Currency Substitution as an Automatic Stabilizer

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Abstract

This paper shows that openness in terms of diversified currency holdings stabilizes a simplified version of Farmer's (1997) indeterminate monetary model. I analytically derive that, when the foreign inflation rate is lower than the domestic long-run inflation rate, the model's unique steady state always displays saddle-path stability. Hence, belief-driven cyclical fluctuations originally present in the domestic country are entirely removed in the presence of diversified currency holdings. When the foreign inflation rate is higher than the domestic long-run inflation rate, then depending on the degrees of currency substitution and relative risk aversion, equilibrium indeterminacy is either impossible or the requisite level of the foreign inflation rate for indeterminacy is too high to square with data. The determinacy result is robust to changing the monetary regime from money growth targeting to flexible inflation targeting, and to whether domestic and foreign currencies display as Edgeworth substitutes or complements, or are additively separable in the household's preferences.

Keywords: Equilibrium (In)determinacy, Currency Substitution, Small Open Economy.

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1 Introduction

It is well-established in the literature that flexible-prices monetary models display belief-driven cyclical fluctuations under high degrees of relative risk aversion. This indeterminacy result is found to be robust to variations in the monetary policy rule and the monetary approaches [Farmer, 1997, 1999]. This paper shows that openness in terms of diversified currency holdings by agents entirely removes cyclical fluctuations of this kind, no matter whether domestic and foreign currencies exhibit substitutability or complementary, or are independent to each other.

As a result of financial liberalization and globalization, there has been increasing opportunities for agents to hold multiple currencies and foreign currency denominated deposits, and use them as a means of payment, a store of value, and/or unit of account. Such a phenomenon, dubbed currency substitution, is now common in the global economy.[1] It is particularly evident in high-inflation countries in which domestic residents switch to holdings of foreign currencies that are not prone to inflationary pressures; e.g., Latin America countries during the 1980s and the 1990s [Savastano, 1992; Airaudo, 2014].

The rise of cryptocurrencies in recent years highlights the issue of currency substitution, considering that these currencies may one day serve as alternative means of payments and reduce the demand for fiat currencies. Despite the significance of currency substitution, there are only a few papers working on the implication of it on a nation's aggregate stability.[2] Earlier works have found exchange rate indeterminacy under substitutable currencies [see Kareken and Wallace (1981), Sargent (1987), King, et al. (1992), Barnett (1992), Giovannini and Turleboom (1994), among others]. Uribe (1997) shows that, when individuals accumulate experience in using the foreign currency as a means of payment, multiple steady states and equilibrium inde-

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[1]See, e.g., Miles (1978), Laffer and Miles (1982), Bufman and Leiderman (1993), Akcay et al. (1997), Selcuk (2003), Prock et al. (2003), and the survey of Giovannini and Turleboom (1994) for empirical evidence of the existence of currency substitution in both developed and emerging market countries.

[2]Chen (1973), Miles (1978), McKinnon (1982), and Chen and Tsaur (1983) show that, where the possibility of currency substitution exists, a flexible exchange rate may no longer provide a cushion against external shocks. McKinnon (1985) stresses the important role of currency substitution on the transmission of monetary policy. Indeed, it has been shown that currency substitution weakens the autonomy of monetary policy and makes it more difficult for the monetary authority to find monetary targets [Vegh, 1989, 1995, 2013; Calvo and Vegh, 1992].

terminacy are more likely to occur. Dupor (2000) demonstrates that the well-known determinacy result under a nominal interest rate peg in a closed economy does not survive in a multi-country, multi-currency endowment-economy setting, no matter whether currencies are perfect substitutes or not substitutable across countries.

Tandon and Wang (1999, 2003) find that the following may give rise to the existence of multiple equilibria: (1) concern by agents for the possibility that the government may impose capital controls that increase the transaction cost of using foreign currencies; or (2) concern by agents that the domestic currency could lose its function as a store of value. Airaudo (2014) presents that equilibrium indeterminacy can occur even if currencies are imperfect substitutes; however, it requires imperfect capital mobility and the adoption of an interest rate-based rule by the central bank.

It is thus clear that, when incorporating particular model features such as imperfect world capital market, capital accumulation, and uncertainties about the stance of government policies, the literature tends to point to the result that currency substitution is detrimental for domestic aggregate stability. This paper singles out the role of currency substitution, and shows that diversified currency holdings themselves serve as an automatic stabilizer that mitigates belief-driven cyclical fluctuations.

To highlight the mechanism that diversified currency holdings create a stabilizing effect that removes the possibility of indeterminacy, I start off with analyzing a simplified version of the closed economy of Farmer (1997) in which agents derive positive utility from holdings of real money balances, and that consumption and real money balances may display as Edgeworth substitutes or complements, or are additively separable in the household's preferences.[3] Simple as it is, the model allows me to analytically derive the necessary and sufficient conditions for the model's steady state to display endogenous cyclical fluctuations or saddle-path stability. Specifically, for endogenous business fluctuations to occur, it requires that the share of consumption in the household's subutility function is lower than a critical value, and that the coefficient of relative risk aversion is above a threshold level; otherwise, equilibrium uniqueness is ensured. Note that the role of the share of consumption in the household's subutility function is not put forth in Farmer (1997, 1999), while that of

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[3]Farmer (1997) considers non-separability between consumption, labor hours, and real money balances.

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the coefficient of relative risk aversion is qualitatively the same as in those papers' numerical results.

Section 3 analyzes a small-open-economy extension of Section 2's closed-economy framework by allowing holdings of both domestic and foreign currencies by domestic agents. The domestic country thus exhibits net financial outflows and a current account surplus. I show that the small-open-economy model exhibits a unique steady state. In addition, when the foreign inflation rate is lower than the domestic long-run inflation rate, saddle-path stability of the model's steady state is guaranteed no matter whether domestic and foreign currencies are Edgeworth substitutes, complements, or independent. Hence, belief-driven cyclical fluctuations originally present in the domestic country are entirely removed in the presence of diversified currency holdings. This determinacy result is distinct from that derived in the literature. For instance, considering a foreign inflation rate lower than the domestic inflation rate, Airaudo (2014) quantitatively shows that equilibrium indeterminacy arises under an imperfect world capital market and a Taylor-type monetary policy rule.

When by contrast the foreign inflation rate is higher than the domestic long-run inflation rate, I analytically derive that belief-driven cyclical fluctuations are impossible when (i) the degree of currency substitution is lower than a critical value, or (ii) the degree of currency substitution exceeds that critical value and the coefficient of relative risk aversion is below a threshold level. Even when the degree of currency substitution exceeds the critical value and the coefficient of relative risk aversion is above the threshold level, the quantitative analysis shows that, to be compatible with indeterminacy, it requires levels of the foreign inflation rate that are proven too high to square with data. Equilibrium indeterminacy is thus extremely hard to occur in this case.

I also find that the higher the degree of openness in terms of diversified currency holdings (measured by a lower share of domestic currency in total liquidity services), the higher the requisite level of the foreign inflation rate for indeterminacy will be. This implies that it is more difficult for equilibrium indeterminacy to occur when the degree of openness increases. Moreover, I show that the minimum level of the foreign inflation rate that guarantees equilibrium indeterminacy increases when the degree of currency substitution and/or the coefficient of relative risk aversion falls.

All of the results presented above demonstrated robustness to changing the monetary policy rule from a constant nominal money growth rate to flexible inflation targeting where, under the latter regime, short-run deviations of the inflation rate from the target level are allowed, and price stability is the central bank's long-run goal. I show that, in a closed-economy setting, the macroeconomic stabilizing properties of flexible inflation targeting are *exactly* the same as those of nominal money growth targeting. In a small-open economy setting, on the other hand, the macroeconomic stabilizing properties of flexible inflation targeting are *qualitatively* the same as those of nominal money growth targeting; in addition, under flexible inflation targeting, the faster the domestic inflation rate converges to the target level, the higher the requisite level of the foreign inflation rate for indeterminacy will be.

The idea that openness serves as a stabilization factor has also been put forth in, e.g., Huang et al. (2017, 2018). Huang et al. (2017) show how integration into the world capital market (especially the structure of the borrowing rate) relieves concerns about aggregate instability resulting from the implementation of Schmitt-Grohé and Uribe's (1997) formulation of the balanced-budget rule. Huang et al. (2018) highlight the roles of international trade factors, including the elasticity of substitution between home and foreign goods and the degree of openness in terms of home consumers' preferences over imported goods, at reducing the scope of indeterminacy. By contrast, there are more papers finding the destabilizing effect of integration into the global economy; e.g., the one-sector model of Chen (2018) under an imperfect world capital market, and the two-sector models of Weder (2001) who incorporates aggregate and sector-specific externalities and Meng and Velasco (2004) who explores the role of factor intensity in the capital good sector.

The remainder of this paper is organized as follows. Section 2 describes the closed-economy model and analytically and graphically examines the model's local dynamics. Section 3 analyzes the theoretical interrelations between currency substitution and the small-open-economy model's local stability properties. Section 4 examines the regime of flexible inflation targeting. Section 5 concludes.

2 Closed Economy

This section analyzes the aggregate stability of a one-sector monetary closed-economy model where households derive utility from holdings of real money balances [Brock, 1974]. I assume that there are no fundamental uncertainties present in the economy.

There is a continuum of identical competitive firms, with the total number normalized to one. The representative firm produces real output y_t according to the linear production technology: $y_t = h_t$, where h_t is labor inputs. Under the assumption that labor market is perfectly competitive, profit maximization of the firm leads to: $w_t = 1$, where w_t is the real wage.

The economy is also populated by a unit measure of identical infinitely-lived households. Each household is endowed with one unit of time and maximizes:

$$\int_0^\infty \overbrace{\left[\frac{v_t^{1-\sigma} - 1}{1 - \sigma} - \frac{h_t^{1+\gamma}}{1 + \gamma} \right]}^{u_t} e^{-\rho t} dt, \tag{1}$$

where the subutility function v_t is given by:

$$v_t = c_t^a m_t^{1-a}, \ \ 0 < a < 1, \tag{2}$$

where c_t is consumption, m_t denotes real money balances, a and $1-a$ are respectively the share of consumption and real money balances in v_t, $\sigma > 0$ measures the relative risk-aversion, $\gamma \geq 0$ denotes the inverse of the intertemporal elasticity of substitution in labor supply, and $\rho > 0$ is the subjective rate of time preference.

The formulation of the household's instantaneous utility function, u_t, given by (1) and with (2), follows that of Obstfeld (1985), and is increasing and strictly concave with respect to consumption, real money balances, and labor hours. When $\sigma = 1$, the household's preference exhibits additive separability between consumption and real money balances, and hence the marginal utility of c_t is independent of m_t ($u_{cm} = 0$). When $\sigma > (<) 1$, the marginal utility of consumption increases (decreases) with respect to real money balances ($u_{cm} < (>) 0$), indicating that c_t and m_t are Edgeworth substitutes (complements), and hence the model becomes an asset substitution model (a transactions service model) [Wang and Yip, 1992]. Under (1), the inverse of the

intertemporal elasticity of substitution in consumption is $\Sigma \equiv 1 - a(1 - \sigma) > 0$, and $\Sigma \gtreqqless 1$, when $\sigma \gtreqqless 1$.

The adoption of the money-in-the-utility-function (MIUF) approach in (1) allows me to derive more general results since under this monetary approach c_t and m_t can exhibit complementarity or substitutability, or be separated from each other in the utility function. Instead, in the cash-in-advance (CIA) approach of Clower (1967), Lucas (1980), and Stocky (1981), more money holdings allow for more consumption purchases. Consumption and real money balances thus exhibit perfect complementarity.

The representative household faces the budget constraint:

$$\dot{m}_t = w_t h_t - c_t - \pi_t m_t + v_t, \tag{3}$$

where π_t denotes the inflation rate, and v_t is real lump-sum transfers from the government.

The first-order conditions for the representative household with respect to the indicated variables and the associated transversality conditions (TVC) are:

$$c_t \quad : \quad \frac{a\left(c_t^a m_t^{1-a}\right)^{1-\sigma}}{c_t} = \lambda_t, \tag{4}$$

$$h_t \quad : \quad \frac{h_t^{\gamma}}{\lambda_t} = w_t, \tag{5}$$

$$m_t \quad : \quad \frac{(1-a)\left(c_t^a m_t^{1-a}\right)^{1-\sigma}}{m_t} - \pi_t \lambda_t = \rho \lambda_t - \dot{\lambda}_t, \tag{6}$$

$$\text{TVC} \quad : \quad \lim_{t \to \infty} e^{-\rho t} \lambda_t m_t = 0, \tag{7}$$

where λ_t is the shadow value of wealth. Equation (4) states that the marginal benefit of consumption equals its marginal cost, which is the marginal utility of having an additional real dollar. Equation (5) equates the marginal disutility of labor to the real wage rate. Equation (6) governs the evolution of the shadow value of wealth, and (7) is the transversality condition.

Note from (5) that the slope of the labor supply curve equals γ, and that the labor demand curve is a horizontal line ($w_t^d = 1$). To guarantee the existence of labor market equilibrium, it thus requires a strictly positive value of $\gamma(> 0)$.

As in Farmer (1997, 1999), I consider in this section the regime of nominal money growth rate targeting (MT). Under this monetary regime, the central bank keeps the nominal money stock growing at the target rate: $\mu_t = \bar{\mu}$, $\forall t$. Nominal money supply thus evolves through time according to:

$$M_t = M_0 e^{\bar{\mu} t}, \quad M_0 > 0 \text{ given,} \tag{8}$$

and the resulting seigniorage is transferred in a lump-sum manner to the household; hence, $v_t = \bar{\mu} m_t$.

Clearing in the money and goods markets implies that:

$$\dot{m}_t = (\mu_t - \pi_t) m_t, \tag{9}$$

and

$$y_t = c_t. \tag{10}$$

To facilitate the analysis of the model's local stability properties, I make the following logarithmic transformation of variables: $\hat{x}_t \equiv \log(x_t)$, where $x_t = \{c_t, m_t, h_t, y_t, \lambda_t\}$. The model's equilibrium conditions can be collapsed into the following single differential equation in the shadow value of wealth $\hat{\lambda}_t$ that describes the equilibrium dynamics:

$$\dot{\hat{\lambda}}_t = \Phi + \frac{\gamma(\sigma-1)(1-a)^2 a^{\frac{1}{(1-a)(1-\sigma)}-1}}{\Delta} \exp\left[\frac{\sigma+\gamma}{(1-a)(\sigma-1)\gamma}\hat{\lambda}_t\right], \tag{11}$$

where $\Delta \equiv (1+\gamma)\Sigma + (1-\sigma)\gamma \gtrless 0$ and $\Phi = \frac{(1-a)(1-\sigma)\gamma(\bar{\mu}+\rho)}{\Delta}$. The remaining endogenous variables can then be derived accordingly. In particular, it can be shown that $\hat{m}_t = \frac{(\Sigma+\gamma)\hat{\lambda}_t - \log(a^\gamma)}{(1-a)(1-\sigma)\gamma}$, $\hat{y}_t = \hat{h}_t = \hat{c}_t = \frac{\hat{\lambda}_t}{\gamma}$, and $\pi_t = \frac{(1-a)(1-\sigma)\gamma\bar{\mu}+(\Sigma+\gamma)\left\{\frac{1-a}{a}\exp[\hat{c}_t-\hat{m}_t]-\rho\right\}}{\Delta}$.

It can be derived that the model exhibits a unique interior steady state that is given by:

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$$\hat{m} = \log\left[a^{a(1-\sigma)-\gamma}\left(\frac{1-a}{\pi+\rho}\right)^{\Sigma+\gamma}\right]^{\frac{1}{\gamma+\sigma}}, \tag{12}$$

$$\hat{y} = \hat{h} = \hat{c} = \log\left[a^{a(1-\sigma)+\sigma}\left(\frac{1-a}{\pi+\rho}\right)^{(1-a)(1-\sigma)}\right]^{\frac{1}{\gamma+\sigma}}, \tag{13}$$

$$\pi = \bar{\mu}, \tag{14}$$

$$\hat{\lambda} = \log\left[\frac{a^{\frac{1}{(1-a)(1-\sigma)}-1}(1-a)}{\pi+\rho}\right]^{\frac{(1-a)(1-\sigma)\gamma}{\gamma+\sigma}}. \tag{15}$$

In terms of the local dynamics around the model's steady state, I linearize (11) around the steady state and find that its local stability property is governed by the eigenvalue:

$$e = \frac{(\gamma+\sigma)(\bar{\mu}+\rho)}{\Delta} \gtrless 0, \quad \text{if} \quad \Delta \gtrless 0. \tag{16}$$

Since the shadow value of wealth λ_t is a non-predetermined jump variable, the unique steady state of the model given by (12)-(15) displays saddle-path stability and equilibrium uniqueness if and only if $\Delta > 0$, which guarantees a positive eigenvalue: $e > 0$. If $\Delta < 0$, then the eigenvalue e is negative, and hence the steady state is a locally indeterminate sink that can be exploited to generate endogenous cyclical fluctuations driven by agents' self-fulfilling expectations or sunspots.

Given the analytical result in (16), Figure 1 depicts the combinations of the preference parameters a and σ that graphically characterize the model's local stability properties, where the positively-sloped convex locus $\tilde{\sigma}$ divides the regions labeled as "sink" and "saddle".[4] Accordingly, I establish the following proposition.

Proposition 1. *In a monetary closed economy under money growth targeting,*

(i) when the share of consumption in the subutility function v_t is higher than the critical value $\tilde{a} \equiv \frac{\gamma}{1+\gamma} \in (0,1)$, there is no possibility of equilibrium indeterminacy;

(ii) when the share a is below the critical level $\tilde{a}$, the economy possesses an (in)determinate steady state if and only if the coefficient of relative risk aversion σ

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[4]I first rewrite Δ as $\Delta = \gamma + (1+\gamma)[1-a+(a-\tilde{a})\sigma]$. It is clear that $a > \tilde{a}$ guarantees a positive Δ. When $a < \tilde{a}$, I derive that $\Delta \gtrless 0$, if $\sigma \lessgtr \tilde{\sigma} > 1$; in addition, (i) $\frac{\partial\tilde{\sigma}}{\partial a} = (\tilde{\sigma}-1)^2 > 0$; (ii)

$\frac{\partial^2\tilde{\sigma}}{\partial a^2} = (\tilde{\sigma}-1)^3 > 0$; (iii) $\tilde{\sigma}(a=0) = 2 + \frac{1}{\gamma}$; and (iv) $\tilde{\sigma}(a \to \tilde{a}) \to \infty$. Hence, the locus of $\tilde{\sigma}$ is a positively-sloped convex curve as depicted in Figure 1.

is below (above) the threshold level $\tilde{\sigma} \equiv 1 + \frac{1}{\tilde{a}-a} > 1$, where $\tilde{\sigma}$ increases with the share a.

Under a more general household utility function where c_t, h_t, and m_t display non-separability in the household's preferences, Farmer (1997) numerically shows that the value of σ larger than unity is necessary to generate indeterminate equilibria. Farmer (1999, section 11.5) then finds that for the CIA economy to display an indeterminate steady state, the value of σ needs to be higher than 2.

The household's instantaneous utility function, given in (1), is a special case of that in Farmer (1997, p.575) since here I consider that only c_t and m_t display non-separability in the household's preferences. Under this formulation of the household's preferences, I am able to *analytically* derive that equilibrium indeterminacy is impossible if $a > \tilde{a} = \frac{\gamma}{1+\gamma}$; this is not put forth in Farmer's (1997, 1999) *quantitative* analysis. When considering the value of $\gamma = 2$ that Chetty et al. (2011, 2012) recommend, I derive that this critical value is $\tilde{a} = \frac{2}{3}$. When $a < \tilde{a} \left(= \frac{2}{3}\right)$, Figure 1 illustrates that the locus of $\tilde{\sigma}$ intersects the vertical axis at the value of 2.5; in addition, raising the value of σ such that it passes through the critical value $\tilde{\sigma}$ turns the steady state from a saddle into a sink. This result is qualitatively the same as those in Farmer (1997, 1999) that I address above, stating that a sufficiently high value of σ is necessary to generate equilibrium indeterminacy.

To understand the intuition behind the above indeterminacy result, I first re-express (11) as:

$$\dot{\hat{\lambda}}_t = \rho - R_t + \pi_t, \tag{17}$$

where R_t is the model's implied nominal interest rate that is given by:

$$R_t = \overbrace{(1-a) \, a^{\frac{1}{(1-a)(1-\sigma)}-1} \exp\left[\frac{\sigma+\gamma}{(1-a)(\sigma-1)}\hat{c}_t\right]}^{u_m}. \tag{18}$$

Specifically, the household's optimization must lead to equality between the opportunity cost (R_t) and the marginal utility (u_m) of holding money balances. In addition, the equilibrium inflation rate π_t is given by:

$$\pi_t = \Phi + \frac{(\Sigma+\gamma) \, R_t}{\Delta}. \tag{19}$$

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I next start the economy from its steady state where $\dot{\hat{\lambda}}_t = 0$ and $\hat{\lambda}_t$ is at its steady-state value $\hat{\lambda}$, and then consider a slight deviation caused by agents' anticipation about a higher future shadow value of wealth. Acting upon this belief, households will decrease today's consumption in exchange for wealth accumulation. Equation (17) indicates that the consequential effect on the evolution of $\hat{\lambda}_t$ is through the impact of $\hat{c}_t$ on R_t and π_t.

I first derive from (18) that:

$$\frac{\partial R_t}{\partial \hat{c}_t} = \frac{(\sigma + \gamma) R_t}{(1 - a)(\sigma - 1)} \gtrless 0, \quad \text{when} \quad \sigma \gtrless 1. \tag{20}$$

When $\sigma > (<) 1$, substitutability (complementarity) between c_t and m_t leads the households to increase (decrease) their money holdings when they reduce consumption. Under diminishing returns of the marginal utility of real money balances, the equilibrium nominal interest rate consequently declines (rises). This tends to raise (reduce) the shadow value of wealth via (17).

I next derive from (19) that:

$$\frac{\partial \pi_t}{\partial \hat{c}_t} = \frac{(\Sigma + \gamma)(\sigma + \gamma) R_t}{\Delta (1 - a)(\sigma - 1)} \gtrless 0, \quad \text{when} \quad \Delta (\sigma - 1) \gtrless 0. \tag{21}$$

Accordingly, I derive that:

$$\text{When} \quad \sigma \; > \; 1, \; \frac{\partial \pi_t}{\partial \hat{c}_t} \gtrless 0, \; \text{if} \; \Delta = (1 + \gamma)\Sigma + (1 - \sigma)\gamma \gtrless 0, \tag{22}$$

$$\text{When} \quad \sigma \; < \; 1, \; \frac{\partial \pi_t}{\partial \hat{c}_t} < 0, \; \text{since} \; \Delta = (1 + \gamma)\Sigma + (1 - \sigma)\gamma > 0. \tag{23}$$

Hence, if $\sigma > 1$, then the drop in $\hat{c}_t$ resulting from agents' animal spirits reduces (raises) π_t, if $\Delta > (<)0$. By contrast, if $\sigma < 1$, then the drop in $\hat{c}_t$ must raise π_t.

Intuitively, there are two opposing forces interact to determine the effect of $\hat{c}_t$ on π_t. First, the decline in $\hat{c}_t$ on the one hand by suppressing aggregate demand and on the other by encouraging labor supply reduces the inflation rate π_t.[5] Second, as addressed above, when $\sigma > (<) 1$, substitutability (complementarity) between c_t and m_t leads to an increase (decrease) in $\hat{m}_t$ when $\hat{c}_t$ falls. By deriving from (4) that

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[5]Equation (4) denotes that the decline in $\hat{c}_t$ leads to a higher $\hat{\lambda}_t$: $\frac{\partial \hat{\lambda}_t}{\partial \hat{c}_t} = -\Sigma < 0$. Equation (5) then implies an increase in the labor supply.

$\frac{\partial \hat{\lambda}_t}{\partial \hat{m}_t} = (1-a)(1-\sigma) < (>)0$, if $\sigma > (<)1$, I can present that: (i) when $\sigma > 1$, since $\hat{m}_t$ rises and given that $\frac{\partial \hat{\lambda}_t}{\partial \hat{m}_t} < 0$, the shadow value of wealth falls; and (ii) when $\sigma < 1$, since $\hat{m}_t$ drops and given that $\frac{\partial \hat{\lambda}_t}{\partial \hat{m}_t} > 0$, the shadow value of wealth also falls in this case. The decline in $\hat{\lambda}_t$ shrinks the labor supply, which reduces aggregate supply and boosts π_t. By combining the two opposing forces, (21) illustrates that π_t may rise or fall, depending on the associated parameters, a, σ, and γ.

It is thus clear that, depending on parameter configurations, R_t and π_t may respond to a change in $\hat{c}_t$ in the same or different directions. First, when $1 < \tilde{\sigma} < \sigma$ such that $\Delta < 0$, since R_t declines while π_t increases, (17) indicates that the next period's shadow value of wealth must rise, validating agents' initial anticipation of a higher future shadow value of wealth.

Second, when $1 < \sigma < \tilde{\sigma}$ such that $\Delta > 0$, both R_t and π_t falls. By using (19), I derive that the real interest rate r_t equals:

$$r_t = R_t - \pi_t = \frac{\gamma(1-a)(1-\sigma)}{\Delta} R_t - \Phi. \tag{24}$$

Under the present case where $1 < \sigma$, $\Delta > 0$, and that R_t falls, (24) shows that the real interest rate r_t rises. As a result, the next period's shadow value of wealth declines, and agents' initial expectations about a higher future shadow value of wealth are therefore not validated as a self-fulfilling equilibrium.

Third, when $0 < \sigma < 1$ (hence $\Delta > 0$), both R_t and π_t rise. It is clear from (24) that the real interest rate r_t rises in this case. Hence, the next period's shadow value of wealth falls, and agents' initial anticipation thus cannot be self-fulfilling.

Finally, when $\sigma = 1$, the household's preference in (1) exhibits additive separability between c_t and m_t. It is straightforward to derive that in this case the shadow value of wealth remains unchanged over time: $\hat{\lambda}_t = \log\left(a^{\frac{\gamma}{1+\gamma}}\right)$, $\forall t$. Thus, belief-driven business fluctuations will not occur.

3   Small Open Economy under Diversified Currency Holdings

The preceding section analytically shows that a closed-economy monetary model displays an indeterminate steady state when the share of consumption in the subutility

function v_t is lower than a critical value, and when the coefficient of relative risk aversion exceeds a threshold level. Building on the closed-economy model, this section allows for holdings of both domestic and foreign currencies by domestic agents; the economy therefore becomes a small-open economy. By means of the framework, I will show how diversified currency holdings serve as an automatic stabilizer that mitigates belief-driven cyclical fluctuations.

I consider that domestic agents produce and consume a single traded good, the foreign price of which is given in the world market. In the absence of any impediments to trade, the law of one price continuously holds. By denoting π^* as the rate of inflation of the good in terms of the foreign currency and ε_t as the rate of depreciation of the domestic currency, the law of one price described in percentage change terms is given by:

$$\pi_t = \pi^* + \varepsilon_t. \tag{25}$$

The subutility function v_t, given by (2), is modified as follows:[6]

$$v_t = c_t^a l_t^{1-a}, \ \ 0 < a < 1, \tag{26}$$

where l_t denotes real liquidity services that are a composite of real domestic currency, m_t, and real foreign currency, f_t, in the constant-elasticity-of-substitution (CES) functional form [Chen et al., 1981; Miles, 1978, 1981; Imrohoroglu, 1996]:

$$l_t = \left[\delta m_t^\theta + (1 - \delta) f_t^\theta \right]^{\frac{1}{\theta}}, \tag{27}$$

where the parameter $\delta \in (0, 1)$ is the share of domestic currency in total liquidity. When $\delta = 1$, I recover the subutility function for the closed economy, given by (2). In this case, there is no incentive for households to hold the foreign currency since they derive zero utility from it. The parameter δ therefore captures the degree of openness in terms of diversified currency holdings, where a lower value of δ represents a higher degree of openness.

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[6]In the literature of currency substitution, examples that adopt the MIUF approach include Livitan (1981) and Calvo (1985), among others; Uribe (1999) and Vegh (1989, 1995) are examples that adopt the CIA and the transaction cost approaches, respectively.

The parameter $\theta < 1$ present in (27) measures the degree of currency substitution. Specifically, the elasticity of substitution between m_t and f_t equals $\frac{1}{1-\theta} > 0$. A larger value of θ thus represents a higher degree of substitution between the currencies. When $\theta \to -\infty$, the elasticity of substitution approaches 0, and l_t approaches the Leontief form: $l_t = \min(m_t, f_t)$; in this case, m_t and f_t display perfect complementarity. When $\theta \to 0$, the elasticity of substitution approaches 1, and l_t approaches the Cobb-Douglas form: $l_t = m_t^\delta f_t^{1-\delta}$; in this case, m_t and f_t are imperfect substitutes. When $\theta \to 1$, the elasticity of substitution approaches infinite, and l_t is linear in m_t and f_t: $l_t = \delta m_t + (1-\delta) f_t$; and m_t and f_t are therefore perfect substitutes.

The instantaneous utility function (1) under the subutility function (26) is increasing and strictly concave with respect to c_t, m_t, f_t, and h_t. When $\sigma = 1$, the household's preference in (1) exhibits additive separability between consumption and real liquidity services; hence, the marginal utility of c_t is independent of m_t and f_t: $u_{cm} = u_{cf} = 0$. When $\sigma > (<) 1$, not only c_t and m_t, but also c_t and f_t are Edgeworth substitutes (complements): $u_{cm} < (>) 0$ and $u_{cf} < (>) 0$, if $\sigma > (<) 1$. Finally, if $\theta > (<) \Sigma$, then m_t and f_t display Edgeworth substitutability (complementarity): $u_{mf} < (>) 0$, if $\theta > (<) \Sigma$.

In the presence of diversified currency holdings, the budget constraint faced by the representative household, (3), is modified as follows:

$$\dot{m}_t + \dot{f}_t = w_t h_t - c_t - \pi_t m_t - \pi^* f_t + v_t. \tag{28}$$

It is straightforward to derive the following first-order conditions for the representative household and the associated TVC:

$$c_t \quad : \quad \frac{a \left(c_t^a l_t^{1-a} \right)^{1-\sigma}}{c_t} = \lambda_t, \tag{29}$$

$$h_t \quad : \quad \frac{h_t^\gamma}{\lambda_t} = w_t, \tag{30}$$

$$m_t \quad : \quad \frac{(1-a) \left(c_t^a l_t^{1-a} \right)^{1-\sigma} \delta m_t^{\theta-1}}{l_t^\theta} - \pi_t \lambda_t = \rho \lambda_t - \dot{\lambda}_t, \tag{31}$$

$$f_t \quad : \quad \frac{(1-a) \left(c_t^a l_t^{1-a} \right)^{1-\sigma} (1-\delta) f_t^{\theta-1}}{l_t^\theta} - \pi^* \lambda_t = \rho \lambda_t - \dot{\lambda}_t, \tag{32}$$

$$\text{TVC} \quad : \quad \lim_{t \to \infty} e^{-\rho t} \lambda_t m_t = \lim_{t \to \infty} e^{-\rho t} \lambda_t f_t = 0. \tag{33}$$

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With (29), equations (31) and (32) lead to the no arbitrage condition between holdings of domestic and foreign currencies, from which the equilibrium domestic inflation rate is determined as:

$$\pi_t = \pi^* + \frac{(1-a)\,c_t}{a}\left(\frac{\alpha_t}{m_t} - \frac{1-\alpha_t}{f_t}\right), \tag{34}$$

where $\alpha_t \equiv \delta\left(\frac{m_t}{l_t}\right)^\theta \in (0,1)$.

The central bank and the government's behaviors are the same as those described in the preceding section. The consolidated budget constraint of the economy is thus:

$$\dot{f}_t = y_t - c_t - \pi^* f_t, \tag{35}$$

which equates net financial outflows to current account surplus.

It is straightforward to show that the model exhibits a unique interior steady state that is given by:

$$\hat{f} = \log\left[\frac{a}{\Pi^\Sigma\,(\Pi + \pi^*)^\gamma}\right]^{\frac{1}{\gamma+\sigma}} + \log\left\{\delta\left[\frac{\Xi\,(\pi^* + \rho)}{\pi + \rho}\right]^\theta + 1 - \delta\right\}^{\frac{(1-a)(1-\sigma)}{(\gamma+\sigma)\theta}}, \tag{36}$$

$$\hat{m} = \hat{f} + \log\left[\frac{\Xi\,(\pi^* + \rho)}{\pi + \rho}\right], \tag{37}$$

$$\hat{c} = \hat{f} + \log(\Pi), \tag{38}$$

$$\hat{y} = \hat{h} = \hat{f} + \log(\Pi + \pi^*), \tag{39}$$

$$\pi = \bar{\mu}, \tag{40}$$

where $\Xi = \left[\frac{\delta}{1-\delta}\left(\frac{\pi^*+\rho}{\pi+\rho}\right)^\theta\right]^{\frac{1}{1-\theta}} > 0$ and $\Pi = \frac{a(\pi^*+\rho)(1+\Xi)}{1-a} > 0$.

In the neighborhood of this steady state, the model's equilibrium conditions can be approximated by the following log-linear dynamical system:

$$\begin{bmatrix} \dot{\hat{m}}_t \\ \dot{\hat{f}}_t \\ \dot{\hat{\lambda}}_t \end{bmatrix} = \underbrace{\begin{bmatrix} \Gamma_1 & \Gamma_2 & \frac{\bar{\mu}-\pi^*}{\Sigma} \\ \frac{\Xi\Pi(1-a)(\sigma-1)}{(1+\Xi)\Sigma} & \frac{\Pi(1-a)(\sigma-1)}{(1+\Xi)\Sigma} - \pi^* & \frac{\pi^*+\Pi}{\gamma} + \frac{\Pi}{\Sigma} \\ \frac{\rho+\bar{\mu}}{1+\Xi}\left(1-\theta+\frac{\sigma\Xi}{\Sigma}\right) - \Gamma_1 & \frac{\rho+\bar{\mu}}{1+\Xi}\left(\theta-1+\frac{\sigma}{\Sigma}\right) - \Gamma_2 & \frac{\pi^*+\rho}{\Sigma} \end{bmatrix}}_{\mathbf{J}^\mu} \begin{bmatrix} \hat{m}_t - \hat{m} \\ \hat{f}_t - \hat{f} \\ \hat{\lambda}_t - \hat{\lambda} \end{bmatrix}, \quad \hat{f}_0 \text{ given,}$$

$$\tag{41}$$

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where

$$\Gamma_1 \equiv \frac{\left(1 - \theta + \frac{\sigma \Xi}{\Sigma}\right) \bar{\mu} + (1 - \theta)(1 + \Xi)\rho - \Xi\left(\theta - 1 + \frac{\sigma}{\Sigma}\right)\pi^*}{1 + \Xi},$$

$$\Gamma_2 \equiv \frac{\left(\theta - 1 + \frac{\sigma}{\Sigma}\right)\bar{\mu} - (1 - \theta)(1 + \Xi)\rho + \left[\frac{\sigma}{\Sigma} + \Xi(1 - \theta)\right]\pi^*}{1 + \Xi}.$$

It follows that the determinant and trace of the model's Jacobian matrix $\mathbf{J}^\mu$ are

$$Det\left(\mathbf{J}^\mu\right) = \frac{(\theta - 1)(\rho + \bar{\mu})(\rho + \pi^*)\left(\frac{\sigma}{\gamma} + 1\right)(\Pi + \pi^*)}{\Sigma} < 0, \qquad (42)$$

and

$$Tr\left(\mathbf{J}^\mu\right) = \underbrace{\left(1 - \theta + \frac{\sigma \Xi}{\Sigma}\right)}_{(+)} \frac{\bar{\mu} - \pi^*}{1 + \Xi} + (2 - \theta)\rho + (1 - \theta)\pi^* \gtreqless 0. \qquad (43)$$

Since the first-order dynamical system (41) possesses one predetermined variable $\hat{f}_t$, and both $\hat{m}_t$ and $\hat{\lambda}_t$ are non-predetermined jump variables, the economy displays saddle-path stability and equilibrium uniqueness if and only if one of the eigenvalues of $\mathbf{J}^\mu$ has a negative real part, and two of the remaining have positive real parts. When two or more eigenvalues have negative real parts, the steady state is a locally indeterminate sink that can be exploited to generate endogenous business cycle fluctuations driven by agents' self-fulfilling expectations or sunspots. The steady state becomes a source when all eigenvalues have positive real parts. In this case, any trajectory that diverges away from the completely unstable steady state may settle down to a limit cycle or to some more complicated attracting sets. In the remainder of this section, I examine the local dynamics of the model's steady state in two parametric configurations.

3.1 When $\pi^* < \bar{\mu}$

Since in the steady state the domestic inflation rate π is equal to the target nominal money growth rate $\bar{\mu}$, in this specification where $\pi^* < \bar{\mu}$, the foreign inflation rate is lower than the long-run domestic inflation rate. It follows immediately from (43) that that the Jacobian $\mathbf{J}^\mu$ displays a positive trace $(Tr\left(\mathbf{J}^\mu\right) > 0)$. Together with the fact in (42) that the model's determinant is negative $(Det\left(\mathbf{J}^\mu\right) < 0)$, this indicates

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that the Jacobian $\mathbf{J}^\mu$ has one stable (negative) root and two unstable (positive) roots. The model's unique steady state thus exhibits saddle-path stability and equilibrium uniqueness. Accordingly, I establish the following proposition.

Proposition 2. *For a small-open monetary economy where diversified currency holdings are allowed and nominal money growth targeting is adopted, if the foreign inflation rate is lower than the long-run domestic inflation rate, then belief-driven cyclical fluctuations originally present in the domestic country are entirely removed.*

3.2 When $\pi^* > \bar{\mu}$

For this parametric configuration, the foreign inflation rate is higher than the long-run domestic inflation rate. I rewrite (43) as:

$$Tr\left(\mathbf{J}^\mu\right) = \underbrace{\left(1 - \theta + \frac{\sigma \Xi}{\Sigma}\right) \frac{\bar{\mu}}{1 + \Xi} + (2 - \theta)\rho}_{(+)} + \underbrace{\frac{\Xi\left(1 - a\right)\left(1 - \theta\right)\pi^*}{(1 + \Xi)\Sigma}}_{(+)} \cdot \Theta, \qquad (44)$$

where

$$\Theta \equiv \frac{(1 - a)(1 - \theta) - [1 - a(1 - \theta)]\sigma}{(1 - a)(1 - \theta)} \gtreqless 0. \qquad (45)$$

It is straightforward to show that, when $\theta < \underline{\theta} \equiv \frac{a-1}{a} < 0$, the bracket $[1 - a(1 - \theta)]$ in the numerator of Θ, given by (45), has a negative sign. It follows that $\Theta > 0$, and hence the Jacobian matrix $\mathbf{J}^\mu$ possesses a positive trace $(Tr\left(\mathbf{J}^\mu\right) > 0)$. Adding this with the fact that $Det\left(\mathbf{J}^\mu\right) < 0$, I find that in the present case where $\theta < \underline{\theta}$, the model's steady state displays only one stable (negative) eigenvalue. Hence, equilibrium uniqueness is ensured. This case is illustrated as Zone I of Figure 2.

I then turn to the case where $\theta > \underline{\theta}$. Here, a positive $[1 - a(1 - \theta)]$ is derived, and hence the numerator of Θ has an ambiguous sign. I then derive from (45) that $\Theta \gtreqless 0$, if $\sigma \lesseqgtr \underline{\sigma} \equiv \frac{(1-a)(1-\theta)}{1-a(1-\theta)}$. Thus, when $\theta > \underline{\theta}$ and $\sigma < \underline{\sigma}$, positive values of both Θ and $Tr\left(\mathbf{J}^\mu\right)$ are guaranteed, implying that the model possesses a determinate steady state. This case is illustrated as Zone II of Figure 2.[7]

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[7]Given that $\underline{\sigma} = \frac{(1-a)(1-\theta)}{1-a(1-\theta)}$, I derive that, under $\theta > \underline{\theta}$: (i) $\frac{\partial \underline{\sigma}}{\partial \theta} = -\frac{\sigma}{(1-\theta)[1-a(1-\theta)]} < 0$; (ii) $\frac{\partial^2 \underline{\sigma}}{\partial \theta^2} = -\frac{2a}{1-a(1-\theta)} \cdot \frac{\partial \underline{\sigma}}{\partial \theta} > 0$; (iii) $\underline{\sigma}(\theta \to \underline{\theta}) \to \infty$; (iv) $\underline{\sigma}(\theta = 1) = 0$; and (v) $\underline{\sigma}(\theta = 0) = 1$. Hence, the locus of $\underline{\sigma}$ is a negatively-sloped convex curve as depicted in Figure 2.

Zone III in Figure 2 represents the case where $\theta > \underline{\theta}$ and $\sigma > \underline{\sigma}$ (hence, $\Theta < 0$), and the steady state displays $Det\left(\mathbf{J}^{\mu}\right) < 0$ and $Tr\left(\mathbf{J}^{\mu}\right) \gtreqless 0$. Since structural and policy parameters enter the Jacobian matrix $\mathbf{J}^{\mu}$ in highly non-linear manners, I am unable to analytically identify the number of (un)stable roots. In what follows, I quantitatively investigate the local stability properties of the steady state locating in Zone III.

Per the parameterization that is commonly adopted in the RBC-based indeterminacy literature, the capital share of national income α is chosen to be 0.3, and the time discount rate ρ is set equal to 0.01. Recall that one needs a strictly positive value of the inverse of the intertemporal elasticity of substitution in labor supply, γ, for guaranteeing the existence of labor market equilibrium; γ is thus calibrated to be 2 (Chetty et al. $2011, 2012$). The domestic inflation rate is set at 4% per year, implying that $\bar{\mu}$ equals $0.04/4$. Finally, under the calibrated value of γ, Figure 1 shows that equilibrium indeterminacy is possible in Section 2's closed economy only when $a < \tilde{a} = \frac{\gamma}{1+\gamma} = \frac{2}{3}$. To test the power of diversified currency holdings in mitigating belief-driven cyclical fluctuations, the preference parameter a is chosen to be 0.5 $(< \frac{2}{3})$.[8] It follows that the critical value of $\underline{\theta} = \frac{a-1}{a}$ is -1; hence, only the cases associated with $\theta > -1$ need to be examined.

Given the baseline parameter values, Figure 3 plots for various combinations of σ and δ the minimum value of the annual foreign inflation rate π^{*} needed for equilibrium indeterminacy as a function of the degree of currency substitution, θ, when the model's steady state is located in Zone III of Figure 2. The two rows of Figure 3 respectively illustrate the case of $\sigma = 10$ and 8. In particular, while the literature finds that the estimate of the coefficient of relative risk aversion, σ, can range from 0 to around 30, the general consensus is that it is below 10.[9] Under the calibrated values of α, ρ, γ, and a, belief-driven cyclical fluctuations are impossible when σ is below 7. I therefore show in the figure the cases of $\sigma = 10$ and 8 only. On the other hand, a lower value of the share of domestic currency in total liquidity, δ, represents a higher degree of openness in terms of diversified currency holdings. The

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[8]One may argue that this value of the preference parameter a is too low in view of the estimates of existing empirical studies. By incorporating other model features, however, the threshold level of a below which equilibrium indeterminacy arises may be increased.

[9]See, e.g., Hall (1988), Beaudry and van Wincoop (1996), Gomes and Paz (2013), and the summary of Agénor and Montiel (2008, p.468).

three columns of Figure 3 thus correspond to the situations in which the degree of openness is low ($\delta = 0.8$), medium ($\delta = 0.5$), and high ($\delta = 0.2$), respectively.

It is obvious that each of the panels in Figure 3 displays a negative nexus between the requisite level of π^* for indeterminacy and the degree of currency substitution, θ. Moreover, for any combination of σ, δ and θ, the value of π^* above which equilibrium indeterminacy occurs is too high to square with data. In particular, in each of the panels in Figure 3, the lowest requisite level of π^* for indeterminacy is observed when $\theta \to 1$. I numerically derive that, when $\theta \to 1$, this level of π^* equals 42% (97%) when $\sigma = 10$ (8), for $\delta = 0.8$, 0.5, and 0.2. This means that, when $\theta \to 1$, the minimum level of π^* that guarantees equilibrium indeterminacy does not change to variations in δ, but increases when σ falls.

When $\theta < 1$, the requisite level of π^* for indeterminacy does change to variations in δ and/or θ. First, under a given level of σ, each of the rows of Figure 3 illustrates that the requisite level of π^* for indeterminacy rises faster as θ declines, if δ takes on a lower value. As a result, taking $\sigma = 10$ and $\theta = 0.75$ as an example, the model's steady state turns from a saddle into a locally indeterminate sink when $\pi^* \geq 49\%$, 50%, or 108%, if $\delta = 0.8$, 0.5, or 0.2. Since an increase in the degree of openness in terms of diversified currency holdings raises the minimum level of π^* that guarantees equilibrium indeterminacy, it makes belief-driven fluctuations more difficult to occur.

Next, under a given level of δ, each of the columns of Figure 3 shows that the requisite level of π^* for indeterminacy rises faster as θ declines, if σ takes on a lower value. Thus, taking $\delta = 0.8$ and $\theta = 0.75$ as an example, the requisite level of π^* for indeterminacy is 49% (113%) if $\sigma = 10$ (8). Recall that belief-driven cyclical fluctuations are impossible when σ is below 7. Because both a high π^* and a high σ are sources of equilibrium indeterminacy, when σ is above 7, the minimum level of π^* that guarantees equilibrium indeterminacy decreases when σ becomes higher. The above results are summarized in the following proposition.

Proposition 3. *For a small-open monetary economy where diversified currency holdings are allowed and nominal money growth targeting is adopted, if the inflation rate of the foreign country is higher than the long-run domestic inflation rate, then:*

(i) belief-driven cyclical fluctuations originally present in the domestic country are entirely removed when $\theta < \underline{\theta} = \frac{a-1}{a} < 0$, or when $\theta > \underline{\theta} = \frac{a-1}{a} < 0$ and

$\sigma < \underline{\sigma} = \frac{(1-a)(1-\theta)}{1-a(1-\theta)} < 1;$

(ii) when $\underline{\theta} < \theta < 1$ and $\sigma > \underline{\sigma}$, indeterminacy occurs at very high foreign inflation rates and the coefficient of relative risk aversion; in this case, the higher the degree of openness in terms of diversified currency holdings and/or the lower the coefficient of relative risk aversion is, the higher the requisite level of the foreign inflation rate will be.

3.3 Discussion

Under the calibrated values of α, ρ, γ, and a, Proposition 1 states that the closed economy displays belief-driven business fluctuations when σ is higher than the threshold level $\tilde{\sigma} = 1 + \frac{1}{\tilde{a}-a} = 7$. Proposition 2 then shows that, by allowing diversified currency holdings by agents, equilibrium indeterminacy becomes impossible when the foreign inflation rate is lower than the domestic long-run inflation rate. This is distinct from the indeterminacy result obtained in the literature. For instance, under an imperfect world capital market and a Taylor-type rule on the nominal government bonds rate, Airaudo's (2014) quantitative analysis considering $\pi^* = 2\% < 6.4\% = \pi$ shows that indeterminacy occurs.

Proposition 3 presents that even when $\pi^* > \pi$, equilibrium indeterminacy is impossible when the model's steady state is located in Zone I or II of Figure 2, and it is extremely unlikely to occur when the steady state is located in Zone III of Figure 2 since the requisite level of π^* for indeterminacy is too high to square with data.

The above discussion demonstrates that diversified currency holdings themselves create a stabilizing effect that eliminates the possibility of endogenous cyclical fluctuations. To understand the intuition behind the determinacy result under currency substitution, I present in what follows the evolution of the shadow value of wealth:

$$\dot{\hat{\lambda}}_t = \rho - \overbrace{\frac{(1-a)(1-\delta)}{a} \exp\left[\frac{\log(a) + [(1-a)(1-\sigma)-\theta\Sigma]\hat{l}_t - \Sigma(1-\theta)\hat{f}_t - \hat{\lambda}_t}{\Sigma}\right]}^{u_f} + \pi^*,$$
(46)

where the second term on the right-hand side is the nominal return on holding the foreign currency, i.e. the marginal utility of the foreign currency, u_f.

I start the economy from its steady state, where $\dot{\hat{\lambda}}_t = 0$ and $\hat{\lambda}_t$ is at its steady-state value $\hat{\lambda}$, and then consider a slight deviation caused by agents' anticipation about a higher future shadow value of wealth. Acting upon this belief, households will decrease today's consumption in exchange for the accumulation of wealth.

When $\sigma > 1$, consumption and the foreign currency are Edgeworth substitutes ($u_{cf} < 0$). Hence, when cutting c_t, households will increase f_t. Since the marginal utility of the foreign currency exhibits diminishing returns, the nominal return on holding the foreign currency falls. This tends to raise the next period's shadow value of wealth. However, currency substitution will offset this positive effect on $\dot{\hat{\lambda}}_t$, thereby preventing agents' anticipation from being self-fulfilling. Specifically, recall that m_t and f_t display substitutability (complementarity), if $\theta > (<)\Sigma$. When $\theta > \Sigma$, substitutability between m_t and f_t leads households to reduce m_t when they increase f_t. Because $u_{mf} < 0$, the decline in m_t raises the nominal return on holding f_t, which offsets the original decline in the nominal return on holding f_t. By contrast, when $\theta < \Sigma$, complementarity between m_t and f_t leads households to raise m_t when they increase f_t. Since $u_{mf} > 0$, the rise in m_t increases the nominal return on holding f_t; the original decline in the nominal return on holding f_t is therefore offset.

When $\sigma < 1$, c_t and f_t display complementarity ($u_{cf} > 0$). Hence, when c_t drops upon agents' animal spirits, f_t falls as well. Diminishing returns of u_f then lead to a higher nominal return on holding f_t, which in turn causes a decline in the next period's shadow value of wealth. Again, this negative effect on $\dot{\hat{\lambda}}_t$ will be offset by currency substitution. Specifically, when $\theta > \Sigma$, substitutability between m_t and f_t leads households to raises m_t when they reduce f_t. Because $u_{mf} < 0$, the increase in m_t shrinks the nominal return on holding f_t. By contrast, when $\theta < \Sigma$, complementarity between m_t and f_t leads households to reduce m_t when they cut f_t. Since $u_{mf} > 0$, the drop in m_t reduces the nominal return on holding f_t.

Finally, when $\sigma = 1$, the household's preference exhibits additive separability between c_t and f_t. Since f_t does not change upon agents' animal spirits, it is clear from (46) that the nominal return on holding the foreign currency (u_f) remains unchanged, and hence belief-driven business fluctuations will not occur.

4 Flexible Inflation Targeting

This section will show that the result whereby diversified currency holdings serve as an automatic stabilizer that mitigates belief-driven cyclical fluctuations is robust to changing the monetary regime from MT to flexible inflation targeting (FIT). In what follows, I first analyze the aggregate stability of a closed economy, and then move to the analysis within a small-open-economy setting.

All model features for the closed and the small-open economies are the same as those in Sections 2 and 3, respectively, except that here the monetary policy rule is no more a constant nominal monetary growth rate. Under FIT, short-run deviations of the inflation rate from the target level are allowed, and price stability is the central bank's long-run goal. By conducting open market operations that adjust the money supply, the central bank gradually achieves the inflation rate target, denoted as $\bar{\pi}$. Following Chen (2018), I capture this feature by postulating that the growth rate of nominal money supply evolves through time according to the law of motion:[10]

$$\dot{\mu}_t = -\phi \left(\pi_t - \bar{\pi} \right), \ \ \phi > 0. \tag{47}$$

The above equation states that when the inflation rate is above/below the target level, the central bank will conduct a sequence of open market sales/purchases that gradually reduces/raises the nominal money growth rate. The coefficient ϕ determines the time needed for the inflation rate to return to the target level.

Under FIT, the closed economy's dynamics are governed by a pair of differential equations:[11]

$$\dot{\hat{\lambda}}_t \ = \ \rho - (1-a)\, a^{\frac{1}{(1-a)(1-\sigma)}-1} \exp\left[\frac{\sigma + \gamma}{(1-a)(\sigma-1)\gamma} \hat{\lambda}_t \right] + \pi_t, \tag{48}$$

$$\dot{\mu}_t \ = \ -\phi \left(\pi_t - \bar{\pi} \right), \ \ \mu_0 \text{ given}, \tag{49}$$

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[10]I do not assume a feedback rule of the central bank's policy rate to the inflation gap. Chen (2015) provides a theoretical framework that incorporates a banking system, a reserves market, and the central bank's open market operations. That paper demonstrates cautions should be made when abstracting from the model the reserves market and using the government bonds rate as a proxy for the federal funds rate when analyzing the macroeconomic stabilizing properties of the nominal interest rate feedback rule.

[11]The derivation details for all equations in this section are available upon request.

where the domestic inflation rate, π_t, is given by:

$$\pi_t = \frac{(1-a)(1-\sigma)\gamma}{\Delta}\mu_t + \frac{\Sigma+\gamma}{\Delta}\left\{(1-a)\,a^{\frac{1}{(1-a)(1-\sigma)}-1}\exp\left[\frac{\sigma+\gamma}{(1-a)(\sigma-1)\gamma}\hat{\lambda}_t\right] - \rho\right\}.$$
(50)

It is straightforward to show that the closed economy under FIT exhibits a unique steady state described by (12), (13), and (15), and that (14) is replaced with $\pi = \bar{\pi}$, since under MT the long-run inflation rate equals the target money growth rate $\bar{\mu}$, while under FIT the long-run inflation rate equals the target inflation rate $\bar{\pi}$. In addition, the determinant and trace of the model's Jacobian matrix are:

$$Det = -\frac{\phi(1-a)(\sigma+\gamma)(\rho+\bar{\pi})}{a\Delta} \gtrless 0, \quad \text{if} \quad \Delta \lessgtr 0,$$
(51)

and

$$Tr = \left[\frac{(\sigma+\gamma)(\rho+\bar{\pi})}{1-a} + \phi(\sigma-1)\gamma\right]\frac{1-a}{\Delta}.$$
(52)

Since the nominal money growth rate, μ_t, exhibits a degree of sluggishness, it is a predetermined variable. Given that $\hat{\lambda}_t$ is a jump variable, the economy thus displays saddle-path stability and equilibrium uniqueness if and only if the two eigenvalues of the Jacobian matrix of the dynamical system (48) and (49) have opposite signs ($Det < 0$). Equation (51) indicates that this is attained by $\Delta > 0$. When $\Delta = (1+\gamma)\Sigma + (1-\sigma)\gamma < 0$ (hence, $\sigma > 1$ must hold), (51) and (52) show that the Jacobian matrix of the dynamical system (48) and (49) exhibit $Det > 0$ and $Tr < 0$; the steady state is therefore a locally indeterminate sink.

It is clear that, in the closed-economy setting, the necessary and sufficient condition for equilibrium indeterminacy and belief-driven business cycles to arise under FIT is exactly the same as that under MT: i.e. $\Delta < 0$. This is due to the fact that a FIT central bank conducts open market operations in a gradual way such that the growth rate of nominal money supply exhibits a degree of inertia. I therefore establish the following proposition.

Proposition 4. *The macroeconomic stabilizing properties of flexible inflation targeting are exactly the same as those of nominal money growth targeting in a closed economy given by Proposition 1.*

When diversified currency holdings are allowed and hence the domestic country becomes a small-open economy, then the steady state under FIT is described by (36)-(39), and that (40) is replaced with $\pi = \bar{\pi}$. In the neighborhood of this steady state, the model's equilibrium conditions can be approximated by the following log-linear dynamical system:

$$
\begin{bmatrix} \dot{\hat{m}}_t \\ \dot{\hat{f}}_t \\ \dot{\hat{\lambda}}_t \\ \dot{\mu}_t \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{J} & \mathbf{x}_1 \\ \mathbf{x}_2 & 0 \end{bmatrix}}_{\mathbf{J}^\pi} \begin{bmatrix} \hat{m}_t - \hat{m} \\ \hat{f}_t - \hat{f} \\ \hat{\lambda}_t - \hat{\lambda} \\ \mu_t - \bar{\pi} \end{bmatrix}, \quad \hat{f}_0, \mu_0 \text{ given}, \tag{53}
$$

where $\mathbf{J}$ is a 3×3 Jacobian matrix that has the same expression as $\mathbf{J}^\mu$, given by (41), except that $\bar{\mu}$ presenting in $\mathbf{J}^\mu$ is replaced with $\bar{\pi}$; $\mathbf{x}_1$ and $\mathbf{x}_2$ denote the vectors of $[1, 0, 0]'$ and $[\phi\Gamma_1, \phi\Gamma_2, \frac{\phi(\bar{\pi} - \pi^*)}{\Sigma}]$, respectively.

It follows that the determinant and trace of the model's Jacobian matrix $\mathbf{J}^\pi$ are:

$$
Det(\mathbf{J}^\pi) = -\phi Det(\mathbf{J}^\mu) > 0 \quad \text{and} \quad Tr(\mathbf{J}^\pi) = Tr(\mathbf{J}^\mu) \gtreqless 0, \tag{54}
$$

implying that the number of eigenvalues of the Jacobian matrix $\mathbf{J}^\pi$ that have negative real parts can be zero, two, or four. Since the dynamical system (53) has two initial conditions for the real stock of foreign currency $\hat{f}_t$ and the nominal money growth rate μ_t, equilibrium determinacy is ensured if and only if $\mathbf{J}^\pi$ has two roots with negative real parts.

Under the baseline parameterization considered in Section 3, it turns out that the local stability properties of the steady state under FIT resemble those under MT (i.e. Figure 3) when $\phi = 45 \times 10^{-7}$. This value of ϕ implies a very sluggish adjustment pattern of the money growth rate; it therefore takes a long time for the domestic inflation rate to converge to the target level.

Figure 4 illustrates the case where ϕ takes on a higher value of 0.01, which represents a lower degree of sluggishness of money supply adjustment than when $\phi = 45 \times 10^{-7}$; the domestic inflation rate therefore converges to the target level at

a faster pace. By comparing Figures 3 and 4, it is clear that an increase in ϕ raises the level of π^* needed for equilibrium indeterminacy to occur for any combination of σ, δ and θ. I therefore derive the following proposition.

Proposition 5. *When diversified currency holdings are allowed, the macroeconomic stabilizing properties of flexible inflation targeting are qualitatively the same as those of nominal money growth targeting; in addition, the faster the domestic inflation rate converges to the target level, the higher the requisite level of π^* for equilibrium indeterminacy will be.*

5   Conclusion

I herein show that diversified currency holdings stabilize a simplified version of Farmer's (1997) indeterminate monetary economy. The determinacy result is robust to changing the monetary regime from MT to FIT, and to whether domestic and foreign currencies are Edgeworth substitutes or complements, or are additively separable in the household's preferences.

The paper can be extended in several directions. For example, it would be worthwhile to incorporate capital accumulation and analyze two-country models of international trade [Nishimura and Shimomura, 2002; Nishimura and Shimomura, 2006; Nishimura et al. 2010; Hu and Mino, 2013; Huang, et al., 2017]. I can also consider features that are commonly adopted in the new-Keynesian literature, such as price stickiness, wage rigidity, and investment adjustment costs, among others. In addition, the particular features of cryptocurrencies other than their function as a means of payment that is considered in this paper can be incorporated. These possible extensions will allow me to further enhance the understanding of the dynamic (in)stability effects of currency substitution. I plan to pursue these research projects in the future.

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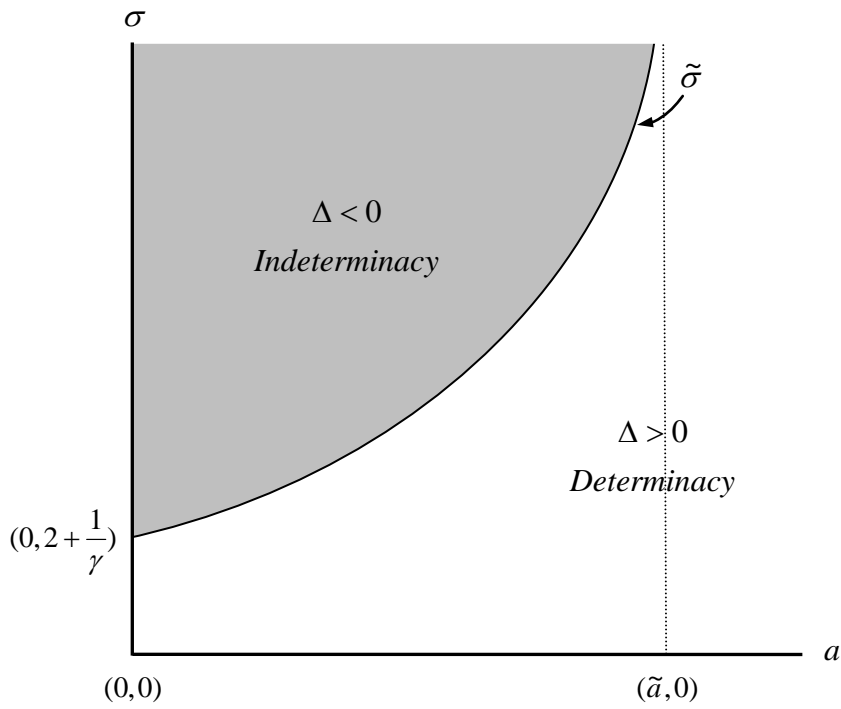
σ

$\tilde{\sigma}$

$\Delta < 0$

Indeterminacy

$\Delta > 0$

Determinacy

$(0, 2 + \dfrac{1}{\gamma})$

$(0,0)$

$(\tilde{a}, 0)$

a

Figure 1: Regions of Equilibrium (In)determinacy: Closed Economy

σ

Zone III

Indeterminacy occurs under very high π^* and σ

Zone I

Determinacy

$(0,1)$

Zone II

Determinacy

$\underline{\sigma}$

$(\underline{\theta}, 0)$

$(0,0)$

$(1,0)$

θ
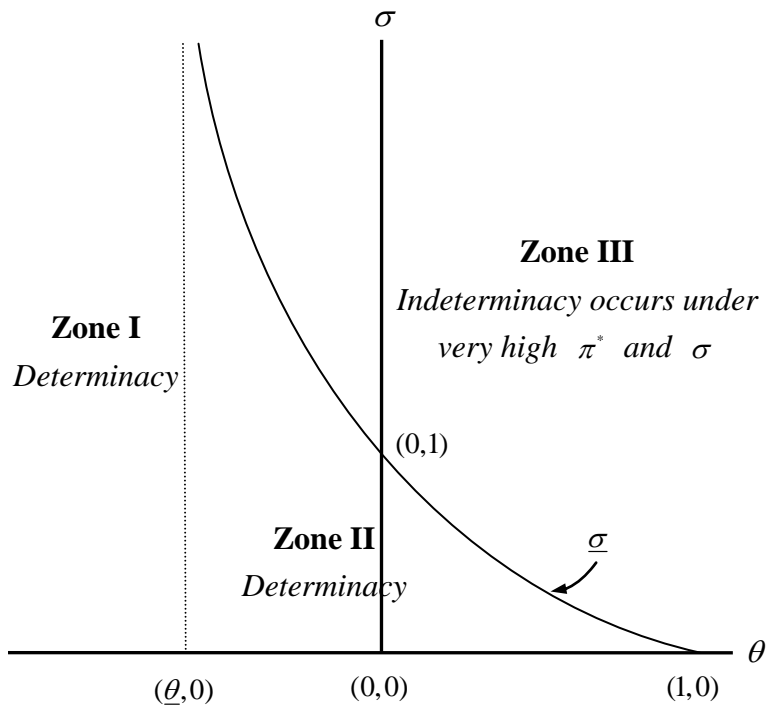
Figure 2: Regions of Equilibrium (In)determinacy: Open Economy and $\pi^* > \overline{\mu}$
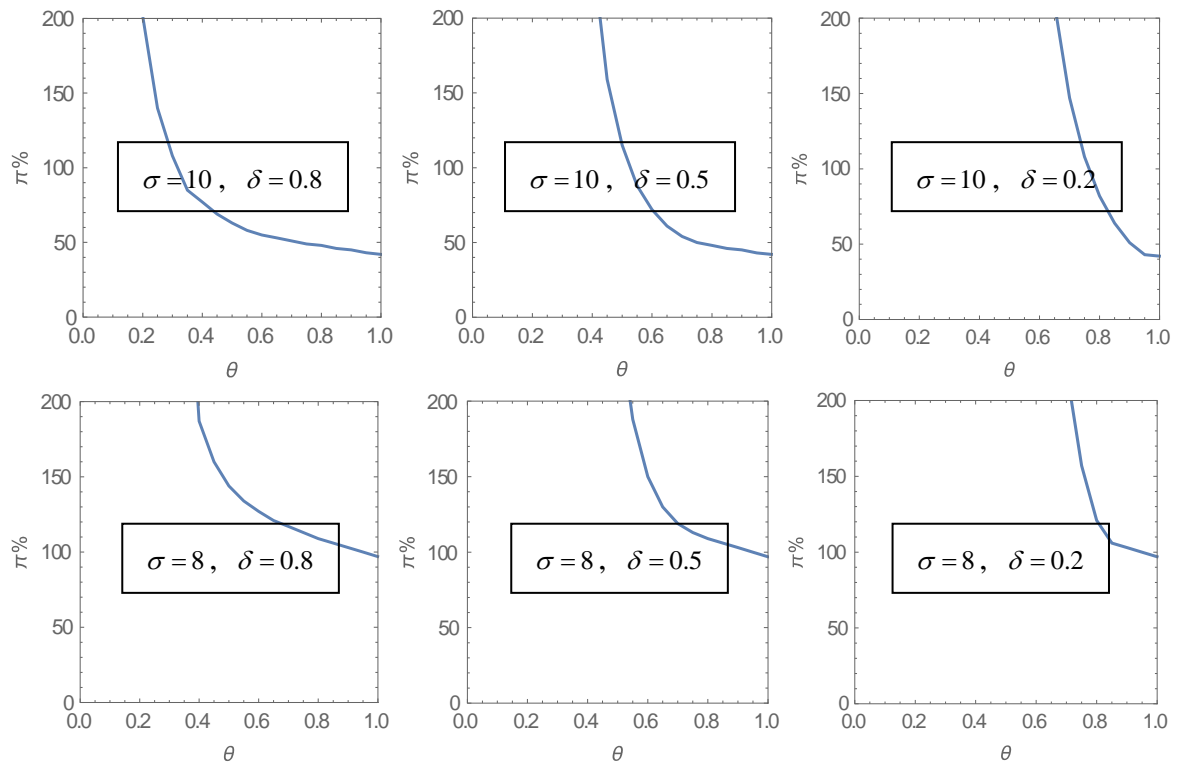
Figure 3: Requisite Annual Inflation Rate for Indeterminacy: MT, Zone III, or FIT with $\phi = 45 \times 10^{-7}$
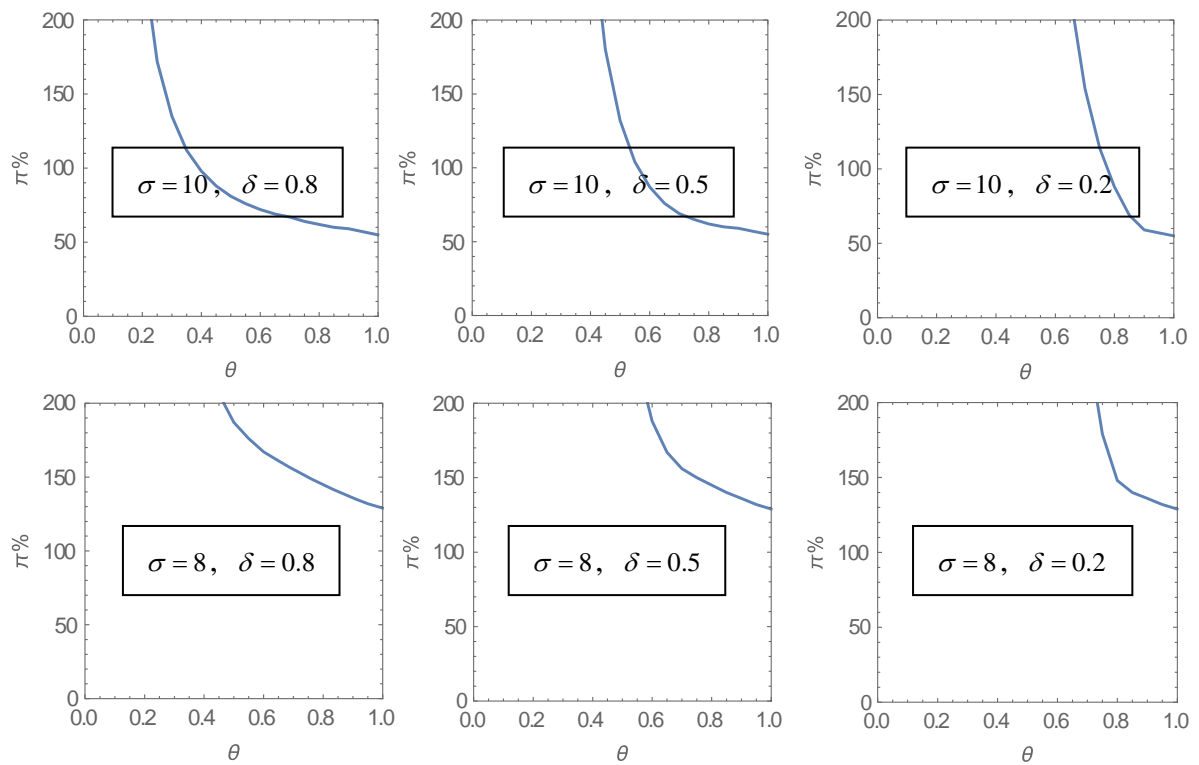

Figure 4: Requisite Annual Inflation Rate for Indeterminacy: FIT with $\phi = 0.01$