Preordering Chinese Restaurant Games

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Outline

Queueing for sharing the resource

A news screenshot of queueing for the library seats.

Sharing problem with externality I

Each person's utility depends on the total number of people in the library. So why do not choose Library B if Library A is very full?

Consider the following problem:
Suppose that there are n potential firms waiting to register for an address in Science Park A or Science Park B which are both under construction.
Each potential firm has a constant water/electric demand and the park may get a shortage of water/electricity resources if there are too many firms in the same area.
The resource total supply of Park A and Park B are different, one is larger and the other is smaller, but the potential firms cannot distinguish them precisely but with a signal of the distribution of resource supply.

Sharing problem with externality II

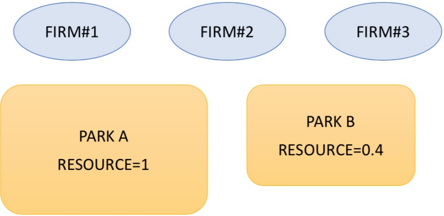
Each potential firm would like to register for the park with rich resource, and with less neighbors (i.e., less firms to share resource with the firm).

Each potential firm chooses a queueing group to register for an address simultaneously, then choose Park A or Park B sequentially. The firms observe the signals and park choices of the former firms in the waiting line, the later the more information about the distribution of park resources, but the former the more first-mover advantage(FMA) to choose a rich park to resister.

Question: Does a potential firm queue for the former(F) position or the later(L)?

A example of park choice I

Complete information:

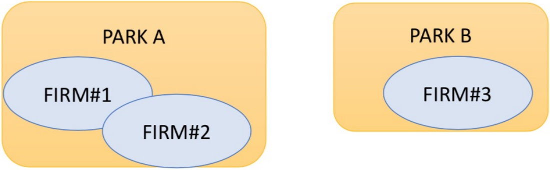


$$u_i(t_i = x, t_j) = \frac{R_x}{n_x},$$

R_x: *size of resource in Park* x; n_x: *number of firms in Park* x.

Figure: Three firms, two tables

A example of park choice II



$$u_1(t_1 = A, t_2 = A, t_3 = B) = \frac{R_A}{n_A} = \frac{1}{2}$$
$$u_2(t_1 = A, t_2 = A, t_3 = B) = \frac{R_A}{n_A} = \frac{1}{2}$$
$$u_3(t_1 = A, t_2 = A, t_3 = B) = \frac{R_B}{n_B} = \frac{0.4}{1} = 0.4$$
$$n_A^* = 2, n_B^* = 1.$$

Motivations I

Queueing with incomplete information and negative population externality is a familiar problem to people. Waiting for public goods like services of Household Resister Offices, emergency department or public baby care centers are those kinds of examples since the customers do not have enough information about the resource sizes provided by different servers.

People also usually join the queue for private goods such as movie tickets, famous restaurant tables, bus, and so on. And the service quality and quantity in all above examples are affected by the total numbers of customers (i.e., the source of externality).

Motivations II

But people may have no idea about the information quality of customers, or do not know which resource is larger for them if there are several servers and people choose the waiting line they prefer to join.

We explore these questions in the sequential game-theoretic model with Bayesian updating featuring:

- The true state of resources distribution is uncertain.
- Players receive their own signals about the true state.
- Every player incurs a constant externality from one more player choosing the same resource with him/her.

Motivations III

- Assume that the system chooses a default preordering rule, and the preordering Chinese restaurant game (PCRG) proceeds for breaking the game into two steps: queueing choice and resource choice.

Motivations IV

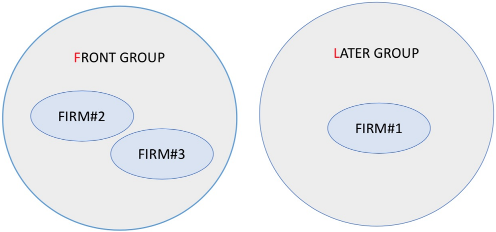


Figure: the firms select their queueing group simultaneously

Motivations V

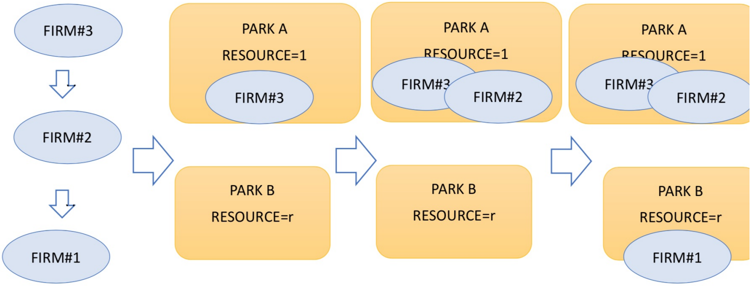


Figure: the firms select their table sequentially

Brief description of results

We provide following results:

- Existence of FF/LL Equilibrium for $|N| = 2$ case, with closed-form solutions.
- The preordering rule FL is not an equilibrium strategy as $|N| \geq 3$.
- (Counter-intuitive) The player with a strong signal moves from F group to L group early than the player with a weak signal as $r \to 0$.
- The queueing equilibrium distribution with two resources structure.
- It is possible that the queueing equilibrium does not exist under a specific set of parameters.

Early works

- Chinese Restaurant Games (Wang et al, 2012)
- Sequential Chinese Restaurant Games (Wang et al, 2013)
- Dynamic Chinese Restaurant Game: Theory and application to cognitive radio networks (Jiang et al, 2014)
- Choose Early or Easily (Kung and Wang, 2015)
- Dynamic decision-making with Bayesian learning: Naive Learning and the Wisdom of Crowd (Golub and Jackson, 2010)
- Preordering concern: Priority Rules and Other Asymmetric Rationing Methods (Moulin, 2000)

Chinese Restaurant Process

Chinese restaurant process, which is a non-parametric learning methods in machine learning (Aldous et al, 1985), provides a non-strategic learning method for unbounded number of objects.

In Chinese restaurant process, there exists infinite number of tables, where each table has infinite number of seats. There are infinite number of customers entering the restaurant sequentially.

When one customer enters the restaurant, he can choose either to share the table with other customers or to open a new table, with the probability being predefined by the process.

Play Strategically

Wang et al(2013) proposed a strategic game called the Chinese Restaurant Game(CRG), for which all players, with limited knowledge on the system state and information held by other players, update their beliefs of true states and make the table decisions.

The players' rewards in the game are determined by the true system state and the number of agents who choose the same table with them.

Benchmark model I

In the previous papers, Wang et al. provided a sequential Chinese Restaurant Game(SCRG) which the players choose their own resource sequentially.
In this paper, an additional preordering rule is added to the SCRG and this game becomes to a two-stage decisions as follows:

1. Queueing Stage: each player choose to follow the rule or not simultaneously.
2. The system assigns an order to each player according the Stage 1 rule and their choices.
3. Table Stage: players updates the belief of true state and choose the resource sequentially (i.e., run a SCRG in the order determined by the Queueing Satge.)

Benchmark model II

Queueing equilibrium: the preordering rule that both player with strong and weak signals do follow.

Question 1: given each set of parameters (table size, population externality, and signal quality of true state), which preordering group (F/L) should the player choose?

In the Bayesian learning process, players updates their information from former players' signals and behavior announced, and the ideal result is that the players with good signal quality (type strong) choose the former(F) positions and the players with bad signal quality (type weak) choose the later(L).

Benchmark model III

Then the type strong players announce their information early, and the type weak players learn and make better decisions.

Question 2: do the FL preordering rule an equilibrium strategy for the players?

Players, beliefs, and payoffs I

- $i = 1, 2, ..., n$: the set of players
- $\theta \in \Theta$: the set of states
- $\mathcal{R} \in \{FF, FL, LF, LL\}$: System announces a predetermined preordering rule $\mathcal{R}$, which is common knowledge among agents.
- $a_i \in \{F, L\}$: Stage 1 actions of players. Each agent chooses a preordering subgroup $a_i \in \{F, L\}$ simultaneously. Then the system assigns each agent a queueing position according to a predetermined rule $\mathcal{R}$.
- $s_i \in \mathcal{I}$: signals of players, which indicates the probability of true state and can be represented by a vector $p = (p_1, p_2, p_3, p_4) \in \Delta(\mathbb{R}^4)$, with $p_1 \geq p_2 \geq p_3 \geq p_4$.

Players, beliefs, and payoffs II

- $\mathcal{I} = \{2, -2, 1, -1\}$: the set of signals, $\mathcal{I} = S \cup W$ with the set of strong signals $S = \{2, -2\}$ and the set of weak signals $W = \{1, -1\}$.
- $g_{k_i}(\theta)$: belief of true state for player i with a position $k_i = 1, 2, ..., n$, which is updated by observing the former players' announced signals.
- $x = 1, 2$ with size $R_x(\theta) \in \{1, r\}$, where $r \in [0, 1]$ denoted the discount rate of certain table.
- $\beta \in [0, 1]$: constant population externality produced by the players.
- $v_i = R_x(\theta) - \beta n_x$: payoff of table share with linear externality, where n_x denotes the total number of players choosing table x.

Players, beliefs, and payoffs III

- $u_i = \frac{R_x(\theta)}{\beta n_x}$: payoff of table share with quotient externality.
- $\mathcal{H}_i$: the set of history for player i containing the common information such as $r, \beta, \mathcal{R}$, prior belief, former players' signals and choices.
- t_x: player's table(resource) choice, $x = 1, 2$.
- $EU_{k_i}(t_x, s_i|\mathcal{H}_i, \mathcal{R})$: the expected utility of player i with preordering position k_i under preordering rule $\mathcal{R}$, and $EU_{k_i}(t_x, s_i|\mathcal{H}_i, \mathcal{R}) = \sum_{\theta\in\Theta} g_{k_i}(\theta)u(R_x(\theta), n_x)$, the form of u_i depends.

Timing

Stage 1 Each player received his/her own signal and choose to follow the preordering rule or not.

Stage 2 According to the order determined by Stage 1 (queueing stage), each player chooses the resource poll sequentially.

Thus, Stage 2 is a standard sequential CRG given the Stage 1 generated ordering, i.e., even if the system's default rule is sending a strong type player to F (front) group, but if the player chooses to join the L (later) group, then the Stage 1 assigns the player into the L group.
Note that all the players in the same group have equal probability to be assigned to each L positions.

The Effect of Preordering Rules I

A preordering rule $\mathcal{R} \in \{FF, FL, LF, LL\}$ indicates that the system divides the set of players into two groups "Former (F)" and "Later (L)" according to the types of players.

For example, FF rule assigns both the strong and weak type players into F group, and the system randomly assigns a position k_i to each player $i \in F$. On the other hand, LF rule assigns the strong type into the later group and the weak type is assigned to the former group, then the system randomly assigns a position $k_i = 1, 2, ..., |F|$ to each player $i \in F$, and assigns a position $k_j = |F| + 1, |F| + 2, ..., n$ to each player $j \in L$.

The Effect of Preordering Rules II

Note that a preordering rule does not assign two persons into one position.
Assume that $N = \{1, 2, 3, 4, 5\}$ and
$s_1 = 2, s_2 = 1, s_3 = -2, s_4 = 2, s_5 = 1$.

A FF rule may produce the following order:
$(1, 2, 3, 4, 5), (2, 3, 4, 5, 1)$, and so on.

A LF rule may produce the following order:
$(2, 5, 1, 3, 4), (5, 2, 4, 1, 3)$ but not $(1, 5, 2, 3, 4)$, and so forth.

Complete Information Case: trivial

In a PCRG with complete information, we observe that

- the preordering rule plays no role in the resource sharing game
- only pooling equilibria (FF/LL) occurred in the queueing stage

And for each set of players $|N| = n$ and $r \in [0, 1]$, denote that

$$n^*(\theta_1) = (k^*, n - k^*); n^*(\theta_2) = (n - k^*, k^*),$$

then for $r \geq \frac{1}{n}$

$$k^* = \arg\min |\frac{1+r}{n} - \frac{1}{k}|, 1 \leq k \leq n;$$

for $r < \frac{1}{n}$, we have $k^* = n$ otherwise.
For the $r \geq \frac{1}{n}$ case, only FF equilibrium occurs since the FMA is $|\frac{1}{k^*} - \frac{r}{n-k^*}| > 0$, and for $r < \frac{1}{n}$, only LL equilibrium occurs since the FMA is zero.

Unknown Distribution of Types

Consider the incomplete information case of PCRG, we have the following assumptions:

1. Player i received his/her own signal s_i.
2. Player i with preordering position k_i expects that he/she observed the former players' signals and choices for all $j \in N$ with $1 \leq k_j < k_i$.
3. According the preordering rule, player i with preordering position k_i does NOT expect that he/she knows the players' type (strong/weak) distribution after player i.

$|N| = 2$ case closed-form solution I

Assume that the resource share has the quotient form $u_i(t_i = x, t_j, j \neq i, i, j \in N) = \frac{R_x(\theta)}{n_x}$, that is, the the share of each player received is dividing the size of certain resource by the number of players.

Let $|N| = 2$ and player i receives the signal $s_i = 2 \in \mathcal{I} = \{2, -2, 1, -1\}$under a predetermined FF preordering rule.

The set of states $\Theta = \{\theta_1, \theta_2\}$ with common prior $Pr(\theta_1) = Pr(\theta_2) = 0.5$, and

$Pr(s_i = 2|\theta_1) = Pr(s_i = -2|\theta_2) = p_1$;
$Pr(s_i = 1|\theta_1) = Pr(s_i = -1|\theta_2) = p_2$;
$Pr(s_i = -1|\theta_1) = Pr(s_i = 1|\theta_2) = p_3$;
$Pr(s_i = -2|\theta_1) = Pr(s_i = 2|\theta_2) = p_4$.

$|N| = 2$ case closed-form solution II

Then in the queueing stage, player i's expected payoff of choosing $a_i = F$ is as follows:

$$EU_i(a_i = F, s_i = 2|FF, \theta, r, \beta, p)$$

$$= Pr(k_i = 1|FF, s_i)EU_1(BR_i = t_i, BR_j = t_j, s_i = 2, s_j \in \mathcal{I}|FF, \mathcal{H}_1)$$

$$+Pr(k_i = 2|FF, s_i)EU_2(BR_i = t_i, BR_j = t_j, s_i = 2, s_j \in \mathcal{I}|FF, \mathcal{H}_2)$$

$$= \frac{1}{2}(\frac{p_1}{p_1 + p_4} \times 1 + \frac{p_4}{p_1 + p_4} \times r)$$

$$+\frac{1}{2}[\sum_{s \in \mathcal{I}} Pr(s_j = s|s_i = 2, FF)EU_2(t_i, t_j, s_i = 2, s_j \in \mathcal{I}|FF, \mathcal{H}_2)],$$

$|N| = 2$ case closed-form solution III

where

$$\sum_{s\in\mathcal{I}} Pr(s_j = s|s_i = 2, FF)EU_2(t_i, t_j, s_i = 2, s_j \in \mathcal{I}|FF, \mathcal{H}_2))$$

$$= Pr(s_j = 2|s_i = 2, FF)EU_2(t_1 = 1, t_2 = 2, s_1 = 2, s_2 = 2)$$

$$+Pr(s_j = -2|s_i = 2, FF)EU_2(t_1 = 2, t_2 = 1, s_1 = -2, s_2 = 2)$$

$$+Pr(s_j = 1|s_i = 2, FF)EU_2(t_1 = 1, t_2 = 2, s_1 = 1, s_2 = 2)$$

$$+Pr(s_j = -1|s_i = 2, FF)EU_2(t_1 = 2, t_2 = 1, s_1 = -1, s_2 = 2)$$

$$= \frac{p_1^2 + p_4^2}{p_1 + p_4}(\frac{p_1^2}{p_1^2 + p_4^2} \times r + \frac{p_4^2}{p_1^2 + p_4^2} \times 1)$$

$$+\frac{2p_1p_4}{p_1 + p_4}(\frac{p_1p_4}{2p_1 + p_4} \times 1 + \frac{p_4p_1}{2p_1p_4} \times r)$$

$|N| = 2$ case closed-form solution IV

$$+\frac{p_1p_2 + p_3p_4}{p_1 + p_4}(\frac{p_1p_2}{p_1p_2 + p_3p_4} \times r + \frac{p_3p_4}{p_1p_2 + p_3p_4} \times 1)$$

$$+\frac{p_1p_3 + p_2p_4}{p_1 + p_4}(\frac{p_1p_3}{p_1p_3 + p_2p_4} \times 1 + \frac{p_2p_4}{p_1p_3 + p_2p_4} \times r).$$

Similarly, player i's expected payoff of choosing $a_i = L$ is as follows:

$$EU_i(a_i = L, s_i = 2|FF, \theta, r, \beta, p)$$

$$= Pr(k_i = 1|FF, s_i)EU_1(BR_i = t_i, BR_j = t_j, s_i = 2, s_j \in \mathcal{I}|FF, \mathcal{H}_1)$$

$$+Pr(k_i = 2|FF, s_i)EU_2(BR_i = t_i, BR_j = t_j, s_i = 2, s_j \in \mathcal{I}|FF, \mathcal{H}_2)$$

$$= 0+1\times\sum_{s\in\mathcal{I}} Pr(s_j = s|s_i = 2, FF)EU_2(t_i, t_j, s_i = 2, s_j \in \mathcal{I}|FF, \mathcal{H}_2).$$

$|N| = 2$ case closed-form solution V

Then the profitable deviation criteria of an $M \in \{S, W\}$ type player i under preordering rule $\mathcal{R} \in \{FF, FL, LF, LL\}$ is denoted by

$$X_i(s_i \in M; \mathcal{R}, \theta, r, \beta, p) \equiv$$

$$EU_i(a_i = F, s_i \in M|\mathcal{R}, \theta, r, \beta, p) - EU_i(a_i = L, s_i \in M|\mathcal{R}, \theta, r, \beta, p).$$

$|N| = 2$ case closed-form solution VI

Set the parameters:
$r = 0.5, \beta = 1, p = (p_1, p_2, p_3, p_4) = (0.75, 0.15, 0.07, 0.03)$.
Note that $EU_i(a_i = F, s_i = 2|FF, \theta, r, \beta, p) \approx 0.94$ and $EU_i(a_i = L, s_i = 2|FF, \theta, r, \beta, p) \approx 0.91$. Thus, $EU_i(a_i = F, s_i = 2|FF, \cdot) - EU_i(a_i = L, s_i = 2|FF, \cdot) > 0$ so there is no profitable deviation for a strong type player i moving from F group to L group.
The weak type expected payoff under certain preordering rule is analogue.

Numerical Experiments I

Fix the parameters (r, β, p), the computational result contains two parts:

- the exact expected utilities of players
- numberical experiments for conforming the above computation which proceeds as follows:
  1. Fix a preordering rule $\mathcal{R} \in \{FF, FL, LF, LL\}$, parameters r, β which are commonly known for all players $i \in N$. Assume that $|N| = n$.
  2. System draws a true state θ randomly.
  3. Given p, the system generates a set of signals $(s_1, ..., s_n)$ to players.
  4. Each player receives its signal and chooses a queueing group (F/L).

Numerical Experiments II

5. System assigns the players to F/L groups according to the default preordering rule if player chooses "not deviate", and assigns the players to the queueing group they chose if player chooses "deviate".
6. System generates an order $(F_1, F_2, ..., F_m, L_1, ..., L_k)$ for $m + k = n$.
7. Players chooses a table t_i for all $i \in N$ according to above orders.
8. Computing the gap between theoretical expected payoff and average experiment payoff.

$|N| = 2$ experiment I

EU gap between the numerical experiment and theoretical expectation: FF rule

$p_1 = 0.85, p_2 = 0.07, p_3 = 0.05, p_4 = 0.03$

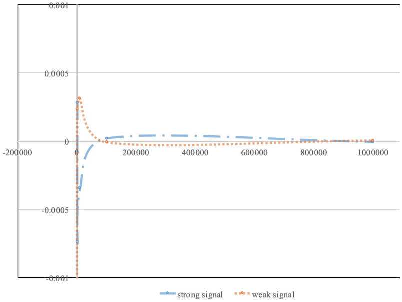


Figure: $r = 0.4$, FF

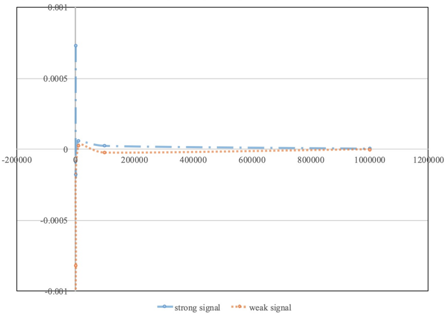


Figure: $r = 0.6$, FF

$|N| = 2$ experiment II

EU gap: FL rule

$p_1 = 0.85, p_2 = 0.07, p_3 = 0.05, p_4 = 0.03$

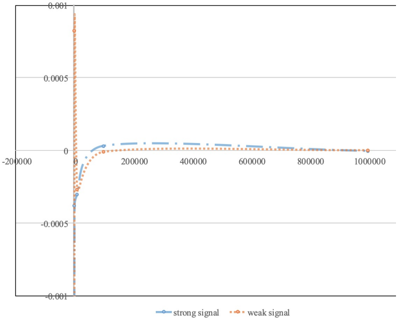


Figure: $r = 0.4$, FL

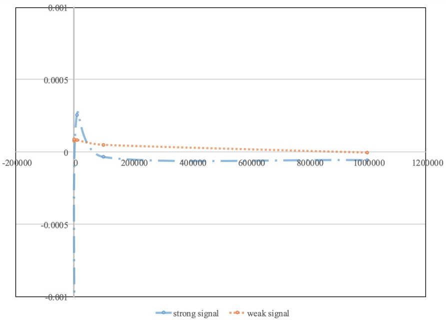


Figure: $r = 0.6$, FL

$|N| = 2$ experiment III

EU gap: LF rule

$p_1 = 0.85, p_2 = 0.07, p_3 = 0.05, p_4 = 0.03$

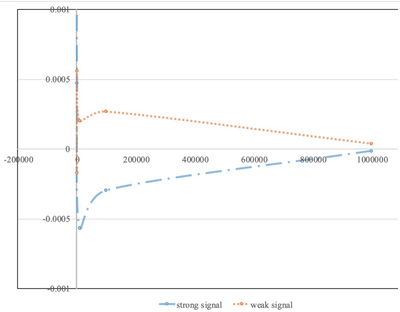


Figure: $r = 0.4$, LF

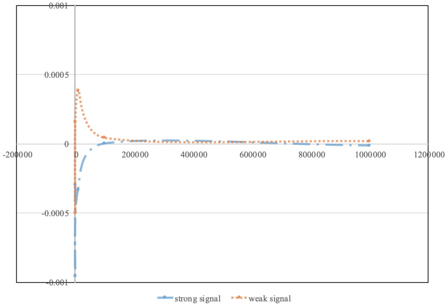


Figure: $r = 0.6$, LF

$|N| = 2$ experiment IV

EU gap: LL rule

$p_1 = 0.85, p_2 = 0.07, p_3 = 0.05, p_4 = 0.03$

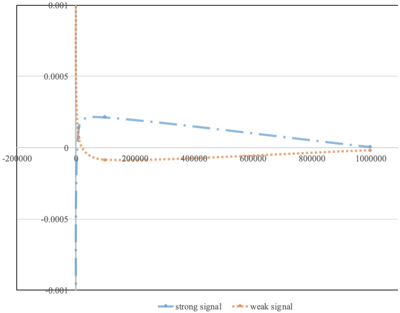


Figure: $r = 0.4$, LL

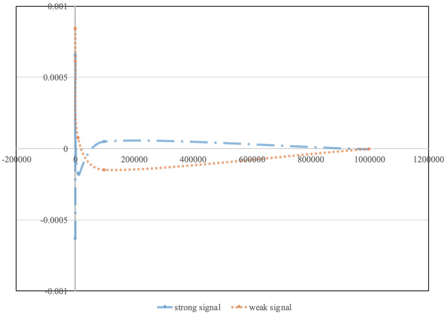


Figure: $r = 0.6$, LL

$|N| = 2$, quotient share

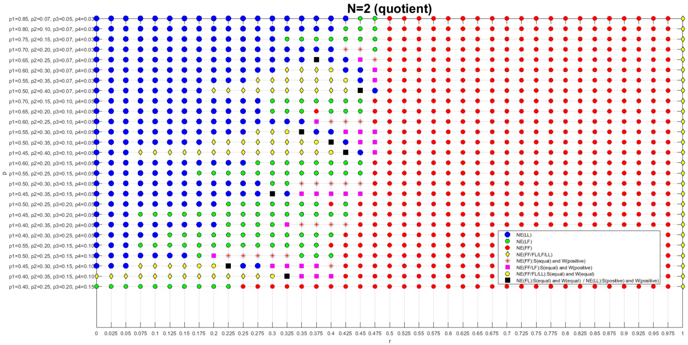


Figure: theoretical expectation EU, $|N| = 2$

Simulation for $|N| = 3$, quotient share

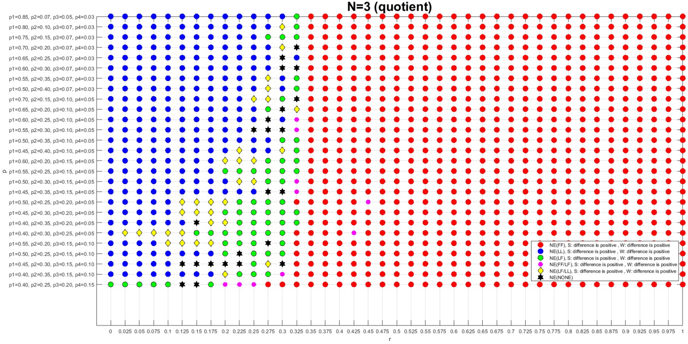


Figure: theoretical expectation EU, $|N| = 3$

Results: $|N| \geq 2$ case I

First we consider the special case $r = 1$ and $r = 0$:

Proposition 1

Let $|N| \geq 2$. For all $p \in \Delta(\mathbb{R}^4)$ and $r = 1$, the preordering rule FF is a queueing equilibrium.

Proof (sketch):
Claim 1. For all $p \in \Delta(\mathbb{R}^4)$ and $r = 1$, each second player's dominant strategy is choosing the table $t_2 \neq t_1$ in the resource stage.
Proof of the Claim 1:

Results: $|N| \geq 2$ case II

Consider (i) $t_2 = t_1$ and (ii) $t_2 \neq t_1$, compute the expected payoff as follows:

$$EU_2(t_2 = t_1 | \mathcal{H}_2, FF, r = 1) = g_2(\theta_1)\frac{R_1(\theta_1)}{2\beta} + g_2(\theta_2)\frac{R_1(\theta_2)}{2\beta}$$

$$= \frac{1}{2\beta} < \frac{1}{\beta} = EU_2(t_2 \neq t_1 | \mathcal{H}_2, FF, r = 1).$$

Thus, choosing F group at Stage 1 to improve the probability to get the FMA is a dominant strategy which is irrelevant to the type of player.

The $|N| > 2$ case follows by the induction arguments.

Results: $|N| \geq 2$ case III

Proposition 2

Let $|N| \geq 2$. For all $p \in \Delta(\mathbb{R}^4)$ and $r = 0$, the preordering rule LL is a queueing equilibrium.

Proof (sketch):
For $r = 0$ case, the FMA=0. That is, the equilibrium player distribution among the of resources is $(2, 0)$ or $(0, 2)$.
Choosing L group at the queueing stage to update the beliefs of true states to select the correct resource is a dominant strategy which is irrelevant to the type of player.
The $|N| > 2$ case follows by the induction arguments.

Results: $|N| \geq 2$ case IV

Now we consider the result of $r \in (0, 1)$:

Proposition 3

Let $|N| \geq 2$. For all $p \in \Delta(\mathbb{R}^4)$ and $\frac{1}{n} < r < 1$ with $|N| = n$. The only preordering equilibrium rule is FF.

Proof (sketch):
Let $|N| = 2$. The analogue of Claim 1 with $r > 1/2$ also implies that the Stage 2 equilibrium outcome is $(n_1(\theta), n_2(\theta)) = (1, 1)$.
Then $a_i^* = F$ for all player i with $s_i \in \mathcal{I}$ by Proposition 1.
By the induction we assume that the Claim 1 holds for $n = k > 2$.
Consider $n = k - 1$.

Results: $|N| \geq 2$ case V

For the additional player i given the other k players choosing F group in Stage 1, player i must be assigned to the last position if he/she chooses L.
Then according to the proof of Proposition 1, the expected payoff of choosing F is strictly larger than the expected payoff of choosing L since $r \neq 1$. This completes the proof by induction arguments.

When N goes to large I

FL rule is not equilibrium for $|N| \geq 3$, and FL equilibrium does not stand-alone as $|N| = 2$:

Proposition 4

For $|N| \geq 3$, $r \in (0,1)$, $\beta \in [0,1]$ and $p \in \Delta(\mathbb{R}^4)$, following the FL rule is not an equilibrium strategy for all players $i \in N$.

Proof:
Consider two cases: $\frac{1}{n} \leq r \leq 1$ and $0 < r < \frac{1}{n}$.
We prove the statement by induction. Assume that the signal quality is perfectly not discriminating, that is, $p = (p_1, p_2, p_3, p_4) = (0.25, 0.25, 0.25, 0.25)$. For the set of true states $\Theta = \{\theta_1, \theta_2\}$ suppose the common prior of state is $g_0(\theta_1) = g_0(\theta_2) = 0.5$ since $p_1 = p_2 = p_3 = p_4 = 0.25$. According

When N goes to large II

to the Bayes rule the belief of first player who owns its signal and the common prior is $g_1(\theta_1) = \frac{p_1 \times g_0(\theta_1)}{p_1 \times g_0(\theta_1) + p_4 \times g_0(\theta_2)} = 0.5$. Similarly, $g_k(\theta_1) = g_k(\theta_2) = 0.5$ for all $k = 1, 2, ..., n$.

(i) Consider the number of players $|N| = 3$ and $1 > r \geq 1/3$. The player i with type strong signal $si \in S$ has the queueing stage expected utility as following form:

$$EU_i(a_i \in \{F, L\} | s_i, \mathcal{R}, r, \beta, \theta)$$

$$= \sum_{k=1}^{3} Pr(k_i = k | s_i, \mathcal{R}) EU_k(t_i^*, t_j^*, s_i, s_j, j \neq i, i, j \in N | \mathcal{R}, \mathcal{H}_{k_i}),$$

When N goes to large III

where $\mathcal{R}$ denotes the default preordering rule with $\mathcal{R} \in \{FF, FL, LF, LL\}$, , k denotes the queueing position assigned to player i for all $i \in N$, and t_i^* denotes the best response choice at the resource stage, which is obtained by recursive induction in a standard SCRG.

Then we have

$$EU_i(a_i = F, s_i|, FL, r, \beta, \theta)$$

$$= a_1 EU_1(t_i^*, t_j^*, j \neq i, |FL, \mathcal{H}_1) + a_2 EU_2(t_i^*, t_j^*, j \neq i, |FL, \mathcal{H}_2)$$

$$+ a_3 EU_3(t_i^*, t_j^*, j \neq i, |FL, \mathcal{H}_3),$$

where for $k = 1, 2, 3$, $a_k = Pr(k_i = k|s_i, FL)$ and the set of history $\mathcal{H}_k$ containing the players' own signal and the expected signals that the other players may received. Since $p = (0.25)$ the

When N goes to large IV

$EU_k(t_i^*, t_j^*, j \neq i, i, j \in N | FL, \mathcal{H}_k)$ does not differ in updating beliefs $g_k(\theta)$ for all $i \in N$, which is abbreviated to $EU1, EU2, EU3$, respectively.
Thus, fix the FL rule, for player i with strong signal type $s_i \in S$, we have

$$EU_i(a_i = F, s_i \in S | FL, r, \beta, \theta) - EU_i(a_i = L, s_i \in S | FL, r, \beta, \theta)$$

$$= (a_1 - a_1')EU1 + (a_2 - a_2')EU2 + (a_3 - a_3')EU3,$$

where a_k denotes the conditional probability of a type strong player i be assigned to the queueing position k as player i belong in the F group under the FL rule. Similarly, a_k' denotes the conditional probability of a type strong player i be assigned to the queueing position k as player i belong in the L group under the FL rule.

When N goes to large V

The FL rule assigns a type strong higher probability to front position, then we have $a_1 \geq a'_1$, $a_2 = a'_2$ and $a_3 \leq a'_3$ for $|N| = 3$. Since $r \geq 1/3$ and $p = (0.25)$, the first-mover advantage in the resource stage is nonnegative. That is, $EU1 \geq EU2 \geq EU3$. Therefore,

$$EU_i(a_i = F, s_i \in S|FL, r, \beta, \theta) - EU_i(a_i = L, s_i \in S|FL, r, \beta, \theta) \geq 0,$$

which means that the type strong player is unprofitable as deviating from FL rule.
On the other hand, the type weak player has the expected utility difference as follows:

$$EU_i(a_i = F, s_i \in W|FL, r, \beta, \theta) - EU_i(a_i = L, s_i \in W|FL, r, \beta, \theta)$$

When N goes to large VI

$$= (b_1 - b_1')EU1 + (b_2 - b_2')EU2 + (b_3 - b_3')EU3,$$

where b_k denotes the conditional probability of a type weak player i be assigned to the queueing position k as player i belong in the F group under the FL rule. Similarly, b_k' denotes the conditional probability of a type weak player i be assigned to the queueing position k as player i belong in the L group under the FL rule. The FL rule assigns a type weak higher probability to later position, then we have $b_1 \geq b_1'$, $b_2 = b_2'$ and $b_3 \leq b_3'$ for $|N| = 3$ case. And $EU_i(a_i = F, s_i \in W|FL, r, \beta, \theta) - EU_i(a_i = L, s_i \in W|FL, r, \beta, \theta) \geq 0$ since $r \geq 1/3$ and $p = (0.25)$ implies that $EU1 \geq EU2 \geq EU3$, which means that the type weak player is profitable as deviating from FL rule.

When N goes to large VII

Therefore, FL rule is not a dominant strategy for players $i \in N$ when $|N| = 3$ and $r \geq 1/3$.

(ii) Consider the number of players $|N| = 3$ and $1/3 > r > 0$. For each type strong player, FL rule implies that $a_1 \geq a'_1, a_2 = a'_2, a_3 \leq a'_3$ and the first-mover advantage is zero since $r < 1/3$, and the updated beliefs $g_1(\hat{\theta} \leq g_2(\hat{\theta} \leq g_3(\hat{\theta})$ for $\hat{\theta} = \max_{\theta \in \Theta} g_k(\theta)$. Thus $EU1 \leq EU2 \leq EU3$ and the resource stage equilibrium player distribution is $(3, 0)$ or $(0, 3)$. And the difference of expected utility with $r < 1/3$ is $EU_i(a_i = F, s_i \in S|FL, r, \beta, \theta) - EU_i(a_i = L, s_i \in S|FL, r, \beta, \theta) \leq 0$, which means that the type strong player has a profitable deviation from FL rule. For each type weak player, FL rule implies that $b_1 \geq b'_1, b_2 = b'_2, b_3 \leq b'_3$ and the first-mover advantage is zero

When N goes to large VIII

since $r < 1/3$, so we have $EU_i(a_i = F, s_i \in W|FL, r, \beta, \theta) - EU_i(a_i = L, s_i \in W|FL, r, \beta, \theta) \geq 0$, which suggests that the type weak player has a profitable deviation from FL rule.
In sum, the FL rule is not an equilibrium strategy at least for one type for all $0 < r < 1$ as $|N| = 3$.

By the induction assumption, suppose that the statement holds for $|N| = n - 1 > 3$. Fix the default preordering rule is FL.

(iii) Consider the number of players $|N| = n$ and $1 > r \geq 1/n$.

When N goes to large IX

The player i with type weak signal $si \in W$ has the queueing stage expected utility as following form:

$$EU_i(a_i \in \{F, L\} | s_i, FL, r, \beta, \theta)$$

$$= \sum_{k=1}^{n} Pr(k_i = k | s_i, FL) EU_k(t_i^*, t_j^*, s_i, s_j, j \neq i, i, j \in N | FL, \mathcal{H}_{k_i})$$

$$\equiv \sum_{k=1}^{n} b_k EUk,$$

where k denotes the queueing position assigned to player i for all $i \in N$, and t_i^* denotes the best response choice at the resource stage, which is obtained by recursive induction in a standard n-person SCRG.

When N goes to large X

Since $p = (0.25)$ the $EU_k(t_i^*, t_j^*, j \neq i, i, j \in N | FL, \mathcal{H}_k)$ does not differ in updating beliefs $g_k(\theta)$ for all $i \in N$, which is abbreviated to $EU1, EU2, EUk, ..., EUn$, respectively.
Thus, fix the FL rule, for player i with strong signal type $s_i \in W$, we have

$$EU_i(a_i = F, s_i \in W | FL, r, \beta, \theta) - EU_i(a_i = L, s_i \in W | FL, r, \beta, \theta)$$

$$= \sum_{k=1}^{n} (b_k - b'_k) EUk$$

where b_k denotes the conditional probability of a type weak player i be assigned to the queueing position k as player i belong in the F group under the FL rule for all $k = 1, 2, ..., n$. Similarly, b'_k denotes the conditional probability of a type weak player i be

When N goes to large XI

assigned to the queueing position k as player i belong in the L group under the FL rule for all $k = 1, 2, ..., n$.
The FL rule always assigns a type weak higher probability to later position, then we have $b_1 \geq b_1'$, $b_2 \geq b_2'$, ..., $b_w = b_w'$,..., $b_n \leq b_n'$ for some positive integer w s.t. $1 < w \leq n - 1$.
And $r \geq 1/3$ and $p = (0.25)$ implies that $EU1 \geq ... \geq EUn$,

$$EU_i(a_i = F, s_i \in W|FL, r, \beta, \theta) - EU_i(a_i = L, s_i \in W|FL, r, \beta, \theta)$$

$$= \sum_{k=1}^{n-1} (b_k - b_k')EUk + (b_n - b_n')EUn \geq 0$$

since by the induction assumption $\sum_{k=1}^{n-1}(b_k - b_k')EUk \geq 0$ and $(b_n - b_n')EUn \geq 0$, which means that the type weak player is profitable as deviating from FL rule for $|N| = n$ case.

When N goes to large XII

(iv) Consider the number of players $|N| = n$ and $1/n > r > 0$. For each type strong player, FL rule implies that $a_1 \geq a_1', a_2 = a_2', ..., a_w = a_w', a_{w+1} \leq a_{w+1}', ..., a_n \leq a_n'$ for some positive integer w s.t. $1 < w \leq n-1$. Note that the first-mover advantage is zero since $r < 1/n$. And $p = (0.25)$ implies that the updated beliefs $g_1(\hat{\theta} \leq ... \leq g_n(\hat{\theta})$ for $\hat{\theta} = \max_{\theta \in \Theta} g_k(\theta), k = 1, 2, ..., n$. Thus $EU1 \leq ... \leq EUn$ and the resource stage equilibrium player distribution is $(n, 0)$ or $(0, n)$. And the difference of expected utility with $r < 1/n$ is

$$EU_i(a_i = F, s_i \in S|FL, r, \beta, \theta) - EU_i(a_i = L, s_i \in S|FL, r, \beta, \theta)$$

$$= \sum_{k=1}^{n-1}(a_k - a_k')EUk + (a_n - a_n')EUn \leq 0$$

When N goes to large XIII

since by the induction assumption $\sum_{k=1}^{n-1}(a_k - a'_k)EUk \leq 0$ and $(a_n - a'_n)EUn \leq 0$, which means that the type strong player is profitable as deviating from FL rule for $|N| = n$ case.
Combine (iii) and (iv) it turns out that the statement holds for $|N| = n$ case.
For the signal distribution is discriminating, that is, for all $p \in \Delta(\mathbb{R}^4)$ we have $p_1 > p_2 \geq p_3 \geq p_4$, and the average EUk maintains the monotonic property as $k = 1, 2, ...$, thus the above (i)-(iv) results hold for the other p s.t. $p_1 > p_2 \geq ... \geq p_4$ and $\sum_{l=1}^{4} p_l = 1$. This completes the proof.

Q.E.D.

When N goes to large XIV

A rare case: players is profitable to deviate from every preordering rule:

Observation 1

There exists at least one set of parameters r, β, p such that every preordering rule $\mathcal{R} \in \{FF, FL, LF, LL\}$ is not a preordering equilibrium.

The non-deviation(ND) inequality holds for some $0 < r < 1/n$ and $p \in \Delta(\mathbb{R}^4)$ s.t.

$$X_i(s_i; \mathcal{R}, \theta, r, \beta, p) \equiv$$

$$EU_i(a_i = D, s_i | \mathcal{R}, \theta, r, \beta, p) - EU_i(a_i = ND, s_i | \mathcal{R}, \theta, r, \beta, p) > 0.$$

When N goes to large XV

For example, the set of parameters $(r, p) = (0.125, 0.4, 0.25, 0.2, 0.15)$ with a quotient share with $|N| = 3$ causes a no equilibrium strategy case.

Applications and further

- Why does the FL equilibrium vanish?
- How to design a game-form to implement the FL equilibrium?
- Extension: resource allocation for more tables?

THANKS FOR ALL ATTENTION.