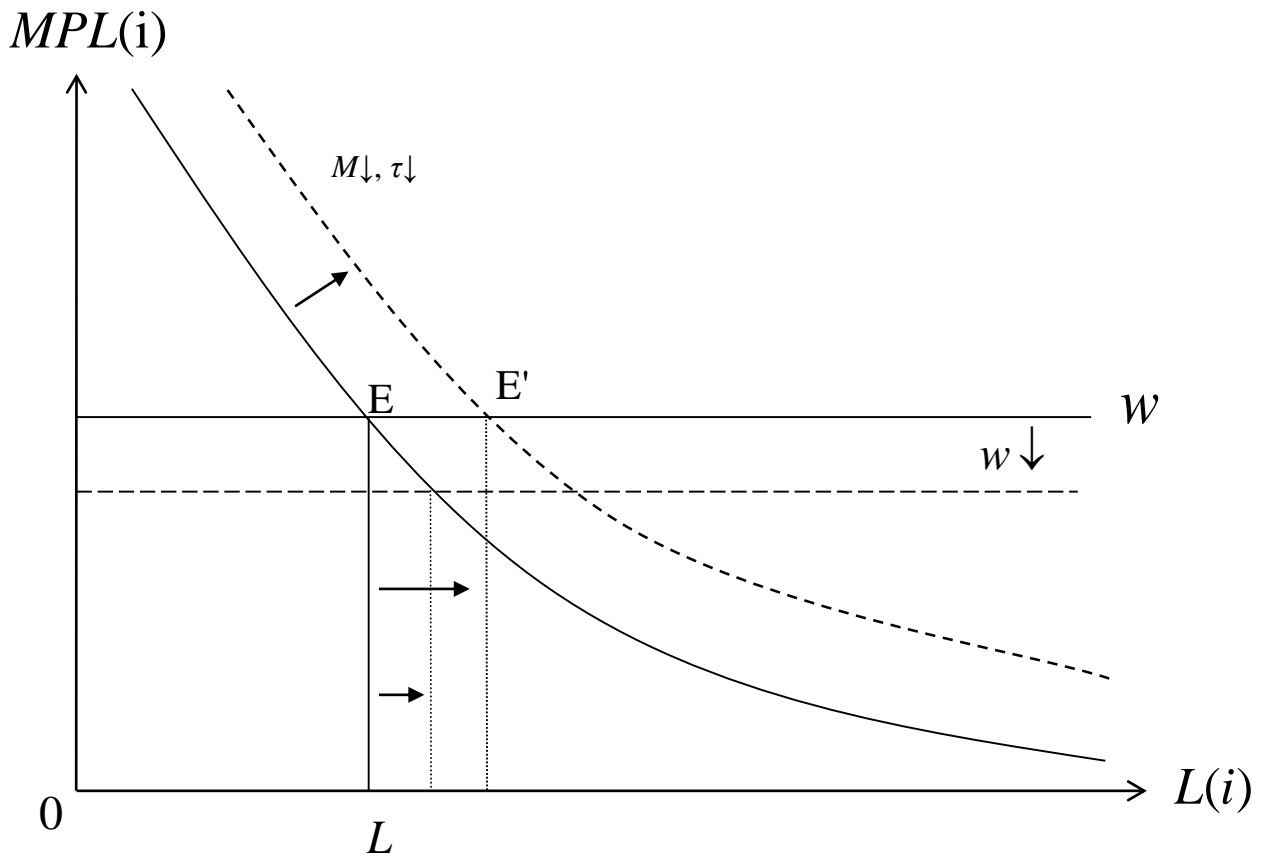$w\downarrow$

$L(i)$

0

L

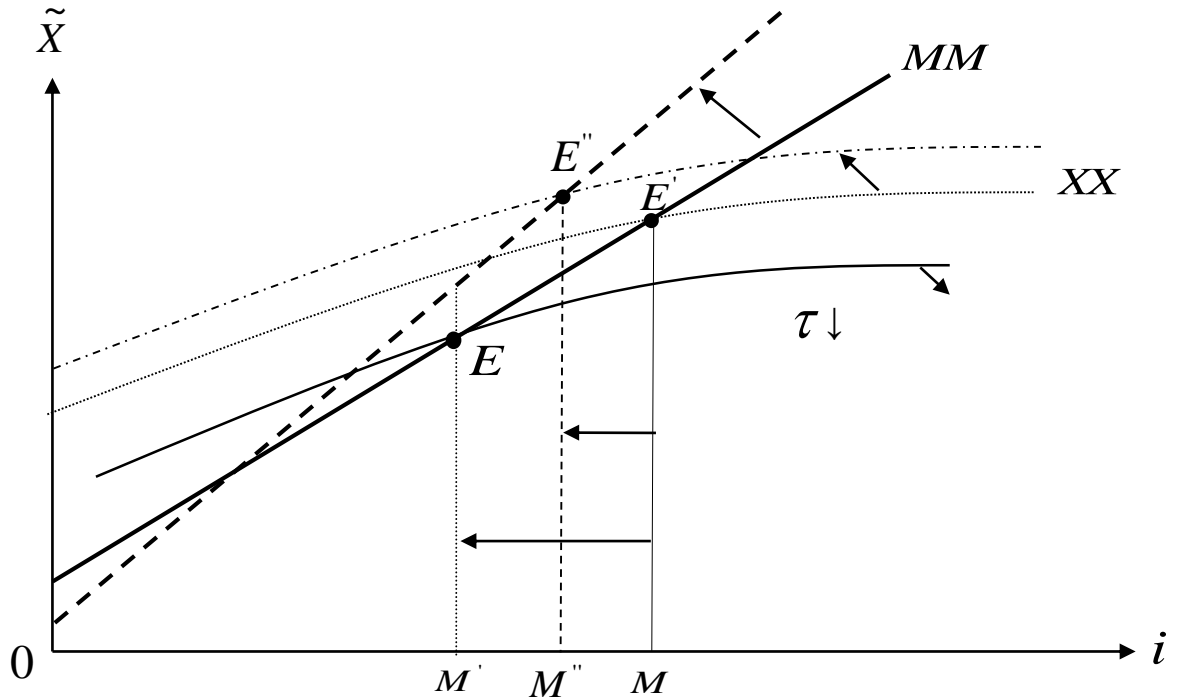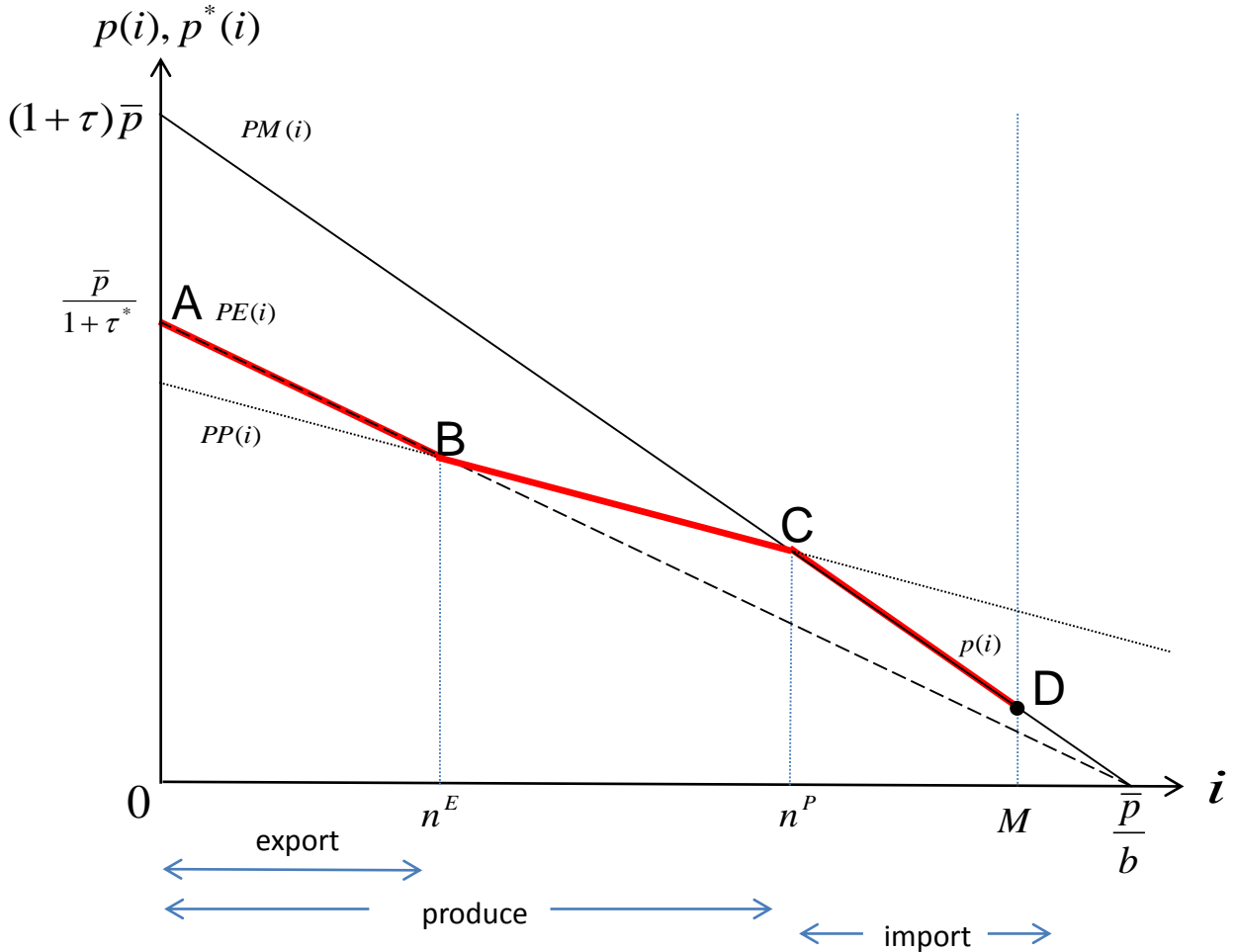## Figure 3. Determination of Length of Production Line

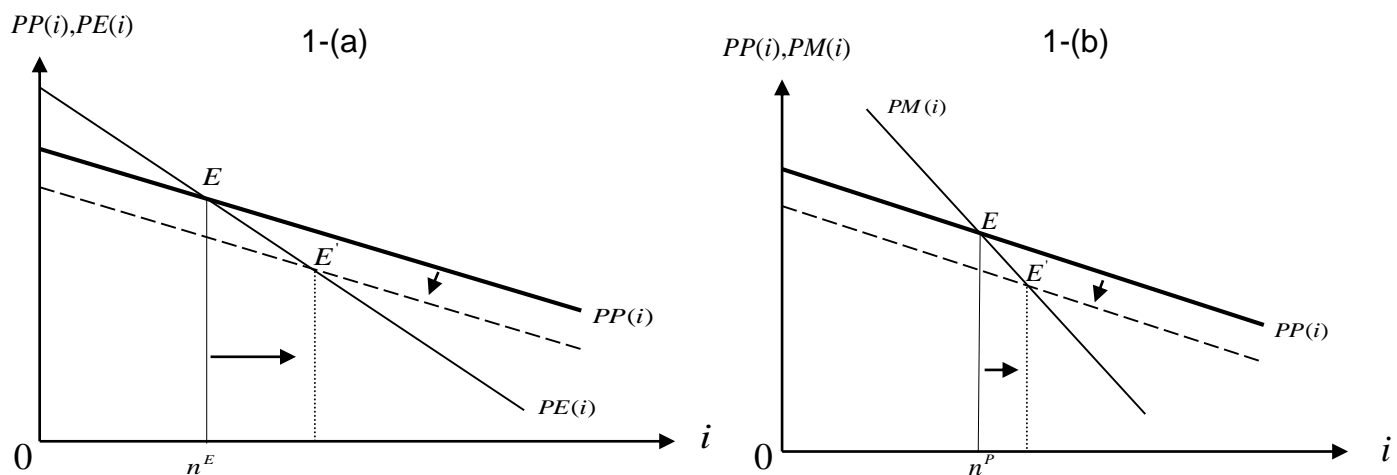## Figure 4. Technology Choice and Trade in Intermediate Goods

Figure 5. Determination of Technology and Trade Pattern

1-(a)

$PP(i), PE(i)$
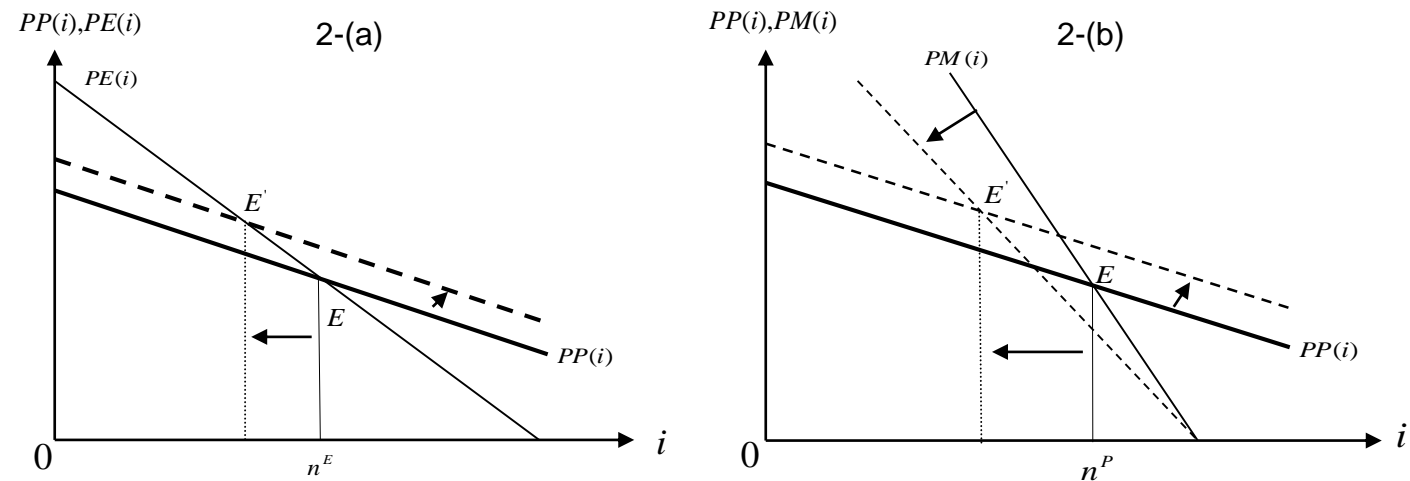
E

$PP(i)$

$PE(i)$

0 n^E i

$M_t \uparrow \Rightarrow \tilde{X}_t \uparrow$

◎ Direct effect : $PP(i) \downarrow \Rightarrow n^E \uparrow$

◎ Indirect effect : $MPL(i) \uparrow \Rightarrow PP(i) \downarrow$

1-(b)

$PP(i), PM(i)$

$PM(i)$

E

E'

$PP(i)$

0 n^P i

$M_t \uparrow \Rightarrow \tilde{X} \uparrow \Rightarrow PP(i) \downarrow \Rightarrow n^P \uparrow, PP(n^P) \downarrow$

$\Delta n^E \to \Delta n^P$

2-(a)

$PP(i), PE(i)$

$PE(i)$

E'

E

$PP(i)$

0 n^E i

τ↓

◎ $PE(i)$ no change

◎ $\tilde{X} \downarrow \Rightarrow MPL(i) \downarrow \Rightarrow PP(i) \uparrow$

2-(b)

$PP(i), PM(i)$

$PM(i)$

E'

E

$PP(i)$

0 n^P i

τ↓

◎ Direct effect : $PM(i) \downarrow, n^P \downarrow$

◎ Indirect effect : $\tilde{X} \downarrow \Rightarrow PP(i) \uparrow$

Figure A1. Determination of aggregate intermediate good usage under exogenous M

$\tilde{X}$

E'

E

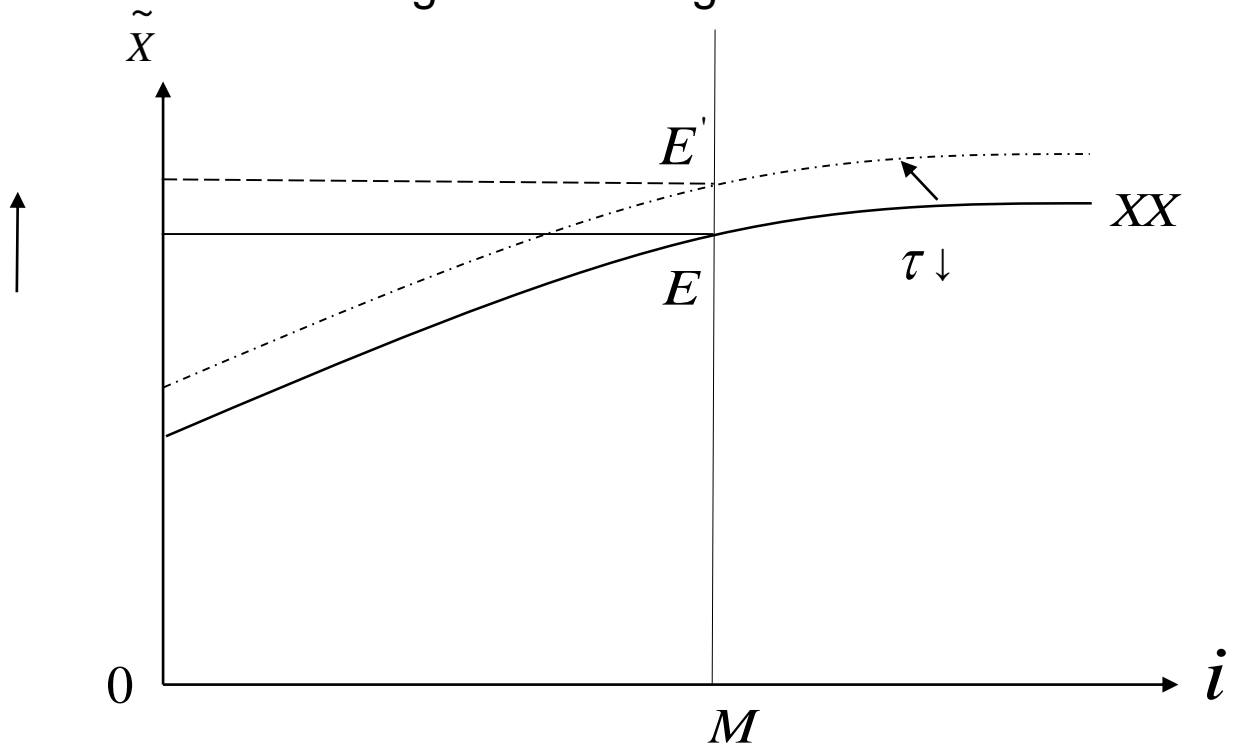XX

$\tau \downarrow$

0

M

i

Figure A2. Technology Choice and Trade in Intermediate Goods under exogenous M

$p(i), p^*(i)$

$(1+\tau)\overline{p}$

$PM(i)$

$\dfrac{\overline{p}}{1+\tau^*}$

A $PE(i)$

$PP(i)$

$\tau \downarrow$

B

C

C'

$p(i)$

D

D'

0

n^E

n^P

M

$\dfrac{\overline{p}}{b}$

i

export

produce

import