Optimal Taxation for R&D and Innovation†

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February 2020

Abstract

This paper studies the optimal taxation with asymmetric information on workers' talent and firms' R&D investment. A task-to-talent model is used to derive the optimal government policies based on the dynamic Mirrlees framework. In this model, the wage premium can be affected by the amount of each sector's production inputs because of the imperfect substitutability among them. To compress the wage inequality, this study suggests that governments reduce the labor taxes of high talent agents and increase capital tax and reduce R&D subsidy of top sector in production. Also, when non-verifiable R&D investment is considered, governments should increase high talent's labor tax and top sector's capital tax and reduce low talent's labor tax and bottom sector's capital tax in response. If R&D investment is observable, the positive externality of R&D only increases subsidy on R&D investment, leaving labor tax and capital tax unaffected, but if R&D investment is not observable, the positive externality reduces the optimal labor tax and capital tax regardless of type.

Keywords: Optimal income taxes, Private information
JEL classification:

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1. Introduction

Many governments nowadays are seeking effective ways to foster technology progress to prosper the economy. Hence, finding an optimal policy for governments to encourage innovation is an important issue for economists. One of a simple and straightforward way might be subsidies on R&D investment, but not all the effort in R&D can be observable by governments. To foster those unobservable R&D investment, what else government policies can encourage firms to invest more on R&D?. Besides, the positive spillovers of R&D indirectly make firms reap other firms' effort on innovation, causing firms save their spending on R&D. How a government can correct such R&D spillovers through policies? The main purpose of this paper is to answer these questions.

In this paper, we adapt talent-to-task framework. Agents are born with heterogeneous talent, which is private information. High talent workers not only have higher productivity on each task, but also have comparative advantage on difficult task than others. Also, our model allows for imperfectly substitutability among inputs of each task in production function. This setting provides the possibility that the relative wage of each type can be altered by the allocation choices of each agent and each sector. Wage premium can lead high talent workers willingly to take over the difficult tasks, but the other side of the coin is that high wage premium can make high types shirk more and benefit more from underreporting their true type, which will generate more social cost. In order to balance the tradeoff of cost and benefit from wage premium, and measure the distortion caused by information frictions, a series of government policies, including taxes on labor and capital tax, and a subsidy on R&D investment has been proposed in this paper. In addition, we discuss the optimal government policy under two scenarios: one is when government can detect the amount of each firm's R&D investment, and subsidy on R&D is feasible; the other is when R&D investment is not observable by governments, and thus R&D subsidy is out of question. We find that when R&D investment is not feasible, the effects on R&D subsidy will shift to taxes on labor and capital, depending on the elasticity of substitution between R&D investment and labor (or capital). Moreover, from our results on optimal policies, one can detect how the government affect spectrum of individuals' task choice through policies, namely sectoral shift effect.

Our paper is closest to the work of Ales, Kurnaz, and Sleet (2015). They construct static model with an inner-outer framework to derive a micro-found production function by the inner problem, and then based on such production function, they solve the planning problem in the outer problem. In this paper, we extend the model to dynamic structure and allow technology to be improved by R&D investment. Thus, our model can incorporate capital wedge and R&D wedge, .which are not available in their work. In addition, their main result on labor wedge can also be seen as a special case in our result on labor wedge: when the task spectrum of each type do not affect the production of final goods. In other words, if we shut down the sectoral shift effect, our labor wedge is back to their labor wedge.

Sectoral shift effect also appears in the work of Rothschild and Scheuer (2013). They also propose an inner-outer problem, but the content is very different from our work and Ales, Kurnaz, and Sleet (2015). Rothschild and Scheuer (2013) study on two-sector model, given the ratio of two sectors' aggregate labor input, the social planning problem is solved in the inner problem, and then find the optimal ratio that can generate the highest social welfare in the outer problem. Their model is also static and do not consider the technology change, so they only focus on labor wedge as well.

R&D wedge is first proposed in the work of Akcigit, Hanley and Stantcheva (2019), to our knowledge. The R&D spillovers are also considered in their framework, but different from our work, they focus on firms' heterogeneous ability on accumulating innovation, rather than workers' talent. They focus on subsidy on R&D investment and highlight the complementary between observable R&D investment and unobservable R&D effort, while we focus on the policy change when R&D subsidy is not feasible.

This paper is organize as follows: In section 2, we provide a two type toy model to explain the mechanism of optimal government in a possible simplest way, and then study a general talent-to-task model in section 3. In section 4, we do the numerical analysis. Finally, we conclude in section 5.

2. An Illustrative Example: A Two-type Model

A. Physical Environment

For sake of introducing the mechanism in a simplest way, we start with a simple two-type ($i \in \{L, H\}$) model. There are two kinds of agents: low talent ($i = L$) and high talent ($i = H$). The portion of type i agent is denoted by π_i and $\pi_L + \pi_H = 1$.

Agents. – Suppose agents live for T periods. All types of agents have an identical preference over consumption, $c_{i,t} > 0$ and work effort, $e_{i,t} > 0$, measured by the following separable utility function:

$$U_i = \sum_{t=1}^{T} \beta^{t-1} \left[u\left(c_{i,t}\right) - h\left(e_{i,t}\right) \right]. \tag{1a}$$

where $0 < \beta < 1$ is the discount factor. The function $u(\cdot)$ satisfies the Inada condition and the function $h(\cdot)$ satisfies $h' \geq 0, h'' \geq 0$.

An agent of talent i has productivity $a_i(v) > 0$ in the sector $v \in \{\underline{v}, \overline{v}\}$. A high talent agent not only have an absolute productive advantage in any sector: $a_H(v) > a_L(v)\ \ \forall\, v \in \{\underline{v}, \overline{v}\}$ but also have a comparative advantage in more complex task, which is the top sector: $\frac{a_H(\overline{v})}{a_L(\overline{v})} > \frac{a_H(\underline{v})}{a_L(\underline{v})}$. For simplicity, in this simple model, we assume that the advantage for the high type is sufficiently large that high type agents will choose to work at the top sector and low type agents will choose to work at the bottom sector. That is, if v_i represents type i agents' working sector, then we restricted $v_H = \overline{v}$ and $v_L = \underline{v}$. Later on, we will consider a general setting called the task-to-talent model that matches the agents' talent to goods sectors. In this simple model, the total labor input of the sector $v \in \{\underline{v}, \overline{v}\}$ is

$$l_t(\overline{v}) = \pi_H a_H(\overline{v}) e_{H,t} \text{ and } l_t(\underline{v}) = \pi_L a_L(\underline{v}) e_{L,t} \tag{1b}$$

Production. – The final good Y_t is produced by using intermediates as inputs. The production technology for the final good is

$$Y_t = \sum_{v \in \{\underline{v}, \overline{v}\}} y_t(v),$$

where $y_t(v)$ is an intermediate produced in sector v. For each sector $v \in \{\underline{v}, \overline{v}\}$, the production technology for the intermediate is:

$$y_t(v) = F^v\left(l_t(v), k_t(v); b_t(v)\right),$$

where $l_t(v)$ denotes the labor input in the sector v; $k_t(v)$ denotes the capital input in sector v, for simplicity reason, capital is assumed fully depreciated, and $b_t(v)$ denotes the technological level in the sector v. The function $F^v(l, k; b)$ satisfies $F_x^v > 0$, $F_{xx}^v < 0$ and $F_{xx'}^v > 0$ for any $x, x' \in \{l, k, b\}$ and $x \neq x'$.[1] Based on the production function, the marginal capital return of sector v is $R_t(v) = \frac{\partial Y_t}{\partial k_t(v)}$, and the marginal labor return of sector v is $w_t(v) = \frac{\partial Y_t}{\partial l_t(v)}$. Thus, the wage rate for a type i who works in the sector v_i is defined as the marginal return of effort, which is $w_{i,t} = w_t(v_i)a_i(v_i)$.

Innovation and R&D Spillovers. **–** The technology level $b_t(v)$ in each sector v evolves from the following innovation accumulation:

$$b_t(v) = A^v\left(b_{t-1}(v), q_t(v), \overline{b}_t\right), \tag{1c}$$

where $q_t(v)$ denotes the R&D investment, and $\overline{b}_t = \frac{1}{2}\left(b_t(\underline{v}) + b_t(\overline{v})\right)$ is the average technology level, which reflects the R&D spillovers in a positive way. The function $A^v\left(b_-, q, \overline{b}\right)$ also satisfies $A_x^v > 0$, $A_{xx}^v < 0$ and $A_{xx'}^v > 0$ for any $x, x' \in \left\{b_-, q, \overline{b}\right\}$ and $x \neq x'$. Later on, we will show that the presence of such spillovers can affect government's policies in different ways, depending on government's observability on R&D investment. The real cost of $q_t(v)$ is $M\left(q_t(v)\right)$, with $M' > 0$ and $M'' \geq 0$.

B. Optimal Policy

Tax Equilibrium of Laissez-faire Economy. . – Given a fixed government spending G_t, an tax equilibrium is, an allocation $\left\{c_{i,t}, e_{i,t}, k_{i,t}(v), q_t(v)\right\}_{i \in \{L,H\}, v \in \{\underline{v}, \overline{v}\}}$, a price profile, $\{w_t(v), R_t(v)\}_{v \in \{\underline{v}, \overline{v}\}}$, and a set of policy functions, including tax functions on labor income and capital income, $T_{l,t}$, $T_{k,t}$, and a subsidy function on R&D investment, $S_{q,t}$, such that

[1] Both of Cobb-Douglas function form and CES function form satisfy this condition.

(i) For each $i = 1, \dots, N$, $\{c_{i,t}, e_{i,t}, k_{i,t}(v)\}$ solves the utility maximization problem, which maximizes (1a) subject to the following budget constraints: for $t = 1,2, \dots, T$

$$c_{i,t} + \sum_{v=\underline{v},\overline{v}} k_{i,t+1}(v) \le \sum_{v=\underline{v},\overline{v}} \left[R_t(v) k_{i,t}(v) - T_{k,t}\left(k_{i,t}(v)\right)\right] + w_{i,t} e_{i,t} - T_{l,t}\left(w_{i,t} e_{i,t}\right),$$

(ii) Given the price profile $\{w_t(v), R_t(v)\}$, the allocation $\left\{e_{i,t}, k_{i,t}(v), q_t(v)\right\}$ solves the competitive firm's maximization problem, which is

$$\max \sum_{v=\underline{v},\overline{v}} F^v\left(l_t(v), k_t(v); b_t(v)\right) - w_t(v) l_t(v) - R_t(v) k_t(v) - M\left(q_t(v)\right) + S_{q,t}\left(q_t(v)\right)$$

subject to (1b), (1c), and $k_t(v) = \sum_{i=L,H} k_{i,t}(v)\pi_i$;

(iii) Goods market is clearing: $G_t + K_{t+1} + \sum_{i=L,H} c_{i,t}\pi_i \le \sum_{v=\underline{v},\overline{v}} F^v(l_t(v), k_t(v); b_t(v))$, where $K_{t+1} = \sum_{v=\underline{v},\overline{v}} k_{t+1}(v)$.

To characterize optimal tax equilibrium, we adapt the conventional approach in Mirrlees framework by considering the associated planning problem, and define wedges as the optimal implicit marginal tax (subsidy) rates to characterize distortions between a laissez-faire economy and the associated planning problem.[2] The planner's problem is stated formally below.

Social Planning Problem

Suppose that a social planner attached Pareto weight g_i to agents of type i, with $g_L + g_H = 1$. The social welfare function is:

$$W = g_L U_L + g_H U_H$$

subject to resource constraints for each period, $t = 1,2, \dots, T$

$$(\chi_t):\ \ Y_t \ge \sum_{i=L,H} c_{i,t}\pi_i + \sum_{v=\underline{v},\overline{v}} k_{t+1}(v) + M\left(q_t(v)\right)$$

and incentive constraint,[3]

$$(\eta):\ \ U_H \ge \sum_{t=1}^{T} \beta^{t-1}\left[u\left(c_{L,t}\right) - h\left(\frac{w_{L,t} e_{L,t}}{w_{H,t}}\right)\right].$$

Characterization of the optimal allocation in terms of wedges. Wedges are usually used to measure the distortion of a planning problem relative to a laissez-faire economy. To this end, in our

[2] The form of optimal tax system that can implement the solution of planner's problem in a decentralized economy is not unique. Literature has proposed many tax implementations to achieve that, e.g. Albanesi and Sleet (2006), Kocherlakota (2005). Chen and Liang (2019) also proposed a tax system that uses wedges directly as linear tax rates to implement the constrained optimal allocation.

[3] Agents have incentives to underreport their types for shirking in traditional Mirrlees model. Hence, in this two type model, we only impose an incentive constraint to prevent high type agents from reporting as low type agents.

model, three kinds of wedges are introduced, which are defined as follows:

(i). Labor wedge, defined as an implicit labor tax so that the efficient condition between consumption and labor holds:

$$\frac{\tau_i^{l_t}}{1-\tau_i^{l_t}} \equiv \frac{w_{i,t}u'(c_{i,t})}{h'(e_{i,t})} - 1.$$

(ii). Capital wedge, defined as an implicit capital tax so that the intertemporal Euler equation (after taxes) holds:

$$\frac{\tau_i^{k_t}(v)}{1-\tau_i^{k_t}(v)} \equiv \frac{\beta R_t(v)u'(c_{i,t})}{u'(c_{i,t-1})} - 1$$

(iii). R&D investment wedge, defined as implicit R&D investment subsidy, which characterizes the gap between marginal cost and marginal benefit of spending in R&D investment:

$$s^{q_t}(v) \equiv M'(q_t(v)) - \frac{\partial F_t^v}{\partial b_t(v)} \cdot \frac{\partial A^v}{\partial q_t(v)}$$

These wedges can be derived by solving the social planning problem. In this paper, we discuss two scenarios: one is the case that R&D investment is observable, the results shown in Proposition 1; the other is the case that R&D investment is not observable by the government, the results shown in Proposition 2.

Proposition 1. Wedges when R&D investment is observable

(i). **Labor wedges** for high type, $i = H$ and low type, $i = L$ are:

$$\frac{\tau_H^{l_t}}{1-\tau_H^{l_t}} = \underbrace{\frac{\beta^{t-1}u'(c_{H,t})\eta}{\chi_t \pi_H} \frac{h'\left(\frac{w_{L,t}}{w_{H,t}}e_{L,t}\right)}{h'(e_{H,t})}\left(\frac{w_{L,t}}{w_{H,t}}\frac{e_{L,t}}{e_{H,t}}\right)\phi_H^t}_{wage\ compression:<0} < 0.$$

$$\frac{\tau_L^{l_t}}{1-\tau_L^{l_t}} = \frac{\beta^{t-1}u'(c_{L,t})\eta}{\chi_t \pi_L}\left[\underbrace{1 - \frac{h'\left(\frac{w_{L,t}}{w_{H,t}}e_{t,L}\right)}{h'(e_{L,t})}\frac{w_{L,t}}{w_{H,t}}}_{Mirrlees:>0} + \underbrace{\frac{h'\left(\frac{w_{L,t}}{w_{H,t}}e_{t,L}\right)}{h'(e_{L,t})}\frac{w_{L,t}}{w_{H,t}}\phi_L^t}_{wage\ compression:>0}\right] > 0$$

where $\phi_i^t \equiv \frac{e_{i,t}}{w_{H,t}/w_{L,t}} \frac{\partial(w_{H,t}/w_{L,t})}{\partial e_{i,t}}$ for $i \in \{L, H\}$ is the cross relative wage elasticity with respect to type i's labor effort. Note that $\phi_H^t < 0$ and $\phi_L^t > 0$.[4]

(ii) **Capital wedges** for top sector, $v = \overline{v}$ and bottom sector, $v = \underline{v}$ are:

$$\frac{\tau_i^{k_t}(\overline{v})}{1-\tau_i^{k_t}(\overline{v})} = \underbrace{\frac{\beta^{t-1}\eta}{\chi_{t-1}} h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\left(\frac{w_{L,t}\cdot e_{L,t}}{w_{H,t}\cdot k_t(\overline{v})}\right)\kappa_{\overline{v}}^t}_{wage\ compression\ :>0} > 0 \quad \text{for } i = L, H$$

[4] All the signs of elasticities are calculated in Appendix A.1.

$$\frac{\tau_i^{k_t}\left(\underline{v}\right)}{1-\tau_i^{k_t}\left(\underline{v}\right)}=\underbrace{\frac{\beta^{t-1}\eta}{\chi_{t-1}}h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}k_t\left(\underline{v}\right)}\right)\kappa_{\underline{v}}^t}_{wage\ compression:<0}<0 \quad \text{for } i=L,H$$

where $\kappa_v^t \equiv \frac{k_t(v)}{w_{H,t}/w_{L,t}}\frac{\partial\left(w_{H,t}/w_{L,t}\right)}{\partial k_t(v)}$ for $v\in\{\underline{v},\overline{v}\}$ is the cross relative wage elasticity with respect to capital. Note that $\kappa_{\overline{v}}^t>0$ and $\kappa_{\underline{v}}^t<0$.

(iii) **R&D investment wedges** for high sector and low sector are:

$$s^{q_t}\left(\overline{v}\right)=\underbrace{\frac{-\beta^{t-1}\eta}{\chi_t}h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}b_t\left(\overline{v}\right)}\right)\varphi_{\overline{v}}^t\left[\frac{\partial A^{\overline{v}}}{\partial q_t\left(\overline{v}\right)}+\frac{\partial A^{\overline{v}}}{\partial\overline{b}_t}\frac{\partial\overline{b}_t}{\partial q_t\left(\overline{v}\right)}\right]}_{wage\ compression:<0}+\underbrace{\frac{\partial F_t^{\overline{v}}}{\partial\overline{b}_t}\frac{\partial\overline{b}_t}{\partial q_t\left(\overline{v}\right)}}_{Pigouvian\ correction:>0}$$

$$s^{q_t}\left(\underline{v}\right)=\underbrace{\frac{-\beta^{t-1}\eta}{\chi_t}h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}b_t\left(\underline{v}\right)}\right)\varphi_{\underline{v}}^t\left[\frac{\partial A^{\underline{v}}}{\partial q_t\left(\underline{v}\right)}+\frac{\partial A^{\underline{v}}}{\partial\overline{b}_t}\frac{\partial\overline{b}_t}{\partial q_t\left(\underline{v}\right)}\right]}_{wage\ compression:\ >0}+\underbrace{\frac{\partial F_t^{\underline{v}}}{\partial\overline{b}_t}\frac{\partial\overline{b}_t}{\partial q_t\left(\underline{v}\right)}}_{Pigouvian\ correction:>0}$$

where $\varphi_v^t \equiv \frac{b_t(v)}{w_{H,t}/w_{L,t}}\frac{\partial\left(w_{H,t}/w_{L,t}\right)}{\partial b_t(v)}$ for $v\in\{\underline{v},\overline{v}\}$ is the cross relative wage elasticity with respect to R&D investment. Note that $\varphi_{\overline{v}}^t>0$ and $\varphi_{\underline{v}}^t<0$.

Proof. See Appendix A.2.

To measure the distortions between planning problem and laissez-faire economy, three kinds of effects are considered in determining the optimal implicit taxes and subsidies, which are:

1) **Mirrlees term:** This term is caused by asymmetric information on agents' productivity. To avoid high type agents shirking and thus encourage high type agents to make sufficient labor effort, this effect increases low type's labor wedge to reduce the benefit from pretending as low type. This term only appears in low type's labor wedge, and does not affect capital wedges and R&D investment wedges.
2) **Wage compression term:** This term is induced by the imperfect substitution among different sectors' labor, capital and R&D investment. Thus, the wage premium, $w_{H,t}/w_{L,t}$ can be affected by the amount of labor, capital and R&D investment of each sector. Also, larger wage inequality makes high type agents have higher motives to underreport, which arise the cost for the planner to provide the correct incentives. To reduce such cost, compressing the wage inequality can help maximize the social welfare in the most efficient way. In such simple two sector model, top sector's labor, bottom sector's capital, and bottom sector's R&D investment can help relieve the wage inequality. Therefore, decreasing top sector's labor tax and bottom sector's capital tax, and increasing bottom sector's subsidy on R&D investment can be seen as efficient ways to reduce the social cost. It is worthy to note that capital wedge do not depend on agents' type in our setting, although it is defined

to be related with agents' marginal rate of substitution between consumption of two adjacent periods. This means that the capital tax only depends on which sector agents invest in rather than who invests the capital.

3) **Pigouvian correction for R&D spillovers term:** In the decentralization economy, firms make their decision without considering the positive R&D spillovers, which makes the R&D investment by firms would be lower than the social optimal level. Hence, the spillovers also generate distortions between planning problem and decentralization economy. To encourage firms to invest more in R&D, increasing subsidy on R&D investment is a straightforward result.

Proposition 2. Wedges when R&D investment is unobservable

(i). **Labor wedge** for high type, $i = H$ and low type, $i = L$ are

$$\frac{\tau_H^{l_t}}{1-\tau_H^{l_t}} = \frac{\beta^{t-1}\eta u'(c_{H,t})}{\chi_t \pi_H h'(e_{H,t})} h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\frac{w_{L,t}e_{L,t}}{w_{H,t}e_{H,t}}\left[\underset{\textit{wage compression}:\,<0}{\phi_H^t} + \underbrace{\varphi_H^t \cdot \xi_{H,\bar{v}}^t}_{\textit{R\&D Unobservability}:\,>0}\right] + \underbrace{\left[\frac{-u'(c_{H,t})}{\pi_H h'(e_{H,t})}\frac{\partial F_t^{\bar{v}}}{\partial \bar{b}_t}\frac{\partial \bar{b}_t}{\partial q_t(\bar{v})}\frac{b_t(\bar{v})/e_{H,t}}{\frac{\partial A^{\bar{v}}}{\partial q_t(\bar{v})}}\xi_{H,\bar{v}}^t\right]}_{\textit{Pigouvian corection}:\,<0}$$

$$\frac{\tau_L^{l_t}}{1-\tau_L^{l_t}} = \frac{\beta^{t-1}u'(c_{L,t})\eta}{\chi_t \pi_L}\left[\underbrace{1-\frac{h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\frac{w_{L,t}}{w_{H,t}}}{h'(e_{L,t})}}_{\textit{Mirrlees}:\,>0} + \frac{h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\frac{w_{L,t}}{w_{H,t}}}{h'(e_{L,t})}\left(\underset{\textit{wage compression}:\,>0}{\phi_L^t} + \underbrace{\varphi_L^t \cdot \xi_{L,\underline{v}}^t}_{\textit{R\&D Unobservability}:\,<0}\right)\right] + \underbrace{\left[\frac{-u'(c_{L,t})}{\pi_L h'(e_{L,t})}\frac{\partial F_t^{\underline{v}}}{\partial \bar{b}_t}\frac{\partial \bar{b}_t}{\partial q_t(\underline{v})}\frac{b_t(\underline{v})/e_{L,t}}{\frac{\partial A^{\underline{v}}}{\partial q_t(\underline{v})}}\xi_{L,\underline{v}}^t\right]}_{\textit{Pigouvian corection}:\,<0}$$

where $\xi_{i,v}^t \equiv \frac{e_{i,t}}{b_t(v)}\frac{\partial b_t(v)}{\partial e_{i,t}} = \frac{e_{i,t}}{b_t(v)}\frac{\partial A^v}{\partial q_t(v)}\frac{\partial q_t(v)}{\partial e_{i,t}} > 0$ for $(i, v) \in \{(L, \underline{v}), (H, \bar{v})\}$ is the elasticity of substitution between R&D investment and labor effort.

(ii) **Capital wedges** for high sector and low sector are: for any $i \in \{L, H\}$

$$\frac{\tau_i^{k_t}(\bar{v})}{1-\tau_i^{k_t}(\bar{v})} = \frac{\beta^{t-1}\eta}{\chi_{t-1}} h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}k_t(\bar{v})}\right)\left(\underset{\textit{wage compression}:\,>0}{\kappa_{\bar{v}}^t} + \underbrace{\varphi_H^t \cdot \zeta_{H,\bar{v}}^t}_{\textit{R\&D Unobservability}:\,>0}\right) + \underbrace{\left[\frac{-\chi_t}{\chi_{t-1}}\cdot\frac{\partial F_t^{\bar{v}}}{\partial \bar{b}_t}\frac{\partial \bar{b}_t}{\partial q_t(\bar{v})}\frac{b_t(\bar{v})/k_t(\bar{v})}{\frac{\partial A^{\bar{v}}}{\partial q_t(\bar{v})}}\zeta_{\bar{v}}^t\right]}_{\textit{Pigouvian correction}:\,<\,0}$$

$$\frac{\tau_i^{k_t}(\underline{v})}{1-\tau_i^{k_t}(\underline{v})} = \frac{\beta^{t-1}\eta}{\chi_{t-1}} h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\frac{w_{L,t}e_{L,t}}{w_{H,t}k_t(\underline{v})}\left(\underset{\textit{wage compression}:\,<0}{\kappa_{\underline{v}}^t} + \underbrace{\varphi_{\underline{v}}^t \cdot \zeta_{\underline{v}}^t}_{\textit{R\&D Unobservability}:\,<0}\right) + \underbrace{\left[\frac{-\chi_t}{\chi_{t-1}}\cdot\frac{\partial F_t^{\underline{v}}}{\partial \bar{b}_t}\frac{\partial \bar{b}_t}{\partial q_t(\underline{v})}\frac{b_t(\underline{v})/k_t(\underline{v})}{\frac{\partial A^{\underline{v}}}{\partial q_t(\underline{v})}}\zeta_{\underline{v}}^t\right]}_{\textit{Pigouvian correction}\,<\,0}$$

where $\zeta_v^t \equiv \frac{k_t(v)}{b_t(v)}\frac{\partial b_t(v)}{\partial k_t(v)} = \frac{k_t(v)}{b_t(v)}\frac{\partial A^v}{\partial q_t(v)}\frac{\partial q_t(v)}{\partial k_t(v)} > 0$ for $v \in \{\underline{v}, \bar{v}\}$ is the elasticity of substitution between R&D investment and capital.

Proof. See Appendix A.2.

When R&D investment is not observable by public, the R&D investment wedge is no longer feasible.

As a results, some adjustments in labor wedges and capital wedges are necessary:

1) **R&D unobservability term:** When R&D investment is unobservable, firms privately choose their R&D investment level. The optimal R&D investment level of each sector can be privately affected by the labor input and the capital input, which causes the labor wedge and capital wedge can indirectly influence firms' spending on R&D investment. This term can be seen as the wage compression term of the R&D investment wedges that shifts to labor wedges and capital wedges. Since labor inputs and R&D investment affect the wage premium in opposite direction, this R&D unobservability term and wage compression term display opposite impact on labor wedge. On the other hand, capital inputs and R&D investment affect the wage inequality in the same direction, and hence in the capital wedge R&D unobservability term and wage compression term have the same signs.
2) **Shifts of Pigouvian correction for R&D spillovers:** When R&D investment is observable, the positive externality of R&D increases the subsidy on R&D investment for the sake of raising the R&D investment to the social optimal level. This R&D spillovers have no impact on labor taxes and capital taxes when R&D investment wedge is feasible. However, when R&D investment is not observable, such positive externality becomes affecting the labor wedge and capital wedge, because labor choices and capital choices can indirectly affect the private R&D investment choices. The government intervenes for the same reason as increasing subsidy on R&D investment. When subsidy on R&D investment is no longer available, such positive R&D spillovers decrease labor wedges and capital wedges.

3. A Talent-to-Task Model

In this section, we extend our model to a framework with task assignment.

A. Physical Environment

Agents. – In this section, the number of agents' type is N-types. That is, $i = 1,2, \dots, N$. The portion of type i is π_i. As before, the utility function U_i is assumed separable on consumption $c_{i,t}$ and work effort $e_{i,t}$:

$$U_i = \sum_{t=1}^{T} \beta^{t-1} \left[u\left(c_{i,t}\right) - h\left(e_{i,t}\right) \right]$$

Talent and task. – The two-sector model now is extended to a continuum of tasks $v \in [\underline{v}, \overline{v}]$ model. The tasks are differentiated by complexity. Different types agents are differentiated by different productivity of each task, which can be seemed as agents' talent. Denote $a_i(v)$ as type i's productivity in task v, and it satisfies the following assumption.

Assumption 1. The talent function $a_i: [\underline{v}, \overline{v}] \rightarrow \mathbb{R}_+$, $i \in \{1, \ldots, N\}$ is differentiable and satisfies

(i) Absolute advantage: for each $i = 1, \ldots, N-1$, $a_{i+1}(v) > a_i(v)$ for any v.

(ii) Comparative advantage: for each $i = 1, \ldots, N-1$, $\frac{a_{i+1}(v)}{a_i(v)} > \frac{a_{i+1}(v')}{a_i(v')}$ for any $v > v'$.

Production. – In this section, the final good's production function Y_t is followed by works of Teulings (1995), Costinot and Vogel (2010), and Ales, Kurnaz and Sleet (2015). All of these papers study in talent-to-task framework. The major difference with their works is that our model incorporates physical capital, k and R&D level, b, which are assumed to affect the weight of each task's labor input in the final good aggregator through function $\Phi(k, b)$. The final good's production function can be formally expressed as follows.

$$Y_t = \left\{ \int_{\underline{v}}^{\overline{v}} \Phi\left(k_t(v), b_t(v)\right) \left[l_t(v) \right]^{\frac{\varepsilon-1}{\varepsilon}} dv \right\}^{\frac{\varepsilon}{\varepsilon-1}} . \tag{2a}$$

where $l_t(v)$ denotes the labor input in the sector v; $k_t(v)$ denotes the capital input in sector v, and $b_t(v)$ denotes the technological level in the sector v. As before, the R&D level, b is accumulated by the function $b_t(v) = A^v(b_{t-1}(v), q_t(v), \bar{b}_t)$, where $q_t(v)$ denotes the R&D investment with cost $M(q_t(v))$, and $M' > 0$ and $M'' \geq 0$. Finally, based on the final good production function, the marginal capital return of sector v is $R_t(v) = \frac{\partial Y_t}{\partial k_t(v)}$, and task-specific wage is $w_t(v) = \frac{\partial Y_t}{\partial l_t(v)}$, and hence, the wage rate for a type i working in the sector v is $w_t(v) a_i(v)$. To maximize one's labor income, type i agent will choose task v that can generate highest marginal return of labor effort. That is,

$$w_{i,t} = \max_v w_t(v) a_i(v) .$$

To characterize the optimal allocation and measure the wedges, followed by the work of Ales, Kurnaz and Sleet (2015), we used the inner-outer method. In the inner step, conditional on effort assignment, $\{e_{i,t}\}$, capital assignment, $\{k_t(v)\}$, and R&D level, $\{b_t(v)\}$, we solve the task assignment problem, and construct an indirect, micro-founded production function of final good. In the outer step, the problem is simply the social planning problem, based on the production function that is derived by the inner problem. And then, using this planning problem to solve the constrained efficient allocation and wedges. Similar to section 2, we also compare the results under two scenarios: when R&D investment is observable and when it is unobservable.

B. Inner Problem

The goal of inner problem is to match the talent i to task v. Let $\lambda_{i,t}(v)$ denote the portion of ith talent agents who work at sector v and thus the labor input in the sector v is:

$$l_t(v) = \sum_{i=1}^{N} \lambda_{i,t}(v) a_i(v) e_{i,t} . \tag{3a}$$

Proposition 3. Given Assumption 1 and the effort, capital, R&D level profile $\{e_{i,t}, k_t(v), b_t(v)\}$, there is a tuple of threshold task $\{\tilde{v}_{i,t}\}_{i=1}^{N-1}$ such that

(i) $\lambda_{i,t}(v) = 0$ for any $v \in [\underline{v}, \tilde{v}_{i-1,t}) \cup (\tilde{v}_{i,t}, \overline{v}]$,

(ii) For any $v \in (\tilde{v}_{i-1,t}, \tilde{v}_{i,t})$

$$\lambda_{i,t}(v) = \frac{[a_i(v)]^{\varepsilon-1}[\Phi(k_t(v), b_t(v))]^{\varepsilon}}{[B(\tilde{v}_{i-1,t}, \tilde{v}_{i,t})]^{\varepsilon}}\pi_i$$

where $B(\tilde{v}_{i-1,t}, \tilde{v}_{i,t}) = \left\{\int_{\tilde{v}_{i-1,t}}^{\tilde{v}_{i,t}} [a_i(v)]^{\varepsilon-1}\left[\Phi\left(k_t(v), b_t(v)\right)\right]^{\varepsilon} dv\right\}^{\frac{1}{\varepsilon}}$,

(iii) $\lambda_{i,t}(v)$ in (ii) is also the solution of the following problem:

$$Y_t = \max_{\lambda_{i,t}(v)}\left\{\sum_{i=1}^{N}\int_{\tilde{v}_{i-1,t}}^{\tilde{v}_{i,t}} \Phi\left(k_t(v), b_t(v)\right)\left[\lambda_{i,t}(v) a_i(v) e_{i,t}\right]^{\frac{\varepsilon-1}{\varepsilon}} dv\right\}^{\frac{\varepsilon}{\varepsilon-1}}$$

subject to $\pi_i - \int_{\tilde{v}_{i-1,t}}^{\tilde{v}_{i,t}} \lambda_{i,t}(v) dv \geq 0$,

(iv) $\{\tilde{v}_{i,t}\}_{i=1}^{N-1}$ satisfies that $\frac{w_{i+1,t}}{w_{i,t}} = \frac{a_{i+1}(\tilde{v}_{i,t})}{a_i(\tilde{v}_{i,t})}$ for $i = 1, \ldots, N-1$.

Proof. See Appendix A.3.

Based on Proposition 3 (i), (ii) and (iii) , the final production function can be written as the following indirect and micro-founded production function

$$\begin{aligned}
Y_t &= \left\{\int_{\underline{v}}^{\overline{v}} \Phi\left(k_t(v), b_t(v)\right)\left[l_t(v)\right]^{\frac{\varepsilon-1}{\varepsilon}} dv\right\}^{\frac{\varepsilon}{\varepsilon-1}} \\
&\overset{\text{by equation } (3a)}{=} \left\{\int_{\underline{v}}^{\overline{v}} \Phi\left(k_t(v), b_t(v)\right)\left[\sum_{i=1}^{N} \lambda_{i,t}(v) a_i(v) e_{i,t}\right]^{\frac{\varepsilon-1}{\varepsilon}} dv\right\}^{\frac{\varepsilon}{\varepsilon-1}} \\
&\overset{\text{by Prop3 (i)}}{=} \sup_{\lambda_{i,t}(v)}\left\{\sum_{i=1}^{N}\int_{\tilde{v}_{i-1,t}}^{\tilde{v}_{i,t}} \Phi\left(k_t(v), b_t(v)\right)\left[\lambda_{i,t}(v) a_i(v) e_{i,t}\right]^{\frac{\varepsilon-1}{\varepsilon}} dv\right\}^{\frac{\varepsilon}{\varepsilon-1}} \\
&\overset{\text{by Prop3 (ii)\&(iii)}}{=} \left\{\sum_{i=1}^{N}\int_{\tilde{v}_{i-1,t}}^{\tilde{v}_{i,t}} \Phi\left(k_t(v), b_t(v)\right)\left[\frac{\left[a_i(v)\right]^{\varepsilon-1}\left[\Phi\left(k_t(v); b_t(v)\right)\right]^{\varepsilon}}{\left[B\left(\tilde{v}_{i-1,t}, \tilde{v}_{i,t}\right)\right]^{\varepsilon}}\pi_i a_i(v) e_{i,t}\right]^{\frac{\varepsilon-1}{\varepsilon}} dv\right\}^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left\{\sum_{i=1}^{N} B\left(\tilde{v}_{i-1,t}, \tilde{v}_{i,t}\right)\left[\pi_i e_{i,t}\right]^{\frac{\varepsilon-1}{\varepsilon}}\right\}^{\frac{\varepsilon}{\varepsilon-1}} = \left\{\sum_{i=1}^{N}\left\{\int_{\tilde{v}_{i-1,t}}^{\tilde{v}_{i,t}}\left[a_i(v)\right]^{\varepsilon-1}\left[\Phi\left(k_t(v), b_t(v)\right)\right]^{\varepsilon} dv\right\}^{\frac{1}{\varepsilon}}\left[\pi_i e_{i,t}\right]^{\frac{\varepsilon-1}{\varepsilon}}\right\}^{\frac{\varepsilon}{\varepsilon-1}}
\end{aligned}$$

In other words, given agents' talent function $a_i(v)$, the final goods' production function depends on each type's labor effort $\{\pi_i e_{i,t}\}_{i=1}^{N}$, each sector's capital input and R&D level, $\{k_t(v), b_t(v)\}$, and the tuple of threshold task $\{\tilde{v}_{i,t}\}_{i=1}^{N-1}$. Therefore, the final good's production function can be written as the following form:

$$Y_t \equiv \hat{Y}_t\left(\left\{\pi_i e_{i,t}\right\}_{i=1}^{N}, \left\{k_t(v), b_t(v)\right\}; \left\{\tilde{v}_{i,t}\right\}_{i=1}^{N-1}\right),$$

where the thresholds $\{\tilde{v}_{i,t}\}$ must satisfy Proposition 3 (iv).

Suppose α_j is defined as $\alpha_j(\tilde{v}_{i,t}) \equiv \frac{a_{j+1}(\tilde{v}_{i,t})}{a_j(\tilde{v}_{i,t})}$, then Proposition 3 (iv) $\frac{w_{i+1,t}}{w_{i,t}} = \frac{a_{i+1}(\tilde{v}_{i,t})}{a_i(\tilde{v}_{i,t})}$ implies that

$$\tilde{v}_{i,t} = \alpha_i^{-1}\left(\frac{w_{i+1,t}}{w_{i,t}}\right).$$

By Assumption 1, we know that $\alpha_i(v)$ is an increasing function, and so is α_i^{-1}. Thus, higher wage premium $\frac{w_{i+1,t}}{w_{i,t}}$ will cause higher task threshold $\tilde{v}_{i,t}$. In other words, higher wage premium $\frac{w_{i+1,t}}{w_{i,t}}$ will push type i agents to raise the upper bound of their task range, which means that agents are willing to take over tasks that are more complex.

C. Outer Problem

The outer problem is simply the social planning problem at the production function, $Y_t = \tilde{Y}_t(\{\pi_i e_{i,t}\}_{i=1}^N, \{k_t(v), b_t(v)\}; \{\tilde{v}_{i,t}\}_{i=1}^{N-1})$, which is induced by the inner problem. As before, denote by g_i as social planner attached Pareto weight to agents of type i, with $\sum_{i=1}^N g_i = 1$. The social welfare function is:

$$W = \sum_{i=1}^N g_i U_i$$

subject to resource constraints for each period, t,

$$(\chi_t):\quad \tilde{Y}_t\left(\left\{\pi_i e_{i,t}\right\}_{i=1}^N, \left\{k_t(v), b_t(v)\right\}; \left\{\tilde{v}_{i,t}\right\}_{i=1}^{N-1}\right) \geq \sum_{i=1}^N c_{i,t}\pi_i + \int_{\underline{v}}^{\overline{v}} \left[k_{t+1}(v) + M\left(q_t(v)\right)\right] dv$$

and local downward incentive constraints,[5] for $i = 2, \dots, N$,

$$(\eta_i):\quad U_i \geq \sum_{t=1}^T \beta^{t-1}\left[u\left(c_{i-1,t}\right) - h\left(\frac{w_{i-1,t} e_{i,t}}{w_{i,t}}\right)\right].$$

Proposition 4. Wedges when R&D investment is observable

(i). **Labor wedge** for type i is:

$$\frac{\tau_i^{l_t}}{1-\tau_i^{l_t}} = \underbrace{\frac{\beta^{t-1}u'(c_{i,t})\eta_{i+1}}{\chi_t \pi_i}\left[1 - \frac{h'\left(\frac{w_{i,t}}{w_{i+1,t}} e_{i,t}\right)\frac{w_{i,t}}{w_{i+1,t}}}{h'(e_{i,t})}\right]}_{Mirrlees} + \frac{u'(c_{i,t})}{\pi_i h'(e_{i,t}) e_{i,t}} \sum_{j=1}^{N-1}\left\{\underbrace{\frac{\beta^{t-1}\eta_{j+1}h'\left(\frac{w_{j,t}}{w_{j+1,t}}e_{j,t}\right)\frac{w_{j,t}}{w_{j+1,t}}e_{j,t}}{\chi_t}}_{wage\ compression} - \underbrace{\frac{\partial \hat{Y}_t}{\partial \tilde{v}_{j,t}} \frac{D\alpha_j^{-1}\left(\frac{w_{j+1,t}}{w_{j,t}}\right)}{\frac{w_{j,t}}{w_{j+1,t}}}}_{\text{sectoral shift}}\right\}\phi_{i,j}^t ,$$

[5] A well-known implication of the Spenec-Mirrlees single crossing condition is that non-local incentive constraints do not bind. Plus, agents are usually assumed having incentives to underreport their types in traditional Mirrlees model. Hence, we only focus on local downward incentive constraints in this section.

where $\phi_{i,j}^{t} \equiv \frac{e_{i,t}}{w_{j+1,t}/w_{j,t}} \frac{\partial\left(w_{j+1,t}/w_{j,t}\right)}{\partial e_{i,t}}$ is the cross relative wage elasticity with respect to type i's labor effort.

(ii) **Capital wedge** for sector v, which is independent on agent's type i is:

$$\frac{\tau_i^{k_t}(v)}{1-\tau_i^{k_t}(v)} = \frac{1}{\chi_{t-1}k_t(v)} \sum_{j=1}^{N-1} \left\{ \underbrace{\beta^{t-1}\eta_{j+1}h'\left(\frac{w_{j,t}e_{j,t}}{w_{j+1,t}}\right)\frac{w_{j,t}e_{j,t}}{w_{j+1,t}}}_{\text{wage compression}} - \underbrace{\chi_t \frac{\partial \hat{Y}_t}{\partial \tilde{v}_{j,t}} \frac{D\alpha_j^{-1}\left(\frac{w_{j+1,t}}{w_{j,t}}\right)}{\frac{w_{j,t}}{w_{j+1,t}}}}_{\text{sectoral shift}} \right\} \kappa_{v,j}^{t}$$

where $\kappa_{v,j}^{t} \equiv \frac{k_t(v)}{w_{j+1,t}/w_{j,t}} \frac{\partial\left(w_{j+1,t}/w_{j,t}\right)}{\partial k_t(v)}$ is the cross relative wage elasticity with respect to capital.

(iii) **R&D investment wedge** for sector v is:

$$s^{q_t}(v) = \frac{1}{b_t(v)\chi_t} \sum_{j=1}^{N-1} \left\{ \underbrace{-\beta^{t-1}\eta_{j+1}h'\left(\frac{w_{j,t}e_{j,t}}{w_{j+1,t}}\right)\frac{w_{j,t}e_{j,t}}{w_{j+1,t}}}_{\text{wage compression}} + \underbrace{\frac{\partial \hat{Y}_t}{\partial \tilde{v}_{j,t}} \frac{D\alpha_j^{-1}\left(\frac{w_{j+1,t}}{w_{j,t}}\right)}{\frac{w_{j,t}}{w_{j+1,t}}}}_{\text{sectoral shift}} \right\} \left[\frac{\partial A_t^v}{\partial q_t(v)} + \frac{\partial A_t^v}{\partial \bar{b}_t} \cdot \frac{\partial \bar{b}_t}{\partial q_t(v)} \right] \varphi_{v,j}^{t} + \underbrace{\frac{\partial \hat{Y}_t}{\partial b_t(v)} \cdot \frac{\partial A_t^v}{\partial \bar{b}_t} \cdot \frac{\partial \bar{b}_t}{\partial q_t(v)}}_{\textit{Pigouvian correction}}$$

where $\varphi_{v,j}^{t} \equiv \frac{b_t(v)}{w_{j+1,t}/w_{j,t}} \frac{\partial\left(w_{j+1,t}/w_{j,t}\right)}{\partial b_t(v)}$ is the cross relative wage elasticity with respect to R&D investment.

Proof. See Appendix A.4.

Similar to the simple two-type model in section 2, wedges include (1) Mirrlees term, which characterizes a distortion induced by asymmetric information on agent's productivity, (2) wage compression term, which characterizes how wage premium change affects social welfare cost for providing correct incentives. (3) Pigouvian correction term, which characterizes a distortion induced by positive R&D spillovers between planning problem and decentralization economy.

If labor effort $e_{i,t}$, capital $k_t(v)$, and R&D level $b_t(v)$ enlarge the wage inequality, $\frac{w_{j+1,t}}{w_{j,t}}$, which implies that the cross relative wage elasticities, $\phi_{i,j}^t$, $\kappa_{v,j}^t$, and $\varphi_{v,j}^t$ are positive, then this higher wage inequality will enhance the cost for agents to reveal their true type. Therefore, wage compression term will increase labor wedge (taxes) and capital wedge (taxes), and decrease R&D investment wedge (subsidy). On the other hand, if factors of production, $e_{i,t}$, $k_t(v)$, $b_t(v)$ relieve the wage inequality, $\frac{w_{j+1,t}}{w_{j,t}}$, then the cross relative wage elasticities, $\phi_{i,j}^t$, $\kappa_{v,j}^t$, and $\varphi_{v,j}^t$ will be negative, which decrease the social cost for providing incentives. As a result, wage compression term will decrease labor wedge (taxes) and capital wedge (taxes), and increase R&D investment wedge (subsidy).

Sectoral Shift Term

In this talent-to-task framework, besides these three terms affecting wedges, there exists one more effect, named "sectoral shift effect", which characterizes a distortion between social optimal choice on task thresholds and individual optimal choice on task thresholds.

In this model, task thresholds $\{\tilde{v}_{j,t}\}$ are individually determined by wage premium $\left\{\frac{w_{j+1,t}}{w_{j,t}}\right\}$ and agent's comparative advantage on productivity $\frac{a_{j+1}(v)}{a_j(v)}$. When wage premium $\left\{\frac{w_{j+1,t}}{w_{j,t}}\right\}$ is affected by factors of production, $e_{i,t}$, $k_t(v)$, $b_t(v)$, it may also indirectly affect individual's optimal choice on task thresholds $\{\tilde{v}_{j,t}\}$, causing agents to shift their task choice.

The optimal social optimal choice on task thresholds is determined by the condition $\frac{\partial \hat{Y}_t}{\partial \tilde{v}^*_{j.t}} = 0$, If the actual task threshold $\tilde{v}_{j,t}$ makes $\frac{\partial \hat{Y}_t}{\partial \tilde{v}_{j,t}} > 0$ (or $\frac{\partial \hat{Y}_t}{\partial \tilde{v}_{j,t}} < 0$), then in order to maximize the production, the planner tends to raise (or compress) wage premium $\frac{w_{j+1,t}}{w_{j,t}}$, which will induces higher (or lower) task threshold $\tilde{v}_{j,t}$. To do so, the government will reduce (or increase) the labor/capital tax and increase (or reduce) the R&D subsidy for any factors of production, $e_{i,t}$, $k_t(v)$, $b_t(v)$ that can increase (or reduce) the wage premium $\frac{w_{j+1,t}}{w_{j,t}}$. That is the reason why the sectoral shift term displays opposite movement with the term $\frac{\partial \hat{Y}_t}{\partial \tilde{v}_{j,t}}\phi^t_{i,j}$ and $\frac{\partial \hat{Y}_t}{\partial \tilde{v}_{j,t}}\kappa^t_{v,j}$ in labor wedge and capital wedge respectively, and be co-movement with the term $\frac{\partial \hat{Y}_t}{\partial \tilde{v}_{j,t}}\varphi^t_{v,j}$ in R&D investment wedge.

Proposition 5. Wedges when R&D investment is unobservable

(i). **Labor wedge** for type i

$$\frac{\tau_i^{l_t}}{1-\tau_i^{l_t}} = \underbrace{\frac{\beta^{t-1}u'(c_{i,t})\eta_{i+1}}{\chi_t\pi_i}\left[1-\frac{h'\left(\frac{w_{i,t}}{w_{i+1,t}}e_{i,t}\right)\frac{w_{i,t}}{w_{i+1,t}}}{h'(e_{i,t})}\right]}_{\text{Mirrlees}} + \frac{u'(c_{i,t})}{\pi_i h'(e_{i,t})e_{i,t}}\sum_{j=1}^{N-1}\left\{\underbrace{\frac{\beta^{t-1}\eta_{j+1}}{\chi_t}h'\left(\frac{w_{j,t}e_{j,t}}{w_{j+1,t}}\right)\frac{w_{j,t}e_{j,t}}{w_{j+1,t}}}_{\text{wage compression}} - \underbrace{\frac{\partial\hat{Y}}{\partial\tilde{v}_j}\frac{D\alpha_j^{-1}\left(\frac{w_{j+1,t}}{w_{j,t}}\right)}{\frac{w_{j,t}}{w_{j+1,t}}}}_{\text{sectoral shift}}\right\}\left[\underbrace{\phi^t_{i,j}}_{\textit{direct effect}} + \underbrace{\int_{\underline{v}}^{\bar{v}}\varphi^t_{v,j}\cdot\xi^t_{i,v}dv}_{\textit{R\&D Unobservability}}\right]$$
$$\underbrace{-\frac{u'(c_{i,t})}{\pi_i h'(e_{i,t})}\frac{d\hat{Y}_t}{d\bar{b}_t}\int_{\underline{v}}^{\bar{v}}\frac{\partial\bar{b}_t}{\partial q_t(v)}\cdot\frac{b_t(v)/e_{i,t}}{\frac{\partial A^v}{\partial q_t(v)}}\xi^t_{i,v}dv}_{\textit{Pigouvian corection}}$$

where $\xi^t_{i,v} \equiv \frac{e_{i,t}}{b_t(v)}\frac{\partial b_t(v)}{\partial e_{i,t}} = \frac{e_{i,t}}{b_t(v)}\frac{\partial A^v}{\partial q_t(v)}\frac{\partial q_t(v)}{\partial e_{i,t}}$ is the elasticity of substitution between R&D investment and labor effort.

(ii) **Capital wedges** for sector v, which is independent on agent's type i is:

$$\frac{\tau_i^{k_t}(v)}{1-\tau_i^{k_t}(v)} = \frac{1}{\chi_{t-1}k_t(v)}\sum_{j=1}^{N-1}\left\{\underbrace{\beta^{t-1}\eta_{j+1}h'\left(\frac{w_{j,t}e_{j,t}}{w_{j+1,t}}\right)\frac{w_{j,t}e_{j,t}}{w_{j+1,t}}}_{\text{wage compression}} - \underbrace{\chi_t\frac{\partial\hat{Y}}{\partial\tilde{v}_j}\frac{D\alpha_j^{-1}\left(\frac{w_{j+1,t}}{w_{j,t}}\right)}{\frac{w_{j,t}}{w_{j+1,t}}}}_{\text{sectoral shift}}\right\}\left[\underbrace{\kappa^t_{v,j}}_{\textit{direct effect}} + \underbrace{\int_{\underline{v}}^{\bar{v}}\varphi^t_{v',j}\cdot\zeta^t_{v,v'}dv'}_{\textit{R\&D Unobservability}}\right] \underbrace{-\frac{\chi_t}{\chi_{t-1}}\frac{d\hat{Y}_t}{d\bar{b}_t}\int_{\underline{v}}^{\bar{v}}\frac{\partial\bar{b}_t}{\partial q_t(v')}\cdot\frac{b_t(v')/k_t(v)}{\frac{\partial A^v}{\partial q_t(v')}}\zeta^t_{v,v'}dv'}_{\textit{Pigouvian corection}}$$

where $\zeta^t_{v,v'} \equiv \frac{k_t(v)}{b_t(v')}\frac{\partial b_t(v')}{\partial k_t(v)} = \frac{k_t(v)}{b_t(v')}\frac{\partial A^{v'}}{\partial q_t(v')}\frac{\partial q_t(v')}{\partial k_t(v)}$ is the elasticity of substitution between R&D investment and capital.

Proof. See Appendix A.4.

Similar to the simple two-type model, in the task-to-talent model, when R&D investment is not observable, R&D wedge is no longer available, and all effects on R&D investment wedge in Proposition 4 will transform into affecting the labor wedge and the capital wedge indirectly through the R&D unobservability term and Pigouvian correct term in Proposition 5. Pigouvian correct term that is only present in R&D investment wedge in Proposition 4 is also affect labor wedge and capital wedge in Proposition 5. This is because the labor effort and capital allocation will indirectly affect agents' private choice on R&D investment.

4. The model at work: quantitative solutions

In this section, we demonstrate our model quantitatively, based on the environment of Section 2 and Section 3 and compute the amount of labor wedge, capital wedge and R&D wedge for each type of agents. To this end, we set specific function forms and corresponding parameter values for preference and production as follows.

Agents are assumed distributed uniformly across an interval of talents, $a_i \in [\underline{a}, \overline{a}]$, which is normalized to $[1,2]$. In our quantitative example, we draw one million agents ($N = 10^6$) from this interval to depict the numerical results in figures, which are shown in Figure 1-Firgure 5. Agents are set to have two periods of working lives, $T = 2$. Each period represents 20 years.

Agent's Preference. Following Ales et al (2015), we assume agent's utility function is of the form

$$U_i = \sum_{t=1}^{T} \beta^{t-1} \left[\log\left(c_{i,t}\right) - \frac{\left(e_{i,t}\right)^{1+\gamma}}{1+\gamma} \right]$$

and set the Frisch elasticity for labor supply to $1/\gamma = 0.75$. The discount rate is set at 4% per annum, which gives $\beta = (0.96)^{20} = 0.442$ for 20 years.

Final goods Production. Final good production is assembled from an interval of intermediate goods (or production sectors), $v_i \in [\underline{v}, \overline{v}]$, which is normalized to $[1,2]$. For simplicity, we assume that there exists an one-to-one correspondence between agents' talent a_i and production sector v_i. The sector v_i represents the sector that the type i agent is working for. Each sector uses effective labor $\pi_i a_i e_{i,t}$, physical capital $k_{i,t}$ and technology level $b_{i,t}$ as inputs to produce intermediate goods. We use Cobb-Douglus function form as the production function for each intermediate good; therefore, the final production function form is of the following form.

$$Y_t = \frac{1}{N}\sum_{i=1}^{N}\left(\pi_i a_i e_{i,t}\right)^{\rho_1}\cdot\left(k_{i,t}\right)^{\rho_2}\cdot\left(b_{i,t}\right)^{\rho_3} \tag{4a}$$

where $\rho_1 = 0.6, \rho_2 = 0.2, \rho_3 = 0.2$.

Innovation and R&D. As for technology level $b_{i,t}$, we set that is accumulated by the following CES form, and is fully depreciated in 20 years, $\delta_b = 1$.

$$b_{i,t} = A\left(b_{t-1}, q_{i,t}, \bar{b}_t\right) = (1-\delta_b)b_{t-1} + \left[\omega_b\left(q_{i,t}\right)^{1-\rho} + (1-\omega_b)\left(\bar{b}_t\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

where $\rho_1 = 1.1$, and $q_{i,t}$ denotes the R&D investment with cost function $M_t\left(q_{i,t}\right) = q_{i,t}^2$. The term $\bar{b}_t$ represents the average technology level, which characterizes the R&D spillovers at period t. In the quantitative result, we discuss and compare both of scenarios that such R&D spillovers exist or not. In the benchmark model, which is assumed that there is no R&D spillovers, and thus we set $\omega_b = 1$. Later, for comparison, we also consider an alternative case with R&D spillovers exists, in which we set $\omega_b = 0.9$.

Optimal tax results. With the specific function form and parameter values, we are ready to solve the planning problem numerically. The planning problem is the same as the ones in Section 2 and Section 3, which maximizes the welfare function $\int U_i di$ subject to resource constraints and incentive compatible constraints.[6] In the benchmark, the R&D investment is assumed to be observable, but for comparison, we also consider the case that R&D investment is unobservable. In this case, subsidy on R&D investment is not available, and firm will invest in R&D to meet the condition: marginal cost equals to marginal revenue for one more unit of R&D investment. In other words, R&D wedge, which is defined as marginal cost minus marginal revenue of R&D investment, must be restricted to zero when R&D investment is unobservable.

Comparison with other production functions.

The final goods production we use in this paper have two features: one is that the inputs of each sectors have imperfect substitutability, and the other is that the production function incorporates endogenous accumulated R&D level. To compare our results with the conventional production functions, besides using (4a) as production technology, we also consider two alternative production technology as follows.

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[6] Suppose that $\tilde{U}(i, i^r)$ denotes the lifetime utility of an agent whose truth type is i and reporting type is i^r. Then, the incentive compatible constraint(ICC) suggest that $U_i = \max_{i^r} \tilde{U}(i, i^r)$. Thus, when type is continuous and utility function is differentiate with i, the ICC implies the envelope condition $\frac{\partial U_i}{\partial i} = \frac{\partial \tilde{U}(i,i^r)}{\partial i}\Big|_{i^r=i}$. To reduce the number of constraints, we follow the approach in Ales et al (2015), which set the range of type as an interval in quantitative analysis and replace the incentive compatible constraints by the envelope conditions.

(1) **Mirrlees** framework: In the classical Mirrlees model, only one sector in producing goods. Although agents have different productivity, a_i the labor or other inputs have perfect substitutability among types. Hence, for comparison, we also plot the wedges under the following production function (dotted red line) in Figure 1.

$$Y_t = \left(\frac{1}{N}\sum_{i=1}^{N}\pi_i a_i e_{i,t}\right)^{\rho_1} \cdot \left(\frac{1}{N}\sum_{i=1}^{N}k_{i,t}\right)^{\rho_2} \cdot \left(\frac{1}{N}\sum_{i=1}^{N}b_{i,t}\right)^{\rho_3} \tag{4b}$$

(2) **Ales et al** (2015) framework: In their paper, they also consider the imperfect substitutability for inputs among sectors, but they do not consider R&D investment in their framework. In Figure 5 (dashed blue line), we extend their static model to dynamic model by shut down channel of R&D investment into the following production function

$$Y_t = \frac{1}{N}\sum_{i=1}^{N}\left(\pi_i a_i e_{i,t}\right)^{\frac{\rho_1}{\rho_1+\rho_2}} \cdot \left(k_{i,t}\right)^{\frac{\rho_2}{\rho_1+\rho_2}} \tag{4c}$$

Finally, in Figure 1, the solid line represents wedges of our benchmark model, which use (4a) as production function.

[Insert Figure 1 here]

As can be seen, R&D wedge is only available in our framework. Mirrlees framework displays zero capital wedge and zero R&D wedge, and the labor wedge is also zero at top and bottom. Ales et al' framework has progressive capital wedge, and our capital wedge is less aggressive than theirs. The labor wedge of Ales et al (2015) is negative in highest type and is positive in lowest type. Our labor wedge has similar trend but is a little bit lower than their labor wedge. Finally, in our benchmark, the numerical result shows that R&D wedge is decreasing with type, and be positive at bottom and negative at top, which is consistent with the result in Proposition 1.

Comparison with non-verifiable R&D and R&D spillovers.

In our benchmark case, we set R&D investment is observable and R&D spillovers are not present. To see the effect of R&D unobservability term and Pigouvian correction term in Proposition 1-2 and 4-5, we must compute the cases when R&D investment is not observable or when R&D spillovers are present. As a result, we discuss the following four situations in this subsection:

Scenario 1: When R&D spillovers are not present and R&D investment is observable. (solid line)
Scenario 2: When R&D spillovers are not present and R&D investment is unobservable. (dashed line)
Scenario 3: When R&D spillovers are present and R&D investment is observable. (dash-dotted line)
Scenario 4: When R&D spillovers are present and R&D investment is unobservable. (dotted line)

Figure 2 displays the wedge results of Scenario 1 and 2; Figure 3 displays the wedge results of Scenario 3 and 4; Figure 4 displays the wedge results of Scenario 1 and 3; Figure 5 displays the wedge results of Scenario 2 and 4. These four cases are consistent with the result in Proposition 1, which prove that labor wedge (tax) and R&D wedge (subsidy, if it is available) are negative for highest type and positive for lowest type, while capital wedge (tax) is positive for highest type and negative for lowest type. Comparing these four cases quantitatively could shed light on the interaction among several theoretical effects in this paper.[7]

The effect of non-verifiable R&D.

In Figure 2, we find that comparing with the case that R&D is observable (solid line), when R&D investment is not observable (dotted line), the labor wedge and capital wedge will be lower in low type and be higher in high type. This result is consistent with what Proposition 2 demonstrates that R&D unobservability term raises high type's labor wedge and capital wedge and reduce the low type's labor wedge and capital wedge. However, when R&D spillover exists, Figure 3 shows that labor wedge and capital wedge will be reduced for almost every agent due to the unobservability R&D. The difference between Figure 2 and Figure 3 is caused by Pigouvian term. Proposition 1 and Proposition 3 show that when R&D is observable, Pigouvian term only appears in R&D wedge, but when R&D is not observable, Pigouvian term will transfer to labor wedge and capital wedge, like Proposition 2 and Proposition 4 shows. These results suggest that when R&D is not observable, governments should absolutely decrease low type's labor tax and capital tax, while for high type, the direction of tax change actually depends on the magnitude of R&D spillovers effect.

[Insert Figure 2 and Figure 3 here]

The R&D spillovers effect.

Figure 4 and Figure 5 compare situations with and without R&D spillovers. In Figure 4, we find that R&D spillovers have greatly increased R&D wedge but have little effect on labor wedge and capital wedge, while in Figure 5, when R&D is unobservable and R&D subsidy is not feasible, R&D spillovers decrease labor wedge and capital wedge significantly. These results suggest that to correct the positive externality of R&D, governments should increase subsidy on R&D investment if it is observable, but if R&D investment is not verifiable, decreasing labor tax and capital tax may indirectly help correct this externality.

[Insert Figure 4 and Figure 5 here]

[7] Figure 1-5 display the labor wedges, capital wedge and R&D wedge in the first period, wedges in other periods also have the similar shapes and trends; therefore, we only display wedges of first period.

5. Concluding Remarks

In this paper, we focus on how the endogenous R&D investment affect the optimal government policies. We use talent-to-task model, where agents are heterogeneous on talent and firms have heterogeneous tasks to integrate the final goods. Agents with different talent are driven by wages premium to take over different complicity tasks. Agents' talent is private information and R&D investment may also be not verifiable by the government, which causes the government has urge to design a series of policies that can not only provide the correct incentive for agents and firms to work and invest in R&D respectively, but also maximizes the social welfare through redistribution.

Three government policy tools are used in this paper: labor wedge (tax), capital wedge (tax), and R&D investment wedge (subsidy). In a simple two-type model without R&D spillovers, we found that the labor wedge is negative for the high talent and positive for the low talent, while the capital wedge is opposite. Capital wedge is positive for top sector and is negative for bottom sector. As for R&D investment wedge, it is negative for top sector and positive for low sector. When R&D spillovers are present, R&D investment wedge increases regardless of sector, and labor wedge and capital wedge are not affected, but in the case when R&D investment are not verifiable by governments, R&D spillovers will reduce the labor wedge and capital wedge regardless of type. If we rule out the R&D spillovers effect, the unobservable R&D investment will increase high type's labor wedge and capital wedge, but reduce low type's labor wedge and capital wedge.

In the general model, we show that the signs of these wedges depend on the fact that these factors of production, such as labor, capital and R&D investment will increase or decrease wage inequality. Higher wage inequality rises the benefit of pretending as low talent, which makes the government has to endure higher social cost for proving the correct incentive for agents to work. To relieve the pressure, government policies must be targeted to compress the wage inequality, which is shown as wage compression term in wedges. On the other hand, higher wage premium encourages agents to take over tasks with higher wages, which is shown as sectoral shift term in wedges. The optimal government policies are set to balance the cost and benefit from wage inequality. Besides wage compression term and sectoral shift terms, wedges also include Mirrlees term, which is a tradition result in Mirrlees literature for providing correct incentives, Pigouvian correction term, which is set to encourage firms to invest in R&D at social optimal level, and R&D unobservability term when R&D investment wedge is not feasible.

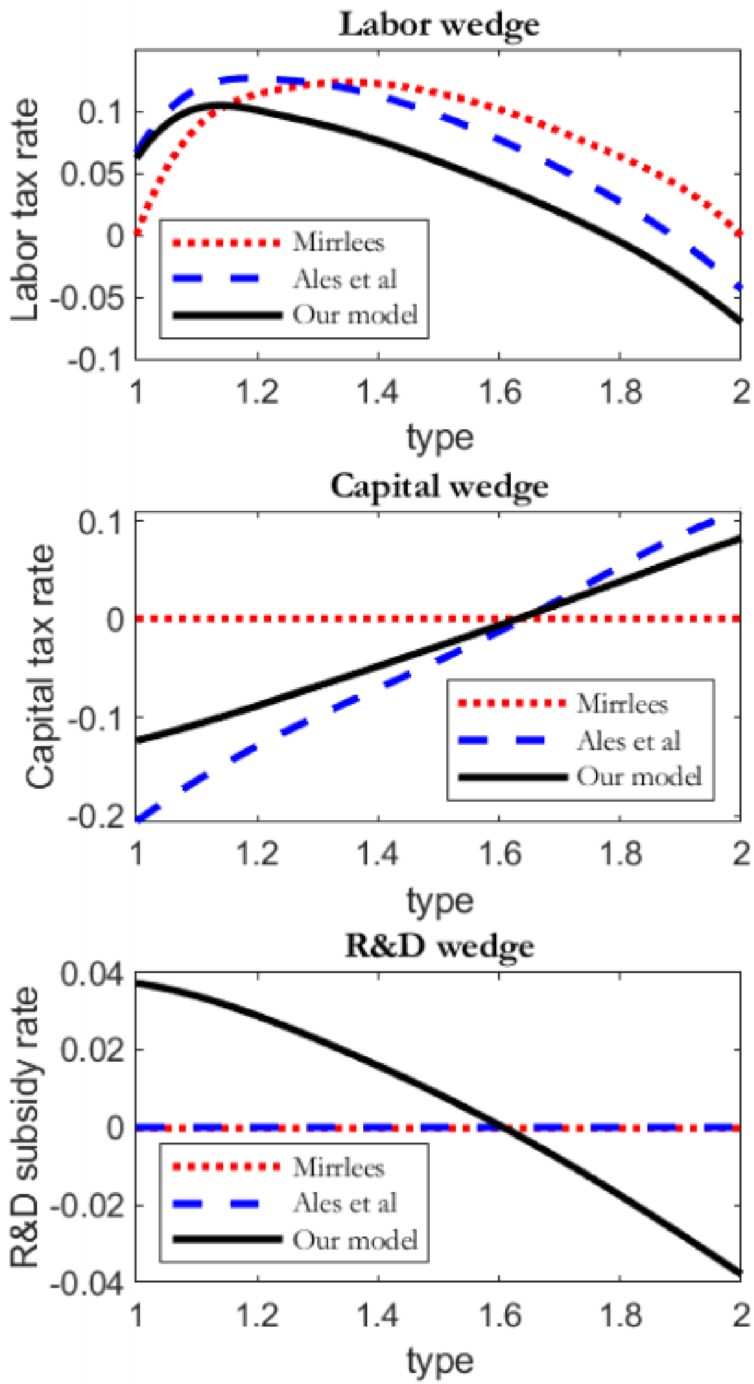

Figure 1 Wedges comparison with different production functions

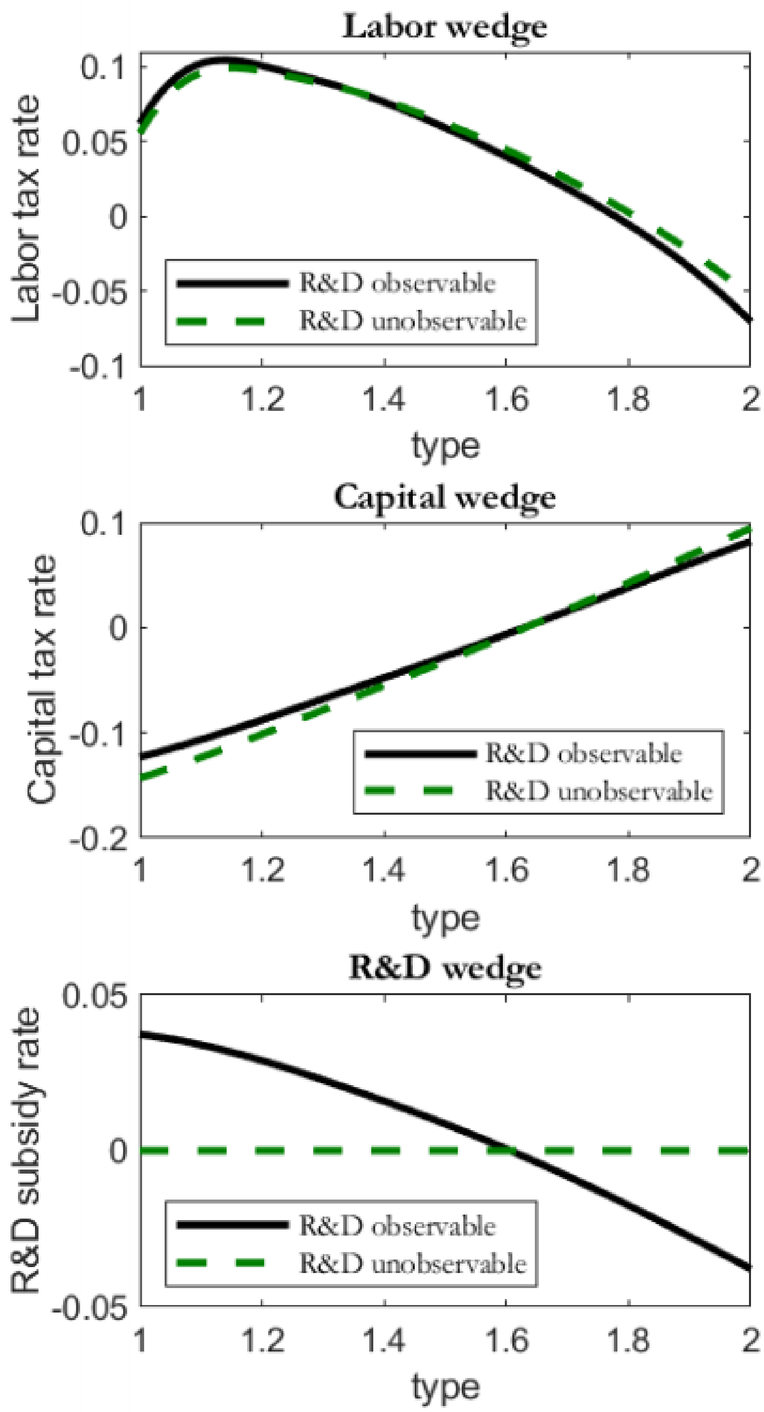

Figure 2. When R&D spillovers are not present

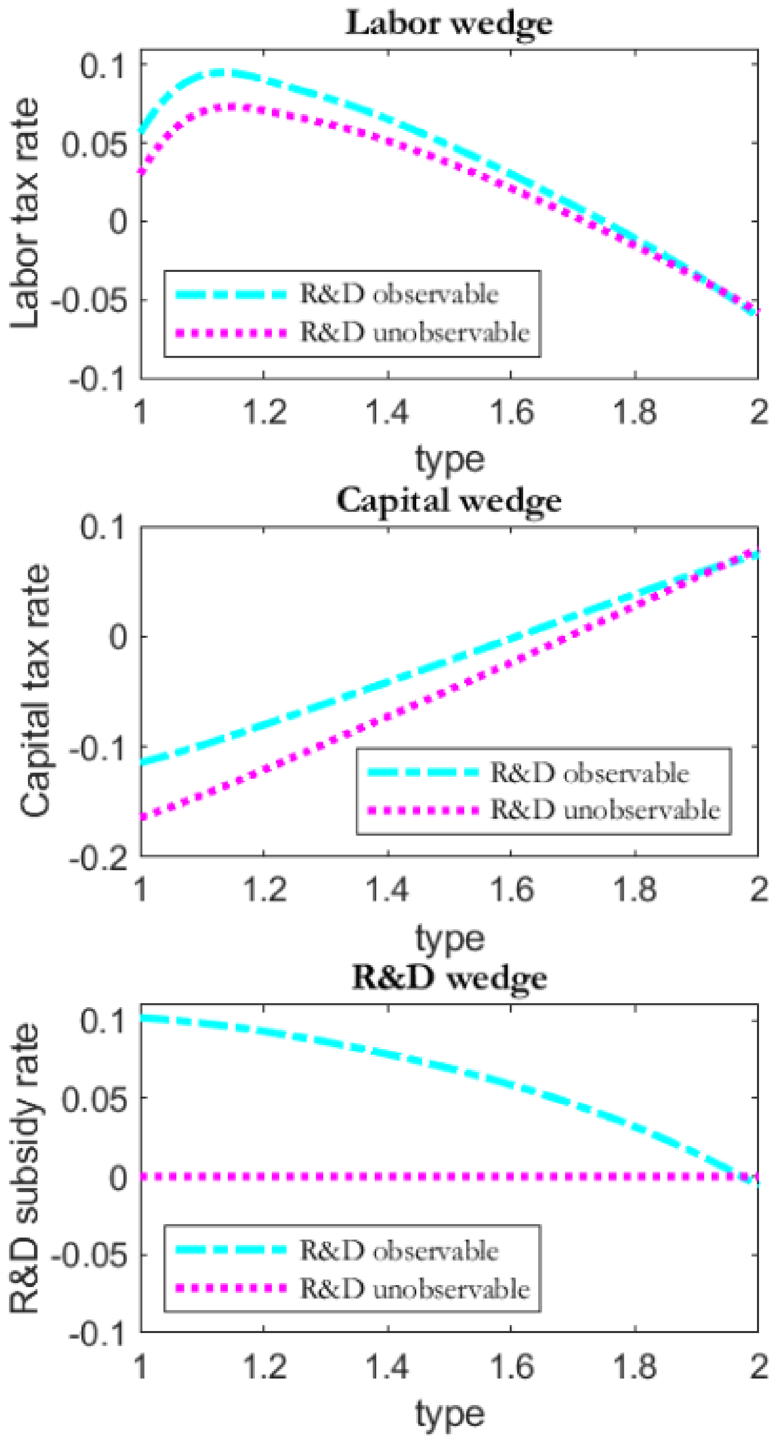

Figure 3. When R&D spillovers are present

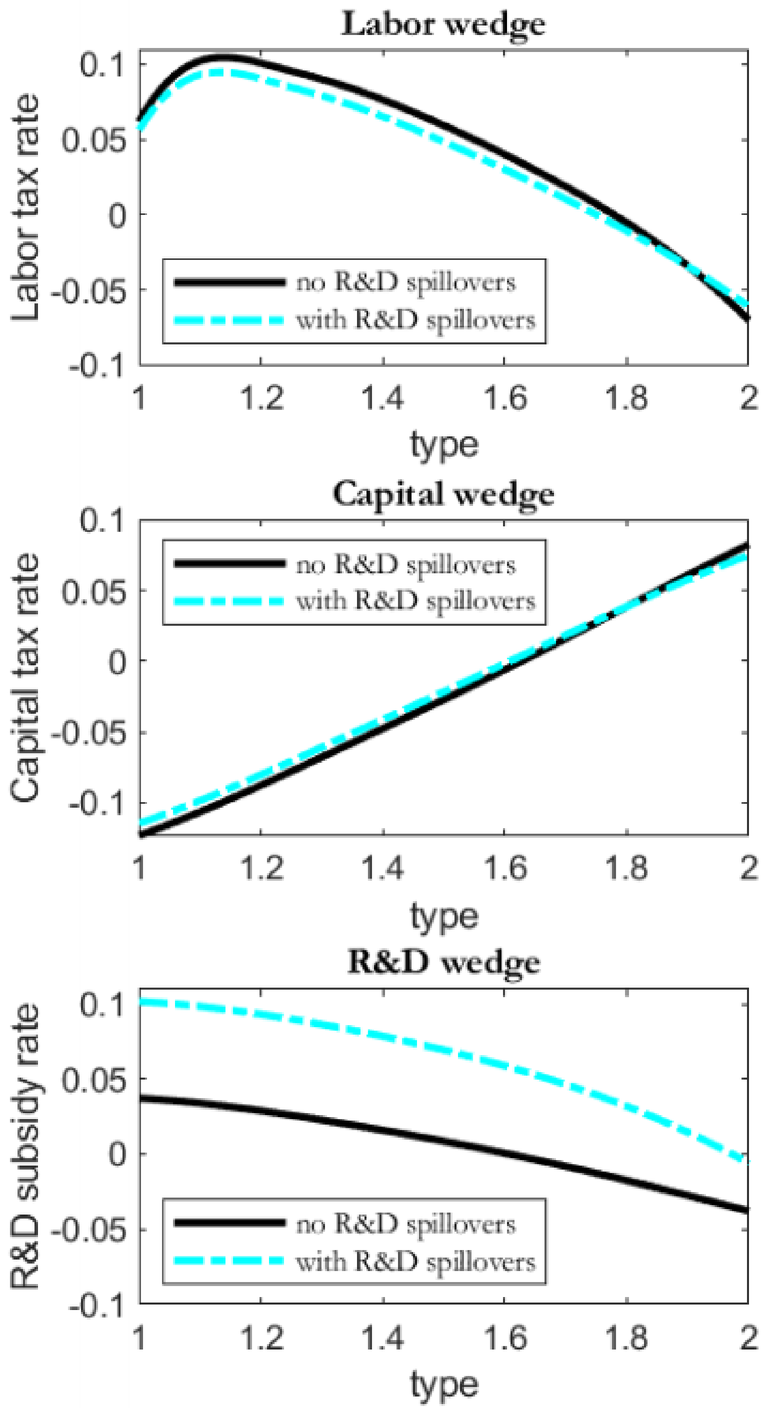

Figure 4. When R&D investment is observable

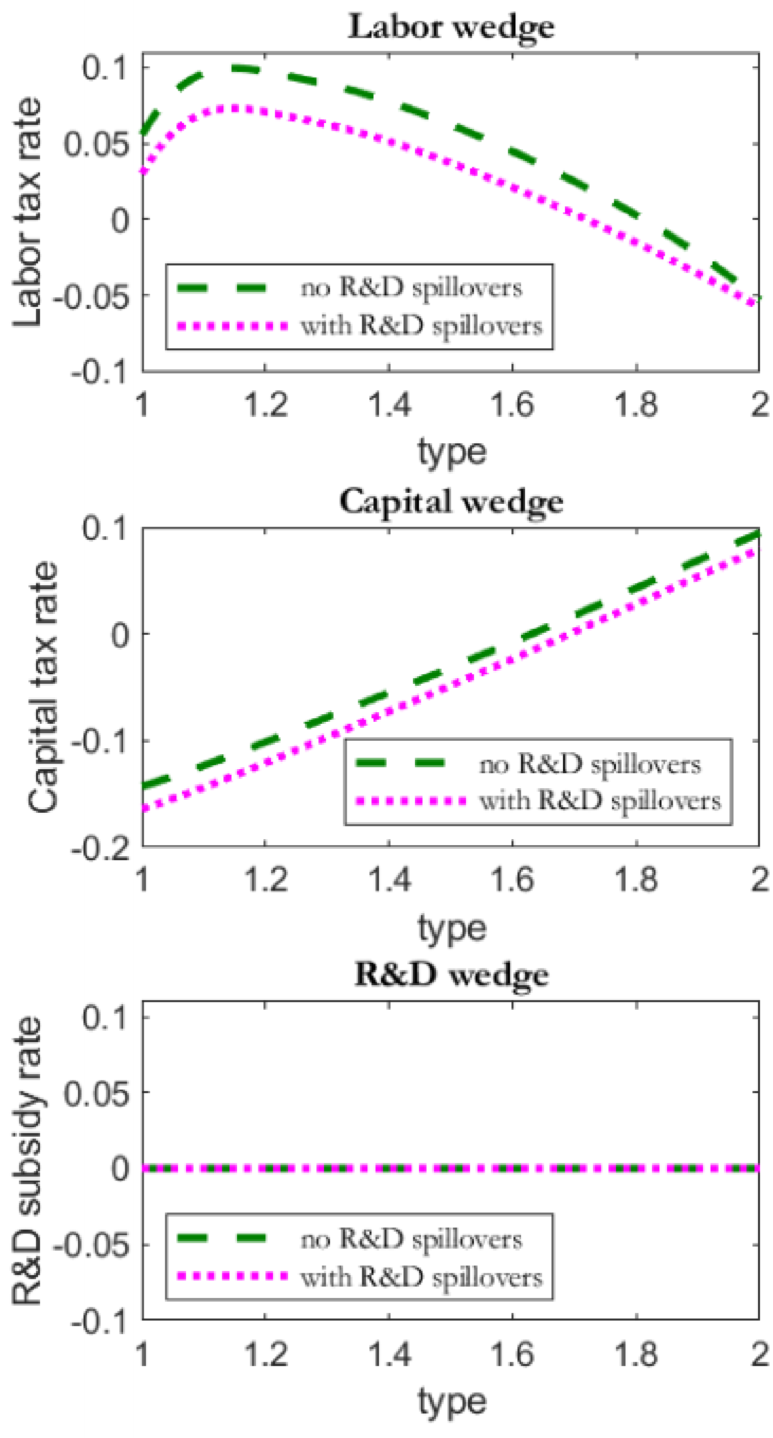

Figure 5. When R&D investment is unobservable

References

Ales, L., M. Kurnaz, and C. Sleet (2015) Technical change, wage inequality and taxes. *American Economic Review* 105(10), 3061–3101.

Albanesi, S. and C. Sleet (2006) Dynamic optimal taxation with private information. *Review of Economic Studies* 73, 1-30.

Akcigit, U., D. Hanley, and S. Stantcheva (2019) Optimal taxation and R&D policies. *Working paper*

Chen, B.-L. and F.-C. Liang (2019) Optimal Mirrleesian taxation and human capital investment. *Working paper*.

Costinot, A., and J. Vogel (2010) Matching and inequality in the world Economy. *Journal of Political Economy* 118 (4): 747–86.

Kocherlakota, N.R. (2005) Zero expected wealth taxes: A Mirrlees approach to dynamic optimal taxation. *Econometrica* 73, 1587-1621

Rothschild, C., and F. Scheuer (2013) Redistributive taxation in the Roy model. *Quarterly Journal of Economics* 128 (2), 623–68.

Teulings, C. (1995) The wage distribution in a model of the assignment of skills to jobs. *Journal of Political Economy* 103 (2): 280–315.

Appendix

A.1. Signs of elasticities in simple two type model.

Based on the production function $Y_t = F^{\underline{v}}(l_t(\underline{v}), k_t(\underline{v}), b_t(\underline{v})) + F^{\overline{v}}(l_t(\overline{v}), k_t(\overline{v}), b_t(\overline{v}))$, the wage rate of high type agent and low type agent are

$$w_{H,t} = a_H(\overline{v}) \cdot F_l^{\overline{v}}\left(l_t(\overline{v}), k_t(\overline{v}), b_t(\overline{v})\right)$$

$$w_{L,t} = a_L(\underline{v}) \cdot F_l^{\underline{v}}\left(l_t(\underline{v}), k_t(\underline{v}), b_t(\underline{v})\right)$$

Hence, the wage premium

$$\frac{w_{H,t}}{w_{L,t}} = \frac{a_H(\overline{v}) \cdot F_l^{\overline{v}}\left(l_t(\overline{v}), k_t(\overline{v}), b_t(\overline{v})\right)}{a_L(\underline{v}) \cdot F_l^{\underline{v}}\left(l_t(\underline{v}), k_t(\underline{v}), b_t(\underline{v})\right)} \tag{5a}$$

1) First, we derive the cross relative wage elasticity with labor effort, which is defined as follows

$$\phi_i^t \equiv \frac{e_{i,t}}{(w_{H,t}/w_{L,t})} \frac{\partial\left(w_{H,t}/w_{L,t}\right)}{\partial e_{i,t}} \quad \text{for } i \in \{L, H\}.$$

Taking derivatives on (5a) with respect to $e_{H,t}$ and $e_{L,t}$, we get

$$\phi_H^t = \frac{e_{H,t}}{(w_{H,t}/w_{L,t})} \frac{\pi_H \left[a_H(\overline{v})\right]^2 F_{ll}^{\overline{v}}\left(l_t(\overline{v}), k_t(\overline{v}), b_t(\overline{v})\right)}{a_L(\underline{v}) F_l^{\underline{v}}\left(\pi_L a_L(\underline{v}) e_{L,t}, k_t(\underline{v}), b_t(\underline{v})\right)} < 0$$

and

$$\phi_L^t = \frac{e_{L,t}}{(w_{H,t}/w_{L,t})} \cdot \frac{\pi_L a_H(\bar{v}) \cdot F_l^{\bar{v}}(l_t(\bar{v}), k_t(\bar{v}), b_t(\bar{v}))}{\left[F_l^{\underline{v}}(\pi_L a_L(\underline{v}) e_{L,t}, k_t(\underline{v}), b_t(\underline{v}))\right]^2} \cdot \left[-F_{ll}^{\underline{v}}(\pi_L a_L(\underline{v}) e_{L,t}, k_t(\underline{v}), b_t(\underline{v}))\right] > 0$$

2) Next, we derive the cross relative wage elasticity with respect to capital, which is defined as follows

$$\kappa_v^t \equiv \frac{k_t(v)}{(w_{H,t}/w_{L,t})} \frac{\partial(w_{H,t}/w_{L,t})}{\partial k_t(v)} \quad \text{for } v \in \{\underline{v}, \bar{v}\}.$$

Taking derivatives on (5a) with respect to $k_t(\bar{v})$ and $k_t(\underline{v})$, we get

$$\kappa_{\bar{v}}^t \equiv \frac{k_t(\bar{v})}{(w_{H,t}/w_{L,t})} \frac{a_H(\bar{v}) \cdot F_{lk}^{\bar{v}}(l_t(\bar{v}), k_t(\bar{v}), b_t(\bar{v}))}{a_L(\underline{v}) \cdot F_l^{\underline{v}}(l_t(\underline{v}), k_t(\underline{v}), b_t(\underline{v}))} > 0$$

and

$$\kappa_{\underline{v}}^t \equiv \frac{k_t(\underline{v})}{(w_{H,t}/w_{L,t})} \frac{a_H(\bar{v}) \cdot F_l^{\bar{v}}(l_t(\bar{v}), k_t(\bar{v}), b_t(\bar{v}))}{a_L(\underline{v}) \cdot \left[F_l^{\underline{v}}(l_t(\underline{v}), k_t(\underline{v}), b_t(\underline{v}))\right]^2} \cdot \left[-F_{lk}^{\underline{v}}(l_t(\underline{v}), k_t(\underline{v}), b_t(\underline{v}))\right] < 0$$

3) Finally, we derive the cross relative wage elasticity with respect to R&D investment, which is defined as follows

$$\varphi_v^t \equiv \frac{b_t(v)}{(w_{H,t}/w_{L,t})} \frac{\partial(w_{H,t}/w_{L,t})}{\partial b_t(v)} \quad \text{for } v \in \{\underline{v}, \bar{v}\}.$$

Taking derivatives on (5a) with respect to $b_t(\bar{v})$ and $b_t(\underline{v})$, we get

$$\varphi_{\bar{v}}^t \equiv \frac{b_t(\bar{v})}{(w_{H,t}/w_{L,t})} \frac{a_H(\bar{v}) \cdot F_{lb}^{\bar{v}}(l_t(\bar{v}), k_t(\bar{v}), b_t(\bar{v}))}{a_L(\underline{v}) \cdot F_l^{\underline{v}}(l_t(\underline{v}), k_t(\underline{v}), b_t(\underline{v}))} > 0$$

and

$$\varphi_{\underline{v}}^t \equiv \frac{b_t(\underline{v})}{(w_{H,t}/w_{L,t})} \frac{a_H(\bar{v}) \cdot F_l^{\bar{v}}(l_t(\bar{v}), k_t(\bar{v}), b_t(\bar{v}))}{a_L(\underline{v}) \cdot \left[F_l^{\underline{v}}(l_t(\underline{v}), k_t(\underline{v}), b_t(\underline{v}))\right]^2} \cdot \left[-F_{lb}^{\underline{v}}(l_t(\underline{v}), k_t(\underline{v}), b_t(\underline{v}))\right] < 0$$

In the case that R&D investment is not observable by the government, firms privately choose R&D investment. Two R&D investment elasticities of substitution characterize how the R&D investment is affected by labor input and capital input, which are defined respectively as follows:

$$\xi_{i,v}^t \equiv \frac{e_{i,t}}{b_t(v)} \frac{\partial b_t(v)}{\partial e_{i,t}} = \frac{e_{i,t}}{b_t(v)} \frac{\partial A^v}{\partial q_t(v)} \frac{\partial q_t(v)}{\partial e_{i,t}} \quad \text{for } i \in \{L, H\} \text{ and } v \in \{\underline{v}, \bar{v}\}$$

and

$$\zeta_v^t \equiv \frac{k_t(v)}{b_t(v)} \frac{\partial b_t(v)}{\partial k_t(v)} = \frac{k_t(v)}{b_t(v)} \frac{\partial A^v}{\partial q_t(v)} \frac{\partial q_t(v)}{\partial k_t(v)} \quad \text{for } v \in \{\underline{v}, \bar{v}\}$$

To derive the signs of these two elasticities, we need to consider the following firms' problem and derive the optimal condition with respect to R&D investment.

$$\max \sum_{v \in \{L,H\}} F^v\left(l_t(v), k_t(v); A^v\left(b_{t-1}(v), q_t(v), \bar{b}_t\right)\right) - \left[w_t(v) l_t(v) + M(q_t(v)) + R_t(v) k_t(v)\right]$$

F.O.C

$$\left[q_t(v)\right]:\quad F_b^v\left(\pi_i a_i(v)e_{i,t},k_t(v),b_t(v)\right)\cdot A_q^v\left(b_{t-1}(v),q_t(v),\bar{b}_t\right)-M'\left(q_t(v)\right)=0 \tag{5b}$$

The optimal R&D investment $q_t(v)$ can be solved by (5b). To see the sign of elasticity of substitution between R&D investment and labor effort, we take derivatives on (5b) with respect to $e_{i,t}$, then we get

$$F_{bl}^v\cdot\pi_i a_i(v)A_q^v+\left[F_{bb}^v\cdot\left(A_q^v\right)^2+F_b^v\cdot A_{qq}^v-M''\left(q_t(v)\right)\right]\cdot\frac{\partial q_t(v)}{\partial e_{i,t}}=0$$

when $(i,v)\in\{(L,\underline{v}),(H,\overline{v})\}$, which implies that

$$\xi_{i,v}^t=\frac{e_{i,t}}{b_t(v)}\frac{\partial A^v}{\partial q_t(v)}\left[\frac{-F_{bl}^v\cdot\pi_i a_i(v)A_q^v}{F_{bb}^v\cdot\left(A_q^v\right)^2+F_b^v\cdot A_{qq}^v-M''\left(q_t(v)\right)}\right]>0\quad\text{for }(i,v)\in\{(L,\underline{v}),(H,\overline{v})\},$$

To see the sign of elasticity of substitution between R&D investment and capital, we take derivatives on (5b) with respect to $k_t(v)$, then we get

$$F_{bk}^v\cdot A_q^v+\left[F_{bb}^v\cdot\left(A_q^v\right)^2+F_b^v\cdot A_{qq}^v-M''\left(q_t(v)\right)\right]\cdot\frac{\partial q_t(v)}{\partial k_t(v)}=0$$

which implies that

$$\zeta_v^t=\frac{k_t(v)}{b_t(v)}\frac{\partial A^v}{\partial q_t(v)}\frac{-F_{bk}^v\cdot A_q^v}{F_{bb}^v\cdot\left(A_q^v\right)^2+F_b^v\cdot A_{qq}^v-M''\left(q_t(v)\right)}>0\quad\text{for }v\in\{\underline{v},\overline{v}\}$$

A.2. Proofs of Proposition 1 and Proposition 2.

Set the Lagrangian

$$\begin{aligned}\mathcal{L}=\max\ &\sum_{i=L,H}g_i\left[\sum_{t=1}^{T}\beta^{t-1}\left[u\left(c_{i,t}\right)-h\left(e_{i,t}\right)\right]\right]\\&+\eta\left\{\sum_{t=1,2}\beta^{t-1}\left[u\left(c_{H,t}\right)-h\left(e_{H,t}\right)\right]-\sum_{t=1,2}\beta^{t-1}\left[u\left(c_{L,t}\right)-h\left(\frac{w_{L,t}}{w_{H,t}}e_{L,t}\right)\right]\right\}\\&+\sum_{t=1}^{T}\chi_t\left[\begin{array}{l}F^{\underline{v}}\left(\pi_L a_L(\underline{v})e_{L,t},k_t(\underline{v});b_t(\underline{v})\right)+F^{\overline{v}}\left(\pi_H a_H(\overline{v})e_{H,t},k_t(\overline{v});b_t(\overline{v})\right)\\-\pi_L c_{L,t}-\pi_H c_{H,t}-k_{t+1}(\underline{v})-k_{t+1}(\overline{v})-M\left(q_t(\underline{v})\right)-M\left(q_t(\overline{v})\right)\end{array}\right]\end{aligned}$$

F.O.C

$$\left[c_{H,t}\right]:\left(g_H+\eta\right)\beta^{t-1}u'\left(c_{H,t}\right)-\chi_t\pi_H=0$$

$$\left[c_{L,t}\right]:\left(g_L-\eta\right)\beta^{t-1}u'\left(c_{L,t}\right)-\chi_t\pi_L=0$$

Case 1: When R&D investment is observable (Proposition 1)

$$\left[e_{H,t}\right]:\left(-g_H-\eta+\eta\frac{h'\left(\frac{w_{L,t}}{w_{H,t}}e_{L,t}\right)}{h'\left(e_{H,t}\right)}\underbrace{\frac{\partial\left(\frac{w_{L,t}}{w_{H,t}}\right)}{\partial e_{H,t}}}_{=\frac{-w_{L,t}}{w_{H,t}}\frac{\phi_H^t}{e_{H,t}}}e_{L,t}\right)\beta^{t-1}h'\left(e_{H,t}\right)+\chi_t\pi_H\underbrace{a_H\left(\bar{v}\right)\cdot\frac{\partial F^{\bar{v}}}{\partial l_t\left(\bar{v}\right)}}_{=w_{H,t}}=0$$

which implies that

$$\frac{\tau_H^{l_t}}{1-\tau_H^{l_t}}=\frac{w_{H,t}u'\left(c_{H,t}\right)}{h'\left(e_{H,t}\right)}-1=\underbrace{\frac{\beta^{t-1}u'\left(c_{H,t}\right)\eta}{\chi_t\pi_H}\left(\frac{h'\left(\frac{w_{L,t}}{w_{H,t}}e_{L,t}\right)}{h'\left(e_{H,t}\right)}\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}e_{H,t}}\right)\phi_H^t\right)}_{\textit{wage compression}:<0}<0.$$

$$\left[e_{L,t}\right]:\left(-g_L+\eta\frac{h'\left(\frac{w_{L,t}}{w_{H,t}}e_{L,t}\right)\frac{w_{L,t}}{w_{H,t}}}{h'\left(e_{L,t}\right)}+\eta\frac{h'\left(\frac{w_{L,t}}{w_{H,t}}e_{L,t}\right)}{h'\left(e_{L,t}\right)}\frac{\partial\left(\frac{w_{L,t}}{w_{H,t}}\right)}{\partial e_{L,t}}e_{L,t}\right)\beta^{t-1}h'\left(e_{L,t}\right)+\chi_t\pi_L\underbrace{a_L\left(\underline{v}\right)\cdot\frac{\partial F^{\underline{v}}}{\partial l_t\left(\underline{v}\right)}}_{=w_{L,t}}=0$$

which implies that

$$\frac{\tau_L^{l_t}}{1-\tau_L^{l_t}}=\frac{w_{L,t}u'\left(c_{L,t}\right)}{h'\left(e_{L,t}\right)}-1=\frac{\beta^{t-1}u'\left(c_{L,t}\right)\eta}{\chi_t\pi_L}\left[\underbrace{1-\frac{h'\left(\frac{w_{L,t}}{w_{H,t}}e_{t,L}\right)}{h'\left(e_{L,t}\right)}\left(\frac{w_{L,t}}{w_{H,t}}\right)}_{\textit{Mirrlees}:>0}+\underbrace{\frac{h'\left(\frac{w_{L,t}}{w_{H,t}}e_{t,L}\right)}{h'\left(e_{L,t}\right)}\left(\frac{w_{L,t}}{w_{H,t}}\right)\phi_L^t}_{\textit{wage compression}:>0}\right]>0.$$

$$\left[k_t\left(\bar{v}\right)\right]:\eta\beta h'\left(\frac{w_{L,t}}{w_{H,t}}e_{L,t}\right)\frac{\partial\left(\frac{w_{L,t}}{w_{H,t}}\right)}{\partial k_t\left(\bar{v}\right)}e_{L,t}-\chi_{t-1}+\chi_t\frac{\partial F^{\bar{v}}}{\partial k_t\left(\bar{v}\right)}=0$$

which implies that

$$\frac{\tau_i^{k_t}\left(\bar{v}\right)}{1-\tau_i^{k_t}\left(\bar{v}\right)}=\frac{\beta R_t\left(\bar{v}\right)u'\left(c_{i,t}\right)}{u'\left(c_{i,t-1}\right)}-1=\underbrace{\frac{\beta^{t-1}\eta}{\chi_{t-1}}h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}k_t\left(\bar{v}\right)}\right)\kappa_{\bar{v}}^t}_{\textit{wage compression}:>0}>0$$

$$\left[k_t\left(\underline{v}\right)\right]:\eta\beta h'\left(\frac{w_{L,t}}{w_{H,t}}e_{L,t}\right)\frac{\partial\left(\frac{w_{L,t}}{w_{H,t}}\right)}{\partial k_t\left(\underline{v}\right)}e_{L,t}-\chi_{t-1}+\chi_t\frac{\partial F^{\bar{v}}}{\partial k_t\left(\underline{v}\right)}=0$$

which implies that

$$\frac{\tau_i^{k_t}\left(\underline{v}\right)}{1-\tau_i^{k_t}\left(\underline{v}\right)}=\frac{\beta R_t\left(\underline{v}\right)u'\left(c_{i,t}\right)}{u'\left(c_{i,t-1}\right)}-1=\underbrace{\frac{\beta^{t-1}\eta}{\chi_{t-1}}h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\frac{\kappa_{\underline{v}}^t}{k_t\left(\underline{v}\right)}}_{\textit{wage compression}:<0}<0$$

$$\left[q_t\left(\bar{v}\right)\right]:\eta\beta^{t-1}h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)e_{L,t}\underbrace{\frac{\partial\left(\frac{w_{L,t}}{w_{H,t}}\right)}{\partial b_t\left(\bar{v}\right)}}_{=\frac{-w_{L,t}}{w_{H,t}}\frac{\varphi_{\bar{v}}^t}{b_t\left(\bar{v}\right)}}\left[\frac{\partial A^{\bar{v}}}{\partial q_t\left(\bar{v}\right)}+\frac{\partial A^{\bar{v}}}{\partial\bar{b}_t}\cdot\frac{d\bar{b}_t}{dq_t\left(\bar{v}\right)}\right]+\chi_t\left[\frac{\partial F^{\bar{v}}}{\partial b_t\left(\bar{v}\right)}\cdot\frac{\partial A^{\bar{v}}}{\partial q_t\left(\bar{v}\right)}+\frac{dF^{\bar{v}}}{d\bar{b}_t}\cdot\frac{d\bar{b}_t}{dq_t\left(\bar{v}\right)}-M'\left(q_t\left(\bar{v}\right)\right)\right]=0$$

which implies that

$$s^{q_t}\left(\bar{v}\right)=M'\left(q_t\left(\bar{v}\right)\right)-\frac{\partial F_t^{\bar{v}}}{\partial b_t\left(\bar{v}\right)}\cdot\frac{\partial A^{\bar{v}}}{\partial q_t\left(\bar{v}\right)}$$
$$=\underbrace{\frac{-\beta^{t-1}\eta}{\chi_t}h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}b_t\left(\bar{v}\right)}\right)\varphi_{\bar{v}}^t\left[\frac{\partial A^{\bar{v}}}{\partial q_t\left(\bar{v}\right)}+\frac{\partial A^{\bar{v}}}{\partial \bar{b}_t}\frac{\partial \bar{b}_t}{\partial q_t\left(\bar{v}\right)}\right]}_{wage\ compression:<0}+\underbrace{\frac{d\bar{b}_t}{dq_t\left(\bar{v}\right)}\frac{dF_t^{\bar{v}}}{d\bar{b}_t}}_{Pigouvian\ correction:>0}$$

$$\left[q_t\left(\underline{v}\right)\right]:\eta\beta^{t-1}h'\left(\tfrac{w_{L,t}e_{L,t}}{w_{H,t}}\right)e_{L,t}\underbrace{\frac{\partial\left(\frac{w_{L,t}}{w_{H,t}}\right)}{\partial b_t\left(\underline{v}\right)}}_{=\frac{-w_{L,t}}{w_{H,t}}\frac{\varphi_{\underline{v}}^t}{b_t\left(\underline{v}\right)}}\left[\tfrac{\partial A^{\underline{v}}}{\partial q_t\left(\underline{v}\right)}+\tfrac{\partial A^{\underline{v}}}{\partial \bar{b}_t}\cdot\tfrac{d\bar{b}_t}{dq_t\left(\underline{v}\right)}\right]+\chi_t\left[\tfrac{\partial F^{\underline{v}}}{\partial b_t\left(\underline{v}\right)}\cdot\tfrac{\partial A^{\underline{v}}}{\partial q_t\left(\underline{v}\right)}+\tfrac{dF^{\underline{v}}}{d\bar{b}_t}\cdot\tfrac{d\bar{b}_t}{dq_t\left(\underline{v}\right)}-M'\left(q_t\left(\underline{v}\right)\right)\right]=0$$

which implies that

$$s^{q_t}\left(\underline{v}\right)=M'\left(q_t\left(\underline{v}\right)\right)-\frac{\partial F_t^{\underline{v}}}{\partial b_t\left(\underline{v}\right)}\cdot\frac{\partial A^{\underline{v}}}{\partial q_t\left(\underline{v}\right)}$$
$$=\underbrace{\frac{-\beta^{t-1}\eta}{\chi_t}h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}b_t\left(\underline{v}\right)}\right)\varphi_{\underline{v}}^t\left[\frac{\partial A^{\underline{v}}}{\partial q_t\left(\underline{v}\right)}+\frac{\partial A^{\underline{v}}}{\partial \bar{b}_t}\frac{\partial \bar{b}_t}{\partial q_t\left(\underline{v}\right)}\right]}_{wage\ compression:\ >0}+\underbrace{\frac{d\bar{b}_t}{dq_t\left(\underline{v}\right)}\frac{dF_t}{d\bar{b}_t}}_{Pigouvian\ correction:>0}$$

Case 2: When R&D investment is unobservable (Proposition 2)

$$\left[e_{H,t}\right]:\left(-g_H-\eta+\eta\frac{h'\left(\frac{w_{L,t}}{w_{H,t}}e_{L,t}\right)e_{L,t}}{h'\left(e_{H,t}\right)}\left[\frac{\partial\left(\frac{w_{L,t}}{w_{H,t}}\right)}{\partial e_{H,t}}+\frac{\partial\left(\frac{w_{L,t}}{w_{H,t}}\right)}{\partial b_t\left(\bar{v}\right)}\cdot\frac{\partial b_t\left(\bar{v}\right)}{\partial e_{H,t}}\right]\right)\beta^{t-1}h'\left(e_{H,t}\right)$$
$$+\chi_t\left[\pi_H\underbrace{a_H\left(\bar{v}\right)\cdot\frac{\partial F^{\bar{v}}}{\partial l_t\left(\bar{v}\right)}}_{=w_{H,t}}+\frac{\partial q_t\left(\bar{v}\right)}{\partial e_{H,t}}\left[\underbrace{\frac{\partial F^{\bar{v}}}{\partial b_t\left(\bar{v}\right)}\cdot\frac{\partial A^{\bar{v}}}{\partial q_t\left(\bar{v}\right)}-M'\left(q_t\left(\bar{v}\right)\right)}_{=0}+\frac{dF^{\bar{v}}}{d\bar{b}_t}\cdot\frac{d\bar{b}_t}{dq_t\left(\bar{v}\right)}\right]\right]=0$$

which implies that

$$\frac{\tau_H^{l_t}}{1-\tau_H^{l_t}}=\frac{w_{H,t}u'\left(c_{H,t}\right)}{h'\left(e_{H,t}\right)}-1$$
$$=\frac{\beta^{t-1}\eta u'\left(c_{H,t}\right)}{\chi_t\pi_H h'\left(e_{H,t}\right)}h'\left(\tfrac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\frac{w_{L,t}e_{L,t}}{w_{H,t}e_{H,t}}\left[\underset{wage\ compression:\ <0}{\phi_H^t}+\underbrace{\varphi_{\bar{v}}^t\cdot\xi_{H,\bar{v}}^t}_{R\&D\ Unobservability:\ >0}\right]+\underbrace{\left[\frac{-u'\left(c_{H,t}\right)}{\pi_H h'\left(e_{H,t}\right)}\frac{dF_t^{\bar{v}}}{d\bar{b}_t}\frac{d\bar{b}_t}{dq_t\left(\bar{v}\right)}\frac{b_t\left(\bar{v}\right)/e_{H,t}}{\frac{\partial A^{\bar{v}}}{\partial q_t\left(\bar{v}\right)}}\xi_{H,\bar{v}}^t\right]}_{Pigouvian\ corection:\ <0}$$

$$\left[e_{L,t}\right]:\left(-g_L+\eta\frac{h'\left(\frac{w_{L,t}}{w_{H,t}}e_{L,t}\right)\frac{w_{L,t}}{w_{H,t}}}{h'\left(e_{L,t}\right)}+\eta\frac{h'\left(\frac{w_{L,t}}{w_{H,t}}e_{L,t}\right)e_{L,t}}{h'\left(e_{L,t}\right)}\left[\frac{\partial\left(\frac{w_{L,t}}{w_{H,t}}\right)}{\partial e_{L,t}}+\frac{\partial\left(\frac{w_{L,t}}{w_{H,t}}\right)}{\partial b_t\left(\underline{v}\right)}\cdot\frac{\partial b_t\left(\underline{v}\right)}{\partial e_{L,t}}\right]\right)\beta^{t-1}h'\left(e_{L,t}\right)$$

$$+\chi_t\left[\pi_L\underbrace{a_L\left(\underline{v}\right)\cdot\frac{\partial F^{\underline{v}}}{\partial l_t\left(\underline{v}\right)}}_{=w_{L,t}}+\frac{\partial q_t\left(\underline{v}\right)}{\partial e_{L,t}}\left[\underbrace{\frac{\partial F^{\underline{v}}}{\partial b_t\left(\underline{v}\right)}\cdot\frac{\partial A^{\underline{v}}}{\partial q_t\left(\underline{v}\right)}-M'\left(q_t\left(\underline{v}\right)\right)}_{=0}+\frac{dF^{\underline{v}}}{d\overline{b}_t}\cdot\frac{d\overline{b}_t}{dq_t\left(\underline{v}\right)}\right]\right]=0$$

which implies that

$$\frac{\tau_L^{l_t}}{1-\tau_L^{l_t}}=\frac{w_{L,t}u'\left(c_{L,t}\right)}{h'\left(e_{L,t}\right)}-1$$

$$=\frac{\beta^{t-1}u'\left(c_{L,t}\right)\eta}{\chi_t\pi_L}\left[\underbrace{1-\frac{h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\frac{w_{L,t}}{w_{H,t}}}{h'\left(e_{L,t}\right)}}_{Mirrlees:>0}+\frac{h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\frac{w_{L,t}}{w_{H,t}}}{h'\left(e_{L,t}\right)}\left(\underbrace{\phi_L^t}_{wage\ compression:>0}+\underbrace{\varphi_{\underline{v}}^t\cdot\xi_{L,\underline{v}}^t}_{R\&D\ Unobservability:<0}\right)\right]\cdot$$

$$+\underbrace{\left[\frac{-u'\left(c_{L,t}\right)}{\pi_L h'\left(e_{L,t}\right)}\frac{dF_t^{\underline{v}}}{d\overline{b}_t}\frac{d\overline{b}_t}{dq_t\left(\underline{v}\right)}\frac{b_t\left(\underline{v}\right)/e_{L,t}}{\frac{\partial A^{\underline{v}}}{\partial q_t\left(\underline{v}\right)}}\xi_{L,\underline{v}}^t\right]}_{Pigouvian\ corection:\ <0}$$

$$\left[k_t\left(\overline{v}\right)\right]:\eta\beta h'\left(\frac{w_{L,t}}{w_{H,t}}e_{L,t}\right)e_{L,t}\left[\frac{\partial\left(\frac{w_{L,t}}{w_{H,t}}\right)}{\partial k_t\left(\overline{v}\right)}+\frac{\partial\left(\frac{w_{L,t}}{w_{H,t}}\right)}{\partial b_2\left(\overline{v}\right)}\cdot\frac{\partial b_2\left(\overline{v}\right)}{\partial k_2\left(\overline{v}\right)}\right]$$

$$-\chi_{t-1}+\chi_t\left[\frac{\partial F^{\overline{v}}}{\partial k_t\left(\overline{v}\right)}+\frac{\partial q_t\left(\overline{v}\right)}{\partial k_t\left(\overline{v}\right)}\left[\underbrace{\frac{\partial F^{\overline{v}}}{\partial b_t\left(\overline{v}\right)}\cdot\frac{\partial A^{\overline{v}}}{\partial q_t\left(\overline{v}\right)}-M'\left(q_t\left(\overline{v}\right)\right)}_{=0}+\frac{dF^{\overline{v}}}{d\overline{b}_t}\cdot\frac{d\overline{b}_t}{dq_t\left(\overline{v}\right)}\right]\right]=0$$

which implies that

$$\frac{\tau_i^{k_t}\left(\overline{v}\right)}{1-\tau_i^{k_t}\left(\overline{v}\right)}=\frac{\beta R_t\left(\overline{v}\right)u'\left(c_{i,t}\right)}{u'\left(c_{i,t-1}\right)}-1$$

$$=\frac{\beta^{t-1}\eta}{\chi_{t-1}}h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}k_t\left(\overline{v}\right)}\right)\left(\underbrace{\kappa_H^t}_{wage\ compression:\ >0}+\underbrace{\varphi_{\overline{v}}^t\cdot\zeta_{H,\overline{v}}^t}_{R\&D\ Unobservability:\ >0}\right)+\underbrace{\left[\frac{-\chi_t}{\chi_{t-1}}\cdot\frac{\partial F_t^{\overline{v}}}{\partial\overline{b}_t}\frac{\partial\overline{b}_t}{\partial q_t\left(\overline{v}\right)}\frac{b_t\left(\overline{v}\right)/k_t\left(\overline{v}\right)}{\frac{\partial A^{\overline{v}}}{\partial q_t\left(\overline{v}\right)}}\zeta_{\overline{v}}^t\right]}_{Pigouvian\ correction:\ <\ 0}$$

$$\left[k_t\left(\underline{v}\right)\right]:\eta\beta h'\left(\frac{w_{L,t}}{w_{H,t}}e_{L,t}\right)e_{L,t}\left[\frac{\partial\left(\frac{w_{L,t}}{w_{H,t}}\right)}{\partial k_t\left(\underline{v}\right)}+\frac{\partial\left(\frac{w_{L,t}}{w_{H,t}}\right)}{\partial b_t\left(\underline{v}\right)}\cdot\frac{\partial b_t\left(\underline{v}\right)}{\partial k_t\left(\underline{v}\right)}\right]$$

$$-\chi_{t-1}+\chi_t\left[\frac{\partial F^{\underline{v}}}{\partial k_t\left(\underline{v}\right)}+\frac{\partial q_t\left(\underline{v}\right)}{\partial k_t\left(\underline{v}\right)}\left[\underbrace{\frac{\partial F^{\underline{v}}}{\partial b_t\left(\underline{v}\right)}\cdot\frac{\partial A^{\underline{v}}}{\partial q_t\left(\underline{v}\right)}-M'\left(q_t\left(\underline{v}\right)\right)}_{=0}+\frac{dF^{\underline{v}}}{d\overline{b}_t}\cdot\frac{d\overline{b}_t}{dq_t\left(\underline{v}\right)}\right]\right]=0$$

which implies that

$$\frac{\tau_i^{k_t}(\underline{v})}{1-\tau_i^{k_t}(\underline{v})}=\frac{\beta R_t(\underline{v})u'(c_{i,t})}{u'(c_{i,t-1})}-1$$

$$=\frac{\beta^{t-1}\eta}{\chi_{t-1}}h'\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}}\right)\left(\frac{w_{L,t}e_{L,t}}{w_{H,t}k_t(\underline{v})}\right)\left(\underbrace{\kappa_{\underline{v}}^t}_{\textit{wage compression}:<0}+\underbrace{\varphi_{\underline{v}}^t\cdot\zeta_{\underline{v}}^t}_{\textit{R\&D Unobservability}:\,<0}\right)+\underbrace{\left[\frac{-\chi_t}{\chi_{t-1}}\cdot\frac{\partial F_t^{\underline{v}}}{\partial\overline{b}_t}\frac{\partial\overline{b}_t}{\partial q_t(\underline{v})}\frac{b_t(\underline{v})/k_t(\underline{v})}{\frac{\partial A^{\underline{v}}}{\partial q_t(\underline{v})}}\zeta_{\underline{v}}^t\right]}_{\textit{Pigouvian correction}\,<\,0}$$

A.3. Proofs of Proposition 3.

Proof of (i): Define

$$\Lambda_{i,t}=\arg\max a_i(v)w_t(v)$$

and set $\underline{v}_{i,t}=\inf\Lambda_{i,t}$ and $\overline{v}_{i,t}=\sup\Lambda_{i,t}$.

Suppose that there is $(\underline{v}_{i,t},\overline{v}_{i,t})\cap(\underline{v}_{i+1,t},\overline{v}_{i+1,t})\neq\emptyset$, which implies that

$$(\underline{v}_{i+1,t},\overline{v}_{i,t})\subseteq(\underline{v}_{i,t},\overline{v}_{i,t})\cap(\underline{v}_{i+1,t},\overline{v}_{i+1,t}),$$

For any $v,v'\in(\underline{v}_{i+1,t},\overline{v}_{i,t})$, on one hand, since . $v,v'\in(\underline{v}_{i,t},\overline{v}_{i,t})$, we have

$$a_i(v)w_t(v)=a_i(v')w_t(v') \tag{6a}$$

On the other hand , since $v,v'\in(\underline{v}_{i+1,t},\overline{v}_{i+1,t})$, we also have

$$a_{i+1}(v)w_t(v)=a_{i+1}(v')w_t(v') \tag{6b}$$

for any $v,v'\in(\underline{v}_{i+1,t},\overline{v}_{i,t})$.

Based on (6a) and (6b), we can derive that

$$1=\frac{a_i(v)w_t(v)}{a_i(v')w_t(v')}=\frac{a_{i+1}(v)w_t(v)}{a_{i+1}(v')w_t(v')}.$$

which implies

$$\frac{a_i(v)}{a_i(v')}=\frac{a_{i+1}(v)}{a_{i+1}(v')}.$$

This contradicts with Assumption 1. Therefore, by contradiction, we complete the proof of $(\underline{v}_{i,t},\overline{v}_{i,t})\cap(\underline{v}_{i+1,t},\overline{v}_{i+1,t})=\emptyset$.

The final step is to eliminate those no-labor-input sectors, and then re-label the range of available sectors such that $\overline{v}_{i,t}=\tilde{v}_{i,t}$. and $\underline{v}_{i,t}=\tilde{v}_{i-1,t}$. ■

Proof of (ii): According to (2a), the wage rate of the sector v is

$$\begin{aligned}w_t(v)&=\frac{\partial Y_t}{\partial l_t(v)}\\&=\left\{\int_{\underline{v}}^{\overline{v}}\Phi(k_t(v),b_t(v))[l_t(v)]^{\frac{\varepsilon-1}{\varepsilon}}dv\right\}^{\frac{1}{\varepsilon-1}}\cdot\Phi(k_t(v),b_t(v))[l_t(v)]^{\frac{-1}{\varepsilon}}\\&=\left[\frac{Y_t}{l_t(v)}\right]^{\frac{1}{\varepsilon}}\Phi(k_t(v);b_t(v))\end{aligned}$$

Based on (i), the labor of sector v can be written as

$$l_t(v) = \lambda_{i,t}(v) a_i(v) e_{i,t} \quad \text{for any } v \in (\tilde{v}_{i-1,t}, \tilde{v}_{i,t}).$$

and thus the wage rate of type i agent is

$$w_{i,t} = a_i(v) w_t(v) = a_i(v) \left[\frac{Y_t}{l_t(v)} \right]^{\frac{1}{\varepsilon}} \Phi\left(k_t(v), b_t(v)\right) \tag{7a}$$

From (7a), we have

$$\lambda_{i,t}(v) = \left[a_i(v)\right]^{\varepsilon-1} \left[\frac{Y_t}{e_{i,t}} \right] \left[\frac{\Phi\left(k_t(v), b_t(v)\right)}{w_{i,t}} \right]^{\varepsilon} \tag{7b}$$

Using (7b) and the fact that $\pi_i = \int_{\tilde{v}_{i-1,t}}^{\tilde{v}_{i,t}} \lambda_{i,t}(v) dv$, we have

$$\begin{aligned} \pi_i &= \int_{\tilde{v}_{i-1,t}}^{\tilde{v}_{i,t}} \lambda_{i,t}(v) dv \\ &= \int_{\tilde{v}_{i-1,t}}^{\tilde{v}_{i,t}} \left[a_i(v)\right]^{\varepsilon-1} \left[\frac{Y_t}{e_{i,t}} \right] \left[\frac{\Phi\left(k_t(v), b_t(v)\right)}{w_{i,t}} \right]^{\varepsilon} dv \\ &= \left[\frac{1}{w_{i,t}} \right]^{\varepsilon} \frac{Y_t}{e_{i,t}} \underbrace{\int_{\tilde{v}_{i-1,t}}^{\tilde{v}_{i,t}} \left[a_i(v)\right]^{\varepsilon-1} \left[\Phi\left(k_t(v), b_t(v)\right)\right]^{\varepsilon} dv}_{=B\left(\tilde{v}_{i-1,t}, \tilde{v}_{i,t}\right)^{\varepsilon}} \end{aligned} \tag{7c}$$

From (7c), the wage rate of type i can be shown as follows

$$w_{i,t} = \left[\frac{Y_t}{\pi_i e_{i,t}} \right]^{\frac{1}{\varepsilon}} B\left(\tilde{v}_{i-1,t}, \tilde{v}_{i,t}\right) \tag{7d}$$

Using (7d) to replace $w_{i,t}$ in (7b), we obtain

$$\lambda_{i,t}(v) = \frac{\left[a_i(v)\right]^{\varepsilon-1} \left[\Phi\left(k_t(v), b_t(v)\right)\right]^{\varepsilon}}{\left[B\left(\tilde{v}_{i-1,t}, \tilde{v}_{i,t}\right)\right]^{\varepsilon}} \pi_i$$

Then, we complete the proof. ■

Proof of (iii): Let μ_i be the multiplier of the constraint. From F.O.C, we have

$$\left[\lambda_{i,t}(v)\right] : \left[Y_t\right]^{\frac{1}{\varepsilon}} \cdot a_i(v) e_{i,t} \left[\lambda_{i,t}(v) a_i(v) e_{i,t}\right]^{\frac{-1}{\varepsilon}} \cdot \left[\Phi\left(k_t(v), b_t(v)\right)\right] - \mu_i = 0$$

which implies

$$\lambda_i(v) = \left[\frac{1}{a_i(v) e_{i,t}} \right]^{1-\varepsilon} \left[\frac{\Phi\left(k_t(v); b_t(v)\right)}{\mu_i} \right]^{\varepsilon} Y_t \tag{8a}$$

Using (8a) and the fact that $\pi_i = \int_{\tilde{v}_{i-1}}^{\tilde{v}_i} \lambda_i(v) dv$, we have

$$\pi_i = \int_{\tilde{v}_{i-1}}^{\tilde{v}_i} \lambda_i(v) dv = Y_t \cdot \left[\frac{1}{\mu_i} \right]^{\varepsilon} \cdot \left[e_{i,t}\right]^{\varepsilon-1} \int_{\tilde{v}_{i-1}}^{\tilde{v}_i} \left[a_i(v)\right]^{\varepsilon-1} \left[\Phi\left(k_t(v); b_t(v)\right)\right]^{\varepsilon} dv \tag{8b}$$

Rearrange (8b), we obtain

$$\mu_i = \left[\pi_i\right]^{\frac{-1}{\varepsilon}} \left[e_{i,t}\right]^{\frac{\varepsilon-1}{\varepsilon}} \left[Y_t \cdot \int_{\tilde{v}_{i-1}}^{\tilde{v}_i} \left[a_i(v)\right]^{\varepsilon-1} \left[\Phi\left(k_t(v); b_t(v)\right)\right]^{\varepsilon} dv\right]^{\frac{1}{\varepsilon}} \tag{8c}$$

Replace μ_i in (8a) by (8c), we get

$$\lambda_i(v) = \frac{\left[a_i(v)\right]^{\varepsilon-1}\left[\Phi\left(k_t(v); b_t(v)\right)\right]^{\varepsilon}}{\int_{\tilde{v}_{i-1}}^{\tilde{v}_i} \left[a_i(v)\right]^{\varepsilon-1}\left[\Phi\left(k_t(v); b_t(v)\right)\right]^{\varepsilon} dv} \pi_i = \frac{\left[a_i(v)\right]^{\varepsilon-1}\left[\Phi\left(k_t(v); b_t(v)\right)\right]^{\varepsilon}}{B\left(\tilde{v}_{i-1}, \tilde{v}_i\right)^{\varepsilon}} \pi_i \qquad \blacksquare$$

Proof of (iv):

Since $\tilde{v}_{i,t}$ is the task threshold between type i and type $i+1$. Working at sector $\tilde{v}_{i,t}$ could be indifference. This means that

$$w_{i,t} = \lim_{v \to \tilde{v}_{i,t}} a_i(v) w_t(v) \quad \text{and} \quad w_{i+1,t} = \lim_{v \to \tilde{v}_{i,t}} a_{i+1}(v) w_t(v) \tag{9a}$$

Since $a_i(v)$ and $w_t(v)$ are differentiable function, and thus

$$w_t\left(\tilde{v}_{i,t}\right) = \frac{w_{i,t}}{a_i\left(\tilde{v}_{i,t}\right)} = \frac{w_{i+1,t}}{a_{i+1}\left(\tilde{v}_{i,t}\right)} \tag{9b}$$

Based on (9b), we find that $\dfrac{w_{i+1,t}}{w_{i,t}} = \dfrac{a_{i+1}\left(\tilde{v}_{i,t}\right)}{a_i\left(\tilde{v}_{i,t}\right)}$ $\blacksquare$

A.4. Proofs of Proposition 4 and Proposition 5.

Set the Lagrangian

$$\begin{aligned}
\mathcal{L} = \max\ & \sum_{i=1}^{N} g_i \sum_{t=1}^{T} \beta^{t-1}\left[u\left(c_{i,t}\right) - h\left(e_{i,t}\right)\right] \\
& + \sum_{i=1}^{N} \eta_i \left\{ \sum_{t=1}^{T} \beta^{t-1}\left[u\left(c_{i,t}\right) - h\left(e_{i,t}\right)\right] - \sum_{t=1}^{T} \beta^{t-1}\left[u\left(c_{i-1,t}\right) - h\left(\frac{w_{i-1,t}}{w_{i,t}} e_{i-1,t}\right)\right]\right\} \\
& + \sum_{t=1}^{T} \chi_t \left[\hat{Y}_t\left(\left\{\pi_i e_{i,t}\right\}_{i=1}^{N}, \left\{k_t(v), b_t(v)\right\}; \left\{ \underbrace{\alpha_i^{-1}\left(\frac{w_{i+1,t}}{w_{i,t}}\right)}_{=\tilde{v}_i} \right\}_{i=1}^{N-1} \right) - \int_{\underline{v}}^{\overline{v}} k_{t+1}(v) - M\left(q_t(v)\right) dv - \sum_{i=1}^{N} \pi_i c_{i,t} \right]
\end{aligned}$$

F.O.C.

$\left[c_{i,t}\right]: \beta^{t-1} u'\left(c_{i,t}\right)\left[g_i + \eta_i - \eta_{i+1}\right] - \chi_t \pi_i = 0$

Case1: When R&D investment is observable

$$\left[e_{i,t}\right]: \beta^{t-1} h'\left(e_{i,t}\right)\left[-g_i-\eta_i+\eta_{i+1}\frac{h'\left(\frac{w_{i,t}}{w_{i+1,t}}e_{i,t}\right)\frac{w_{i,t}}{w_{i+1,t}}}{h'\left(e_{i,t}\right)}\right]$$
$$+\sum_{j=1}^{N-1}\left\{\beta^{t-1}\eta_{j+1}h'\left(\tfrac{w_{j,t}}{w_{j+1,t}}e_{j,t}\right)e_{j,t}+\chi_t\frac{\partial \hat{Y}_t}{\partial \tilde{v}_j}\frac{d\left(\alpha_j^{-1}\right)}{d\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}\right\}\frac{\partial\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}{\partial e_{i,t}}+\chi_t\underbrace{\frac{\partial \hat{Y}_t}{\partial e_{i,t}}}_{=\pi_i w_{i,t}}=0$$

Using the fact that $\frac{\partial\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}{\partial e_{i,t}}=\frac{-w_{j,t}}{w_{j+1,t}e_{i,t}}\phi_{i,j}^t$ and $\frac{d\left(\alpha_j^{-1}\right)}{d\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}=\frac{-D\alpha_j^{-1}\left(\frac{w_{j+1,t}}{w_{j,t}}\right)}{\left(\frac{w_{j,t}}{w_{j+1,t}}\right)^2}$, the first-order conditions with respect to $c_{i,t}$ and $e_{i,t}$ imply that

$$\frac{\tau_i^{l_t}}{1-\tau_i^{l_t}}=\frac{w_{i,t}u'\left(c_{i,t}\right)}{h'\left(e_{i,t}\right)}-1==\underbrace{\frac{\beta^{t-1}u'\left(c_{i,t}\right)\eta_{i+1}}{\chi_t\pi_i}\left[1-\frac{h'\left(\frac{w_{i,t}}{w_{i+1,t}}e_{i,t}\right)\frac{w_{i,t}}{w_{i+1,t}}}{h'\left(e_{i,t}\right)}\right]}_{Mirrlees}+\frac{u'\left(c_{i,t}\right)}{\pi_i h'\left(e_{i,t}\right)e_{i,t}}\sum_{j=1}^{N-1}\left\{\underbrace{\frac{\beta^{t-1}\eta_{j+1}h'\left(\frac{w_{j,t}}{w_{j+1,t}}e_{j,t}\right)\frac{w_{j,t}}{w_{j+1,t}}e_{j,t}}{\chi_t}}_{wage\ compression}-\underbrace{\frac{\partial\hat{Y}_t}{\partial\tilde{v}_{j,t}}\frac{D\alpha_j^{-1}\left(\frac{w_{j+1,t}}{w_{j,t}}\right)}{\frac{w_{j,t}}{w_{j+1,t}}}}_{\text{sectoral shift}}\right\}\phi_{i,j}^t$$

$$\left[k_t\left(v\right)\right]: \sum_{j=1}^{N-1}\left\{\beta^{t-1}\eta_{j+1}h'\left(\tfrac{w_{j,t}}{w_{j+1,t}}e_{j,t}\right)e_{j,t}+\chi_t\frac{\partial \hat{Y}}{\partial \tilde{v}_j}\frac{d\left(\alpha_j^{-1}\right)}{d\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}\right\}\frac{\partial\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}{\partial k_t\left(v\right)}+\chi_t\cdot\frac{\partial \hat{Y}_t}{\partial k_t\left(v\right)}-\chi_{t-1}=0$$

Using the fact that $\frac{\partial\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}{\partial k_t(v)}=\frac{-w_{j,t}}{w_{j+1,t}k_t(v)}\kappa_{v,j}^t$ and $\frac{d\left(\alpha_j^{-1}\right)}{d\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}=\frac{-D\alpha_j^{-1}\left(\frac{w_{j+1,t}}{w_{j,t}}\right)}{\left(\frac{w_{j,t}}{w_{j+1,t}}\right)^2}$, the first-order conditions with respect to $c_{i,t}$ and $k_t\left(v\right)$ imply that

which implies that

$$\frac{\tau_i^{k_t}\left(v\right)}{1-\tau_i^{k_t}\left(v\right)}=\frac{\beta R_t\left(v\right)u'\left(c_{t,i}\right)}{u'\left(c_{t-1,i}\right)}-1=\frac{1}{\chi_{t-1}k_t(v)}\sum_{j=1}^{N-1}\left\{\underbrace{\beta^{t-1}\eta_{j+1}h'\left(\frac{w_{j,t}e_{j,t}}{w_{j+1,t}}\right)\frac{w_{j,t}e_{j,t}}{w_{j+1,t}}}_{\text{wage compression}}-\underbrace{\chi_t\frac{\partial\hat{Y}}{\partial\tilde{v}_j}\frac{D\alpha_j^{-1}\left(\frac{w_{j+1,t}}{w_{j,t}}\right)}{\frac{w_{j,t}}{w_{j+1,t}}}}_{\text{sectoral shift}}\right\}\kappa_{v,j}^t$$

$$\left[q_t\left(v\right)\right]: \sum_{j=1}^{N-1}\left\{\beta^{t-1}\eta_{j+1}h'\left(\tfrac{w_{j,t}}{w_{j+1,t}}e_{j,t}\right)e_{j,t}+\chi_t\frac{\partial \hat{Y}}{\partial \tilde{v}_j}\frac{d\left(\alpha_j^{-1}\right)}{d\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}\right\}\frac{\partial\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}{\partial b_t\left(v\right)}\cdot\left[\frac{\partial A_t^v}{\partial q_t\left(v\right)}+\frac{\partial A_t^v}{\partial \bar{b}_t}\cdot\frac{\partial \bar{b}_t}{\partial q_t\left(v\right)}\right]$$
$$+\chi_t\cdot\left[\frac{\partial \hat{Y}_t}{\partial b_t\left(v\right)}\cdot\left[\frac{\partial A_t^v}{\partial q_t\left(v\right)}+\frac{\partial A_t^v}{\partial \bar{b}_t}\cdot\frac{\partial \bar{b}_t}{\partial q_t\left(v\right)}\right]-M'\left(q_t\left(v\right)\right)\right]=0$$

Using the fact that $\frac{\partial\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}{\partial q_t(v)}=\frac{-w_{j,t}}{w_{j+1,t}q_t(v)}\varphi_{v,j}^t$ and $\frac{d\left(\alpha_j^{-1}\right)}{d\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}=\frac{-D\alpha_j^{-1}\left(\frac{w_{j+1,t}}{w_{j,t}}\right)}{\left(\frac{w_{j,t}}{w_{j+1,t}}\right)^2}$, the first-order conditions with respect to $c_{i,t}$ and $q_t\left(v\right)$ imply that

$$s^{q_t}(v) = M'(q_t(v)) - \frac{\partial \hat{Y}_t}{\partial b_t(v)} \cdot \frac{\partial B_t^v}{\partial q_t(v)}$$

$$= \frac{1}{b_t(v)\chi_t} \sum_{j=1}^{N-1} \left\{ \underbrace{-\beta^{t-1}\eta_{j+1} h'\left(\frac{w_{j,t}e_{j,t}}{w_{j+1,t}}\right)\frac{w_{j,t}e_{j,t}}{w_{j+1,t}}}_{\text{wage compression}} + \underbrace{\frac{\partial \hat{Y}}{\partial \tilde{v}_j} \frac{D\alpha_j^{-1}\left(\frac{w_{j+1,t}}{w_{j,t}}\right)}{\frac{w_{j,t}}{w_{j+1,t}}}}_{\text{sectoral shift}} \right\} \varphi_{v,j}^t \cdot \left[\frac{\partial A_t^v}{\partial q_t(v)} + \frac{\partial A_t^v}{\partial \bar{b}_t} \cdot \frac{\partial \bar{b}_t}{\partial q_t(v)} \right] + \underbrace{\frac{\partial \hat{Y}_t}{\partial b_t(v)} \cdot \frac{\partial A_t^v}{\partial \bar{b}_t} \cdot \frac{\partial \bar{b}_t}{\partial q_t(v)}}_{Pigouvian\ correction}$$

Case2: When R&D investment is unobservable

$$[e_{i,t}] : \beta^{t-1}h'(e_{i,t}) \left[-g_i - \eta_i + \eta_{i+1} \frac{h'\left(\frac{w_{i,t}}{w_{i+1,t}} e_{i,t}\right)\frac{w_{i,t}}{w_{i+1,t}}}{h'(e_{i,t})} \right]$$

$$+ \sum_{j=1}^{N-1} \left\{ \beta^{t-1}\eta_{j+1} h'\left(\frac{w_{j,t}}{w_{j+1,t}} e_{j,t}\right) e_{j,t} + \chi_t \frac{\partial \hat{Y}}{\partial \tilde{v}_j} \frac{d(\alpha_j^{-1})}{d\left(\frac{w_{j,t}}{w_{j+1,t}}\right)} \right\} \left[\frac{\partial\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}{\partial e_{i,t}} + \int_{\underline{v}}^{\bar{v}} \frac{\partial\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}{\partial b_t(v)} \cdot \frac{\partial b_t(v)}{\partial e_{i,t}} dv \right]$$

$$+ \chi_t \left[\underbrace{\frac{\partial \hat{Y}_t}{\partial e_{i,t}}}_{=\pi_i w_{i,t}} + \int_{\underline{v}}^{\bar{v}} \left(\underbrace{\frac{\partial \hat{Y}_t}{\partial b_t(v)} \cdot \frac{\partial A_t^v}{\partial q_t(v)} - M'(q_t(v))}_{MR-MC=0} + \frac{d\hat{Y}_t}{d\bar{b}_t} \cdot \frac{\partial \bar{b}_t}{\partial q_t(v)} \right) \cdot \frac{\partial q_t(v)}{\partial e_{i,t}} dv \right] = 0$$

Using the fact that $\frac{\partial\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}{\partial e_{i,t}} = \frac{-w_{j,t}}{w_{j+1,t}e_{i,t}} \phi_{i,j}^t$, $\frac{\partial\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}{\partial q_t(v)} = \frac{-w_{j,t}}{w_{j+1,t}q_t(v)} \varphi_{v,j}^t$, $\frac{\partial q_t(v)}{\partial e_{i,t}} = \frac{b_t(v)/e_{i,t}}{\frac{\partial A^v}{\partial q_t(v)}} \xi_{i,v}^t$ and $\frac{d(\alpha_j^{-1})}{d\left(\frac{w_{j,t}}{w_{j+1,t}}\right)} = \frac{-D\alpha_j^{-1}\left(\frac{w_{j+1,t}}{w_{j,t}}\right)}{\left(\frac{w_{j,t}}{w_{j+1,t}}\right)^2}$, the first-order conditions with respect to $c_{i,t}$ and $e_{i,t}$ imply that

$$\frac{\tau_i^{l_t}}{1-\tau_i^{l_t}} = \frac{w_{i,t}u'(c_{i,t})}{h'(e_{i,t})} - 1$$

$$= \underbrace{\frac{\beta^{t-1}u'(c_{i,t})\eta_{i+1}}{\chi_t \pi_i} \left[1 - \frac{h'\left(\frac{w_{i,t}}{w_{i+1,t}} e_{i,t}\right)\frac{w_{i,t}}{w_{i+1,t}}}{h'(e_{i,t})} \right]}_{\text{Mirrlees}} + \frac{u'(c_{i,t})}{\pi_i h'(e_{i,t}) e_{i,t}} \sum_{j=1}^{N-1} \left\{ \underbrace{\frac{\beta^{t-1}\eta_{j+1}}{\chi_t} h'\left(\frac{w_{j,t}e_{j,t}}{w_{j+1,t}}\right)\frac{w_{j,t}e_{j,t}}{w_{j+1,t}}}_{\text{wage compression}} - \underbrace{\frac{\partial \hat{Y}}{\partial \tilde{v}_j} \frac{D\alpha_j^{-1}\left(\frac{w_{j+1,t}}{w_{j,t}}\right)}{\frac{w_{j,t}}{w_{j+1,t}}}}_{\text{sectoral shift}} \right\} \left[\underbrace{\phi_{i,j}^t}_{direct\ effect} + \underbrace{\int_{\underline{v}}^{\bar{v}} \varphi_{v,j}^t \cdot \xi_{i,v}^t dv}_{R\&D\ Unobservability} \right]$$

$$\underbrace{- \frac{u'(c_{i,t})}{\pi_i h'(e_{i,t})} \frac{d\hat{Y}_t}{d\bar{b}_t} \int_{\underline{v}}^{\bar{v}} \frac{\partial \bar{b}_t}{\partial q_t(v)} \cdot \frac{b_t(v)/e_{i,t}}{\frac{\partial A^v}{\partial q_t(v)}} \xi_{i,v}^t dv}_{Pigouvian\ corection}$$

$$[k_t(v)] : \sum_{j=1}^{N-1} \left\{ \beta^{t-1}\eta_{j+1} h'\left(\frac{w_{j,t}}{w_{j+1,t}} e_{j,t}\right) e_{j,t} + \chi_t \frac{\partial \hat{Y}}{\partial \tilde{v}_j} \frac{d(\alpha_j^{-1})}{d\left(\frac{w_{j,t}}{w_{j+1,t}}\right)} \right\} \left[\frac{\partial\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}{\partial k_t(v)} + \underbrace{\int_{\underline{v}}^{\bar{v}} \frac{\partial\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}{\partial b_t(v')} \cdot \frac{\partial b_t(v')}{\partial k_t(v)} dv'}_{R\&D\ Unobservability} \right]$$

$$+ \chi_t \cdot \left[\frac{\partial \hat{Y}_t}{\partial k_t(v)} + \int_{\underline{v}}^{\bar{v}} \left(\underbrace{\frac{\partial \hat{Y}_t}{\partial b_t(v')} \cdot \frac{\partial A^{v'}}{\partial q_t(v')} - M'(q_t(v'))}_{MR-MC=0} + \frac{d\hat{Y}_t}{d\bar{b}_t} \cdot \frac{\partial \bar{b}_t}{\partial q_t(v')} \right) \cdot \frac{\partial q_t(v')}{\partial k_t(v)} \right] - \chi_{t-1} = 0$$

Using the fact that $\frac{\partial\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}{\partial k_t(v)} = \frac{-w_{j,t}}{w_{j+1,t}e_{i,t}}\kappa_{v,j}^t$, $\frac{\partial\left(\frac{w_{j,t}}{w_{j+1,t}}\right)}{\partial q_t(v)} = \frac{-w_{j,t}}{w_{j+1,t}q_t(v)}\varphi_{v,j}^t$, $\frac{\partial q_t(v')}{\partial k_t(v)} = \frac{b_t(v')/k_t(v)}{\frac{\partial A^{v'}}{\partial q_t(v')}}\zeta_{v,v'}^t$ and $\frac{d\left(\alpha_j^{-1}\right)}{d\left(\frac{w_{j,t}}{w_{j+1,t}}\right)} = \frac{-D\alpha_j^{-1}\left(\frac{w_{j+1,t}}{w_{j,t}}\right)}{\left(\frac{w_{j,t}}{w_{j+1,t}}\right)^2}$,

the first-order conditions with respect to $c_{i,t}$ and $k_t(v)$ imply that

$$\frac{\tau_i^{k_t}(v)}{1-\tau_i^{k_t}(v)} = \frac{\beta R_t(v)u'\left(c_{t,i}\right)}{u'\left(c_{t-1,i}\right)} - 1$$

$$= \frac{1}{\chi_{t-1}k_t(v)}\sum_{j=1}^{N-1}\left\{\underbrace{\beta^{t-1}\eta_{j+1}h'\left(\frac{w_{j,t}e_{j,t}}{w_{j+1,t}}\right)\frac{w_{j,t}e_{j,t}}{w_{j+1,t}}}_{\text{wage compression}} - \underbrace{\chi_t\frac{\partial\hat{Y}}{\partial\tilde{v}_j}\frac{D\alpha_j^{-1}\left(\frac{w_{j+1,t}}{w_{j,t}}\right)}{\frac{w_{j,t}}{w_{j+1,t}}}}_{\text{sectoral shift}}\right\}\left[\underset{\textit{direct effect}}{\kappa_{v,j}^t} + \underbrace{\int_{\underline{v}}^{\bar{v}}\varphi_{v',j}^t\cdot\zeta_{v,v'}^t dv'}_{\textit{R\&D Unobservability}}\right] - \underbrace{\frac{\chi_t}{\chi_{t-1}}\frac{d\hat{Y}_t}{d\bar{b}_t}\int_{\underline{v}}^{\bar{v}}\frac{\partial\bar{b}_t}{\partial q_t(v')}\cdot\frac{b_t(v')/k_t(v)}{\frac{\partial A^{v}}{\partial q_t(v')}}\zeta_{v,v'}^t dv'}_{\textit{Pigouvian corection}}$$