On Stars and Galaxies: Exploiting Social Influence in Networks

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Motivation

We are swayed by friends, colleagues, lifestyle bloggers: behavior adopted by peers is more attractive.

How can external actor exploit social influence to coordinate agents on certain behavior and which network architectures are most susceptible to the external influence?

Problem

Principal

- wants agents to act, e.g., *to vote for a proposal, vaccinate.*
- spends resources to make them act, e.g., *lobbying, educational campaigns.*

How intensively should the principal target each individual in a social network?

Which networks are more susceptible to external influence?

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Literature

- Social networks:
  Mutuswami and Winter (2002), Jackson and Yariv (2007), Ballester et al. (2006), Ghiglino and Goyal (2010), Patacchini and Zenou (2012), Bramoullé et al. (2014), ...
- Mechanisms to induce coordination:
  Segal (2003), Winter (2004, 2006), Halac, Kremer, and Winter (2020), Halac, Lipnowski and Rappoport (2020).
- Viral marketing and strategic diffusion:
  Hartline et al. (2008), Galeotti and Goyal (2009), Arthur et al. (2009), Kleinberg and Tardos (2016).
- Optimizing over a network structure:
  Chwe (2000), Guimerà, R. et al. (2002), Goyal and Vigier (2014), Dziubiński and Goyal (2017).

Model

- n individuals in a social network G: i and j are friends iff $g_{ij} = 1$, and $g_{ij} = 0$ otherwise.
- Each i acts ($x_i = 1$) or not ($x_i = 0$).
- Payoff from not acting is 0;
- Payoff from acting is

$$U_i(x, G) = -c_i + t_i + \text{Social Influence},$$

  - $c_i > 0$ is an individual cost of an action;
  - t_i influence of the principal;
- To induce action without social influence we need $t_i \geq c_i$.

Incentives to act are higher if more of your friends act.

$$\text{Social influence on } i = f(d_i) \sum_j g_{ij} x_j$$

- $f(\cdot)$ is weakly decreasing – the dilution of social influence: *someone with many friends is swayed less by each of them.*

Problem: induce coordination

Given a social network G and an influence mechanism $t = (t_1, \ldots, t_n) \in \mathbb{R}^n$, agents play a strategic game.

1. t is *incentive inducing* (INI) if $x_i = 1$ for all i in unique Nash equilibrium of the game induced by t.
2. t^* is *optimal* if it is the cheapest among all INI mechanisms.
3. A *network is susceptible* if there is no other network where the Principal can induce everyone to act at a lower total reward.

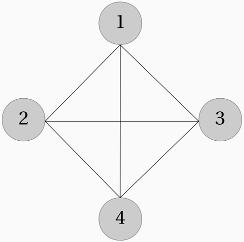
Example

- Individuals directly care about a relative proportion and/or an absolute number of active neighbors (Ghiglino and Goyal, 2010, Jackson and Zenou, 2012, Patacchini and Zenou, 2012, Liu et al., 2014).
- Then an influence of each friend on i is

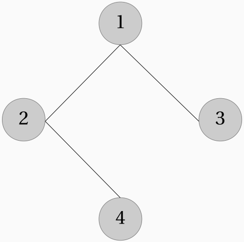
$$f(d_i) = \alpha + \frac{1}{d_i},$$

  $\alpha \geq 0$ and d_i is the number of i's friends.
- If $\alpha = 0$, only the relative proportion matters; as $\alpha \to \infty$, only the absolute number matters.

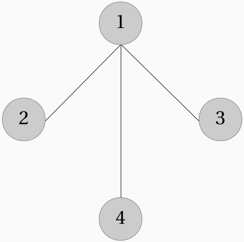
- There are four agents.
- $f(d_i) = \alpha + \frac{1}{d_i}$ and cost of acting are the same $c_i = c$.
- Consider the networks:



(a) Complete



(b) Line

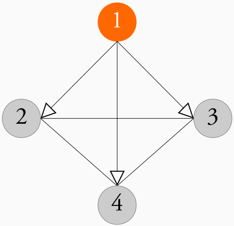


(c) Star

1. Find an optimal influence mechanism in each network.
2. Which network is easier to manipulate - requires lower total reward to induce action?

Example: complete network

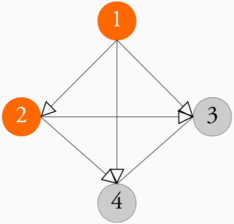
At least one agent must be offered $t_i \geq c$, otherwise there is a NE where noone acts.



Remember that payoff of i is:

$$U_i(x, G) = x_i \left(\left[\alpha + \frac{1}{d_i} \right] \sum_j g_{ij} x_j + t_i - c \right)$$

At least one of the remaining agents must be offered $t_i \geq c - \alpha - 1/3$, otherwise there is a NE where only 1 acts.



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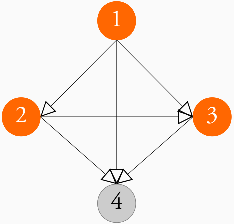
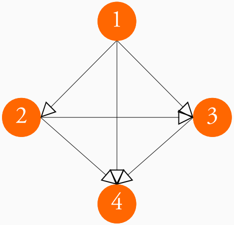
At least one of the remaining agents must be offered $t_i \geq c - 2\alpha - 2/3$, otherwise there is a NE where only 1 and 2 act.



Remember that payoff of i is:

$$U_i(x, G) = x_i \left(\left[\alpha + \frac{1}{d_i} \right] \sum_j g_{ij} x_j + t_i - c \right)$$

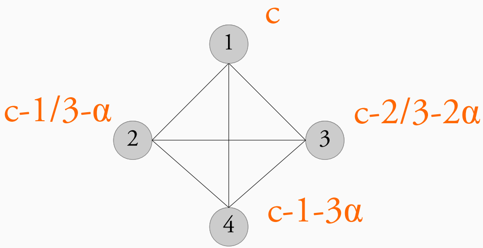
Must offer the remaining agent $t_i \geq c - 3\alpha - 1$, otherwise there is a NE where only 1, 2, and 3 act.



Remember that payoff of i is:

$$U_i(x, G) = x_i \left(\left[\alpha + \frac{1}{d_i} \right] \sum_j g_{ij} x_j + t_i - c \right)$$

The total reward in an optimal mechanism in a complete network is: $4c - 6\alpha - 2$.



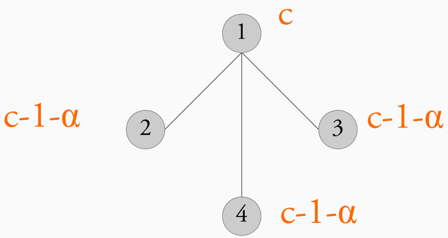
Note:

- agents are symmetric in a network, but rewards are distinct.
- rewards induce dominance cascade

Example: star

Suppose $f(d_i) = \alpha + 1/d_i$, $c_i = c$ and consider a star network:

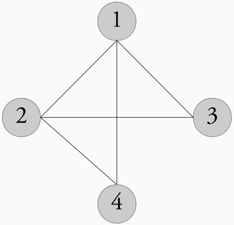
- Whereas in the complete network the sequence of agents in the dominance cascade did not matter, generally it matters.
- The optimal influence mechanism corresponds to a dominance cascade where agent 1 acts independently of the others.



- The corresponding total reward is $4c - 3\alpha - 3$.

Example: severed link

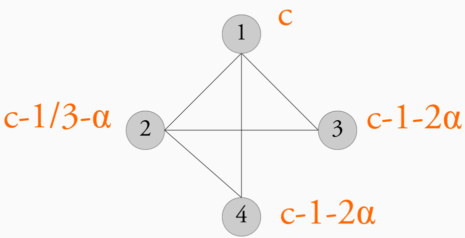
Let $f(d) = \alpha + 1/d$, $c_i = c$ and severe the link between 3 and 4:



- Sequence $1 \to 2 \to 3 \to 4$ results in the lowest total reward:

$$4c - 5\alpha - 7/3.$$

- Here agent 3 can be paid less than in complete network, and agent 4 must be paid more.

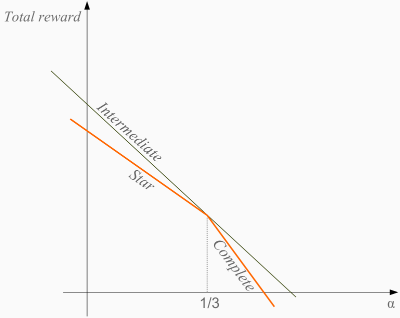


Example: networks requiring the lowest reward

Complete: $4c - 6\alpha - 2$.

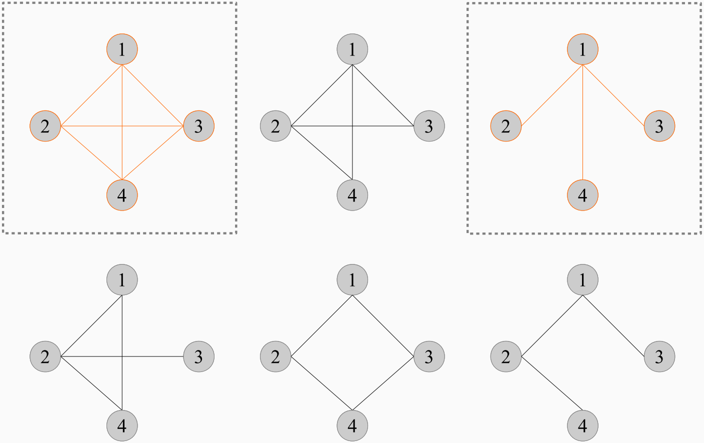
Star: $4c - 3\alpha - 3$.

Intermediate: $4c - 5\alpha - 7/3$.



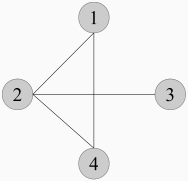
- The complete network requires a lower reward if $\alpha > 1/3$.
- The star requires a lower reward if $\alpha < 1/3$.
- All three networks require the same reward if $\alpha = 1/3$.

Example: all susceptible networks for $f(d_i) = \alpha + \frac{1}{d_i}$



Generically, a star or a complete network is susceptible.

A permutation of agents π is *non-increasing* if degrees of connected agents do not increase, i.e. for all i and j such that $g_{ij} = 1$ and $d_i > d_j$, we have $\pi(i) < \pi(j)$, where $\pi(i)$ is a place of agent i in permutation π.



- $(2, 1, 4, 3)$ and $(2, 4, 1, 3)$ are the only non-increasing permutations.

Optimal influence mechanism

Proposition
An influence mechanism $t = (t_1, \ldots, t_n)$ *is optimal if and only if there exists non-increasing permutation* π *of agents such that for all* i*,*

$$t_i = c_i - f(d_i) \sum_{j:\pi(j)<\pi(i)} g_{ij}.$$

- Agents of high degree centrality receive higher powered incentives than the rest and take the role of network leaders allowing the principal to utilize their social influence on others.

To characterize susceptible networks we assume:

(Benefit) Adding an active friend never decreases an agent's incentives to act. For each $m \geq 1$, and $k \leq m$ we have

$$f(m+1)(k+1) \geq f(m)k.$$

Which can be rewritten as: $f(m+1)(1+1/m) \geq f(m)$, for all m.

(Convexity) Influence function is convex. For each $m \geq 1$, we have

$$f(m) - f(m+1) \geq f(m+1) - f(m+2).$$

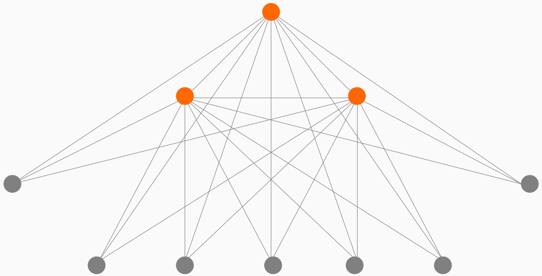
(Strong Convexity) Influence function is strongly convex. For each $m \geq 1$, we have

$$f(m) - f(m+1) \geq (f(m+1) - f(m+2))\,(1 + \frac{1}{n/2}),$$

where n is the number of agents.

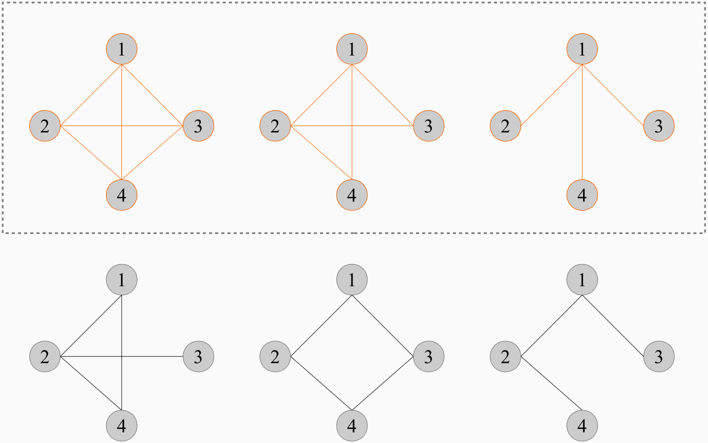
Galaxy is a graph such that all nodes can be partitioned into:

- *Stars* which are connected to all the nodes,
- *Periphery* nodes which are connected only to stars.



- Above is a galaxy with 3 stars and 7 periphery agents.
- Star and complete networks are special cases of a galaxy.

Galaxies in the example



Susceptible networks

Theorem
Suppose assumptions B and SC hold. Then a susceptible network is a galaxy.

- The easiest networks to manipulate are also *the most unequal in terms of their degree distributions* among all similarly dense networks.
- The stars receive high differentiated rewards, whereas the periphery agents receive the lower identical rewards (because they are not connected between themselves).

What do we know already about Galaxies?

Theory

- Galiotti and Goyal (2010): decentralized network formation in the context of the production of local public good leads to a galaxy structure.
- Herskovic and Ramos (2020): a model of decentralized informational network formation in the context of financial markets (Beauty Contest) leads to a galaxy structure.

Evidence

- Lada and Adar (2003): MIT and Stanford students interact through their webpages information dissemination is dominated by influencers and sequences are short.
- Goel at al (2012): online (Twitter) 94%-99% of information dissemination is happening within 1 degree of seed node.

Contributions

- Formulated a model of heterogeneous social influence and a tractable problem of maximizing its spread with focus on coordination problems.
- Introduced galaxies – a novel type of hub-periphery networks.
- A step toward understanding how social networks can be used to shape behavior.