Financial Frictions and Uncertainty Shocks

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Abstract

This paper investigates the role of credit constraints in the propagation of uncertainty shocks within a business cycle framework. Traditional business cycle models struggle to account for observed declines in output, consumption, investment, and labor hours in response to heightened uncertainty. To address this gap, we introduce a collateral-based credit constraint for both impatient households and entrepreneurs, with borrowing limitations connected to the value of their collateral assets. As uncertainty escalates, households and entrepreneurs demand a higher risk premium for collateral, leading to a decrease in its overall demand. This reduced demand, in turn, prompts households to curtail their labor supply, leading to a decrease in output. Our study emphasizes that collateral adjustments following uncertainty shocks can induce macroeconomic co-movements in a real business cycle model, even in the absence of nominal rigidities. Furthermore, we discover that a lower loan-to-value (LTV) ratio can help alleviate the adverse impacts of uncertainty shocks. This research offers a new perspective on how financial constraints shape macroeconomic dynamics in times of heightened uncertainty, providing valuable insights for policymakers and economists.

Keywords: Uncertainty, co-movement problem, financial friction
JEL classification: E21, E32, E44

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1 Introduction

In recent years, a growing body of literature has delved into the ramifications of uncertainty shocks on macroeconomic dynamics. These investigations all demonstrate that increased uncertainty is linked to decreases in overall output, consumption, investment, and working hours (Jurado, Ludvigson and Ng 2015; Fernández-Villaverde et al. 2015; Baker, Bloom and Davis 2016; Basu and Bundick 2017; Carriero, Clark and Marcellino 2018; Oh 2020; Cross et al. 2023). However, when researchers incorporate uncertainty shocks into standard business cycle models, the resulting predictions often diverge from the empirical evidence outlined above. This discrepancy arises because increased uncertainty in the model amplifies the precautionary motives of economic agents, leading the representative household to reduce consumption and increase labor supply. While current technology and capital stock remain unchanged, the higher labor supply boosts output. The rise in output, coupled with reduced consumption, implies an increase in investment. This theoretical explanation for increased output, labor supply, and investment contradicts empirical findings. This inconsistency between model predictions and empirical findings is commonly known as the co-movement problem associated with uncertainty shocks.[1]

Based on the analysis provided earlier, it becomes clear that the key challenge in this literature is to get labor hours to fall when uncertainty rises. The existing literature primarily focuses on factors affecting labor demand, such as irreversible hiring (Leduc and Liu 2016), non-convex adjustment costs (Bloom et al. 2018), risky hiring (Arellano, Bai and Kehoe 2019), and precautionary pricing (Basu and Bundick 2017; Born and Pfeifer 2021), to explain the reduction in labor hours during uncertain times. This emphasis leaves a gap in understanding the supply-side factors behind the decline in labor hours following uncertainty shocks.

The main distinction between the shift in the labor supply and demand channels lies in their impacts on real wages and labor hours. Specifically, the labor demand channel would lead to a positive co-movement in both real wages and labor hours, while the labor supply channel would lead to a negative co-movement in real wages and labor hours following the shocks. A recent study by Cross et al. (2023), utilizing Bayesian vector autoregression, indicates that average hours fall, while hourly earnings rise following uncertainty shocks (see Figure 7 on page 13 in Cross et al. (2023)).[2] In addition, recent research by Lee,

[1]The co-movement problem is linked to the insights of Barro and King (1984), who show that the one-sector growth model generates business-cycle-like co-movement patterns only when contemporaneous shocks to total factor productivity (TFP) occur. Other types of shocks struggle to replicate the observed patterns of positive co-movement in the data.

[2]In contrast, Fernández-Villaverde et al. (2015) and Carriero, Clark and Marcellino (2018) identify a decline in labor hours but have not found a statistically significant difference in the response of real

Park and Shin (2021) highlights the significant influence of both supply-side and demand-side factors on aggregate working hours throughout the business cycle. The existing literature's focus on demand-side factors has resulted in a gap in understanding the supply-side elements contributing to the reduction in labor hours post-uncertainty shocks. Our approach aims to bridge this gap by establishing a connection between the decrease in the supply of working hours and the stylized procyclical household debt documented by Campbell and Hercowitz (2011).

Our paper builds upon the framework introduced by Iacoviello (2005) to incorporate two essential components. Firstly, it considers the heterogeneity in time discount rates among a patient household, an impatient household, and an entrepreneur, leading to borrowing and lending dynamics and emphasizing the significance of debt. Secondly, it integrates the concept of borrowers using durable assets such as housing or physical capital as collateral to address repayment concerns, in line with Kiyotaki and Moore (1997).[3] By imposing collateral constraints on both impatient households and entrepreneurs, we show that our model is capable of generating a simultaneous decline in key macroeconomic variables in response to a rise of uncertainty, even in the absence of nominal rigidity.

In our economic model, we find that patient households closely resemble the representative agent in the standard business cycle model. They increase their work hours and reduce their consumption in response to heightened uncertainty. In contrast, impatient households choose to reduce their work hours. The reduction in work hours is primarily attributed to the diminishing borrowing incentives, which stem from the use of collateral, and the increased risk associated with collateral holdings during heightened uncertainty. It's crucial to note that while both patient and impatient households actively participate in the housing market, a notable distinction lies in their housing finance strategies. Specifically, patient households acquire properties without employing leverage, while impatient households use mortgages for their purchases. On one hand, this leveraged approach enables impatient households to acquire larger properties with a relatively modest initial down payment. On the other hand, this strategy also exposes them to higher risks, especially during periods of uncertainty. As overall uncertainty increases, the expected housing resale value becomes more volatile, so impatient households demand

wages following uncertainty shocks, as their confidence intervals all contain zeros.

[3]Housing markets played a pivotal role in the Great Recession, characterized by significant price fluctuations observed in the United States. During this period, housing prices surged by approximately 80% from 2000 to their peak in 2007, followed by a subsequent decline of roughly 25% until reaching their lowest point in 2012. The interconnection of macroeconomics, finance, and housing is well documented in the literature. For an extensive review, refer to Davis and Van Nieuwerburgh (2015). Specifically, household debts are constrained by the collateral values, which depend on the relative price of housing. When housing prices decline due to changes in uncertainty-induced risk premiums in housing markets, it results in a tightening of households' borrowing constraints. This, in turn, impedes households' capacity to purchase homes, further exacerbating the decline in housing prices and amplifying the recession.

a greater risk premium compared to patient households who buy homes without loans. As a result, impatient households choose to downsize in order to decrease their housing exposure. This downsizing prompts impatient households to reallocate their expenditures from housing towards consumption. Furthermore, the decreased housing expenses resulting from downsizing also contribute to a decrease in their working hours, as they no longer need to exert as much effort to afford larger housing.[4] If the reduction in labor hours among impatient households outweighs the increase among patient households, aggregate labor hours decrease, leading to a subsequent decline in aggregate production, as housing and capital are predetermined.

This reduction in aggregate labor hours also affects the marginal product of housing and capital, as labor complements both in the production process. Consequently, entrepreneurs decrease their demand for housing and investment. Furthermore, increased uncertainty leads to a decreased incentive in using these assets as collateral to secure external loans. Consequently, financial constraints exacerbate the decline in entrepreneurs' demand for production factors, leading to reduced declines in both housing and physical capital. Therefore, a simple variant of the flexibly priced business cycle model, incorporating a collateral constraint for both impatient households and entrepreneurs, can reproduce boom-bust business cycles in response to the rise in uncertainty.

In our model, the uncertainty shock leads to a redistribution of housing from impatient to patient households. On one hand, impatient households choose to downsize their housing to reduce their exposure to fluctuations in housing resale value. On the other hand, housing serves as both a durable consumption good and a means of future savings for patient households. Since the shadow value of long-lived durables remains relatively stable after the uncertainty shock, patient households are less sensitive to the timing of their durable goods purchases. In response to the uncertainty shock, the relative price of housing falls, prompting patient households to increase their housing holdings.

Furthermore, the financial frictions related to impatient households effectively encompass the mechanisms of the financial labor supply accelerator as outlined in Campbell and Hercowitz (2011) model. Within the representative household framework, they establish a long-term negative relationship between the minimum down payment required for collateral and household working hours. In our model, patient households have a higher down payment requirement compared to impatient households. This difference arises because only impatient households utilize leverage to acquire houses, while patient households do not employ this financial strategy. During periods of heightened uncertainty, the decline in aggregate labor hours aligns with a housing transition from impatient households to

[4] Consequently, there is a positive correlation between labor supply and household debt. The strength of this relationship is positively influenced by the down payment rate.

patient households. This transition of housing effectively raises the overall down payment required for housing purchases. As a result, we illustrate the negative relationship between down payments and labor supply in an economy comprising both patient and impatient households.

A recent paper by Chatterjee, Gunawan and Kohn (2023) also examines the interaction between credit constraints and uncertainty shocks. They focus on credit shocks that directly affect the firm's borrowing constraints. In contrast, we concentrate on uncertainty shocks that affect aggregate TFP. Specifically, their credit uncertainty shocks affect the firm's borrowing, directly impacting labor demand. In contrast, our approach involves the households' borrowing constraints, which influence household labor supply behavior. While our model incorporates both household's and entrepreneur's borrowing constraints, the key constraint that resolves the co-movement problem is the household's borrowing constraint. This becomes evident when we deactivate the household's borrowing constraint while keeping the entrepreneur's borrowing constraints active in section 4.1.1; we are unable to resolve the puzzle. Therefore, two key differences between their study and our work lie in both the type of second-moment shocks and the primary mechanism at play.

Prior research in the literature has shown that the incorporation of financial frictions offers a solution to the co-movement problem. A series of noteworthy studies have explored the interplay between uncertainty and financial frictions. For instance, Gilchrist, Sim and Zakrajšek (2014) have delved into the relationship between uncertainty, investment, and credit spreads, highlighting the amplifying effect of financial frictions on uncertainty's impact through credit spreads. Christiano, Motto and Rostagno (2014) have introduced agency issues related to financial intermediation in a monetary dynamic general equilibrium model, emphasizing the pivotal role of volatility shocks in steering the business cycle. Arellano, Bai and Kehoe (2019) have developed a DSGE model that incorporates labor and financial market frictions, unveiling how uncertainty shocks elevate default risk and credit spreads, leading to workforce reductions by firms. Furthermore, Ottonello and Winberry (2020) have explored the connection between financial frictions, firm diversity, and the influence of monetary policy on firm investment. In line with this body of research, our paper takes a step toward comprehending the impact of household heterogeneity in the transmission of uncertainty shocks. Our model predicts that household indebtedness is a critical factor contributing to the heightened effects of uncertainty.

Our model also complements the existing studies by showing how financing frictions may amplify or propagate output fluctuations in response to different types of aggregate shocks. Bernanke and Gertler (1989); Kiyotaki and Moore (1997); and Carlstrom and Fuerst (1997) show that financial frictions can amplify the output fluctuation in response to technology shocks. Iacoviello (2005); and Iacoviello and Neri (2010) note that financial frictions can amplify and propagate policy shocks. In our research, we propose that financial frictions can exacerbate and transmit uncertainty shocks. Specifically, we assess the effects of uncertainty shocks in our baseline model, which includes financial frictions, and compare them to a model without financial frictions but with nominal price rigidities. Our analysis reveals that the reduction in aggregate production is more pronounced in our benchmark model than in the model featuring price stickiness but lacking financial frictions.

In terms of the co-movement problem associated with the uncertainty shocks, most existing studies require nominal rigidity to generate co-movement among key macroeconomic aggregates in response to an uncertainty shock; however, our model can reproduce the boom-bust business cycles even without nominal rigidity.[5] As Born and Pfeifer (2021) exhibits that the nominal rigidity in the price setting behavior is inconsistent with the data on uncertainty shocks, we believe our research is beneficial to the field of study.[6]

The structure of the paper is as follows. In Section 2, we present our baseline model, which includes credit-constrained households and entrepreneurs. Section 3 forms the core of our quantitative analysis. We begin with the estimation of the stochastic process of uncertainty shocks and then utilize these estimated parameters in simulations to examine the effects of uncertainty shocks. In Section 4, we conduct several exercises to enhance our understanding of how key mechanisms work in the model and conduct a comparative analysis between our baseline model and a dynamic New Keynesian model with sticky prices to assess their relative importance. Finally, Section 5 provides our concluding remarks.

[5] For example, Basu and Bundick (2017); Born and Pfeifer (2014); Leduc and Liu (2016); and Cesa-Bianchi and Fernandez-Corugedo (2018) all rely on nominal rigidity to reproduce the boom-bust business cycles following the uncertainty shocks. In particular, Born and Pfeifer (2014) study the impacts of the uncertainty policy risk in a model with sticky prices and wages. They find that the aggregate effect of policy risk is minor on the business cycle. Leduc and Liu (2016) show that combining both nominal rigidity and labor market search frictions can explain the rise in unemployment and the fall in inflation following the rise of uncertainty. Cesa-Bianchi and Fernandez-Corugedo (2018) investigate the impact of macro and micro uncertainty within a DSGE model with sticky prices and financial frictions where "macro uncertainty" is referred to as the uncertainty about aggregate shocks, such as the time-varying variance of TFP, and "micro uncertainty" is referred to as the uncertainty about idiosyncratic shocks, such as the cross-sectional dispersion of firm-level productivity. They show that microeconomic uncertainty shocks have a bigger impact on growth than macro uncertainty shocks.

[6] Born and Pfeifer (2021) assess whether the model channel of sticky prices and wages is consistent with data following uncertainty shocks. They find that the response to uncertainty shocks is consistent with the sticky wage setting, but not price setting.

2 The Model

Our model builds upon the framework established by Iacoviello (2005) to examine the impacts of uncertainty shocks. Within this model, time is discrete and indexed by t. There are three types of agents: patient households, impatient households, and entrepreneurs. Impatient households and entrepreneurs possess lower discount factors in comparison to patient households, leading them to borrow from the latter. Consequently, we refer to impatient households and entrepreneurs as "borrowers" and patient households as "savers." These borrowers encounter credit constraints similar to the framework proposed by Kiyotaki and Moore (1997), utilizing durable assets such as housing or physical capital as collateral to address repayment concerns that arise due to costly enforcement. Our analysis delves into the behaviors of these groups to examine the transmission mechanisms associated with uncertainty shocks within the model. Below, we introduce the problems faced by each agent in turn.

2.1 Patient Households

There is a continuum of mass one of savers that choose consumption, $c_{s,t}$, bonds, $b_{s,t}$, housing, $h_{s,t}$, and working hours, $n_{s,t}$, to maximize their lifetime utility:[7]

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta_s^t \left[\Gamma_{c,s} \cdot \frac{(c_{s,t} - \phi_c c_{s,t-1})^{1-\sigma_c} - 1}{1-\sigma_c} + J \cdot \frac{h_{s,t}^{1-\sigma_h} - 1}{1-\sigma_h} - \kappa \cdot \frac{n_{s,t}^{1+\eta}}{1+\eta} \right],$$

where E_0 represents the expectation operator conditional on information in period 0, β_s is the savers' discount factor, σ_c, σ_h and η determine the curvature of the utility function with respect to consumption, housing, and labor hours, respectively. Parameters J and κ reflect the preferences associated with housing and work. Additionally, ϕ_c measures the strength of consumption habit, and $\Gamma_{c,s} \equiv (1-\phi_c)/(1-\beta_s\phi_c)$ is a scaling factor ensuring the patient households' marginal utility of consumption is $1/c_s$ in the steady state.

Savers face a budget constraint given by:

$$c_{s,t} + q_t h_{s,t} + b_{s,t} \leq w_{s,t} n_{s,t} + q_t h_{s,t-1} + \frac{R_{t-1}}{\Pi_t} \cdot b_{s,t-1} + div_t, \tag{1}$$

where $b_{s,t}$ represents the bond holdings of savers, $w_{s,t}$ represents the real wage rate, q_t represents the relative price of housing, R_t is the nominal interest rate, Π_t is the inflation rate, and div_t is the dividend earned from owning retailers' businesses.

The first-order conditions associated with savers' problems with consumption, labor

[7] See, for example, Chambers, Garriga and Schlagenhauf (2009); Choa and Francis (2011); and Cao and Nie (2017).

hours, housing, and bond holdings are:

$$\lambda_{s,t} = uc_{s,t}, \tag{2}$$

$$\lambda_{s,t} w_{s,t} = -un_{s,t}, \tag{3}$$

$$\lambda_{s,t} q_t = \beta_s \mathrm{E}_t \left(\lambda_{s,t+1} q_{t+1}\right) + uh_{s,t}, \tag{4}$$

$$\lambda_{s,t} = \beta_s \mathrm{E}_t \left(\lambda_{s,t+1} \frac{R_t}{\Pi_{t+1}}\right), \tag{5}$$

where $\lambda_{s,t}$ is the Lagrange multiplier associated with the savers' budget constraints, Eq. (1), $uc_{s,t}$, $uh_{s,t}$, and $un_{s,t}$ are the first-order derivatives of the savers' utility function with respect to $c_{s,t}$, $h_{s,t}$, and $n_{s,t}$, respectively, which can be expressed as follows:

$$\begin{aligned} uc_{s,t} &= \Gamma_{c,s}\left\{(c_{s,t} - \phi_c c_{s,t-1})^{-\sigma_c} - \beta_s \phi_c \mathrm{E}_t \left[(c_{s,t+1} - \phi_c c_{s,t})^{-\sigma_c}\right]\right\}, \\ uh_{s,t} &= J h_{s,t}^{-\sigma_h}, \\ un_{s,t} &= -\kappa n_{s,t}^{\eta}. \end{aligned}$$

Combing Eq. (2) and Eq. (4), we obtain the optimal condition of housing for patient households as follows:

$$q_t = \mathrm{E}_t(\Lambda^s_{t+1,t} q_{t+1}) + mrs^s_{hc,t}, \tag{6}$$

where $\Lambda^s_{t+1,t} \equiv \beta_s \lambda_{s,t+1}/\lambda_{s,t}$ is savers' stochastic discount factor (SDF) and $mrs^s_{hc,t} \equiv uh_{s,t}/\lambda_{s,t}$ is savers' marginal rate of substitution (MRS) for housing with respect to non-durable goods. Because housing is a durable good, patient households select their housing so that the current housing purchase cost equals the combined benefit of the expected discounted resale value (calculated as the resale housing price multiplied by their stochastic discount factor) and the MRS between housing and non-durable goods.

Combing Eq. (2) and Eq. (3) yields the optimal labor-leisure condition, balancing the cost and benefit of working:

$$uc_{s,t} w_{s,t} = -un_{s,t}. \tag{7}$$

The patient household can utilize both housing and bonds as savings instruments. Therefore, the no-arbitrage condition dictates that the one-period return on bonds must be equal to the return from holding housing, i.e.,

$$\mathrm{E}_t\left(\frac{\Lambda^s_{t+1,t} R_t}{\Pi_{t+1}}\right) = \mathrm{E}_t\left(\frac{\Lambda^s_{t+1,t} q_{t+1}}{q_t}\right) + \frac{mrs^s_{hc,t}}{q_t}. \tag{8}$$

2.2 Impatient Households

There is a continuum of impatient households with mass one who choose consumption, $c_{b,t}$, bonds, $b_{b,t}$, housing, $h_{b,t}$, and working hours, $n_{b,t}$, to maximize their expected lifetime utility:

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta_b^t \left[\Gamma_{c,b} \cdot \frac{(c_{b,t} - \phi_c c_{b,t-1})^{1-\sigma_c} - 1}{1-\sigma_c} + J \cdot \frac{h_{b,t}^{1-\sigma_h} - 1}{1-\sigma_h} - \kappa \cdot \frac{n_{b,t}^{1+\eta}}{1+\eta} \right],$$

where β_b is the discount factor of impatient households. We assume $\beta_b < \beta_s$ to ensure that the credit constraints for impatient households are binding in equilibrium. In addition, $\Gamma_{c,b} \equiv (1-\phi_c)/(1-\beta_b\phi_c)$ denotes the scaling factor that ensures the impatient households' marginal utility of consumption equates to $1/c_b$ in the steady state.

The budget constraint for impatient households is:

$$c_{b,t} + q_t h_{b,t} + b_{b,t} \leq w_{b,t} n_{b,t} + q_t h_{b,t-1} + \frac{R_{t-1}}{\Pi_t} \cdot b_{b,t-1}, \tag{9}$$

where $b_{b,t}$ represents the bond holdings of the impatient households, and $w_{b,t}$ is the wage rate for the impatient households.

Furthermore, patient households are unable to enforce debt repayment unless the debt is supported by the borrower's housing as collateral. This implies that houses have a dual role for impatient households, serving as both residences and collateral assets. Specifically, the impatient households face borrowing limits, which are related to a fraction $m_b \in [0,1]$ of their housing value:

$$-b_{b,t} \leq m_b \mathrm{E}_t \left(\frac{\Pi_{t+1}}{R_t} \cdot q_{t+1} \right) h_{b,t}. \tag{10}$$

The fraction m_b is known as the loan-to-value (LTV) ratio. It reflects the extent of financial market frictions, which can stem from factors such as credit market tightness due to financial innovation or regulations.

We then derive the first-order conditions associated with impatient households' problems for $c_{b,t}$, $n_{b,t}$, $h_{b,t}$, and $b_{b,t}$:

$$\lambda_{b,t} = uc_{b,t}, \tag{11}$$

$$\lambda_{b,t} w_{b,t} = -un_{b,t}, \tag{12}$$

$$\lambda_{b,t} q_t = \mathrm{E}_t(\beta_b \lambda_{b,t+1} q_{t+1} + \rho_{b,t} m_b q_{t+1} \Pi_{t+1}) + uh_{b,t}, \tag{13}$$

$$\lambda_{b,t} = \beta_b \mathrm{E}_t \left(\lambda_{b,t+1} \frac{R_t}{\Pi_{t+1}} \right) + \rho_{b,t} R_t, \tag{14}$$

where $\lambda_{b,t}$ denotes the Lagrange multiplier associated with the budget constraint, Eq. (9), and $\rho_{b,t}$ is associated with the collateral constraint, Eq. (10). $uc_{b,t}$, $uh_{b,t}$, and $un_{b,t}$ denote respectively the first derivatives of the impatient households' utility with respect to $c_{b,t}$, $h_{b,t}$, and $n_{b,t}$, which can be expressed as follows:

$$uc_{b,t} = \Gamma_{c,b}\Big\{(c_{b,t} - \phi_c c_{b,t-1})^{-\sigma_c} - \beta_b \phi_c \mathrm{E}_t[(c_{b,t+1} - \phi_c c_{b,t})^{-\sigma_c}]\Big\}, \tag{15}$$

$$uh_{b,t} = J h_{b,t}^{-\sigma_h}, \tag{16}$$

$$un_{b,t} = -\kappa n_{b,t}^{\eta}. \tag{17}$$

The main difference between the optimal conditions of the patient households and the impatient households lies in their housing choices. In particular, combining Eq. (13) and Eq. (14), we can derive the optimal housing condition for impatient households as follows,

$$q_t = \mathrm{E}_t\left[\left((1 - m_b)\cdot \Lambda^b_{t+1,t} + m_b \cdot \frac{\Pi_{t+1}}{R_t}\right)\cdot q_{t+1}\right] + mrs^b_{hc,t}. \tag{18}$$

where $\Lambda^b_{t+1,t} \equiv \beta_b \lambda_{b,t+1}/\lambda_{b,t}$ is impatient households' SDF and $mrs^b_{hc,t} \equiv uh_{b,t}/\lambda_{b,t}$ is their MRS for housing with respect to non-durable goods.[8] Much like the optimal housing condition for patient households as shown in Eq. (6), impatient households make their housing choice to ensure that the current housing price is equivalent to the sum of the discounted resale housing value and the MRS for housing with non-durable goods. However, in contrast to patient households, who exclusively rely on their SDF to evaluate the expected resale housing value, impatient households utilize a weighted average of their SDF and the inverse of the real interest rate to evaluate the expected resale housing price. The reason the inflation term is included in the equation is because they use houses as collateral for borrowing, and nominal debt is tied to the value of their housing resale.[9]

[8] An alternative way to express the optimal condition of housing for impatient households is as follows,

$$\lambda_{b,t}\left[q_t - m_b \mathrm{E}_t\left(\frac{\Pi_{t+1}}{R_t}\cdot q_{t+1}\right)\right] = \beta_b(1 - m_b)\mathrm{E}_t(\lambda_{b,t+1}q_{t+1}) + uh_{b,t}. \tag{19}$$

The term $[q_t - m_b \mathrm{E}_t(\Pi_{t+1}/R_t \cdot q_{t+1})]$ represents the "down payment" faced by impatient households. When impatient households are allowed to use housing as collateral to borrow, they only need to pay a fraction of the housing price in the current period. In contrast, the saver does not borrow, so their down payment for housing today is simply q_t.

[9] The optimal housing condition for the saver can be decomposed as

$$q_t = \mathrm{E}_t(\Lambda^s_{t+1,t})\cdot \mathrm{E}_t(q_{t+1}) + \mathrm{Cov}_t(\Lambda^s_{t+1,t}, q_{t+1}) + mrs^s_{hc,t}.$$

Eq. (18) can be further decomposed as

$$\begin{aligned} q_t = & \left[(1-m_b)\mathrm{E}_t(\Lambda^b_{t+1,t}) + m_b\mathrm{E}_t(\Pi_{t+1})/R_t\right] \cdot \mathrm{E}_t(q_{t+1}) \\ & + (1-m_b)\mathrm{Cov}_t(\Lambda^b_{t+1,t}, q_{t+1}) + m_b\mathrm{Cov}_t(\Pi_{t+1}, q_{t+1})/R_t + mrs^b_{hc,t}. \end{aligned} \tag{20}$$

From the decomposition, inflation appears in both the product of the expected terms and the covariance terms. The product of expected terms accounts for the inflation's impact on the expected resale value of housing prices. The term associated with the covariance of the inflation rate and resale housing prices represents the risk premiums associated with housing purchases. Similarly, the terms linked to the covariance of resale housing prices and the SDF determine the risk premiums of housing purchases. If there is an increase in the expected real interest rate or a decrease in either of the covariance terms, the demand for housing will decrease, as the willingness to pay for housing falls.

Furthermore, by imposing these borrowing limits, one can derive the consolidated budget constraint through a combination of Eq. (9) and Eq. (10) to yield:

$$c_{b,t} + \left[q_t - m_b\mathrm{E}_t\left(\frac{\Pi_{t+1}}{R_t} \cdot q_{t+1}\right)\right] h_{b,t} \leq w_{b,t}n_{b,t} + (1-m_b) \cdot q_t \cdot h_{b,t-1}. \tag{21}$$

It's worth noting that the borrowing constraint, while not directly affecting the optimal labor-leisure conditions for impatient households, does influence the choice set of labor and non-durable consumption. Specifically, the change in relative expenditure on housing, which impacts borrowing limits, contributes to the shift in the consolidated budget constraint, thereby affecting decisions related to labor and non-durable consumption.

2.3 The Entrepreneurs

There is a continuum of mass one of entrepreneurs who produce homogeneous wholesale goods according to the following Cobb-Douglas production function:

$$y_t = A_t k^{\mu}_{t-1} h^{\nu}_{e,t-1} n^{1-\mu-\nu}_t, \tag{22}$$

where A_t represents the aggregate TFP, k_{t-1} represents the capital stock, $h_{e,t-1}$ represents the entrepreneurs' real estate holdings, n_t denotes the aggregate labor input, μ measures the share of capital, ν measures the share of housing, and $1-\mu-\nu$ represents the share of aggregate labor hours in the production function. Furthermore, the aggregate labor input, $n_t = n^{\alpha}_{s,t} n^{1-\alpha}_{b,t}$, is a combination of labor inputs from both patient households, $n_{s,t}$, and impatient households, $n_{b,t}$.

Entrepreneurs encounter credit constraints similar to those faced by impatient households. Entrepreneurs have the flexibility to use both houses and physical capital as collateral, relative to impatient households. Moreover, we allow different LTV ratios for housing and capital, denoted as $m_e^h \in [0,1]$ for housing loans and $m_e^k \in [0,1]$ for capital loans. Consequently, the borrowing limit can be expressed as follows:

$$-b_{e,t}^h \leq m_e^h \mathrm{E}_t \left(\frac{\Pi_{t+1}}{R_t} \cdot q_{t+1} \right) h_{e,t}, \tag{23}$$

$$-b_{e,t}^k \leq m_e^k \mathrm{E}_t \left(\frac{\Pi_{t+1}}{R_t} \right) k_t. \tag{24}$$

In Eq. (23), $b_{e,t}^h$ represents the loans secured by the value of housing, while in Eq. (24), $b_{e,t}^k$ represents the loans secured by the value of capital. Entrepreneurs' overall loan position is defined as $b_{e,t} \equiv b_{e,t}^h + b_{e,t}^k$.[10] These borrowing constraints reflect the limitations on entrepreneurs' borrowing capacity based on the expected discounted value of their collateral and the applicable LTV ratios.

The entrepreneur starts each period with an initial loan of $b_{e,t-1}$ and earns $(1/X_t) \cdot y_t$ by selling their output to retailers. Specifically, the $(1/X_t)$ denotes the wholesale goods price, and X_t represents the retailers' markup.[11] Entrepreneurs then acquire new debt of $b_{e,t}$, consume $c_{e,t}$, invest i_t, adjust their real estate holdings $q_t(h_{e,t} - h_{e,t-1})$, and repay their previous debt $(R_{t-1}/\Pi_t) \cdot b_{e,t-1}$. Additionally, they pay wages to both patient households $(w_{s,t}n_{s,t})$ and impatient households $(w_{b,t}n_{b,t})$. Therefore, entrepreneurs' budget constraints can be expressed as:

$$c_{e,t} + i_t + q_t(h_{e,t} - h_{e,t-1}) + w_{s,t}n_{s,t} + w_{b,t}n_{b,t} + b_{e,t} = \frac{1}{X_t} \cdot y_t + \frac{R_{t-1}}{\Pi_t} \cdot b_{e,t-1}, \tag{25}$$

The law of motion for capital is:

$$k_t = i_t + (1-\delta)k_{t-1}, \tag{26}$$

where k_t represents the capital stock, and δ is the depreciation rate.

Let β_e be the discount factor for entrepreneurs. We assume the entrepreneurs' discount factor satisfies $\beta_e < \beta_s$ to ensure that the credit constraints for the entrepreneurs are

[10] For the case where $m_e^h = m_e^k = m_e$, Eqs. (23)–(24) can be combined into a single credit constraint, $b_{e,t} = m_e \mathrm{E}_t \left[\Pi_{t+1}/R_t (q_{t+1}h_{e,t} + k_t) \right]$.

[11] While our model does not depend on price stickiness to address the co-movement puzzle, we consider the possibility of price stickiness for the purpose of model comparison. Introducing monopolistically competitive retailers allows us to effortlessly transition between a flexible and a sticky price economy by modifying the parameter associated with price stickiness.

binding in equilibrium. Additionally, their lifetime utility function is given by:

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta_e^t \left[\Gamma_{c,e} \cdot \frac{(c_{e,t} - \varepsilon_c c_{e,t-1})^{1-\sigma_c} - 1}{1-\sigma_c} \right].$$

where $\Gamma_{c,e} \equiv (1-\phi_c)/(1-\beta_e\phi_c)$ is the scaling factor that ensures the marginal utility of consumption equals $1/c_e$ in the steady state.

Finally, entrepreneurs maximize their lifetime utility subject to the budget constraint, Eq. (25), the law of motion of the capital stock, Eq. (26), and the credit constraints, Eqs. (23)–(24). The associated first-order conditions are:

$$\lambda_{e,t} = uc_{e,t}, \tag{27}$$

$$\lambda_{e,t} q_t = \mathrm{E}_t \left[\beta_e \lambda_{e,t+1} \left(\frac{\nu y_{t+1}}{X_{t+1} h_{e,t}} + q_{t+1} \right) + m_e^h \rho_{e,t}^h q_{t+1} \Pi_{t+1} \right], \tag{28}$$

$$\lambda_{e,t} = \mathrm{E}_t \left(\beta_e \lambda_{e,t+1} \frac{R_t}{\Pi_{t+1}} \right) + \rho_{e,t}^h R_t, \tag{29}$$

$$\lambda_{e,t} = \beta_e \mathrm{E}_t \left\{ \lambda_{e,t+1} \left[\frac{\mu y_{t+1}}{X_{t+1} k_t} + (1-\delta) \right] + m_e^k \rho_{e,t}^k \Pi_{t+1} \right\}, \tag{30}$$

$$\lambda_{e,t} = \mathrm{E}_t \left(\beta_e \lambda_{e,t+1} \frac{R_t}{\Pi_{t+1}} \right) + \rho_{e,t}^k R_t, \tag{31}$$

$$w_{s,t} = \frac{\alpha(1-\mu-\nu) y_t}{X_t n_{s,t}}, \tag{32}$$

$$w_{b,t} = \frac{(1-\alpha)(1-\mu-\nu) y_t}{X_t n_{b,t}}, \tag{33}$$

where $\lambda_{e,t}$ denotes the Lagrange multiplier for budget constraint, Eq. (25). $\rho_{e,t}^h$ and $\rho_{e,t}^k$ stand for the Lagrange multipliers associated with credit constraints, Eqs. (23)–(24), respectively. $uc_{e,t}$ represents the marginal utility of consumption of entrepreneurs, which is defined as

$$uc_{e,t} \equiv \Gamma_{c,e} \left\{ (c_{e,t} - \phi_c c_{e,t-1})^{-\sigma_c} - \beta_e \phi_c \mathrm{E}_t [(c_{e,t+1} - \phi_c c_{e,t})^{-\sigma_c}] \right\}.$$

In contrast to a standard model without borrowing constraints, the first-order conditions for an entrepreneur's housing and capital now incorporate terms that represent the shadow value associated with the relaxation of the borrowing constraint due to an additional unit of housing or capital, respectively. Consequently, the optimal choice of capital and housing ensures that the entrepreneur equates today's prices of capital and housing to the sum of the discounted resale values of housing and capital, along with the discounted marginal product of each factor. Similar to the impatient households, the borrowed amount is tied to the collateral value associated with housing and capital. As a result, the effective discounted resale value is influenced by both their SDF and the real interest rate, impacting the repayment amount. Specifically, combining Eqs. (28)–(31):

$$q_t = \mathrm{E}_t \left\{ \Lambda^e_{t+1,t} \cdot \nu y_{t+1}/(X_{t+1} h_{e,t}) + \left[(1 - m^h_e) \cdot \Lambda^e_{t+1,t} + m^h_e \cdot \frac{\Pi_{t+1}}{R_t} \right] q_{t+1} \right\}, \tag{34}$$

$$1 = \mathrm{E}_t \left\{ \Lambda^e_{t+1,t} \cdot \mu y_{t+1}/(X_{t+1} k_{e,t}) + \left[(1 - \delta - m^h_e) \cdot \Lambda^e_{t+1,t} + m^h_e \cdot \frac{\Pi_{t+1}}{R_t} \right] \right\}, \tag{35}$$

where $\nu y_{t+1}/(X_{t+1} h_{e,t})$ and $\mu y_{t+1}/(X_{t+1} k_{e,t})$ stand for the marginal product of housing and the marginal product of capital, respectively. $\Lambda^e_{t+1,t} \equiv \beta_e \lambda_{e,t+1}/\lambda_{e,t}$ is entrepreneurs' SDF.

The next subsection introduces the final goods producer and the retailers. The introduction of these entities enables us to compare our model with the sticky price model that we will present later.

2.4 Final Goods Producer

There is a representative final goods producer that acquires a continuum of differentiated intermediate goods from retailers and subsequently transforms them into the final products. The production technology is governed by the following constant elasticity of substitution (CES) function:

$$Y_t = \left(\int_0^1 Y_t(z)^{\frac{\eta_y - 1}{\eta_y}} dz \right)^{\frac{\eta_y}{\eta_y - 1}}, \tag{36}$$

where Y_t represents the quantity of final goods, $Y_t(z)$ is the intermediate good purchased from retailer z, and η_y signifies the elasticity of substitution among the various intermediate goods.

Let $P_t(z)$ be the price of intermediate good; we can express the aggregate price of the final good, P_t, as follows:

$$P_t = \left(\int_0^1 P_t(z)^{1-\eta_y} dz \right)^{1/(1-\eta_y)}. \tag{37}$$

Supposing the market for final goods is competitive, we can express the final goods firm's problem as

$$\max_{Y_t(z)} P_t Y_t - \int_0^1 P_t(z) Y_t(z) dz.$$

The corresponding optimality condition can be expressed as follows:

$$Y_t(z) = \left[\frac{P_t(z)}{P_t}\right]^{-\eta_y} Y_t. \tag{38}$$

2.5 Retailers

In our benchmark model, a continuum of monopolistically competitive retailers, indexed by $z \in [0,1]$, each purchase homogeneous goods from entrepreneurs at price $P_t^w \equiv P_t/X_t$, modify them into differentiated goods, $Y_t(z)$, without any cost, and sell $Y_t(z)$ to the final goods firm at price $P_t(z)$. Here, we assume that retailers have the capability to adjust their prices every period without incurring any associated costs. Later, when we extend our analysis to incorporate price stickiness, we will introduce a quadratic adjustment cost. However, in this context, retailer z's problem is:

$$\max_{\{P_t(z)\}_{j=t}^{\infty}} \left[\frac{P_t(z) - P_t^w}{P_t} \cdot Y_t(z)\right],$$

subject to its demand function Eq. (38). The first-order condition associated with retailer z's problem is:

$$(1-\eta_y) \cdot \left[\frac{P_t(z)}{P_t}\right]^{1-\eta_y} + \eta_y \cdot \frac{P_t^w}{P_t} \cdot \left[\frac{P_t(z)}{P_t}\right]^{-\eta_y} = 0. \tag{39}$$

Given that all retailers face the same profit maximization problem, they all choose the same price, $P_t(z) = P_t$, and produce the same quantity, $Y_t(z) = Y_t$. Hence, we get:

$$0 = (1-\eta_y) + \frac{\eta_y}{X_t} \tag{40}$$

2.6 Source of the Uncertainty

We assume that the log of TFP shock A_t follows an AR(1) process, which takes the form:

$$\log A_t = \rho_a \log A_{t-1} + \exp(V_t)\sigma_A \varepsilon_{A,t}, \tag{41}$$

where $\varepsilon_{A,t} \sim N(0,1)$, ρ_a is the persistence of stochastic process to A_t, and σ_A is the standard deviation of innovations to A_t. Furthermore, the autoregressive process of productivity features time-varying volatility. In particular, the log of the standard deviation, V_t, of the innovations to productivity follows the autoregressive process:

$$V_t = \rho_v V_{t-1} + \sigma_V \varepsilon_{V,t}, \tag{42}$$

where $\varepsilon_{V,t} \sim N(0,1)$, ρ_v is the persistence of stochastic process to V_t, and σ_V is the standard deviation of innovations to V_t.

Two independent innovations, $\varepsilon_{A,t}$ and $\varepsilon_{V,t}$, affect the productivity. The first innovation denotes the productivity shocks, which change the productivity itself, while the second innovation denotes the volatility shock, which affects the spread of values for productivity. The volatility shock in the productivity implies that all firms are affected by more volatile shocks. Given the timing assumption, firms learn in advance the dispersion of shocks from which they will draw in the next period. This timing assumption captures the notion of uncertainty that firms face about future business conditions.

2.7 Monetary Policy and Market Clearing Conditions

We assume that the monetary authority follows a simple interest rate rule, which responds to changes in inflation as follows:[12]

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\Pi}, \tag{43}$$

where R and Π are, respectively, the steady-state nominal interest rate and inflation rate, and ϕ_Π is the policy parameter.

Housing market equilibrium requires

$$h_{s,t} + h_{b,t} + h_{e,t} = 1, \tag{44}$$

Bonds market equilibrium requires

$$b_{s,t} + b_{b,t} + b_{e,t} = 0, \tag{45}$$

Goods market equilibrium requires

$$c_t + i_t = y_t \tag{46}$$

where $c_t = c_{s,t} + c_{b,t} + c_{e,t}$ denotes the aggregate consumption.

We define the gross domestic product of this economy as the sum of aggregate output and the imputed housing services of owner-occupied homes:

$$gdp_t \equiv y_t + h_{s,t} \cdot mrs^s_{hc,t} + h_{b,t} \cdot mrs^b_{hc,t}. \tag{47}$$

[12]See Monacelli (2009), and Mendicino and Punzi (2014).

3 Quantitative Analysis

This section is divided into four parts. Firstly, we describe the solution method utilized to solve the model. Secondly, we estimate the exogenous process of uncertainty shocks. Thirdly, we calibrate the model parameters to ensure that they accurately reproduce the fundamental characteristics of the U.S. economy. Finally, we employ the calibrated model to analyze the impacts of uncertainty shocks.

3.1 Solution Method

We employ the third-order perturbation approximation with the pruning method introduced by Andreasen, Fernández-Villaverde and Rubio-Ramírez (2017) to address our dynamic stochastic general equilibrium model. This choice stems from the need for at least a third-order approximation of the policy functions to analyze the impulse response to a second-moment shock, as indicated by Fernández-Villaverde et al. (2011). In addition, we prune terms with higher-order effects beyond the third order to prevent higher-order approximations from generating explosive sample paths.

Following the approach of Basu and Bundick (2017), we set the exogenous shocks to zero and stimulate the economy for a sufficiently long period until all endogenous variables have converged to their stochastic steady-state. Then, we introduce the uncertainty shock and compute its impulse responses as a percentage deviation from the steady state. Specifically, we implement the simulation using Dynare software.

3.2 Estimation of Exogenous Processes

An important aspect of our analysis is the estimate of the key parameters associated with the stochastic process of uncertainty shocks, including ρ_A, ρ_V, σ_A, and σ_V. Following Cesa-Bianchi and Fernandez-Corugedo (2018), we estimate the process of productivity and uncertainty shocks using the time series data of aggregate TFP for the U.S. business sector. The sample periods ranges from 1970Q1 to 2019Q1.[13]

By fitting an AR(1) process to the log-deviations of TFP from a linear trend, we estimate the parameter that captures the persistence of the productivity process, ρ_A, and the standard deviation of its innovations, σ_A, which are 0.9533 and 0.0074, respectively. These values are in line with the findings of Cesa-Bianchi and Fernandez-Corugedo (2018) and Fernández-Villaverde et al. (2011).

We then compute the standard deviations of the TFP innovations with an eight quarter rolling window to get the proxy for the time-varying volatility of the TFP innovations,

[13]The data is available at https://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tfp/ (see Fernald 2014; Fernald and Matoba 2009; Basu, Fernald and Kimball 2006).

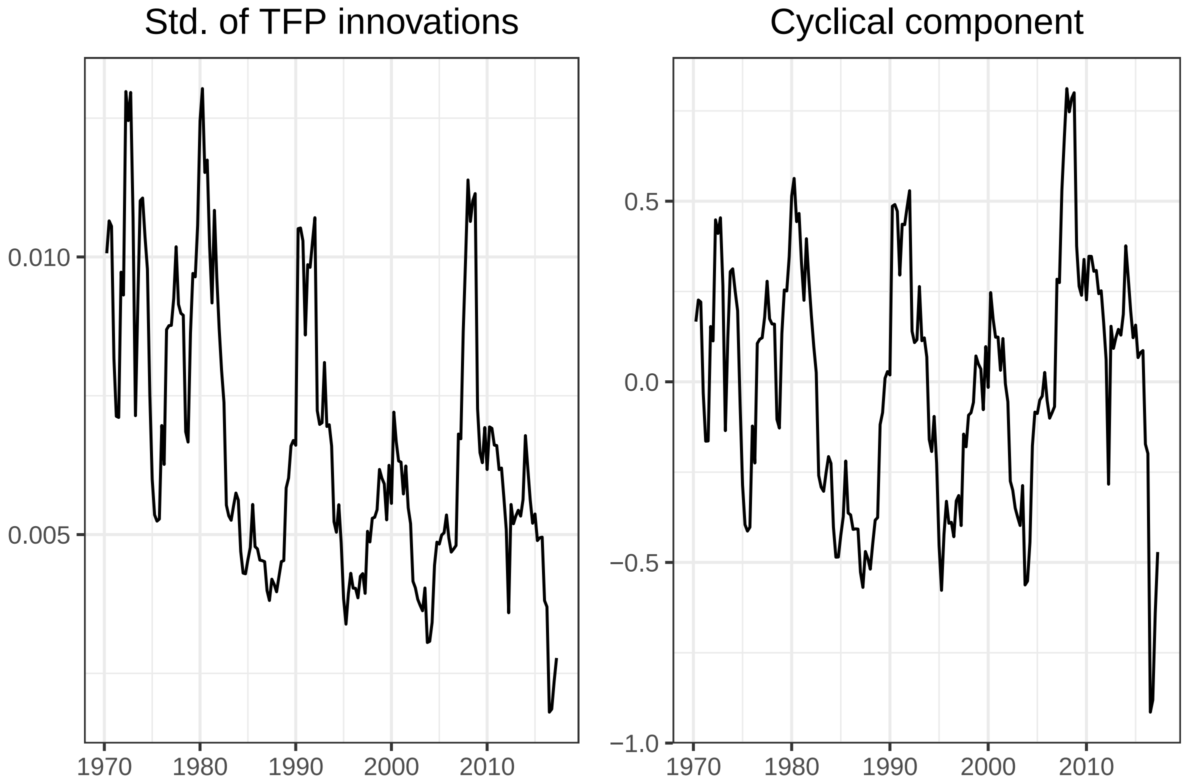

Figure 1: Time-varying volatility of TFP innovations

which yields the value of ρ_V being 0.8993 and σ_V being 0.1475. These parameter values are similar to the estimation from Cesa-Bianchi and Fernandez-Corugedo (2018) and Caldara et al. (2012).[14]

Figure 1 plots the standard deviation and cyclical component associated with σ_t^{TFP}. In particular, the left panel of Figure 1 demonstrates standard deviations of σ_t^{TFP}, which exhibits a downward trend as in Cesa-Bianchi and Fernandez-Corugedo (2018). Hence, we take the log-deviations of σ_t^{TFP} from a linear trend as a proxy for V_t. These cyclical components of σ_t^{TFP} are shown in the right panel of Figure 1.

3.3 Calibration

We now calibrate the key parameters of our model. Since our model is quarterly, we choose a savers' discount factor of $\beta_s = 0.9925$, which corresponds to a 3% annual real interest rate. The impatient households' and entrepreneurs' discount factors are set to $\beta_b = 0.94$ and $\beta_e = 0.94$, respectively, as in Iacoviello (2015). We also follow Choa and Francis (2011) in setting the parameters $\sigma_c = 3.0$ and $\sigma_h = 1.5$ to match the increasing

[14] An alternative way to construct a measure of uncertainty shocks is based on a Bayesian approach that computes the likelihood function with flat priors and samples from the posterior with a Markov Chain Monte Carlo method. The details can be found in Fernández-Villaverde et al. (2015).

ratio of housing to non-housing consumption as income increases. Furthermore, when σ_h exceeds σ_c, changes in the housing stock have a milder effect on utility compared to changes in consumption. Such a feature allows households use housing stocks as a buffer against unexpected shocks. For a detailed discussion, please refer to Zanetti (2014). Both the inverse of the Frisch elasticity of labor supply, η, and the weight of work, κ, are chosen as 1, based on Guerrieri and Iacoviello (2017).

The weight of housing, J, is set to 0.0757, which gives a real estate wealth to annual output ratio of 3.1, consistent with Iacoviello (2015). The maximum LTV ratios for borrowers and entrepreneurs are both set to 0.9, in line with Iacoviello (2015), and Guerrieri and Iacoviello (2017). The strength of consumption habit, ϕ_c, is set to 0.6842, following the research of Guerrieri and Iacoviello (2017).

We choose a depreciation rate of $\delta = 0.025$, corresponding to an annual depreciation rate of 10%. The shares of capital and real estate to output are fixed at $\mu = 0.3$ and $\nu = 0.03$, respectively, as in Iacoviello (2005). Savers' wage share, α, is set to 0.67, consistent with Iacoviello (2015). Finally, the monetary policy rule is set to $\phi_\Pi = 1.5$, a standard choice in the literature akin to Monacelli (2009). To enhance clarity, we provide a summary of all parameter values in Table 1.

Table 1: Parameters Values

Parameter	Description	Value	Source/Target
β_s	Discount factor for savers	0.9925	3% annual real interest rate
β_b	Discount factor for impatient households	0.94	Iacoviello (2015)
β_e	Discount factor for entrepreneurs	0.94	Iacoviello (2015)
σ_c	Risk aversion for consumption	3.0	Choa and Francis (2011)
σ_h	Risk aversion for housing	1.5	Choa and Francis (2011)
η	Inverse of Frisch elasticity	1	Guerrieri and Iacoviello (2017)
J	housing preference	0.0723	Steady-state housing wealth to annual output ratio of 3.1
m_b, m_e^h, m_e^k	LTV ratio	0.9	Iacoviello (2015)
δ	Depreciation rate	0.025	10% annual depreciation rate
μ	Capital share	0.3	Iacoviello (2005)
ν	Housing share	0.03	Iacoviello (2005)
α	Savers wage share	0.67	Iacoviello (2015)
ϕ_c	Habit in consumption	0.6842	Guerrieri and Iacoviello (2017)
ϕ_p	Price adjustment cost	117.3594	Implied average duration of 4 quarters
ϕ_k	Investment adjustment cost	2.5	Christiano, Eichenbaum and Evans (2005)
ϕ_Π	Response of inflation	1.5	Monacelli (2009)
ρ_A	AR(1) TFP shock	0.9533	Data
σ_A	Std. TFP shock	0.0074	Data
ρ_V	AR(1) TFP uncertainty shock	0.8993	Data
σ_V	Std. TFP uncertainty shock	0.1475	Data

3.4 Quantitative Analysis

To analyze the impacts of uncertainty shocks, we follow Fernández-Villaverde et al. (2011) to compute impulse response functions (IRFs) to a mean preserving shock to the variance of TFP. Since we are interested in investigating which model ingredients are key to propagating the uncertainty shocks during the business cycles, we focus on the responses of output, consumption, investment, housing holdings, employment, and total loans throughout our analysis.

3.4.1 The Dynamic Adjustment in Response to Uncertainty Shocks

We commence our analysis in a frictionless economy characterized by the absence of credit constraints and flexible prices. In contrast to our benchmark model, encompassing both patient and impatient households, this alternative model exclusively consists of patient households. This setup is akin to a conventional Representative Agent (RA) model, featuring solely patient households. In this frictionless economy, there is also

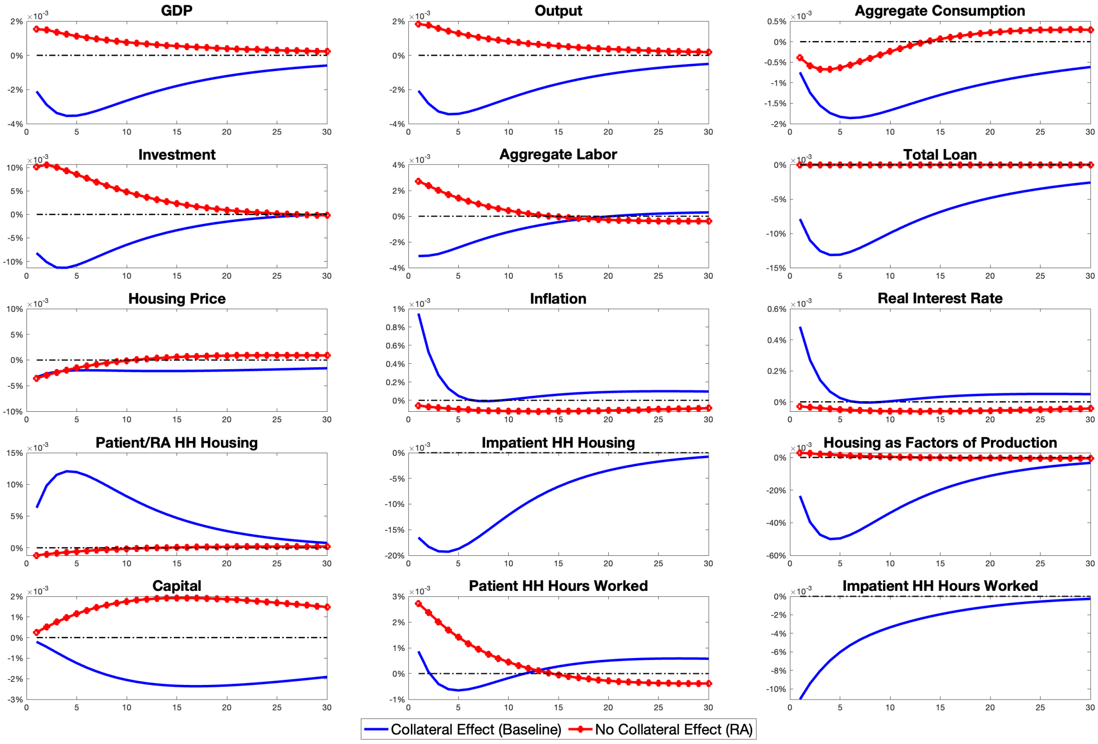

Figure 2: IRFs to a TFP uncertainty shock: RA model vs baseline model.

a representative entrepreneur, owned by the patient households, who utilizes the same technology as outlined in Eq. (22) for the production of homogeneous goods. Notably, this entrepreneur accumulates capital and housing for production, free from any borrowing constraints. This feature distinguishes it from our baseline model, which does impose such constraints. Additionally, new market clearing conditions now become $h_{s,t}+h_{e,t}=1$, $b_{s,t}=0$, and $c_{s,t}+i_t=y_t$. For a detailed presentation of equations related to the model without collateral constraints, please refer to Appendix A.

In this frictionless economy, the representative household exhibits precautionary saving motives affecting both consumption and labor hours. This leads to a decrease in consumption and an increase in labor hours in response to heightened future uncertainty. Given that the predetermined capital and present TFP do not change, the increased labor hours contribute to higher aggregate production. This rise in production, combined with reduced consumption, leads to increased investment. Consequently, the flexible-price model with standard preferences but without collateral constraints experiences the co-movement problem. Figure 2 illustrates the impulse response functions (IRFs) of the frictionless economy in reaction to a one-standard deviation increase in TFP uncertainty shock.

We then transition to our baseline model, which encompasses both patient and impatient households as well as entrepreneurs. This model incorporates credit constraints under flexible price setting. Specifically, both impatient households and entrepreneurs face collateral constraints, with LTV ratios set at $m_b = m_e^h = m_e^k = 0.9$. This modification enables us to explore the impact of these financial constraints on the propagation of uncertainty shocks within the context of business cycles. Unlike the previous frictionless model, which lacks financial frictions, the inclusion of financial frictions among impatient households and entrepreneurs addresses the co-movement problem that the typical business cycle model fails to explain.

In our benchmark model, the behavior of patient households closely resembles that of their counterparts in a frictionless economy, primarily driven by their precautionary saving motives. Patient households respond to uncertainty shocks by reducing their non-durable consumption and increasing their labor supply. Contrasting with patient households, the optimal decisions regarding non-durable consumption and labor for impatient households are intricately linked to their collateral (housing) decisions, which are now contingent on their SDF and the prevailing real interest rate when they sell their collateral, as illustrated in Eq (20). Specifically, the precautionary saving motive is primarily influenced by the SDF, with only a minor impact on the housing decision, characterized by a weight of $(1 - m_b) = 0.1$. On the other hand, the real interest rate at the time when they sell their housing plays a significantly more substantial role in determining the housing decision among impatient households, with a weight of $m_b = 0.9$. This difference in weight highlights that the housing decisions of impatient households are primarily influenced by the resale risk associated with the real interest rate, rather than the precautionary motive. Furthermore, this risk can be decomposed into the product of the inverse of real interest rate expectation and housing resale price expectation as well as covariance terms between housing resale price and the inverse of real interest rate. Since nominal interest rate is determined in current period, the change in real interest rate in mainly captured by the change in future inflation rate. The heightened uncertainty leads to a more pronounced negative correlation between future inflation and housing prices (see Figure 3), which lowers the impatient households' willingness to pay for housing today, resulting in reduced demand for housing. This, in turn, prompts impatient households to downsize their housing, transitioning from larger houses to smaller ones.

The impact of the uncertainty shock on housing distribution is significant. On one hand, heightened uncertainty reduces the incentives for impatient households to borrow against their housing since the risk of using it as collateral for a loan increases. On the other hand, housing plays a dual role for patient households, serving both as a durable consumption and a means of future savings. It's worth noting that, following the uncertainty shock, the shadow value of long-lived durables remains relatively stable, making

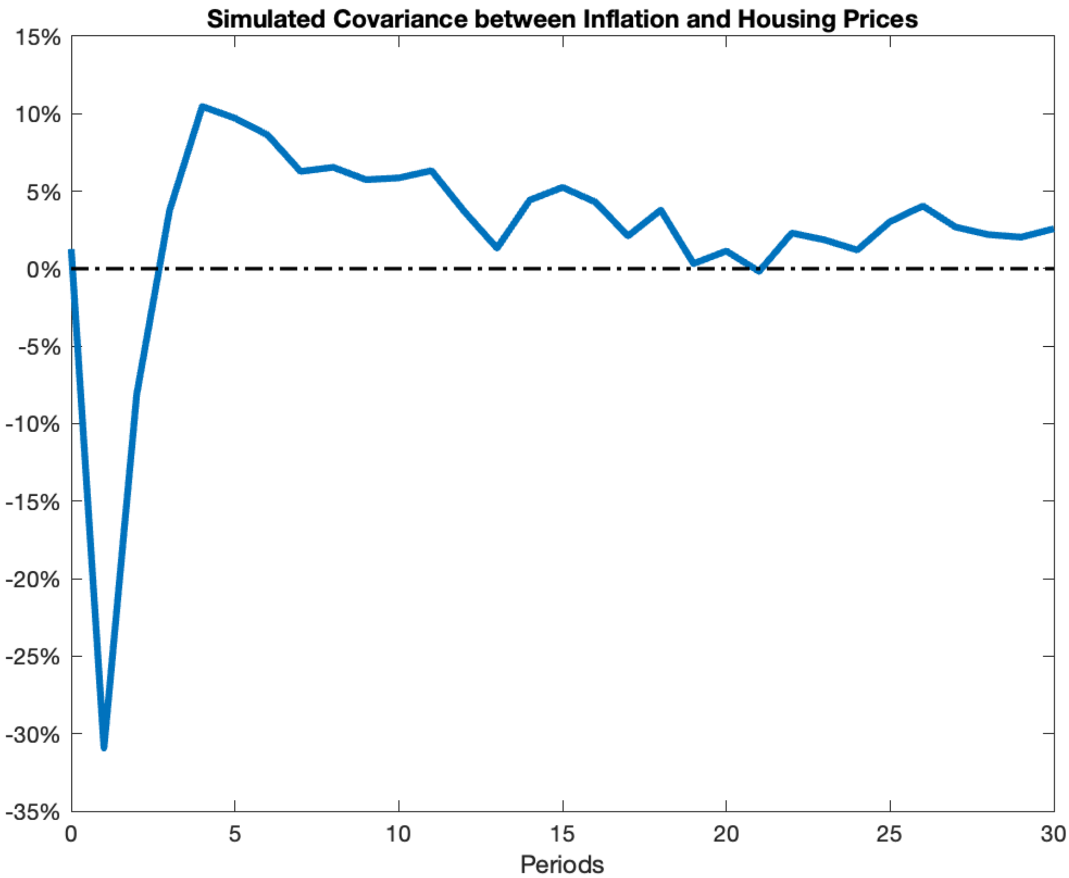

Figure 3: Percentage Deviation in Simulated Covariance between Inflation and Housing Prices over 10,000 Simulations

patient households less sensitive to the timing of their durable goods purchases. Therefore, in response to a decrease in the relative price of housing following the uncertainty shock, patient households increase their housing holdings. In summary, the uncertainty shock triggers a redistribution of housing from impatient households to patient households.

Why do housing prices fall following uncertainty shocks? This phenomenon can be attributed to differences in the financing methods for housing purchases and the redistribution of housing from impatient to patient households. Specifically, impatient households utilize leverage for their housing purchases, whereas patient households do not. Therefore, the transfer of ownership from impatient households to patient households leads to a more significant decrease in housing demand among the former than the increase in demand among the latter. Consequently, the aggregate demand for housing (from both patient and impatient households) declines after the shock. This substantial reduction in housing aggregate demand, coupled with a constant housing aggregate supply, results in declining housing prices.

While the change in housing expenditure may not directly influence the optimal trade-off between non-durable consumption and leisure, it does impact the choice set of labor and non-durable consumption, as depicted in Eq. (21). In response to increasing uncertainty, impatient households reduce their housing holdings, $h_{b,t}$, to mitigate their risk exposure associated with using housing as collateral. Consequently, their expenditure on down payments for housing decreases. This results in an increase in non-durable consumption for them, assuming that working hours remain constant. However, working hours will adjust accordingly to ensure that the marginal cost of leisure equals the marginal benefit of leisure. Therefore, the rise in non-durable consumption is accompanied by a decrease in labor hours. In other words, heightened risk prompts impatient households to reallocate their expenditures from housing (future non-durable consumption) to immediate non-durable goods consumption and leisure. Consequently, the diminished demand for housing among impatient households indirectly influences their labor supply decisions, resulting in reduced labor supply.

The aggregate labor supply includes changes in the labor supply of both patient and impatient households. If the reduction in labor supply among impatient households exceeds the increase in labor supply among patient households, it leads to a decrease in aggregate labor supply. Conversely, labor demand, measured by the marginal product of labor, remains stable following uncertainty shocks as housing and capital are predetermined and the level of productivity is constant. Therefore, the primary factor contributing to the decline in aggregate labor lies in the reduction of labor supply in terms of labor market adjustments.

The decrease in labor supply among impatient households also allows our model to generate a positive correlation between labor supply and household debt, as documented in previous studies (e.g. Fortin 1995; Del Boca and Lusardi 2003). This finding is similar to the mechanism of the financial labor supply accelerator proposed by Campbell and Hercowitz (2011), who established a long-run relationship between the increase in the minimum down payment required for collateral and a decrease in households' hours worked within a representative agent framework. In our model, there is a difference in the financial methods of housing purchase between patient and impatient households. In particular, patient households purchase houses without loans, necessitating a higher down payment, while impatient households use leverage for home acquisition, leading to a lower down payment. Consequently, when houses transition from impatient households to patient households, the total down payment required for housing purchases effectively increases. Therefore, we demonstrate the positive (negative) relationship between leverage (down payment) and labor supply in an economy with both patient and impatient households.

Given that the levels of housing and capital stock for production are predetermined, such a decrease in aggregate labor hours leads to a subsequent decline in overall output. Moreover, as the heightened uncertainty is expected to last over multiple periods, the reduction in aggregate labor supply is likely to persist. This persistent decrease in labor hours, in turn, leads to a sustained reduction in the expected marginal productivity of capital, thus resulting in an extended decrease in investment in both housing and physical capital. Furthermore, Eqs. (34)–(35) illustrate that the same reasoning for the decrease in housing stock among impatient households applies to entrepreneurs' choices regarding housing and physical capital. In particular, the motivation to amass additional collateral to ensure loans diminishes when the value of collateral becomes increasingly uncertain due to the impact of uncertainty shocks. Consequently, the entrepreneurs' demand for both investment and housing further decreases. Credit-market frictions exacerbate the fluctuations in the demand for factors of production by borrowing-constrained firms, essentially embodying the mechanism described in the financial accelerator concept proposed by Bernanke, Gertler and Gilchrist (1996), Kiyotaki and Moore (1997), and Bernanke, Gertler and Gilchrist (1999).

In summary, this increase in uncertainty leads to a decrease in consumption, investment, aggregate labor hours, and output. Consequently, our baseline model effectively reproduces the co-movement among key macroeconomic aggregates following heightened uncertainty. These findings are illustrated in Figure 2, where we also present the dynamics associated with the frictionless economy. It's worth noting that our model with financial frictions can address the co-movement puzzle, even in the absence of nominal rigidities.

3.4.2 The Importance of the Risk Premium Channel

To highlight the importance of the risk premium in addressing the co-movement puzzle, we examine an alternative scenario where both patient households and entrepreneurs employ the steady-state housing price, denoted as $\overline{q}$, to evaluate the collateral value. Consequently, the borrowing constraints associated with housing are modified as follows:

$$b_{b,t} \leq m_b \mathrm{E}_t \left(\frac{\Pi_{t+1}}{R_t} \cdot \overline{q} \right) h_{b,t},$$
$$b^h_{e,t} \leq m^h_e \mathrm{E}_t \left(\frac{\Pi_{t+1}}{R_t} \cdot \overline{q} \right) h_{e,t}.$$

When housing prices are held constant, the risk premium channel becomes effectively inactive, as $\mathrm{Cov}_t(\Lambda^b_{t+1,t}, \overline{q}) = \mathrm{Cov}_t(\Lambda^e_{t+1,t}, \overline{q}) = \mathrm{Cov}_t(\Pi_{t+1}, \overline{q})/R_t = 0$. The corresponding

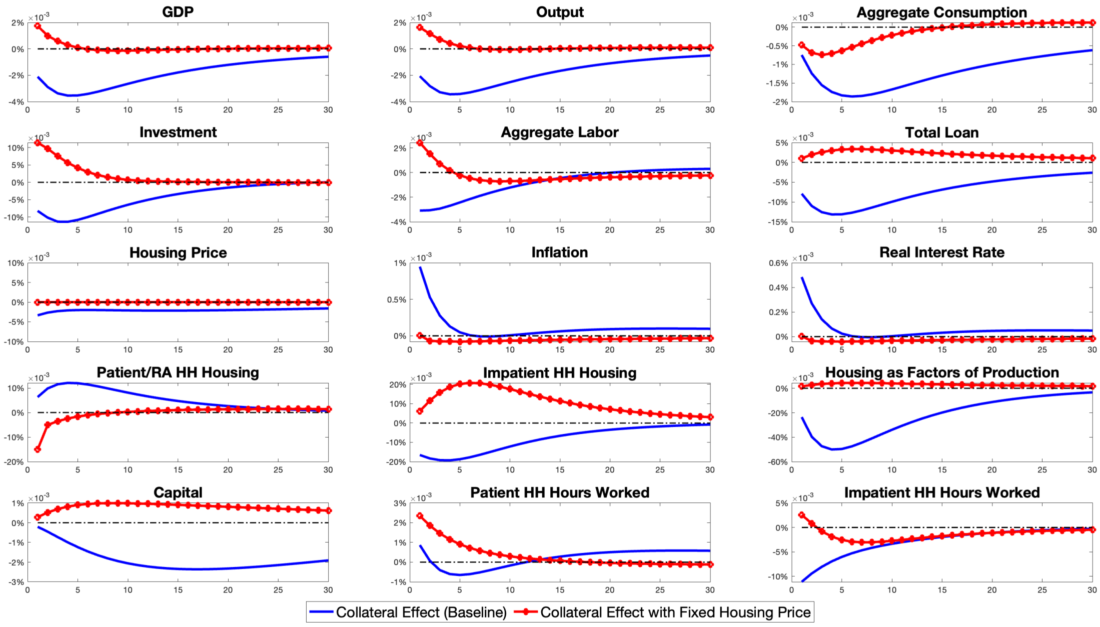

Figure 4: IRFs to a TFP uncertainty shock in alternative credit constraint with a fixed price of q.

optimal housing condition in Eqs. (20) and (34) now becomes:

$$q_t = \mathrm{E}_t \left(\Lambda^b_{t+1,t} \cdot q_{t+1} \right) + m_b \cdot \mathrm{E}_t \left(\frac{\Pi_{t+1}}{R_t} - \Lambda^b_{t+1,t} \right) \cdot \bar{q} + mrs^b_{hc,t},$$

$$q_t = \mathrm{E}_t \left(\Lambda^e_{t+1,t} \cdot mph_{t+1} + \Lambda^e_{t+1,t} \cdot q_{t+1} \right) + m^h_e \cdot \mathrm{E}_t \left(\frac{\Pi_{t+1}}{R_t} - \Lambda^e_{t+1,t} \right) \cdot \bar{q}.$$

As the risk premium terms associated with housing resale prices vanish, the incentive for impatient households to decrease their demand for housing following uncertainty shocks also disappears. Consequently, their labor supply will not decrease as it did previously. Therefore, the increase in aggregate labor hours among patient and impatient households leads to an overall rise in aggregate labor hours, which in turn leads to the higher aggregate production, given that current technology and capital stock remain unchanged. This boost in output, combined with reduced consumption, results in an increase in investment, a pattern similar to the outcomes in the frictionless economy model. Figure 4 presents the simulation results of the case with and without risk premium in our baseline model.

4 Understanding the Key Features of the Model

In this section, we further examine a series of quantitative exercises to better understand how the various elements of our model contribute to our results. Specifically, credit constraints affect activities of both entrepreneurs and impatient households. We first evaluate the relative importance of households' and entrepreneurs' credit constraint in resolving the puzzle. We also examine the stringency of credit constraints to see how these shocks influence the dynamics adjustment following the shocks. Furthermore, we examine the role of habit formation in the model. Finally, we compare our quantitative results with that in the typical sticky price model.

4.1 The Relative Importance of Households' and Entrepreneurs' Credit Constraints

In this subsection, we examine the relative importance of the financial frictions associated with impatient households and entrepreneurs. In particular, we examine scenarios where only one of these economic agents faces these constraints, as opposed to situations where both patient households and entrepreneurs encounter borrowing restrictions.

4.1.1 The Economy with Entrepreneurs' Credit Constraints Only

We start by analyzing the scenario in which only the entrepreneurs are subject to the borrowing constraints. The behavior of the entrepreneurs in this model economy is identical to that in our baseline model. However, when we remove the credit constraints for impatient households, we set their discount factors at the same level as that of patient households, i.e., $\beta_b = \beta_s$. Consequently, patient and impatient households become indistinguishable and can be effectively merged into a single category of patient households. This change leads to an aggregate labor input of $n_t = n_{s,t}$. Therefore, the setting of the patient household is very similar to its counterpart in the frictionless economy we introduce earlier. One key difference is that the entrepreneur is not owned by the patient households, and thus their valuation for future profits is different than their counterparts in the frictionless economy setting. The new market clearing conditions now become $h_{s,t} + h_{e,t} = 1$, $b_{s,t} + b_{e,t} = 0$, and $c_{s,t} + c_{e,t} + i_t = y_t$.

In this economy, the uncertainty shock triggers an increase in labor supply due to precautionary saving motives, resulting in a rise in aggregate production. Since labor is complementary to capital, the increased labor supply also boosts the marginal productivity of capital, consequently increasing entrepreneurs' demand for capital. Additionally, the financial accelerator effect further amplifies this demand for capital, leading to an increase in investment. Regarding the demand for housing by entrepreneurs, the augmented labor supply elevates the marginal product of housing, thereby increasing the demand for housing. However, the heightened uncertainty surrounding the resale value of housing diminishes their demand for it. Together, the decrease in housing demand resulting from increased uncertainty outweighs the demand increase due to its complementary relationship with production, leading to a net reduction in housing demand for entrepreneurs. The IRFs for this scenario are depicted by the red dotted lines in Figure 5. It's important to note that the model without the credit constraints for impatient households incorrectly predicts an expansionary effect from the uncertainty shock and fails to resolve the co-movement puzzle.

4.1.2 The Economy with Impatient Households' Credit Constraints Only

Next, we shift our focus to an economy in which entrepreneurs' credit constraints have been removed, while impatient households still encounter credit constraints. Here, the behavior of patient and impatient households is identical to our baseline model. However, the behavior of entrepreneurs is identical to our previous frictionless economy model setting. This setting assumes that entrepreneurs use the same technology described in Eq. (22) and are owned by the patient households. Furthermore, in this case, entrepreneurs are not subject to any borrowing constraints when accumulating capital and housing. Additionally, new market clearing conditions now becomes $h_{s,t} + h_{b,t} + h_{e,t} = 1$, $b_{s,t} + b_{b,t} = 0$, and $c_{s,t} + c_{b,t} + i_t = y_t$.

During times of heightened uncertainty, the resale housing value becomes more volatile, prompting impatient households to downsize their homes in an effort to reduce their housing exposure. This downsizing leads to a reduction in working hours among impatient households, which in turn causes a decrease in aggregate labor hours, resulting in a decline in aggregate production.

Since the fall in aggregate labor hours also leads to a decrease in the expected marginal productivity of capital, investment will fall in response. However, in the absence of the financial frictions associated with entrepreneurs, the decline in investment is less severe, leading to a milder recession. The simulation results for this scenario are depicted by the green lines in Figure 5.

In summary, financial frictions related to impatient households play a crucial role in addressing the co-movement issue following uncertainty shocks. In contrast, financial frictions associated with entrepreneurs only act as a mechanism that amplifies the reduction in investment, thereby intensifying the recession when compared to scenarios without these financial constraints.

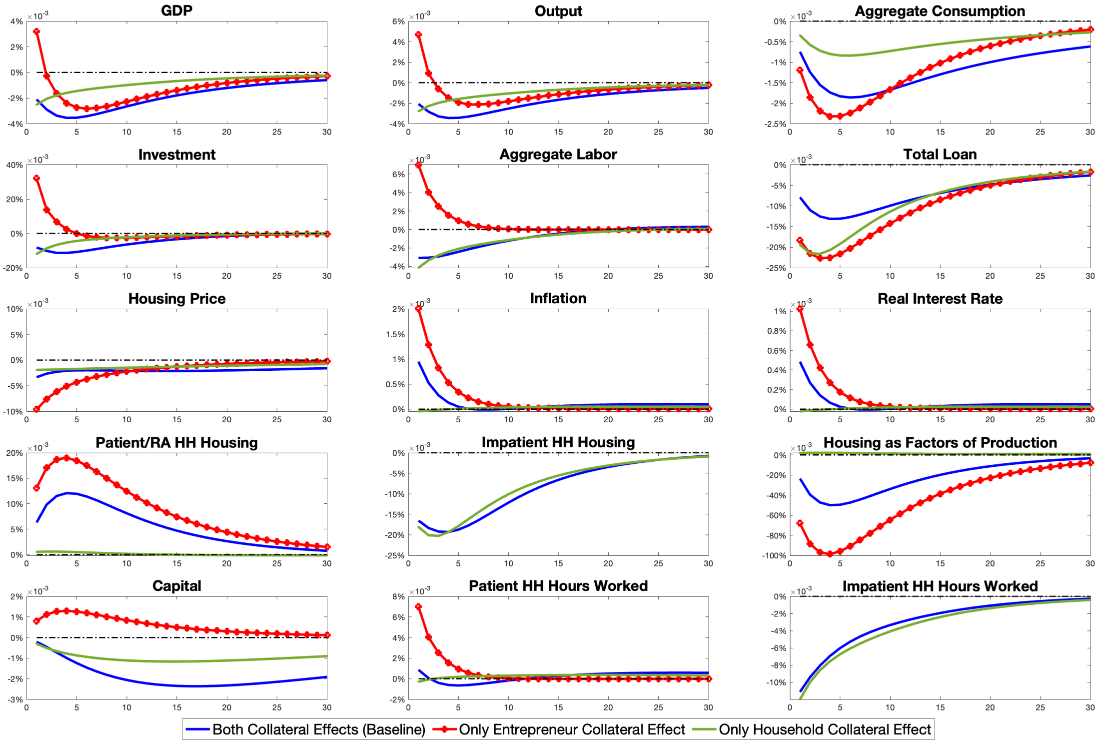

Figure 5: IRFs to a TFP uncertainty shock under different types of collateral constraints

4.2 Varying Degrees of Financial Frictions and Their Effects

In this exercise, we manipulate the stringency of borrowing constraints for both impatient households and entrepreneurs by adjusting their LTV ratios. Our objective is to gain insight into how these constraints influence the amplification of uncertainty shocks and their subsequent impact on macroeconomic dynamics. Without loss of generality, we examine a scenario where LTV ratios are consistent among the agents, denoted as $m_b = m_e^k = m_e^h = M$. In this context, a higher M value signifies a more relaxed borrowing constraint, allowing both impatient households and entrepreneurs to access more substantial collateral with a relatively smaller initial down payment, thereby leading to increased leverage. Consequently, when impatient households respond to increased uncertainty by reducing their housing holdings, the decrease in aggregate labor supply becomes more significant with larger LTV ratios. This, in turn, leads to a deeper recession.

Figure 6 presents the Impulse Response Functions (IRFs) under three different levels of financial friction, specifically with M values set at 0.875, 0.9, and 0.925. The simulation results demonstrate that a lower LTV ratio corresponds to smaller fluctuations in aggregate output, while a higher LTV ratio leads to a more substantial decline in aggregate production. From this perspective, deregulation amplifies the responses in aggregate production, aligning with prior studies by Liou (2013); Born (2011); and Chang (2011).

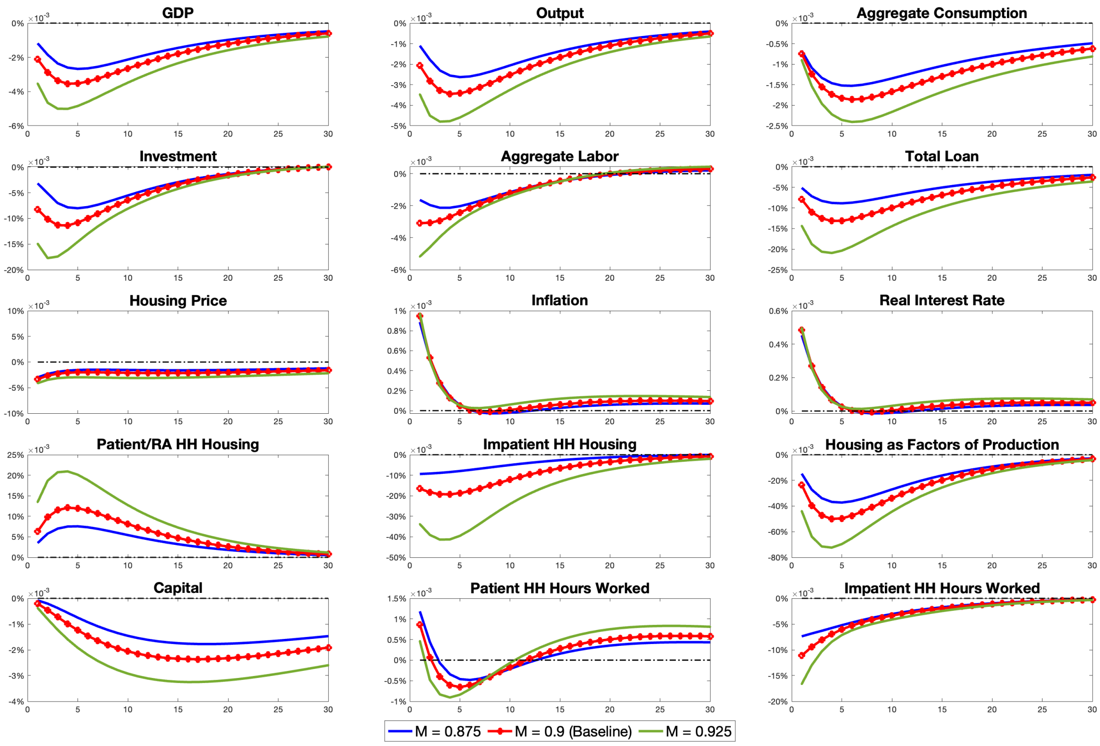

Figure 6: IRFs to a TFP uncertainty shock under varying M.

As a result, policymakers may consider regulating LTV ratios to mitigate extensive fluctuations in aggregate production.

4.3 Model without Consumption Habit

In this subsection, we analyze the model in which consumption habits are removed for all economic agents. We present their corresponding IRFs in Figure 7. We also incorporate the IRFs from our baseline model for a comparative assessment. From the Figure, we observe that the model without habit formation can still effectively address the comovement puzzle and generate boom-bust cycles that align with the empirical dynamics triggered by uncertainty shocks. However, the inclusion of consumption habits can magnify the adverse effects of uncertainty shocks, primarily through the substitution behavior of impatient households. From our previous analysis, we know the decrease in borrowing incentives following uncertainty shocks leads to a shift in consumption expenditure from housing to non-durable goods, and thus the decrease in current labor hours. This change in current consumption also depends on the intertemporal substitution from future periods to the current period amid heightened uncertainty. The presence of consumption habits discourages substitution in current consumption, causing impatient households to

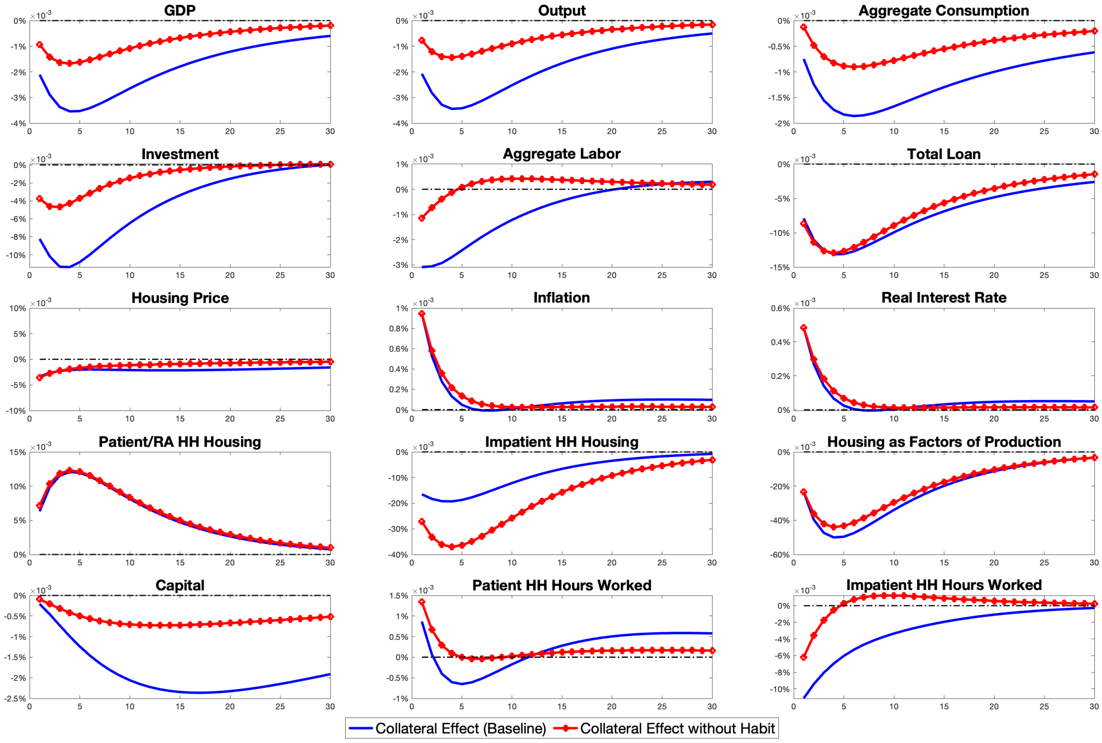

Figure 7: IRFs to a TFP uncertainty shock: baseline model vs baseline model without habit.

substitute more in leisure compared to scenarios without consumption habits. Besides resulting in a more significant reduction in labor supply, the sluggish adjustment in consumption renders the decrease in labor supply more persistent, ultimately contributing to a more profound and prolonged recession.

4.4 Comparison with Sticky Price Model

In this subsection, our primary objective is to conduct a quantitative comparison between our baseline model and the conventional sticky price model concerning their respective responses to uncertainty shocks. To address this, we introduce a theoretical framework that encompasses nominal frictions in the context of representative households. This model also incorporates real frictions like consumption habits and investment adjustment costs, which are extensively discussed in the literature. Following this, the section delves into a comparison of the IRFs between our baseline model and the one featuring price stickiness.

4.4.1 The Model with Sticky Price and Investment Adjustment Cost

In the model with price stickiness, there is a representative household that maximizes its lifetime utility by choosing consumption, housing, labor hours and saving just like the patient households in our baseline model. Moreover, the setting of entrepreneur is almost identically to our frictionless economy model. However, due to the investment adjustment cost, the law of motion for capital, Eq. (26), now becomes:

$$k_t = (1-\delta)k_{t-1} + \left[1 - \frac{\phi_k}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right] i_t,$$

where $\phi_k \geq 0$ represents the investment adjustment cost parameter. Regarding the behavior of retailers, they purchase wholesale goods at the price $P_t^w \equiv P_t/X_t$ from entrepreneurs. These goods are then transformed into various intermediate goods, which are subsequently sold to the final goods producer. However, in contrast to our baseline model setting where retailers can freely change their prices each period without incurring any expenses, they now encounter quadratic adjustment costs in their price-setting behavior, akin to Rotemberg (1982). Specifically, these adjustment costs take the form $\Phi_{y,t} \equiv \frac{\phi_p}{2}\left[\frac{P_t(z)}{P_{t-1}(z)} - \Pi\right]^2 Y_t$, where ϕ_p is the adjustment cost parameter that determines the level of nominal price rigidity, and Π represents the steady state inflation level. Therefore, retailer z's problem now becomes:

$$\max_{\{P_{t+j}(z)\}_{j=t}^{\infty}} \mathrm{E}_t \sum_{j=0}^{\infty} \frac{\beta^j \lambda_{s,t+j}}{\lambda_{s,t}} \cdot \left[\frac{P_{t+j}(z) - P_{t+j}^w}{P_{t+j}} \cdot Y_{t+j}(z) - \Phi_{y,t}\right],$$

subject to its demand function Eq. (38).

The first-order condition associated with retailer z's problem is:

$$\begin{aligned}(1-\eta_y) \cdot \left[\frac{P_t(z)}{P_t}\right]^{1-\eta_y} + \eta_y \cdot \frac{P_t^w}{P_t} \cdot \left[\frac{P_t(z)}{P_t}\right]^{-\eta_y} - \phi_p \left[\frac{P_t(z)}{P_{t-1}(z)} - \Pi\right] \cdot \frac{P_t(z)}{P_{t-1}(z)} \\ + \mathrm{E}_t \left\{\frac{\beta_s \lambda_{s,t+1}}{\lambda_{s,t}} \cdot \frac{Y_{t+1}}{Y_t} \cdot \phi_p \left[\frac{P_{t+1}(z)}{P_t(z)} - \Pi\right] \cdot \frac{P_{t+1}(z)}{P_t(z)}\right\} = 0.\end{aligned} \tag{48}$$

Since all retailers face the same profit maximization problem, they all choose the same price, $P_t(z) = P_t$, and produce the same quantity, $Y_t(z) = Y_t$. Hence, we get:

$$\phi_p(\Pi_t - \Pi) \cdot \Pi_t = (1-\eta_y) + \frac{\eta_y}{X_t} + \mathrm{E}_t \left[\frac{\beta_s \lambda_{s,t+1}}{\lambda_{s,t}} \cdot \frac{Y_{t+1}}{Y_t} \cdot \phi_p(\Pi_{t+1} - \Pi) \cdot \Pi_{t+1}\right]. \tag{49}$$

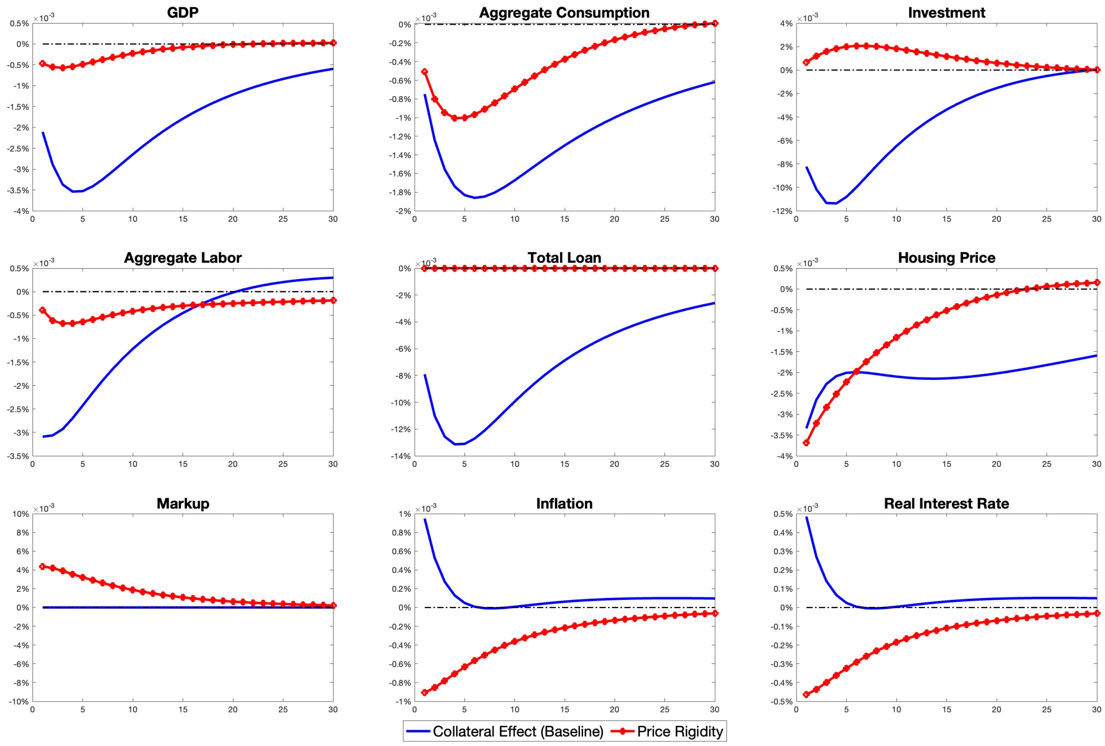

Figure 8: IRFs to a TFP uncertainty shock – Collateral Effect vs. Nominal Rigidity

Furthermore, the wholesale goods market clearing condition implies that

$$y_t = \int_0^1 Y_t(z)dz = Y_t.$$

4.4.2 The Quantitative Comparison

We select the price adjustment cost parameter (ϕ_p) to match the slope of the New Keynesian Phillips Curve typically found in a Calvo model with an average price duration of three quarters. Additionally, we fix the investment adjustment cost parameter (ϕ_k) at a value of 2.5. Figure 8 illustrates a comparison of the IRFs between our baseline model and the RA model with sticky price and investment adjustment costs.

As explained by Basu and Bundick (2017), uncertainty shocks lead households to reduce their consumption and increase their labor supply due to precautionary saving motives. The subsequent increase in labor supply exerts downward pressure on wages and firms' marginal costs, ultimately causing prices to decrease in a flexible price model to maintain constant markups. However, in the presence of sticky prices, the adjustment of prices is sluggish, resulting in an increase in markups. Additionally, as noted by Born and Pfeifer (2021), uncertainty shocks lead to heightened variability in aggregate productivity.

This variability prompts retailers to set higher prices, driven by a precautionary pricing motive when they have the flexibility to adjust their prices. This is because lowering prices under these circumstances could result in significant losses due to rising marginal costs and the inability to adapt prices accordingly. Through both setting higher prices when they have the opportunity and the increase in the markup channel, uncertainty shocks diminish the demand for output and labor, ultimately leading to a recession, as illustrated in Figure 8.

In the sticky price model, this markup channel generates a fall in aggregate production mainly by lowering labor demand. However, when households' precautionary labor supply is strong, the reduction in hours worked is limited, resulting in relatively small uncertainty shock effects. In contrast, the financial labor supply accelerator channel in our baseline model resolves the co-movement puzzle by reducing labor supply. As entrepreneurs' housing and physical capital are predetermined, the current labor demand remains fixed. Consequently, a decrease in labor supply can readily translate into a reduction in hours worked, leading to larger uncertainty shock effects.

5 Conclusion

This study examines the impacts of uncertainty shocks in an economy characterized by patient households, impatient households, and entrepreneurs. By introducing borrowing constraints linked to collateral values for both households and entrepreneurs, this model can effectively address the co-movement puzzle in response to the rise in uncertainty, whereas standard business cycle models fail to resolve this puzzle. The pivotal aspect of our model's ability to resolve this co-movement challenge lies in the context of housing redistribution amid heightened uncertainty. Specifically, the increased risk associated with housing holdings prompts impatient households to reduce their housing size, and thus the ability to borrow. This, in turn, leads to a redirection of expenditure from housing to non-durable consumption, and thus impacting the labor-leisure choices of impatient households, which results in a decrease in their labor supply. If the decrease in labor supply among impatient households outweighs the corresponding increase among patient households, the result is a decrease in aggregate labor hours. Given the predetermined nature of capital and housing, this decrease in labor hours consequently leads to a decline in aggregate production that the standard business cycle model fails to deliver. Our modeling approach does not rely on nominal rigidities, and we show that our model demonstrates a notably larger output response compared to the model incorporating nominal rigidities. In addition, the link between house downsizing and the labor supply for impatient households also allows our model to establish a positive long-run relationship between household debt and aggregate labor supply in the economy with heterogeneous households over the business cycle. This research enhances our understanding of the complex relationship between financial frictions and macroeconomic dynamics in response to increasing uncertainty.

Appendix

A Standard DSGE Model

In this appendix, we explain how to modify our baseline model into a frictionless economy. Specifically, we eliminate the credit constraints for both impatient households and entrepreneurs while setting their discount factors to be equal to that of patient households, denoted as $\beta_b = \beta_e = \beta_s$. Consequently, patient and impatient households become indistinguishable and can be collectively represented as patient households. This leads to an aggregate labor input of $n_t = n_{s,t}$. Furthermore, we assume entrepreneurs are owned by savers. Below, we introduce the problems faced by patient households and entrepreneurs in turn.

A.1 Patient Households

There is a continuum of mass one of patient households that choose consumption, $c_{s,t}$, housing, $h_{s,t}$, and working hours, $n_{s,t}$, to maximize their lifetime utility:

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta_s^t \left[\Gamma_{c,s} \cdot \frac{(c_{s,t} - \phi_c c_{s,t-1})^{1-\sigma_c} - 1}{1-\sigma_c} + J \cdot \frac{h_{s,t}^{1-\sigma_h} - 1}{1-\sigma_h} - \kappa \cdot \frac{n_{s,t}^{1+\eta}}{1+\eta} \right].$$

Patient households are subject to a budget constraint represented by:

$$c_{s,t} + q_t h_{s,t} + b_{s,t} \leq w_{s,t} n_{s,t} + q_t h_{s,t-1} + \frac{R_{t-1}}{\Pi_t} \cdot b_{s,t-1} + div_t + \pi_t,$$

where π_t represents profits earned from the wholesale goods firm.

The first-order conditions associated with savers' problems with consumption, labor hours, housing, and bond holdings are:

$$\begin{aligned}
\lambda_{s,t} &= uc_{s,t}, \\
\lambda_{s,t} w_{s,t} &= -un_{s,t}, \\
\lambda_{s,t} q_t &= \beta_s \mathrm{E}_t \left(\lambda_{s,t+1} q_{t+1} \right) + uh_{s,t}, \\
\lambda_{s,t} &= \beta_s \mathrm{E}_t \left(\lambda_{s,t+1} \frac{R_t}{\Pi_{t+1}} \right),
\end{aligned}$$

where $\lambda_{s,t}$ is the Lagrange multiplier associated with savers' budget constraint. $uc_{s,t}$, $uh_{s,t}$, and $un_{s,t}$ are the first-order derivatives of the savers' utility function with respect to $c_{s,t}$, $h_{s,t}$, and $n_{s,t}$:

$$\begin{aligned} uc_{s,t} &\equiv \Gamma_{c,s}\left\{(c_{s,t}-\phi_c c_{s,t-1})^{-\sigma_c} - \beta_s\phi_c \mathrm{E}_t\left[(c_{s,t+1}-\phi_c c_{s,t})^{-\sigma_c}\right]\right\}, \\ uh_{s,t} &\equiv Jh_{s,t}^{-\sigma_h}, \\ un_{s,t} &\equiv -\kappa n_{s,t}^{\eta}. \end{aligned}$$

A.2 The Entrepreneur

There is an entrepreneur that produces homogeneous wholesale goods according to the Cobb-Douglas production function, Eq. (22). The entrepreneur's period profit is defined as:

$$\pi_t \equiv \frac{P_t}{X_t}\cdot y_t - i_t - q_t(h_{e,t}-h_{e,t-1}) - w_{s,t}n_{s,t}.$$

We assume patient households own this entrepreneur. This modification leads entrepreneurs to use patient households' SDF to evaluate their profits. Therefore, the entrepreneur's problem is to maximize

$$\max \mathrm{E}_0 \sum_{t=0}^{\infty} \beta_s^t \cdot \lambda_{s,t}\cdot \pi_t,$$

subject to the Cobb-Douglas production function, Eq. (22), and the law of motion for capital. The associated first-order conditions are:

$$\begin{aligned} \lambda_{s,t}q_t &= \mathrm{E}_t\left[\beta_s\lambda_{s,t+1}\left(\frac{\nu y_{t+1}}{X_{t+1}h_{e,t}} + q_{t+1}\right)\right], \\ \lambda_{s,t} &= \beta_s \mathrm{E}_t\left\{\lambda_{s,t+1}\left[\frac{\mu y_{t+1}}{X_{t+1}k_t} + (1-\delta)\right]\right\}, \\ w_{s,t} &= \frac{(1-\mu-\nu)y_t}{X_t n_{s,t}}. \end{aligned}$$

A.3 Market Clearing Conditions

The new market clearing conditions become:

$$\begin{aligned} h_{s,t} + h_{e,t} &= 1, \\ c_{s,t} + i_t &= y_t. \end{aligned}$$

B Translation of Calvo Price Duration into Implied Rotemberg Adjustment Cost Parameter

This section first introduces retailers who reoptimize prices à la Calvo, deriving its associated New Keynesian Phillips Curve. Then we show how to translate Calvo price duration into implied Rotemberg adjustment cost parameter by comparing their New Keynesian Phillips Curves.

B.1 Calvo Price Setting

Building on the work of Calvo (1983) and Yun (1996), we assume that retailers can reset their price with probability $1-\tilde{\phi}_p$. Retailer z's problem is:

$$\max_{P_t^\star(z)} \mathrm{E}_t \sum_{j=0}^{\infty} \frac{(\beta\tilde{\phi}_p)^j \lambda_{s,t+j}}{\lambda_{s,t}} \cdot \left[\frac{P_{t+j}^\star(z) - P_{t+j}^w}{P_{t+j}} \cdot Y_{t+j}(z)\right],$$

subject to its demand function Eq. (38). The first-order condition associated with retailer z's problem is:

$$\mathrm{E}_t \sum_{j=0}^{\infty} \frac{(\beta\tilde{\phi}_p)^j \lambda_{s,t+j}}{\lambda_{s,t}} \cdot \left\{(1-\eta_y)\cdot\left[\frac{P_t^\star(z)}{P_{t+j}}\right]^{1-\eta_y} + \eta_y \cdot \frac{P_{t+j}^w}{P_{t+j}} \cdot \left[\frac{P_t^\star(z)}{P_{t+j}}\right]^{-\eta_y}\right\} \cdot Y_{t+j} = 0. \quad \text{(A.1)}$$

Given that all retailers face the same profit maximization problem, they all choose the same price, $P_t^\star(z) = P_t^\star$. Hence, we get:

$$P_t^\star = \frac{\eta_y}{\eta_y - 1} \cdot \frac{\mathrm{E}_t \sum_{j=0}^{\infty} (\beta\tilde{\phi}_p)^j \lambda_{s,t+j} \cdot \frac{P_{t+j}^{\eta_y}}{X_{p,t+j}} \cdot Y_{t+j}}{\mathrm{E}_t \sum_{j=0}^{\infty} (\beta\tilde{\phi}_p)^j \lambda_{s,t+j} \cdot P_{t+j}^{\eta_y - 1} \cdot Y_{t+j}}. \quad \text{(A.2)}$$

Finally, the price for the final goods, Eq. (37), can be rewritten as the following equation:

$$P_t = \left[(1-\tilde{\phi}_p)\cdot(P_t^\star)^{1-\eta_y} + \tilde{\phi}_p \cdot P_{t-1}^{1-\eta_y}\right]^{\frac{1}{1-\eta_y}}. \quad \text{(A.3)}$$

B.2 The Equivalence of Rotemberg and Calvo

Upon linearizing Eq. (49), we arrive at the following:

$$\widehat{\Pi}_t = \beta_s \mathrm{E}_t\left[\widehat{\Pi}_{t+1}\right] - \frac{\eta_y - 1}{\Pi^2 \phi_p} \cdot \widehat{X}_t. \quad \text{(A.4)}$$

The first-order Taylor expansions of Eq. (A.2) and Eq. (A.3) yield the following results:

$$\widehat{P}_t^\star = (1-\beta\tilde{\phi}_p)(\widehat{P}_t - \widehat{X}_t) + \beta\tilde{\phi}_p \mathrm{E}_t \widehat{P}_{t+1}^\star;$$
$$\widehat{\Pi}_t = (1-\tilde{\phi}_p)(\widehat{P}_t^\star - \widehat{P}_{t-1}).$$

Upon combining these two equations, we have the inflation equation:

$$\widehat{\Pi}_t = \beta_s \mathrm{E}_t \left[\widehat{\Pi}_{t+1}\right] - \frac{(1-\tilde{\phi}_p)(1-\beta\tilde{\phi}_p)}{\tilde{\phi}_p} \cdot \widehat{X}_t. \tag{A.5}$$

By comparing the inflation equations derived from Rotemberg's and Calvo's price-setting mechanisms, specifically Eq. (A.4) and Eq. (A.5), we can solve for the parameter value that translates a Calvo price-setting duration into an equivalent Rotemberg price adjustment cost parameter:

$$\phi_p = \frac{\tilde{\phi}_p(\eta_y - 1)}{(1-\tilde{\phi}_p)(1-\beta\tilde{\phi}_p)\Pi^2}. \tag{A.6}$$

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