A Risk-Based Liquidity Theory of International Currency*

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Abstract

This paper develops a risk-based liquidity theory of international currency, grounded in the endogenous ranking of currencies as media of exchange. These rankings emerge from their asymmetric hedging properties against counterparties' future income shocks within an open-economy search model featuring multiple currencies. The model rationalizes various currency payment patterns as equilibrium outcomes, including single-currency dominance and the coexistence of multiple currencies. Empirical evidence supports the model's prediction that the share of the dollar in international trade settlements and currency holdings is more prevalent in countries with more procyclical valuations of local currencies against the US dollar. The calibrated three-region model successfully replicates the observed dominance of the US dollar in the U.S. and Latin American regions due to its superior insurance properties against local economic conditions, alongside the local adoption of the euro in the Eurozone. Furthermore, the model underscores the significant welfare costs of de-dollarization policies, which result from the deterioration of terms of trade caused by the exclusion of optimal foreign currency payment arrangements, particularly as risk aversion increases.

Keywords: International currencies, medium of exchange, monetary search, liquidity, risk

JEL Codes: D51, D83, F4, E41

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1 Introduction

Since the Global Financial Crisis, the US dollar has steadily increased its share across various global sectors, including bond and loan contracts, foreign exchange trade volume, and central bank reserves (see Maggiori, Neiman, and Schreger (2019)). Many observers attribute this phenomenon to the safety of the dollar, stemming from the superior performance of dollar-denominated assets, particularly when such assets were most valued during economic crises. However, what remains less understood is why this perceived safety contributes to the dollar's dominance in international trade settlement as a medium of exchange. While some observers, such as Ito and Chinn (2015), argue that the "flight to safety" effect benefited dollar-denominated assets and, consequently, led to more dollar invoicing in international trade, there has been a lack of formal theoretical or empirical studies that fully formalize the link between safety and the emergence of the dollar as the dominant international medium of exchange.

The objective of this paper is to develop a theory of international medium of exchange based on endogenous ranking arising from the safety of alternative currencies and address the following fundamental questions in international macroeconomics and finance: (i) under what conditions is a country's currency, such as the US dollar, widely used as a vehicle for international and domestic transactions and, widely held in other countries, particularly in emerging economies—a phenomenon known as dollarization? (ii) What are the implications of the US dollar's safety relative to local currencies—its countercyclical valuation relative to local and global economic conditions—for endogenous currency choices as media of exchange? (iii) What are the welfare gains of having a safe currency circulated as an international medium of exchange, and the costs of de-dollarization policies that restrict foreign currency transactions?

To address these questions, I develop a general equilibrium framework with multiple currencies in an open-economy search model, incorporating: (i) the conditions that allow for the emergence of an international medium of exchange; (ii) endogenous currency valuations and all currency payment patterns as equilibrium outcomes of private citizens' optimal choices[1], including national currency dominance, coexistence of multiple currencies, and dollarization—supported by novel cross-country evidence; (iii) a quantitative model calibrated to empirical measurements of the safety properties of alternative currencies derived from exchange rate data, capable of replicating the payment and currency holdings in deposit of the US, Eurozone, and Latin America; and (iv) an evaluation of the welfare benefits and costs of de-dollarization policies for both global and local economies.

The search-theoretic framework is particularly crucial for understanding international currency use, as it allows private agents to endogenously select currencies based on their role as mediums of exchange, rather than assuming this choice exogenously, as is common in much of the international macroeconomic literature. This paper advances our understanding of the internationalization of currencies by providing a microfounded framework that explains global currency settlement and currency holding patterns through the lens of currency safety. In doing so, it contributes to the crucial understanding of what determines which currency the world adopts, as aspired by Maggiori, Neiman, and Schreger (2019).

The model incorporates currency payment patterns by allowing private agents to choose which currencies to accept. A key innovation of this model is that the ranking of mediums of exchange is determined by the safety defined as the hedging property of a currency against future aggregate income shocks hitting the counterparty—a seller of goods during economic crises. This is a novel aspect in international monetary economics, incorporating the notion of "safety" as the hedging property. Sellers prefer to receive a currency as the medium of exchange that better hedges against potential future income risks. Consequently, buyers also prefer to hold and pay with the currency that offers better hedging against these risks, as it allows them to obtain more favorable terms of trade. This leads to the endogenous determination of the medium of exchange.

This hedging concept for currencies builds on foundational insights from a substantial

[1] Hence, the model resolves the nominal exchange rate indeterminacy problem by Kareken and Wallace (1981).

body of international macroeconomics and finance literature emerging since the Global Financial Crises, particularly the role of stores of value, as pioneered by Gourinchas, Rey, and Govillot (2017)[2]. These studies highlight the superior insurance properties of the US dollar, which tends to appreciate against other currencies during periods of global economic stagnation. Their contributions, followed by numerous studies, emphasize the cross-country insurance (hedging) role of the US dollar as a safe currency in the form of the transfer from the U.S. to the rest of the world, achieved through the revaluation of US dollar-denominated assets and liabilities.

Recent literature, including Bocola and Lorenzoni (2020), Drenik, Kirpalani, and Perez (2021), Chirstiano, Dalgic, and Nurbekyan (2022), Oskolkov and Sorá (2023), and Dalgic (2024), extends this analysis to examine the dollarization of domestic financial flows. This research explores within-country insurance and redistributive effects between domestic savers and borrowers, as local currencies in emerging markets frequently depreciate against the US dollar during economic crises, such as currency and banking crises. I build on these insights into "safe currencies" to analyze the determinants of international medium of exchange.

In the two-country, two-currency, infinite-horizon model, agents interact in decentralized markets, trading with individuals from either the same or a different country. These interactions are followed by participation in a Walrasian currency exchange market in each period, in the spirit of Lagos and Wright (2005). The environment is characterized by search frictions, private trading histories, and imperfect recognizability of assets. Each country issues one currency, and citizens in each country face aggregate income shocks in the subsequent currency market, with these shocks being imperfectly correlated across countries.

Trade involves the exchange of local goods for a portfolio of currencies, without restrictions on which currencies can be used by private citizens. Sellers prefer to receive currency portfolios that provide better hedging against future adverse shocks (i.e., currencies with more countercyclical revaluations relative to the shock)[3] and are willing to offer more favor-

[2]See also Obstfeld, Shambaugh, and Taylor (2010), Bernanke (2017), and Maggiori (2017)

[3]This paper remains agnostic about the specific sources of future income shocks that sellers face, following

able terms of trade in exchange for such portfolios. Depending on the insurance properties of alternative currencies, various equilibrium patterns of currency usage emerge.

By formalizing the role of currency in payments, the model provides a channel through which monetary policy can influence inflations, trade, and welfare. Currency substitution, for instance, arises endogenously not only from expected local inflation but also from the weaker hedging properties of the local currency. This model prediction is well-supported by cross-country evidence, where dollarization measures of currency holdings in local deposit, as outlined in Levy-Yeyati (2006), and international trade settlement/invoicing currency shares from Boz, Casas, Georgiadis, Gopinath, Mezo, Mehl, and Nguyen (2020) are strongly correlated with the procyclicality of local currency valuations relative to the US dollar with respect to local real GDP growth.

Average inflation rates significantly predict currency holding dollarization, aligning with earlier findings in the literature, such as Levy-Yeyati (2006) and Chirstiano, Dalgic, and Nurbekyan (2022), but notably, do not explain trade currency shares. This evidence challenges the traditional view in international monetary economics such as Zhang (2014), which has predominantly emphasized currency substitution channels driven by average inflation rates.

This paper also models the strategic interaction among monetary authorities to provide insights into the choice of state-dependent inflation in interdependent economies. In a simple dynamic policy game, policymakers have incentives to earn seigniorage from foreign citizens by deviating from the Friedman rule on average and increasing expected local inflation. Importantly, they also seek to maximize the insurance properties of the local currency in terms of hedging to protect local citizens from shocks, consistent with the inflation choices made by a global planner who maximizes total welfare by jointly setting inflation rates in

the approach of Jacquet and Tan (2012) and Drenik, Kirpalani, and Perez (2021). In open-economy models with heterogeneous agents, sellers' exposure to future aggregate shocks is often modeled through their lower risk aversion relative to savers or the uninsurable risks faced by entrepreneurs. These frameworks are commonly used to study the redistributive effects of financial contracts denominated in nominal units. Notable examples include Bocola and Lorenzoni (2020), Chirstiano, Dalgic, and Nurbekyan (2022), Oskolkov and Sorá (2023), and Dalgic (2024).

both countries. This result, new and difficult to achieve in previous studies without shocks, underscores the role of state-dependent monetary policy in currency competition.

Finally, I calibrate a quantitative three-region version of the model to empirical correlations among exchange rates, real GDP growth, inflation rates, and trade flow data for the U.S., the Eurozone, and Latin America. The model successfully replicates key empirical patterns of currency usage across these regions. The US dollar emerges as a medium of exchange and is held not only within the U.S. but also in the Latin America region. In contrast, the euro serves predominantly as the local currency within the Eurozone.

The model further predicts that the US dollar acts as a vehicle currency, facilitating international trade between the Eurozone and Latin America, even when U.S. agents are not directly involved in these transactions. Notably, while the euro's lower inflation rates—and consequently higher average currency returns—would theoretically favor its global dominance, the empirical observation of the US dollar's dominance is explained by its superior insurance (hedging) properties. These properties lead agents in Latin America to prefer the US dollar as an international medium of exchange, overturning the prediction based solely on inflation differentials.

I then conduct a counterfactual analysis to examine the welfare consequences of de-dollarization policies, which prohibits the usage and holding of the US dollar in the Latin America region. While the U.S. emerges as the sole loser from such policies due to reduced seigniorage revenue collected from Latin America, the Latin America region experiences mixed effects. Although it benefits from lower inflation tax transfers to the U.S., the policy significantly deteriorates its terms of trade, as it eliminates access to the US dollar, a superior medium of exchange for international trade. This new channel of welfare cost—linked to the loss of an efficient medium of exchange—grows with increasing risk aversion, substantially offsetting the welfare gains from reduced inflation tax. The Eurozone remains unaffected by this policy, as the euro is exclusively used by Eurozone agents.

Global welfare declines due to the deterioration in terms of trade and the absence of net global gains from redistributed seigniorage transfers. The welfare gains from maintaining the usage of an international medium of exchange with superior insurance properties are comparable in magnitude to those reported in the existing literature, such as Zhang (2014), which examines the welfare implications of international currencies characterized by lower inflation. This analysis underscores the trade-offs associated with de-dollarization policies, particularly in regions that rely heavily on foreign currencies for efficient trade.

Related literature: Theories of international currencies date back to foundational work by Menger (1982), Kindleberger (1967), Swoboda (1969), and Krugman (1984). This paper contributes to this literature by providing a new microfoundation, emphasizing the role of international currencies in facilitating cross-border settlement within a search-theoretic framework to study the emergence of international medium of exchange.

The study of international medium of exchange within search theory also has a rich tradition, beginning with the pioneering work of Matsuyama, Kiyotaki, and Matsui (1993). Subsequent two-country, two-currency search models include contributions by Zhou (1997), Trejos and Wright (2001), Trejos (2003), Allen and Shi (2003), Camera and Winkler (2003), Li and Matsui (2009), and Liu and Shi (2010). Zhang (2014) offers a comprehensive literature review on currency substitution in one-country models. However, these earlier studies either impose restrictions on asset divisibility or fail to generate the full spectrum of acceptance patterns explored in this paper as equilibrium outcomes.

More recently, Zhang (2014) and Gomis-Porqueras, Kam, and Waller (2017) developed two-country, two-currency monetary models with divisible monies and goods, inspired by Lagos and Wright (2005). These models determine nominal exchange rates and rationalize various currency settlement patterns in equilibrium, such as national currency dominance and the coexistence of multiple currencies. In their framework, deterministic inflation serves as the primary driver of currency acceptance patterns, with lower inflationary currencies generally dominating others.

This paper contributes to this line of research by: (i) presenting empirical evidence that inflation rates do not robustly predict currency payment patterns in international trade, whereas the hedging properties of local currencies against the local business cycles risk emerge as a more critical predictor; and (ii) introducing a search-theoretic model that incorporates the asymmetric hedging properties of alternative currencies, generating a broader range of equilibrium currency payment patterns—including not only national currency dominance and the coexistence of multiple currencies but also dollarization.

Regarding policy, Li and Matsui (2009) and Zhang (2014) also examine currency competition among welfare-maximizing monetary authorities, who choose state-independent inflation rates in non-stochastic economies. This paper proposes an alternative and novel insurance (hedging) role for state-dependent monetary policy that affect the currency acceptance patterns and terms of trade, and characterizes how this channel affects welfare through seigniorage and terms of trade within and across borders.

This paper also builds on the general equilibrium models of bilateral trading and financial contracts developed by Jacquet and Tan (2012) and Drenik, Kirpalani, and Perez (2021), where specific monies or assets provide superior insurance properties compared to others, making them more effective mediums of exchange or units of account in closed-economy settings. Unlike their studies, this paper introduces multiple currencies in a multi-country, multi-currency framework, treating these currencies asymmetrically based on their hedging properties.

Crucially, a contribution of this paper is to solve and characterize for all possible equilibria under various parameter configurations, generating diverse currency circulation patterns and characterizing optimal monetary policies. Furthermore, I empirically document that currencies with superior hedging properties are more prevalently used in international trade settlement and invoicing, and held in local deposits across countries.

More broadly, theories of dominant currencies for units of account and/or mediums of exchange often emphasize strategic complementarities and the existence of multiple equilibria. These include (i) increasing returns to scale, as examined by Gopinath and Stein

(2021), Chahrour and Valchev (2022), and Coppola, Krishnamurthy, and Xu (2024), and (ii) strategic complementarities in price setting within sticky price models, as explored by Engel (2006), Gopinath, Itskhoki, and Rigobon (2010), and Mukhin (2022).

On the other hand, this paper attributes the emergence of a dominant currency to its observed insurance (hedging) characteristics, which are consistent with the cross-country evidence of the dollar share in international trade settlement shares and currency holdings. It demonstrates that state-dependent currency returns play a crucial role in the emergence of international mediums of exchange and their associated welfare implications.

Regarding the insurance/hedging properties of currencies, a substantial body of literature in international macroeconomics and finance has studied the role of the US dollar as a safe store of value for cross-country risk sharing, as highlighted by Gourinchas, Rey, and Govillot (2017), Obstfeld, Shambaugh, and Taylor (2010), Bernanke (2017), and Maggiori (2017), among others. Additionally, research such as Bocola and Lorenzoni (2020), Chirstiano, Dalgic, and Nurbekyan (2022), Oskolkov and Sorá (2023), and Dalgic (2024) has explored its role in within-country risk sharing.

In contrast, this paper focuses on the implications of safe currencies for the emergence of international mediums of exchange in a multi-country framework. It further provides novel insights into the global welfare consequences of de-dollarization policies, particularly their effects on terms of trade.

The structure of this paper: The rest of this paper is organized as follows: Sections 2 and 3 describe the baseline two-country model with multiple currencies and section 4 defines and characterizes the equilbria. Section 5 provides empirical support for theoretical predictions. Section 6 develops a three-region model with multiple currencies calibrated to data, compares the key model predictions with the data, and evaluate the welfare consequences of de-dollarization policies. Finally, section 7 concludes.

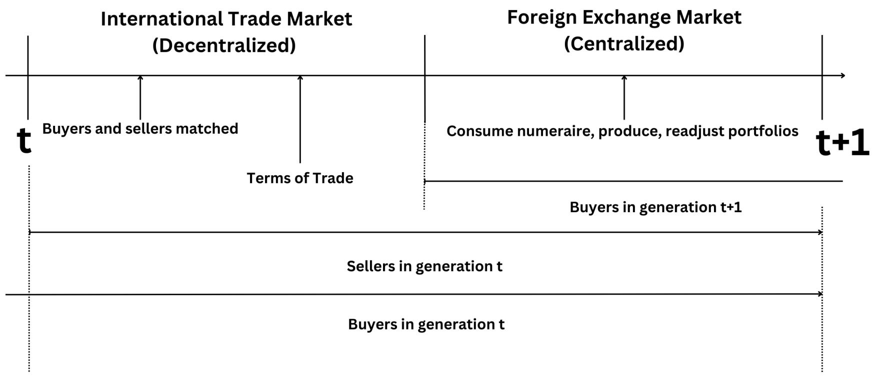

Figure 1: Timing of events

2 Environment

Time is discrete and continues indefinitely. There are two countries, 1 and 2. Each period consists of two sub-periods: the first is for decentralized trade in local and foreign goods (DM) involving international trades, and the second is for settlement and currency exchange (CM). In each period, the state of the world is stochastic and independent and identically distributed (iid) over time. The state (shock) for a given period is revealed in the CM before the markets open.

Each country is populated by overlapping generations of agents. All agents of generation t with mass 2 ($2n$) are born at the beginning of the CM of period t in country 1 (2), after the state of the world is revealed, where $n \in (0, 1)$ denotes relative country size. They die at the end of the CM of period $t + 1$ (see Figure 1). For convenience, we refer to agents as being "young" in the first period of their lives and "old" in the last period of their lives.

There are two types of agents in each generation from each country, which I will refer to as buyers and sellers, based on the roles they play in the DM. These agents are evenly divided between these two roles: in each period t, sellers born in period $t - 1$ from country $i \in \{1, 2\}$ can produce output $c_t^{s,i,t-1}$, but do no want to consume, while buyers want to

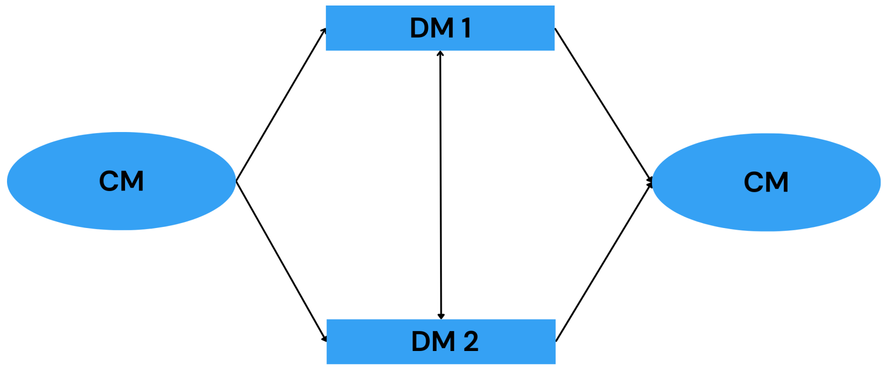

Figure 2: Market structure

consume but cannot produce. Sellers have immobile factors of production and are unable to produce the goods of the other country.

In the second sub-period, all trade occurs in a frictionless competitive market (CM). The type of an agent is denoted by $(x, j, t) \in \{b, s\} \times \{1, 2\} \times \mathbb{N}$, where the first element is either buyer or seller (b or s), the second element is the origin of country, and the last element is the period when he is born.

In each period τ, both old agents and young buyers can consume a numeraire good, $C_\tau^{(p,j,t)}$, which is produced according to a linear production function in labor, and supply hours of work in the CM, $N_\tau^{(p,j,t)}$, implying that the real wage rate is equal to one. On the other hand, young sellers are not allowed to trade in the CM[4]. Figure 1 and 2 summarizes the timing of events and market structure, respectively.

For tractability, utilities for both buyers and sellers are additively separable and quasi-linear in hours. For buyers with type (b, j, t),

$$\mathbb{E}_t[U(C_t^{b,j,t}) - \theta^b N_t^{b,j,t} + \beta\{u(c_{t+1}^{b,j,t}) + U(C_{t+1}^{b,j,t}) - \theta^b N_{t+1}^{b,j,t}\}], \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor and θ^b is the deterministic marginal disutility of

[4]The young sellers' inability to trade in the CM is not consequential in this model. As seen below, the buyers' choice of medium of exchange is independent of buyers' currency position.

supplying labor in CMs. Similarly for sellers,

$$\mathbb{E}_t[\beta\{-c_{t+1}^{s,j,t} + U(C_{t+1}^{s,j,t}) - \theta_{t+1}^{s,j,t} N_{t+1}^{s,j,t}\}]. \tag{2}$$

In contrast to buyers, sellers are exposed to future labor disutility cost, $\theta_{t+1}^{s,j,t}$, iid over time in the last CM of their lives in the spirit of Jacquet and Tan (2012). Those iid shocks are the only state variables in this economy.

I assume that u and U are twice continuously differentiable with $U' > 0$, $u' > 0$, $U'' < 0$, and $u'' < 0$. Furthermore, I assume that U and u satisfy the Inada conditions. Finally, there exist $c^{**} \in (0, \infty)$ such that $u'(c^{**}) = 1$ and $C^{**} \in (0, \infty)$ such that $U'(C^{**}(\theta)) = \theta$ for all θ. In the following, I assume that $u(c) = c^{1-\gamma}/(1-\gamma)$.

Discussion of labor disutility shock $\theta_{t+1}^{s,j,t}$: *The labor disutility shock can be interpreted as labor income shocks that co-move with aggregate national economic fluctuations. In the rest of the paper, I refer to the labor disutility shock and labor earning shock interchangeably. In emerging market economies, these aggregate fluctuations are frequently linked to economic crises, such as banking and currency crises. Following the approach of Chirstiano, Dalgic, and Nurbekyan (2022), this paper remains agnostic about the specific origins of these aggregate shocks, focusing instead on their economic implications for hedging motive and the choice of medium of exchanges. Chirstiano, Dalgic, and Nurbekyan (2022) empirically documents that unlike the external financial flows, the domestic deposit dollarization does not cause currency or banking crises in emerging market countries, which is consistent with the this paper's treatment of the labor earning shock as exogenous with respect to the endogenous decisions such as currency portfolio choice.*

Discussion on heterogeneity: *In the model, sellers are assumed to be more exposed to shocks than buyers, creating a key source of heterogeneity that drives the hedging motive for sellers. This type of heterogeneity between buyers and sellers has also been employed in the study of currency choice in financial contracts between borrowers and lenders, as ex-*

plored by Drenik, Kirpalani, and Perez (2021). Similarly, in a search-theoretic framework, this assumption has been used by Jacquet and Tan (2012) to investigate the monetary policy transmission mechanism to asset prices via the liquidity channel. They argue that the heterogeneous exposure to aggregate shocks aligns with the risks faced by entrepreneurs, who encounter undiversifiable income shocks, compared to buyers, who represent households. Entrepreneurs, being more vulnerable to aggregate shocks, embody the spirit of Heaton and Lucas (2000) and Moskowitz and Vissing-Jorgensen (2002), which applied similar assumptions to address the equity premium and risk-free rate puzzles. Alternatively, building on the seminal work by Constantinides and Duffie (1996), Heaton and Lucas (1996) and Heaton and Lucas (1997) study the asset pricing implications of nontradable labor income risk.

In the recent international macroeconomics literature, this assumption has been utilized to analyze various phenomena, as seen in the works of Chirstiano, Dalgic, and Nurbekyan (2022), Oskolkov and Sorá (2023), and Dalgic (2024). These studies highlight the relevance of such heterogeneity in explaining observed the dollarization of the domestic financial flows and analyzing the welfare consequences of financial de-dollarization policies in emerging market countries.

Discussion on the role of buyers: *The buyers in this model can be interpreted as a composite entity encompassing households, who save their labor income in the centralized market (CM), and firms, which borrow funds from households to finance international trade expenses for imports in the decentralized market (DM). Chahrour and Valchev (2022) explicitly model this type of environment, highlighting the complementarity between households' portfolio choices and firms' selection of collateral assets for borrowing. This interaction underpins the emergence of a dominant store of value and medium of exchange in international trade.*

While this paper abstracts from the detailed modeling of such strategic complementarities, it focuses instead on the direct implications of exporters' hedging motives against future aggregate risks. By doing so, the analysis isolates the novel role of insurance properties in *determining currency usage patterns in trade.*

Each country issues its own fiat currency, $i \in 1, 2$, which is both perfectly divisible and storable. Currency $m_i \in \mathbb{R}_+$ is valued at ϕ_i, the price of money in terms of the numeraire. The nominal exchange rate is defined as the price of currency 2 in terms of currency 1: $e \equiv \phi_2/\phi_1$. Since market clearing in the CM implies that the law of one price holds, agents can trade currencies at the market-clearing exchange rate. Hence, the CM also functions as a foreign exchange market.

Money supplies, M_i, grow or shrink each period, independent and identically distributed (iid) over time, by a factor of γ_i, where $\gamma_i \equiv M_i'/M_i$. Variables with a prime denote next period's parameters or choices. Changes in the money supply are implemented through lump-sum monetary transfers or taxes in domestic currency within the CM to each country's young buyers, denoted by $T_{j,t}$. The government budget constraint for each country $j \in 1, 2$ is given by:

$$\phi_{j,t}(M_{j,t} - M_{j,t-1}) = T_{j,t}. \tag{3}$$

I seek a stationary equilibrium where the stock of real balances remains constant over time and across different states. Specifically, if we denote $M_{j,t}(x_t)$ as the quantity of money and $\phi_{j,t}(x_t)$ as the price of money in terms of the numeraire in the CM during period t when the state of the world is x_t, I look for a stationary equilibrium where $\phi_{j,t}(x_t)M_t(x_t) = \phi_{j,t+1}(x_{t+1})M_{j,t+1}(x_{t+1})$ for all t and all states, implying that $\gamma_{j,t+1}^{-1} = \phi_{j,t+1}/\phi_{j,t}$.

Discussion on market structure: *Adopting the trading structure of Lagos and Wright (2005) allows for a clear distinction between the two roles a currency can serve: as a medium of exchange and as a store of value. Unlike existing studies in international macroeconomics and finance, which primarily focus on the optimal saving choices of alternative currencies with different insurance properties—such as Chirstiano, Dalgic, and Nurbekyan (2022), Oskolkov and Sorá (2023), and Dalgic (2024)—or on currencies as units of account Drenik, Kirpalani, and Perez (2021), this paper centers on the insurance properties of currencies that* *determine their selection as a medium of exchange, rather than as a store of value.*

In the model, buyers choose a currency portfolio specifically for use as the medium of exchange in trades. Consequently, sellers, anticipating future labor income shocks in the subsequent CM, value the hedging properties of the medium of exchange they receive. This means that the currency portfolio traded in the DM also serves as a store of value for sellers. Through this mechanism, the insurance properties of currencies emerge as the fundamental driver in determining the choice of medium of exchange in the model.

3 Model

This section describes the equilibrium of the two-country, two-currency model. I solve the agents' individual problems using a backward induction approach.

3.1 Old agents' value functions in CM

In the CM, let real balance holdings of monies for an old agent with type (x, j, t) holding be $\mathbf{q} \equiv (q_1, q_2) \equiv (\phi_1 m_1, \phi_2 m_2) \in \mathbb{R}^2_+$ and let $W^{x,j,t}_{t+1}$ and $V^{x,j,t}_{t+1}$ denote value functions for this agent in CM and DM in period $t+1$, respectively.

At the start of the CM, such an agent faces the following maximization problem:

$$W^{x,j,t}_{t+1}(\mathbf{q}) = \max_{C_{t+1}, N_{t+1}} \{U(C_{t+1}) - \theta^{x,j,t}_{t+1} N^{x,j,t}_{t+1}\} \tag{4}$$

subject to

$$C^{x,j,t}_{t+1} = N^{x,j,t}_{t+1} + \gamma^{-1}_{1,t+1} q_{1,t} + \gamma^{-1}_{2,t+1} q_{2,t}. \tag{5}$$

Then the optimality condition implies the labor supply:

$$N^{x,j,t}_{t+1} = C^{**}(\theta^{x,j,t}_{t+1}) - \gamma^{-1}_{1,t+1} q_{1,t} - \gamma^{-1}_{2,t+1} q_{2,t}, \tag{6}$$

where $U'(C^{**}(\theta)) = \theta$. An old agent's expected utility at the beginning of the CM is the sum of net consumption in the CM and the valuation of real balances in domestic and foreign currencies. The expected value function, conditional on the current currency portfolio without knowing the state realization in period $t+1$, is given by

$$\mathbb{E}_t W_{t+1}^{x,j,t}(\mathbf{q}) = \mathbb{E}_{t+1-}[U(C^{**}(\theta_{t+1}^{x,j,t})) - \theta_{t+1}^{x,j,t} C^{**}(\theta_{t+1}^{x,j,t})] + v_1^{x,j} q_1 + v_2^{x,j} q_{2,t}, \tag{7}$$

where E_{t+1-} is the conditional expectations on not knowing the state realization and the state-independent valuation of a currency c is written as

$$v_c^{x,j} = \underbrace{\mathbb{E}_{t+1-}\gamma_{c,t+1}^{-1}}_{Inflation\ Cost} \times \mathbb{E}_{t+1-}\theta_{t+1}^{x,j,t} + \underbrace{Cov_{t+1-}(\gamma_{t+1}^{-1}, \theta_{t+1}^{x,j,t})}_{Insurance}, \tag{8}$$

where Cov_{t+1-} is the covariance operator conditional on not knowing the state realization in period $t+1$. In line with Jacquet and Tan (2012) and Drenik, Kirpalani, and Perez (2021), the currency valuation is determined not only by the expected inflation cost but also by its insurance/hedging property: an agent value a currency more if it is revaluated when his labor earning deteriorates. As I discuss earlier, this modeling is a parsimonious approach to incorporate the sellers' exposure to the aggregate shock hitting the country.

Note that $\mathbb{E}_t W_{t+1}^{x,j,t}$ is linear in total expected wealth $v_1^{x,j} q_1 + v_2^{x,j} q_{2,t}$. Consequently, the CM expected value function is linear in expected total wealth: $\mathbb{E}_t W_{t+1}^{x,j,t}(\mathbf{q}) = \mathbb{E}_t W_{t+1}^{x,j,t}(\mathbf{0}) + v_1^{x,j} q_1 + v_2^{x,j} q_{2,t}$.

3.2 Terms of trade in DM

This section characterizes the terms of trade in the DM, assuming agents are restricted to holding non-negative quantities of each currency and taking as given their currency portfolios.

Suppose that a buyer from country j meets a seller from country i in period-$t+1$ DM. Let $\mathbf{q}_t^{b,j,t}$ denote the buyer's currency portfolio. The value functions of such a buyer, conditional on meeting this seller, is given by

$$V_{i,t+1}^{b,j,t}(\mathbf{q}_t^{b,j,t}) = u(c_{t+1}^{j,i}) + \mathbb{E}W_{t+1}^{b,j,t}(\mathbf{q}_t^{b,j,t} - \mathbf{d}_t^{b,j,t}), \tag{9}$$

where $\mathbf{d}_t^{b,j,t}$ is the currency transfer from the buyer to the seller in the meeting. The seller's value function, conditional on having this meeting in DM, is given by

$$V_{j,t+1}^{s,i,t} = -c_{t+1}^{j,i} + \mathbb{E}W_{t+1}^{s,i,t}(\mathbf{d}_t^{b,j,t}). \tag{10}$$

Note that the seller does not have any currency since they are not allowed to trade in CM when they are young[5]. Combined with the linearity of the value function in the next DM, the seller will participate in a trade if the terms of trade satisfy $v_1^{s,i}d_{1,t+1}^{j,i} + v_2^{s,i}d_{1,t+1}^{j,i} \geq c_{t+1}^{j,i}$.

The buyer makes a take-it-or-leave-it offer (TIOLI) to the seller. The TIOLI offer maximizes the buyer's surplus by choosing the DM consumption and currency portfolio transfer, subject to the seller's incentive compatibility (IC) and feasibility constraint of the currency transfer:

$$\max_{c_{t+1}^{j,i}, d_{1,t+1}^{j,i}, d_{2,t+1}^{j,i}} V_{t+1}^{b,j,t}(\mathbf{q}_t^{b,j,t}) - \mathbb{E}_t W_{t+1}^{b,j,t}(\mathbf{q}_t^{b,j,t}) = \max_{c_{t+1}^{j,i}, d_{1,t+1}^{j,i}, d_{2,t+1}^{j,i}} u(c_{t+1}^{j,i}) - (v_1^{b,j}d_{1,t+1}^{j,i} + v_2^{b,j}d_{1,t+1}^{j,i}) \tag{11}$$

subject to:

$$v_1^{s,i}d_{1,t+1}^{j,i} + v_2^{s,i}d_{1,t+1}^{j,i} \geq c_{t+1}^{j,i}, \qquad \textit{(seller's IC)} \tag{12}$$

and for $c \in \{1, 2\}$:

$$0 \leq d_{c,t+1}^{j,i} \leq q_{c,t}^{b,j,t} \qquad \textit{(feasibility).} \tag{13}$$

Note that the seller's IC constraint always binds at optimum. Since the problem is concave, the first-order conditions (FOCs) are necessary and sufficient. The marginal gains from trading the good are infinitely large at $c = 0$, which implies that trade will occur provided

[5] Even if the seller holds currency portfolio in DM, his marginal valuations for currencies will not be affected in the current setting.

that the buyer holds a non-empty portfolio. The FOC for real balance transfer of currency $c \in \{1, 2\}$ implies

$$\underbrace{v_c^{b,j}}_{\text{Marginal Cost of Real Balance Transfer}} \geq \underbrace{v_c^{s,i} u'(c_{t+1}^{j,i}) - v_c^{s,i} \lambda_c^{j,i}}_{\text{Marginal Benefit of Real Balance Transfer}}, \tag{14}$$

with equality if $d_{c,t+1}^{j,i} > 0$, where $v_c^{s,i} \lambda_c^{j,i} \geq 0$ is Lagrange multiplier on the upper bound on the feasibility constraint. Equivalently,

$$\epsilon_c^{j,i} \geq u'(c_{t+1}^{j,i}) - \lambda_c^{j,i}, \tag{15}$$

where $\epsilon_c^{j,i} = v_c^{b,j}/v_c^{s,i}$ is the relative marginal benefit of holding real balance of currency c for the buyer to the seller, or the marginal cost of transferring that currency. Therefore, the buyer does not transfer currency c, even with a positive amount, if the relative marginal benefit of holding it (on the left hand side) exceeds the marginal benefit of consuming the DM goods (on the right hand side).

To simplify the notation for characterizing the terms of trade, focus on the domestic meeting between a buyer and seller from country 1, where currency 1 is the national currency and currency 2 is the foreign currency. The country superscripts are suppressed to reduce the notational complexity.

3.2.1 Indeterminacy with common relative currency valuations

This section characterizes the terms of trade when two currencies have the same relative valuation $\epsilon_1 = \epsilon_2 = \epsilon$, following the spirit of Jacquet and Tan (2012), in the context of multiple currencies rather than a single currency and Lucas tree. The relative cost of transferring those currencies is exactly equalized from the buyer's perspective. Therefore, the composition of the optimal currency portfolio transfer is indeterminate as shown in the following proposition:

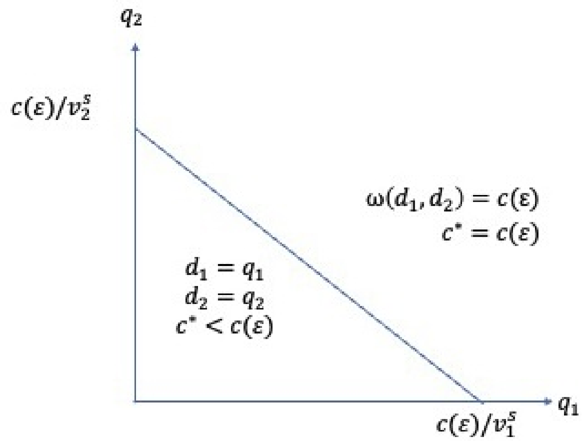

Figure 3: Terms of trade when symmetric currencies

Proposition 1. *Let $\omega_j(d_1, d_2)$ be the expected value of a transfer (d_1, d_2) for a seller, $v_1^s d_1 + v_2^s d_2$ and $c(\epsilon)$ be the DM consumption of a buyer such that $u'(c(\epsilon)) = \epsilon$. Then the terms of trade in the DM (c^*, d_1, d_2) satisfy $c^* = \omega(d_1, d_2) = \min\{\omega_j(q_1, q_2), c(\epsilon)\}$.*

Proof. See section A.1 in the Appendix A. □

Figure 3 depicts the terms of trade graphically. If the buyer holds sufficient real balances such that $\omega(q_1, q_2) \geq c(\epsilon)$, the buyers purchases $c(\epsilon)$ amounts of goods from the seller by transferring the real balance worth $c(\epsilon)$ for a seller. In particular, when unconstrained, the buyer is indifferent as to which currency to use as a medium of exchange. To obtain an additional unit of goods, the buyer incurs a cost of ϵ in utility, which is independent of the choice of payment currency. If the buyer does not hold real balances to purchase this unconstrained optimal amounts, then he transfers his entire portfolio holdings to the seller to obtain the maximum level of consumption within his budget.

3.2.2 Terms of trade with different relative valuations

Now I turn to the case where two alternative currencies have different relative valuations. In this case, the buyer has a strict preference for using one of the two currencies as a medium of exchange.

Proposition 2. *Suppose $\epsilon_1 > \epsilon_2$ and buyers' currency portfolio is given by (q_1, q_2). Then there exist $q_1(\epsilon_1)$ and $q_1(\epsilon_2)$ with $q_1(\epsilon_1) > q_1(\epsilon_2)$ such that*

$$(c^*(q_1, q_2), d_1, d_2) = \begin{cases} (c(\epsilon_1), q_1(\epsilon_1), 0), & \text{if} \quad q_1 > q_1(\epsilon_1) \\ (v_1^s q_1, q_1, 0), & \text{if} \quad q_1 \in [q_1(\epsilon_2), q_1(\epsilon_1)] \\ (c(\epsilon_2), q_1, (c(\epsilon_2) - v_1^s q_1)/v_2^s), & \text{if} \quad q_1 < q_1(\epsilon_2) \text{ and } \omega(q_1, q_2) \geq c(\epsilon_2) \\ (v_1^s q_1 + v_2^s q_2, q_1, q_2), & \text{otherwise.} \end{cases} \tag{16}$$

Proof. See section A.2 in Appendix A. □

The result of this proposition is graphically described in Figure 4. When the buyer holds a sufficient amount of the real balance of the seller's preferred currency 1, $q_1 > q_1(\epsilon_1)$, he purchases the unconstrained optimal amount of goods $c(\epsilon_1)$. If the real balance falls short of this level, consumption is constrained, and the buyer transfers the entire real balance of currency 1 but does not use the less preferred currency 2 as long as consumption remains above $c(\epsilon_2)$. This occurs because, when the buyer uses the currency 2 as a medium of exchange, the marginal cost of obtaining additional units of obtaining additional units of consumption, ϵ_2, is still higher than the marginal utility of consumption. As the consumption level falls below $c(\epsilon_2)$, the buyer begins to transfer the currency 2 to the seller.

3.2.3 Relative valuations and liquidity

In this model, the notion of liquidity capture the idea that, beyond physical or legal frictions limiting the trading of an asset, whether an asset is actively traded also reflects the choices made by agents, in the spirit of Jacquet and Tan (2012). Since I do not assume any type of trading restrictions or transactions costs in either sub-period and old agents die after the first

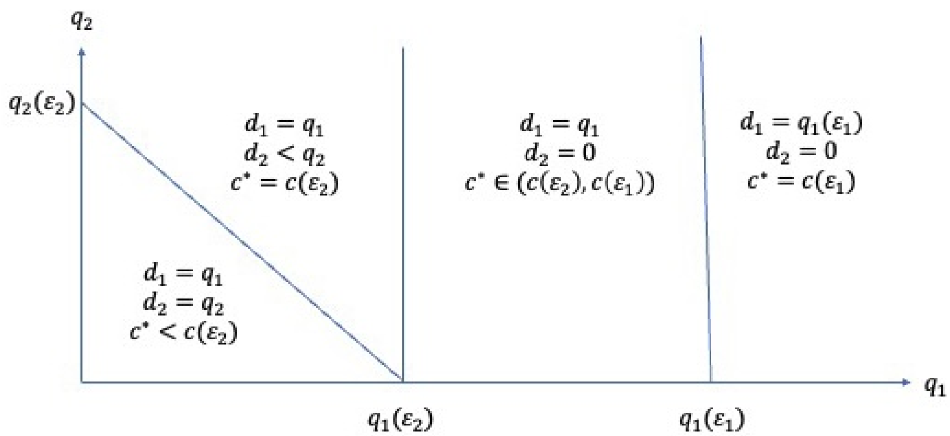

Figure 4: Terms of trade when asymmetric currencies

sub-period, both currencies are equally liquid in the CM. Therefore, I focus on the liquidity properties of assets in the DM. Let $c^*(q_1.q_2)$ denote the quantity consumed in the DM by a buyer with portfolio (q_1, q_2) as given in propositions 1 and 2, I define for currency $c \in \{1, 2\}$,

$$\mathcal{L}^c(q_1, q_2) = \frac{u'(c^*(q_1, q_2))\frac{\partial c^*(q_1,q_2)}{\partial d_c}}{v_c^b} - 1. \tag{17}$$

Definition 1.

1. *If $\mathcal{L}^c \geq 0$, then currency c is said to provide liquidity to buyers, while if $\mathcal{L}^i < 0$, then asset c is said to not provide liquidity to buyers.*

2. *If $\mathcal{L}^c > \mathcal{L}^{c'} > 0$, then currency c is said to provide more liquidity than currency c'.*

If $\mathcal{L}^c \geq 0$, it implies that $u'(c^*(q_1, q_2))\frac{\partial c^*(q_1,q_2)}{\partial d_c} > v_c^b$, meaning that if buyers were given an extra unit of currency c, they would spend it in the DM meeting. Conversely, if $\mathcal{L}^c < 0$ on the contrary, agents strictly prefer not to sell currency c for consumption in the DM meeting, meaning that the currency does not provide liquidity to agents.

Lemma 1. *For currency $c \in \{1, 2\}$, $\mathcal{L}^c = \frac{u'(c^*)}{\epsilon_c} - 1$.*

Proof. The proof can be immediately completed by using the expression for the DM consumption in proposition 1 or 2 to compute the partial derivative $\frac{\partial c^*(q_1,q_2)}{\partial d_c}$. □

When buyers enter the DM with very little in their portfolio $\omega(q_1, q_2) < c(\epsilon_2)$, the marginal utility of using each asset in the DM exceeds its discounted expected marginal value in the next CM, even if buyers spend all of their portfolio in the DM. As the value of buyers' portfolio increases, the liquidity provided by each currency falls because buyers can purchase more DM consumption. If $\epsilon_1 < \epsilon_2$, when the value of buyers' currency portfolio exceeds $q_1(\epsilon_2)$, currency 2 stops prividing liquidity, but currency 1 still provides liquidity. In this case $\epsilon_1 < \epsilon_2$, in equilibrium, currency 1 always provide liquidity, whereas it is possible for currency 2 not to provide liquidity.

Proposition 3.

1. *If $\epsilon_1 = \epsilon_2 = \epsilon$, then $\mathcal{L}^1 = \mathcal{L}^2 > 0$ if and only if $\omega(q_1, q_2) < c(\epsilon)$, and $\mathcal{L}^1 = \mathcal{L}^2 = 0$, otherwise.*

2. *If $\epsilon_1 < \epsilon_2$, then:*

   (a) *for currency 1, $\mathcal{L}^1 > 0$ if and only if $q_1 < q_1(\epsilon_1)$ and $\mathcal{L}^c = 0$, otherwise. For currency 2, $\mathcal{L}^2 > 0$ if and only if $\omega(q_1, q_2) < c(\epsilon_2)$; $\mathcal{L}^2 = 0$, if and only if $q_1 \leq q_1(\epsilon_2)$ and $\omega(q_1, q_2) \geq c(\epsilon_2)$; and $\mathcal{L}^2 < 0$, otherwise.*

   (b) *$\mathcal{L}^1 > \mathcal{L}^2$ for all (q_1, q_2).*

Proof. The proof relies on the result from lemma 1 and the first-order conditions (15). □

In case 1, Figure 5 provides a graphical summary of part (a) in proposition 3, where both currencies provide the same degree of liquidity, but both will strictly provide liquidity if and only if the value of buyer's portfolio is small. On the other hand, as seen in Figure 6, for case (b), the currency 1 always provides more liquidity than currency 2, when there is a difference in relative valuation in favor of currency 1. Since the liquidity service provided by a currency depends on agents' optimal portfolio decisions as well as the terms of trade for the

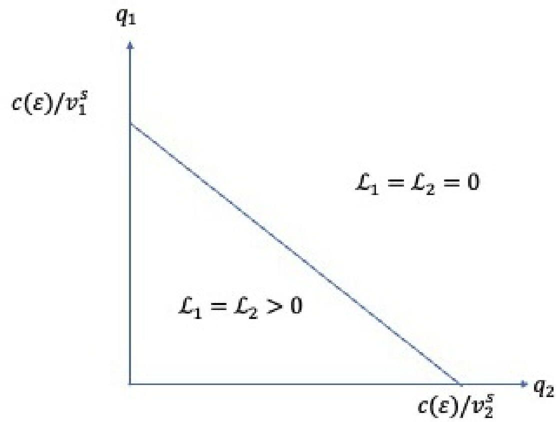

Figure 5: Liquidity in symmetric case

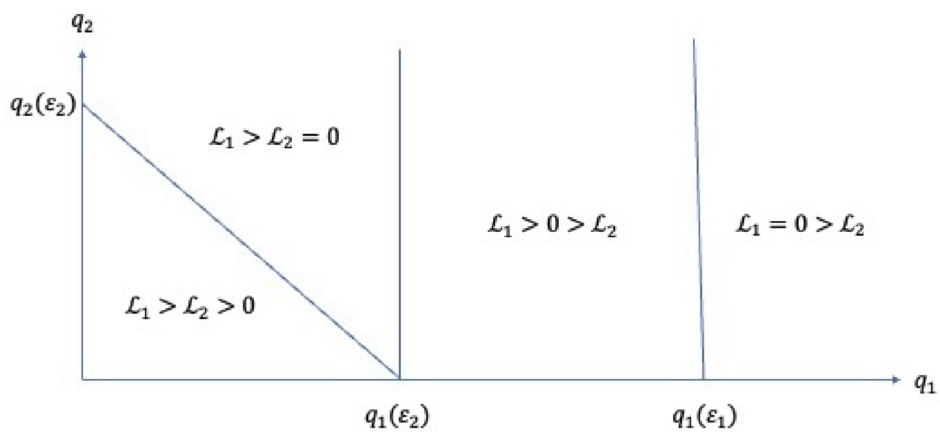

Figure 6: Liquidity in asymmetric case

DM, the liquidity provided by each currency is endogenous, as are the liquidity differences across currencies.

3.3 Value functions at the beginning of DM

Now I describe the value functions of buyers and sellers at the beginning of the decentralized market (DM) before knowing whom they meet, given the currency portfolio chosen in the previous centralized market (CM). Let $\lambda^b_{j,i}$ ($\lambda^s_{j,i}$) denote the probability of a buyer (seller) from country j meeting a seller (buyer) from country i in the DM.

The value function of a buyer from country j born in period t is given by

$$V_{t+1}^{b,j,t}(\mathbf{q}) = \lambda_{j,1} V_{1,t+1}^{b,j,t}(\mathbf{q}) + \lambda_{j,2} V_{2,t+1}^{b,j,t}(\mathbf{q}) + (1 - \lambda_{j,1} - \lambda_{j,2}) \mathbb{E}_t W_{t+1}^{b,j,t}(\mathbf{q}). \tag{18}$$

The value function of a seller from country j born in period t is given by

$$V_{t+1}^{s,j,t} = \lambda_{j,1}^{s} V_{1,t+1}^{s,j,t} + \lambda_{j,2}^{s} V_{2,t+1}^{s,j,t} + (1 - \lambda_{j,1}^{s} - \lambda_{j,2}^{s}) W_{t+1}^{s,j,t}. \tag{19}$$

Note that each subcomponent in the value function $V_i^{x,j,t}$ incorporates the possibility of alternative meetings with different types of agents where the liquidity provided by a currency differs depending on the counterparty.

3.4 Currency portfolio choice for young buyers in CM

In this section, I characterize the currency portfolio choice for young buyers in period t who choose the medium of exchange used in the subsequent DM, and I define the stationary monetary equilibrium.

The problem of a young buyer born in period t in country j in the CM is to choose consumption $C_t^{b,j,t}$, labor supply $N_t^{b,j,t}$, and currency real balances $\mathbf{q}_t^{b,j,t}$ to bring into the DM, aiming to maximize his lifetime expected discounted utility. A young buyer's problem in the CM is the same every period since the shock is iid and I focus on monetary policy rules where the inflation rate between t and $t+1$ depends on the state of the world in period $t+1$. Consequently, the supply and demand of currencies is the same every period in all states of the nature in terms of the numeraire.

The maximization problem of a young buyer (b, j, t) is formulated as

$$W_t^{b,j,t} = \max_{C_t^{b,j,t}, N_t^{b,j,t}, \mathbf{q}_t^{b,j,t}} U(C_t^{b,j,t}) - N_t^{b,j,t} + \beta V_{t+1}^{b,j,t}(\mathbf{q}_t^{b,j,t}) \tag{20}$$

subject to

$$C_t^{b,j,t} + q_{1,t}^{b,j,t} + q_{2,t}^{b,j,t} = N_t^{b,j,t} + T_{j,t}. \tag{21}$$

From the quasi-utility function, it follows that the optimal labor supply is $N_t^{b,j,t} = C^{**}(1) + + q_{1,t}^{b,j,t} + q_{2,t}^{b,j,t} - T_{j,t}$. Then the currency portfolio choice problem is given by

$$\max_{\mathbf{q}_t^{b,j,t}} -q_{1,t}^{b,j,t} - q_{2,t}^{b,j,t} + \beta V_{t+1}^{b,j,t}(\mathbf{q}_t^{b,j,t}). \tag{22}$$

The first-order condition for currency c real balance in the CM is given by

$$1 \geq \beta \partial_{q_{c,t}^{b,j,t}} V_{t+1}^{b,j,t}(\mathbf{q}_t^{b,j,t}) \tag{23}$$

with equality if $q_{c,t}^{b,j,t} > 0$. The following proposition characterizes this optimality condition in terms of the liquidity:

Proposition 4. *Let $l_{j,i}^c = \max\{0, \mathcal{L}_{j,i}^c\}$ be the liquidity premium currency c carries, where $\mathcal{L}_{j,i}^c$ is the liquidity of currency c in the meeting between a buyer from country j and a seller from country i. The FOC in equation (23) can be written as*

$$\theta^b \geq \beta v_c^{b,j} \times [1 + \lambda_{j,1} \times l_{j,1}^c(\boldsymbol{q}_t^{b,j,t}) + \lambda_{j,2} \times l_{j,2}^c(\boldsymbol{q}_t^{b,j,t})]. \tag{24}$$

Proof. See section A.3 in the Appendix A. □

Note that the optimal currency choice depends on the meeting probability with sellers and liquidity premia in those meetings. The liquidity $\mathcal{L}_{j,i}^c$ appears in the first-order conditions for the currency choice of buyers if and only if the currency strictly provides liquidity to the buyer in one of the DM meetings. If a currency is illiquid in a particular meeting, it does not appear in the first-order conditions because buyers do not wish to use any of it for purchasing consumption in that meeting−they prefer to hold onto it until the next CM.

Discussion of excluding young sellers from CM: *Young sellers are not allowed to trade* *in the CM in the current setting, which eliminates the possibility of them accumulating assets to insure themselves from future CM labor disutility risks. Although this assumption seems to be theoretically restrictive, the liquidity of currencies is not affected if young sellers are allowed to trade in CM. The reason is that even if a young seller accumulate large amount of currencies in the CM, he will still have preferences for currency portfolio transferred from buyers since the disutility from the labor supply is linear in the last CM of his life.*

4 Equilibrium

This section provides the definition of the stationary equilibrium that is of my interest and its characterizations of the patterns of currency portfolio holding and means of payment across different meetings under alternative parameter configurations. Finally, I discuss the state-dependent optimal monetary policies for the global planner who maximizes the total welfare of all countries and local planners who maximizes the welfare of their national citizens in a simple policy game in the spirit of Zhang (2014).

Definition 2 (Stationary Monetary Equilibrium)**.** *A stationary monetary equilibrium is a list of quantities traded* $\{C_t^{b,j,t}, C_{t+1}^{b,j,t}, c_t^{j,i}, \boldsymbol{d}_t^{j,i}\}$*, labor supply* $(\{N_t^{b,j,t}\}, \{N_{t+1}^{b,j,t}, \{N_{t+1}^{s,j,t}\})$*, and currency portfolios* $\{\boldsymbol{q}_t^{b,j,t}\}$ *for all* t *and* $j \in \{1, 2\}$ *s.t.*

1. *The first-order conditions for currency choices, equation 24, are satisfied;*
2. *The terms of trade* $(c_t^{j,i}, \boldsymbol{d}_t^{j,i})$ *satisfy the optimal TIOLI offers in propositions 1 or 2;*
3. *The choice in the CMs is consistent with optimality conditions.*

4.1 Equilibrium characterizations

I illustrate various equilibrium patterns of medium of exchange under the parameter values reported in Table 1, where two currencies are identical with respect to the expected valuations for buyers and the buyer meets sellers from different countries with an equal probability (0.5).

Parameter	Description	Value
γ	Relative risk aversion	2
β	Discount factor	0.9
$v_1^{b,1}$	Buyer's valuation of currency 1	0.9
$v_2^{b,1}$	Buyer's valuation of currency 2	0.9
$\lambda_{1,1}$	Probability of meeting seller 1	0.5
$\lambda_{1,2}$	Probability of meeting seller 2	0.5
$\epsilon_1^{1,2}$	Relative transfer cost of currency 1 to seller 2	2
$\epsilon_2^{1,1}$	Relative transfer cost of currency 2 to seller 1	2

Table 1: Parameter values in numerical experiment

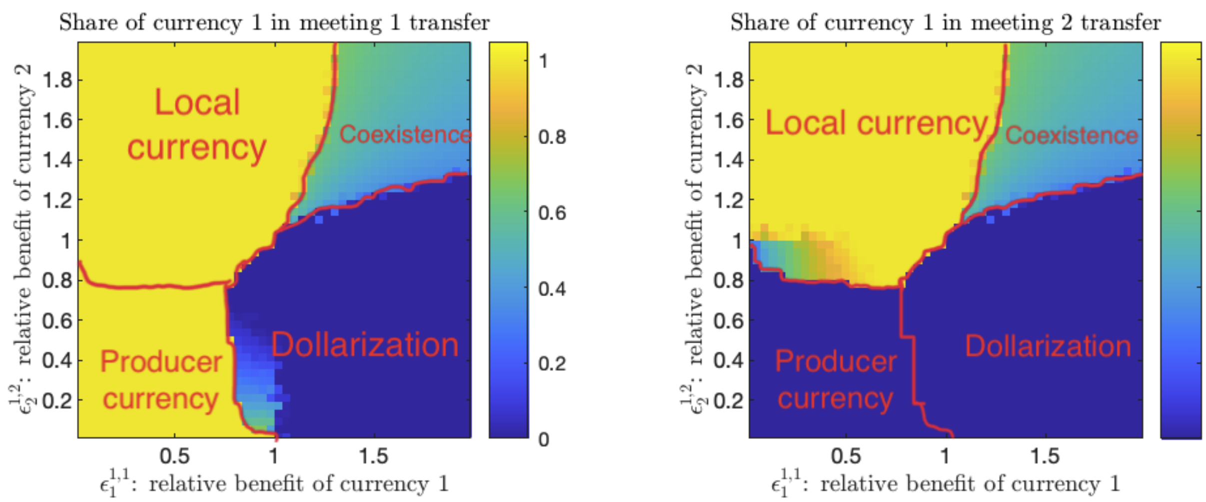

Figure 7: Currency 1 shares in DM transfers

I vary the currency c's relative valuations in the meeting with sellers from country c, issuing this currency, $\epsilon_c^{1,c}$, while assuming that it is better than another currency in such a meeting. Specifically, currency 1 is more valued than currency 2 in the meeting with country 1 sellers ($\epsilon_1^{1,1} < \epsilon_2^{1,1}$), i.e. in local meetings, and vice versa in the meeting with country 2 sellers ($\epsilon_2^{1,2} < \epsilon_1^{1,2}$).

The choice of these parameters is not intended to replicate empirical patterns but rather to demonstrate the model's ability to generate diverse equilibrium patterns of currency usage. In the subsequent section, I extend the analysis to a three-country model, where parameter values calibrated from data are used to evaluate the model's ability to reproduce empirical patterns of currency usage.

Figure 7 visualizes the share of real balance transfers of currency 1 in both domestic and foreign meetings, quantifying the degree of dollarization in terms of means of payment. The model generates all possible payment patterns in the DM. When the national currency is cheap to transfer in domestic meetings (lower $\epsilon_1^{1,1}$) and the foreign currency is costly to transfer in foreign meetings (higher $\epsilon_2^{1,2}$), the buyer always uses the national currency as the means of payment. This pattern is referred to as the local currency regime, as all DM transactions are processed using the buyer's national currency.

Conversely, when the national currency is costly to transfer in domestic meetings and the foreign currency is cheap to transfer in foreign meetings, the buyer always uses the foreign currency as the means of payment. This pattern represents the dollarization of means of payment, as all DM transactions are conducted using the foreign currency.

In the producer currency regime, both the national and foreign currencies are relatively cheap to transfer in domestic and foreign meetings, respectively, as sellers prefer their national currencies. Thus, the means of payment depends on the nationality of sellers (producers), distinguishing it from the local currency regime.

Finally, when both currencies are costly to transfer in their respective meetings, the buyer uses both currencies in all DM meetings, representing the coexistence of multiple currencies as mediums of exchange.

Figure 8 illustrates the liquidity premia of currencies across different meetings. In regions 1 and 3 of the upper-left panel, the liquidity premium of the national currency in domestic meetings increases as the national currency becomes more costly to transfer (higher $\epsilon_1^{1,1}$). This occurs because DM consumption decreases, thereby raising the marginal value of transferring the currency. However, in region 2, when currency 1 becomes sufficiently costly to transfer, the liquidity premium declines. This is because the national currency becomes so devalued for the domestic seller that transferring it results in very small marginal amounts of DM consumption, reducing its utility.

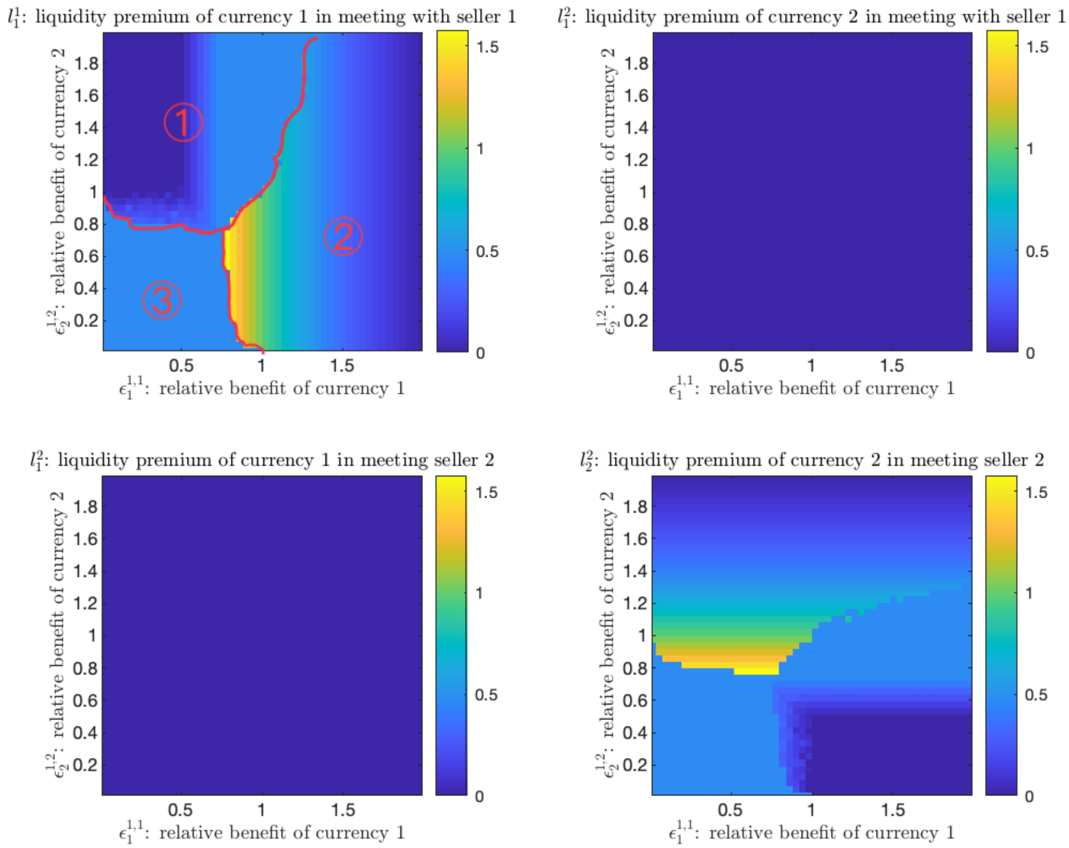

Figure 8: Liquidity premia of currencies

Conversely, as shown in the lower-left panel, the liquidity premium of currency 1 in foreign meetings remains negligible. Even when currency 1 is used as a means of payment in certain regions, the high transfer cost in foreign meetings prevents it from providing significant liquidity premia compared to its use in domestic meetings.

Figure 9 illustrates the share of real balance holdings of currency 1 relative to currency 2, quantifying the degree of dollarization as a store of value. The numbers in this figure are approximately a linear combination of the two panels in Figure 7, reflecting the fact that buyers save with the primary intention of spending their currency portfolio in the subsequent DM meetings, rather than saving for the next CM.

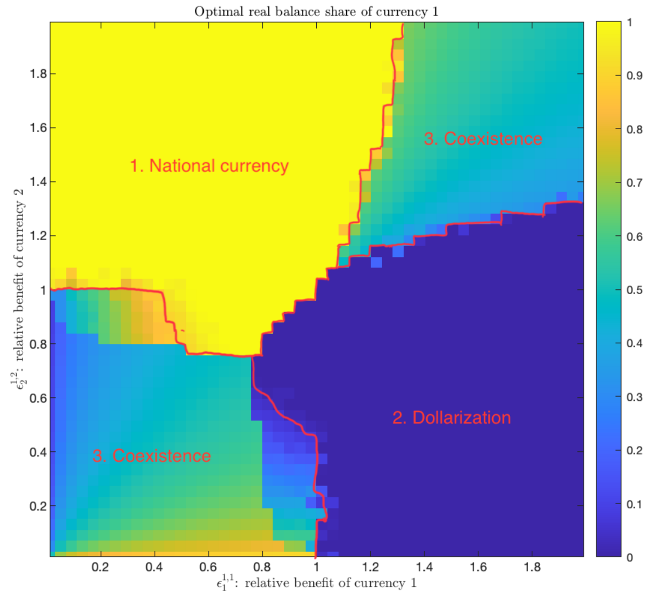

Figure 9: Currency 1 share in CM real balance

The figure delineates four distinct regions, representing all possible currency holding patterns. In region 1, buyers hold only the national currency because it is significantly less costly to use in domestic DM meetings, and the foreign currency does not offer a substantial cost advantage in foreign DM meetings. This equilibrium regime is termed the national currency regime.

In contrast, in region 2, buyers exclusively hold the foreign currency. When both currencies are either relatively cheap or costly to transfer, buyers diversify their holdings, obtaining both currencies in the CM. This pattern, observed in region 3, leads to the coexistence regime of multiple currencies.

4.2 Optimal policy

This section characterizes the optimal policies by national governments that maximize the total welfare of their citizens and by a global government that maximizes the total welfare of all citizens across the entire economy. The following proposition characterizes the welfare of citizens in each country:

Proposition 5. *The monetary policies setting the state-dependent money supply at* $t+1$*,* $(\gamma_{1,t+1}, \gamma_{2,t+1})$ *affect the expected welfare of country* j*'s citizens through the following channels: gains from trade, foreign currency holding costs, and seigniorage revenues as* $\beta \to 1$*:*

$$\mathcal{W}_j = \underbrace{S_j(q_t^{b,j,t})}_{\text{Gains from trade}} + \underbrace{\theta^b \sum_{c \neq j} (\mathbb{E}[\gamma_{c,t}^{-1}] - 1) q_{c,t}^{b,j,t}}_{\text{Foreign currency holding costs}} + \underbrace{\theta^b \mathbb{E}[1 - \gamma_{j,t}^{-1}] \sum_{c \neq j} q_{j,t}^{b,c,t}}_{\text{Seigniorage revenue from other countries}}, \tag{25}$$

where

$$S_j(q_t^{b,j,t}) = \sum_i \lambda_{j,i} [u(c_{t+1}^{j,i}) - \sum_c v_c^{b,j} d_{c,t}^{j,i}].$$

Therefore, the global planner maximizes the sum of those welfare criteria in two coutries:

$$\mathcal{W}_g = \sum_j \mathcal{W}_j = \sum_j S_j(q_t^{b,j,t}). \tag{26}$$

Proof. See section A.4 in the Appendix A. □

Note that the variances of currency returns $\gamma_{c,t+1}^{-1}$ do not directly affect the welfare but rather influence and mitigate the restrictions on the covariances between the currency return and shocks $\theta_{t+1}^{s,i}$, which have a direct impact on welfare. This leads to the following proposition for the optimal policy of the global planner:

Proposition 6. *Suppose that* $Var(\gamma_{j,t+1}) \leq \overline{M}_j$ *for some* $\overline{M}_j > 0$*. The global planner chooses the maximum variances of the currency returns without any additional restrictions. To maximize gains from trade in DM, the global planner adopts the Friedman rule on average s.t.* $\mathbb{E}[\gamma_{c,t+1}^{-1}] = 1/\beta$ *and maximizes the covariances between the currency returns and shocks,* $Cov(\gamma_{c,t+1}^{-1}, \theta_{t+1}^{s,i})$*.*

Proof. See section A.5 in the Appendix A. □

I am studying the Nash equilibrium of a simple one-shot policy game in the spirit of Zhang (2014), where all national governments, acting as first movers, simultaneously choose their domestic state- and time-dependent money supply rules (i.e., inflation rates) in the first stage. Then, in the second stage, private agents, as second movers, choose their actions. The national governments are aware of how their money supply rules affect the subsequent actions of private agents through inflation rates when deciding on their policies.

The national governments will adopt a policy regarding the second moments of their national currency return (inverse inflation rate) from the global planner's policy but deviate from the Friedman rule on average due to the incentive to collect seigniorage from other countries:

Proposition 7. *Suppose that* $Var(\gamma_{j,t+1}) \leq \overline{M}_j$ *for some* $\overline{M}_j > 0$. *National governments choose the maximum variances of the currency returns without any additional restrictions. To maximize gains from trade in DM, the national governments maximize the covariances between the currency returns and shocks,* $Cov(\gamma_{c,t+1}^{-1}, \theta_{t+1}^{s,i})$ *However, it might deviate from the Friedman rule on average:* $\mathbb{E}[\gamma_{c,t+1}^{-1}] < 1/\beta$.

Proof. See section A.6 in the Appendix A. □

These two proposition show that both the global planner and national governments align in terms of providing the maximum level of insurance. Therefore, even national governments should have sufficient incentives to make their currencies as safe as those envisioned by the global planner. Moreover, the national government does not exclude the usage of foreign currencies as store of values or medium of exchanges to eliminate the inflation taxes paid for foreign countries, if foreign countries are better medium of exchanges in some meetings than the national currency.

5 Empirical Analysis

In this section, I provide empirical support for the hedging motive in the theory, highlighting it as a key determinant of the choice of medium of exchange and currency holding. The optimality condition for the currency choice of buyer j in eq.(24) suggests that the holding of a particular currency depends positively on (i) the expected currency return (inverse inflation rate) $v_c^{b,j}$ and (ii) the liquidity premia $l_{j,i}^c$, which are interacted with meeting probabilities $\lambda_{j,i}$.

As shown in Figure 9, the holding of a currency decreases as the marginal cost of transferring the currency $\epsilon_c^{j,i}$ increases, i.e., it increases in the hedging benefits for sellers that the buyer potentially meets in the decentralized market (DM), denoted by $Cov(\gamma_{c,t+1}^{-1}, \theta_{t+1}^{s,i})i$. The hedging benefit for a particular seller i has a larger impact on the buyer's currency holding if the meeting probability with that seller, $\lambda j, i$, is higher. Similarly, Figure 7 shows that a particular currency c will be used more as a medium of exchange in a meeting with seller i if the cost to transfer it ($\epsilon_c^{j,i}$) is lower, indicating that the currency provides better hedging benefits for the seller.

In the remainder of this section, I test these theoretical implications of currency hedging benefits in conjunction with inflation rates and meeting probabilities. I apply them to the international trade settlement/invoicing currency choice and deposit dollarization. The hedging implications for the choice of trade settlement currency in the cross-section of countries are novel, while the implications for deposit dollarization have been previously explored in the literature Levy-Yeyati (2006), Bocola and Lorenzoni (2020), Chirstiano, Dalgic, and Nurbekyan (2022), and Dalgic (2024).

5.1 Measurement and data

I measure the sellers' labor income shocks, $\theta_{t+1}^{s,i}$, using real GDP and aggregate income data from each country. This approach is motivated by the theory and findings in the literature that agents have a significant hedging motive against adverse aggregate income shocks during recessions. This has been studied in countries where deposit and credit dollarization occur, particularly in emerging economies, as discussed by Chirstiano, Dalgic, and Nurbekyan (2022), Drenik, Kirpalani, and Perez (2021), and Dalgic (2024). These studies argue that depositors in such countries hold foreign currency (typically the US dollar) as insurance against income risk during recessions, when their local currencies tend to devalue against the US dollar.

Moreover, a large body of literature, including works by Gourinchas, Rey, and Govillot (2017), Obstfeld, Shambaugh, and Taylor (2010), and Maggiori (2017), emphasizes the hedging role of currencies, particularly the US dollar, in response to both global and national aggregate shocks. This literature helps explain the large demand for the US dollar as both a private and public store of value globally.

To measure sellers' hedging motives, I use its connection to country 1's nominal GDP in the model, evaluated with currency 1 as the numeraire[6]:

$$\begin{aligned} \textit{Nominal GDP}_i = & \underbrace{\lambda^s_{1,1}\left(\frac{d_1^{1,1}}{\phi_1}+\frac{d_2^{1,1}}{\phi_2}\right)}_{\textit{Value added in the DM meeting with domestic buyers}} \\ & + \underbrace{\lambda^s_{1,2}\left(\frac{d_1^{2,1}}{\phi_1}+\frac{d_2^{2,1}}{\phi_2}\right)}_{\textit{Value added in the DM meeting with foreign buyers}} \\ & + \underbrace{\frac{2C^{b,1}}{\phi_{1,t}}}_{\textit{Young and old buyers' CM consumption}} + \underbrace{\frac{C^{s,1}(\theta^{s,1})}{\phi_{1,t}}}_{\textit{Old sellers' CM consumption}} . \end{aligned} \tag{27}$$

Note that only the last term involving old sellers' CM consumption is stochastic in this economy. The real GDP is defined as *Real GDP* $\equiv$ *Nominal GDP* $\times \phi_i$ and the real GDP growth is given by

$$\begin{aligned} \Delta \log RGDP_{i,t+1} \equiv \log RGDP_{i,t+1} - \log RGDP_{i,t} &\approx \frac{RGDP_{i,t+1}-RGDP_{i,t}}{RGDP_{i,t}} \\ &= \frac{C^{s,i}(\theta^{s,i}_{t+1})-C^{s,i}(\theta^{s,i}_{t})}{C^{s,i}(\theta^{s,i}_{t})} \approx \log C^{s,i}(\theta^{s,i}_{t+1}) - \log C^{s,i}(\theta^{s,i}_{t}) \\ &= -\frac{1}{\gamma}(\log\theta^{s,i}_{t+1}-\log\theta^{s,i}_{t}), \end{aligned} \tag{28}$$

[6]In the rest of this this paper, the time subscript will not explicitly indicated unless they are noted.

where the last equality follows from the assumption that the utility function is CRRA. Noting that the depreciation rate of currency i against currency j is $\Delta e_{ji,t+1} \equiv \log e_{ji,t+1} - \log e_{ji,t} = \log \gamma_{i,t+1} - \log \gamma_{j,t+1}$, the covariance between the real GDP growth rate and local currency depreciation rate against the US dollar is given by

$$Cov(\log RGDP_{i,t+1}, \Delta \log e_{USDi,t+1}) = \frac{1}{\gamma} Cov\bigg(\log \theta^{s,i}_{t+1}, \log \gamma_{i,t+1} - \log \gamma_{j,t+1} \bigg), \tag{29}$$

where $\log \theta^{s,i}_t$ drops out since it is orthogonal to the future inflation rates $\log \gamma_{c,t+1}$. This equation shows that the measurable covariance between real GDP growth and the currency depreciation rate serves as a good proxy for the relative hedging property of the local currency compared to the US dollar against the sellers' income shock. A positive covariance implies that the local currency provides better hedging against income shocks than the US dollar, which translates to a lower transfer cost for the currency in decentralized markets (DM).

I use the export/import settlement and invoicing currency share dataset from Boz et al. (2020) as proxies for the medium of exchange in international trade in the model. While it would be ideal to separate the settlement and invoicing currencies in the data, Goldberg and Tille (2008) document that the currency used in trade invoicing is typically the same as the one used for actual payments. Therefore, this approach remains consistent with the model.

As a proxy for the currency portfolio shares of buyers, I employ the extended dataset of deposit dollarization from Levy-Yeyati (2006), where the deposit dollarization for country i and year t, as

$$\textit{deposit dollarization}_{i,t} \equiv \frac{\textit{value of foreign currency deposits held by domestic residents in domestic banks}}{\textit{total deposits held by domestic residents in domestic banks}},$$

where both the numerator and denominator are expressed in local currency units. I follow the literature (see Levy-Yeyati (2006) and Chirstiano, Dalgic, and Nurbekyan (2022)) in referring to 'foreign currency' as the dollar.

Finally, the trade share data from the OECD Inter-Country Input-Output (ICIO) table are used as proxies for the meeting probabilities with US sellers in the DM (Decentralized

Market) in the model.

The merged dataset spans annual data from 1995 to 2009, with the number of countries depending on the availability of each data series at any given time. I compute the time-series average for each variable of dollarization indices to construct the cross-country dataset, following the methodology outlined in the literature Levy-Yeyati (2006), Chirstiano, Dalgic, and Nurbekyan (2022), and Dalgic (2024). The average inflation and trade flows with the US is simply a time-series average of inflation rates and of export to and import from the US. The correlations between real GDP growth rate and local currency depreciation rate are computed by using the entire annual time series between 1995 and 2009.

5.2 Econometric specification and results

The econometric specification in this paper follows Levy-Yeyati (2006) and Chirstiano, Dalgic, and Nurbekyan (2022):

$$\begin{aligned} \textit{Dollarization index}_i = \alpha + \beta_1 \cdot \underbrace{corr(\Delta \log \textit{Real GDP}_i, \overbrace{\Delta \log e_{US,i}}^{\textit{Local currency depreciation rate against dollar}})}_{\textit{Exchange rate cyclicality}} + \\ + \beta_2 \cdot \underbrace{\gamma_i}_{\textit{Average local inflation rate}} + \beta_3 \cdot \textit{US import share}_i + \beta_4 \cdot \textit{US export share}_i + u_i, \end{aligned}$$

where u_i represents the regression residual.

Table 2 presents the regression results, which align with the theoretical predictions. Consistent with the empirical findings of Levy-Yeyati (2006) and Chirstiano, Dalgic, and Nurbekyan (2022), the second column demonstrates that more procyclical local exchange rates against the US dollar are significantly associated with higher degrees of deposit dollarization ($\beta_1 < 0$). The average local inflation rate also predicts higher dollarization ($\beta_2 > 0$), which is consistent with the theory. However, as empirically documented by Levy-Yeyati (2006) and Chirstiano, Dalgic, and Nurbekyan (2022), once additional controls are introduced, the average inflation rate becomes statistically insignificant.

The third and last columns reveal novel empirical patterns for international settlement

Variable	deposit dollarization	import USD share	export USD share
Intercept	0.045	0.325***	0.284***
$corr(\Delta \log GDP_i, \Delta \log e_{US,i})$	−0.408***	−0.258**	−0.378**
Average inflation rate γ_i	1.54***	0.821	1.506**
US import share$_i$	-0.927	1.41***	-
US export share$_i$	1.146	-	1.518***
R-squared	0.4016	0.4806	0.4631
Adj. R-squared	0.3095	0.4207	0.4012

Table 2: Empirical results

Notes: * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$. The definitions and measurements of variables are described in section 5.1. The standard errors are computed by the standard Newey-West estimators.

currencies, consistent with the theory. More procyclical exchange rates of local currencies against the US dollar predict higher shares of US dollar invoicing/settlement in international trade ($\beta_1 < 0$). Additionally, larger trade flows with the U.S. contribute to higher US dollar shares, aligning with the theoretical prediction that if the U.S. agents prefer to use the US dollar as a medium of exchange as they do indeed in reality, increased trade with U.S. buyers and sellers will raise the aggregate US dollar share ($\beta_3 > 0$, $\beta_4 > 0$). On the other hand, higher local average inflation does not significantly increase the US dollar share in international trade settlement.

These empirical findings for both deposit and international trade settlement patterns suggest that the riskiness of local currencies may also drive the choice of medium of exchange, aligning with the theory presented in this paper, rather than solely serving as stores of value, as explored in the existing literature (e.g., Levy-Yeyati (2006), Chirstiano, Dalgic, and Nurbekyan (2022), and Dalgic (2024)). These studies focus on both the positive and normative implications of deposit dollarization in emerging countries, viewing it as a store of value and an intra-country redistributive instrument during economic crises. They emphasize the welfare cost of deposit de-dollarization, which removes better insurance mechanisms within countries, and also empirically argue that although the high riskiness of local currencies is driven by economic crises, intra-national deposit dollarization does not systematically cause such crisis episodes.

This paper extends that analysis by highlighting the welfare benefits of deposit dollarization through improved terms of trade due to the superior medium of exchange. Specifically, foreign currency holding in deposits facilitates the use of a more effective medium of exchange with better hedging properties. Indeed, Section 4.2 demonstrates that welfare-maximizing monetary policy should focus on enhancing the insurance properties of national currencies against such shocks, rather than entirely restricting deposit dollarization. In the next section, I quantify the welfare impact of this channel in a quantitative model, calibrated to match the empirical insurance properties of alternative currencies and to reflect the key features of the international monetary system.

6 Quantitative Analysis

In this section, I extend the baseline model to a three-country open economy framework to quantitatively assess the model's ability to replicate the trade settlment and currency holding patterns observed in the data. Specifically, I calibrate the model to match the business cycle moments and empirical currency return properties of three regions: the U.S., the Eurozone, and Latin America. I then evaluate how well the calibrated model replicates observed currency choices in international trade. Furthermore, I offer an interpretation through the model of why the U.S. dollar is widely used in international trade, even in transactions where the U.S. is not directly involved.

Building on this quantitative model, I quantify the welfare gains for the global economy from having a safe international currency, such as the U.S. dollar. This is done by comparing welfare outcomes with a counterfactual scenario in which the Latin American region implements deposit de-dollarization policies, banning dollar deposits. The welfare differences are expressed in terms of annual consumption-equivalent terms. According to Proposition 5,

Parameter	Value	Source	Description
n_{US}	0.393	US GDP share	US country size
n_{EU}	0.408	Euro area GDP share	Euro area size
n_{LM}	0.198	LM GDP share	LM region size
$\lambda_{US,EU}$	0.143	Import share of US from EU	US buyer meeting prob. w. EU seller
$\lambda_{US,LM}$	0.173	Import share of US from LM	US buyer meeting prob. w. LM seller
$\lambda_{EU,US}$	0.092	Import share of EU from US	EU buyer meeting prob. w. US seller
$\lambda_{EU,LM}$	0.023	Import share of EU from LM	EU buyer meeting prob. w. LM seller
$\lambda_{LM,US}$	0.377	Import share of LM from US	LM buyer meeting prob. w. US seller
$\lambda_{US,LM}$	0.136	Import share of LM from EU	LM buyer meeting prob. w. EU seller
$\mathbb{E}(\gamma^{-1}_{dollar})$	0.9750	Average reciprocal of inflation in US	USD annual return
$\mathbb{E}(\gamma^{-1}_{euro})$	0.9754	Average reciprocal of inflation in EU	Euro annual return
$\mathbb{E}(\gamma^{-1}_{peso})$	0.9397	Average reciprocal of inflation in LM	LM annual return

Table 3: Calibrated parameters

Parameters	Value	Source	Description
γ	10	Mean GDP share of US seigniorage (0.1-0.2%)	Relative risk aversion parameter
$\epsilon^{\cdot,US}_{dollar}$	0.9992	$\gamma \cdot Cov(\Delta \log RGDP_{US,t}, \log \gamma^{-1}_{dollar,t})$	Transfer cost of USD to US seller
$\epsilon^{\cdot,US}_{euro}$	1.0063	$\gamma \cdot Cov(\Delta \log RGDP_{US,t}, \log \gamma^{-1}_{euro,t})$	Transfer cost of Euro to US seller
$\epsilon^{\cdot,US}_{peso}$	1	$\gamma \cdot Cov(\Delta \log RGDP_{US,t}, \log \gamma^{-1}_{peso,t})$	Transfer cost of peso to US seller
$\epsilon^{\cdot,EU}_{dollar}$	1.0027	$\gamma \cdot Cov(\Delta \log RGDP_{EU,t}, \log \gamma^{-1}_{dollar,t})$	Transfer cost of USD to EU seller
$\epsilon^{\cdot,EU}_{euro}$	0.9992	$\gamma \cdot Cov(\Delta \log RGDP_{EU,t}, \log \gamma^{-1}_{euro,t})$	Transfer cost of euro to EU seller
$\epsilon^{\cdot,EU}_{peso}$	1.0022	$\gamma \cdot Cov(\Delta \log RGDP_{EU,t}, \log \gamma^{-1}_{peso,t})$	Transfer cost of peso to EU seller
$\epsilon^{\cdot,LM}_{dollar}$	0.9768	$\gamma \cdot Cov(\Delta \log RGDP_{LM,t}, \log \gamma^{-1}_{dollar,t})$	Transfer cost of USD to LM seller
$\epsilon^{\cdot,LM}_{euro}$	0.9836	$\gamma \cdot Cov(\Delta \log RGDP_{LM,t}, \log \gamma^{-1}_{euro,t})$	Transfer cost of euro to LM seller
$\epsilon^{\cdot,LM}_{peso}$	09971	$\gamma \cdot Cov(\Delta \log RGDP_{LM,t}, \log \gamma^{-1}_{peso,t})$	Transfer cost of peso to LM seller

Table 4: Calibrated transfer costs of currencies in DM

the superior safety of the U.S. dollar in the form of hedging property influences a country's welfare through seigniorage transfers between countries and the improved terms of trade resulting from liquidity provision to citizens.

6.1 Calibration

To calibrate the model, the global economy is divided into three trading blocs: the United States, the Eurozone, and Latin America. After discussing the parameters that are conventionally calibrated in the existing literature, I outline the calibration procedure for the insurance properties of the three alternative currencies, which are directly estimated from the data. I then evaluate how well the model replicates the currency usage patterns observed in the data. All data used are on an annual basis, covering the period from 2000 to 2009, unless otherwise specified. This time frame was chosen based on the availability of deposit dollarization data and euro exchange rate data.

The functional forms for the utility function is the standard CRRA utility function $U(c) = u(c) = c^{1-\gamma}/(1-\gamma)$. where the constant relative risk aversion (CRRA) parameter is calibrated to match the share of seigniorage revenue in the U.S. GDP $0.1\% - 0.2\%$ as reported by Portes and Rey (2002) and Goldberg (2011). The US seigniorage revenue is increasing in relative risk aversion parameter γ since higher risk aversions increase the demand for the US dollar as the superior hedging currency. The discount factor is set to $\beta = 0.966$, consistent with an annual real interest rate of 3.5%. The marginal labor utility cost for buyers is unity following the literature Lagos and Wright (2005).

Since the model assumes that gross money growth rates are equal to gross inflation rates in a stationary equilibrium, the expected inflation rates $\mathbb{E}(\gamma_{US}^{-1}), \mathbb{E}(\gamma_{EU}^{-1})$, and $\mathbb{E}(\gamma_{LM}^{-1})$ are set to the inverse of the average annual inflation rates for the period 1999 to 2009. The inflation rates are approximately 0.9750 for the U.S., 0.9754 for the Eurozone, and 0.9397 for the Latin American region, using data from the World Bank.[7]

The parameters governing the Eurozone and Latin American regions are computed as real GDP-weighted averages of member countries' values. The euro delivers the highest currency return, followed by the dollar, and then the Latin American peso by a wide margin. This ranking implies that, in the absence of insurance motives, the euro would be the most preferable store of value, and thus the preferred medium of exchange.

The next set of parameters concerns the model's meeting parameters for each country pair, denoted as $\lambda_{j,i}$. These six international meeting parameters, $\lambda_{j,i}$, are calibrated using bilateral trade data for the period between 2000 and 2009 from the OECD Inter-Country Input-Output (ICIO) table. The share of each region's population is determined by its share

[7] Note that the U.S. inflation rate is more conservative in this paper, as it spans data up to 2009, whereas Zhang (2014) includes data only up to 2005.

of Real GDP. The size of each region n_j is the real GDP share in the three regions in this economy.

The final set of parameters involves the costs associated with transferring a currency in the decentralized market (DM), denoted as $\epsilon_c^{j,i}$. This requires the measurement of the covariance between labor earning risk and currency returns, $Cov(\theta_{t+1}^{s,i}, -\log\gamma_{c,t+1})$. They[8] are calibrated using data on foreign exchange rate growth, inflation rates and real GDPs. The measurement of $\log\theta_{t+1}^{s,i}$ from the real GDP data follows the procedure outlined in Section 5.1. Both $\theta_{i,t+1}^{s,i}$ and $\gamma_{c,t+1}$ are assumed to be jointly log-normally distributed.

To measure the currency returns $\gamma_{j,t+1}^{-1}$ for sellers in country i, I use the following approach: when $j \neq i$, the currency return is given by $\gamma_{j,t+1}^{-1} = \exp(\log\Delta e_{ji,t+1} - \log\gamma_{i,t+1})$, where $\gamma_{i,t+1}$ represents the inflation rate of country i. For the domestic currency, the return is simply the inverse of the inflation rate. This measurement approach is motivated by the fact that foreign currency returns (or appreciation rates) tend to deviate from the foreign inflation rates, as is well known in the international macroeconomics and finance literature. Therefore, foreign currency returns based on foreign inflation rates would not provide precise estimates of the returns that domestic agents can actually obtain.

Table 3 and Table 4 summarize the baseline calibration results for the three-region model. This calibration indicates that the US dollar is the most effective as an insurance currency; however, it is dominated by the euro in terms of expected currency return.

6.2 Quantitative results and welfare benefits of de-dollarization policy

Table 5 shows the quantitative implications of the extended model, compared with the data. The US dollar is used as the medium of exchange and is held by agents in both the U.S. and Latin America. This captures the phenomenon of dollarization common in many Latin

[8] It is important to note that, in the model, buyers are assumed to be identical, regardless of their country of origin. Consequently, there remain only nine $\epsilon_c^{j,i}$'s to calibrate.

	Description	Model	Data
US	USD deposit share	1	-
	USD share in imports	1	0.94
	USD share of EU import	1	-
	USD share of LM import	1	-
	Seigniorage in RGDP	0.11 %	0.1-0.2%
EU	USD deposit share	0	0.036
	Euro deposit share	1	-
	Euro share in imports	1	0.7
	Euro share of US import	1	-
	Euro share of LM import	1	-
LM	USD deposit share	1	0.3
	USD share in LM transactions	1	-
	USD share in imports	1	0.97
	USD share of US import	1	-
	USD share of EU import	1	-

Table 5: Quantitative results

American and Eastern European economies. On the other hand, the euro functions as the local currency, being used exclusively within the Eurozone, despite the fact that the expected return for euro is higher than the dollar. The dollar dominance comes from its superior insurance property, consistent with the argument by Ito and Chinn (2015) that the flight-to-quality effect is the key driver of surge of the US dollar as international medium of exchange. Rationalizing the observation by Gourinchas, Rey, and Sauzet (2019) and Boz et al. (2020), these model implications align with the key empirical patterns observed in the current international monetary system, where the US dollar remains the dominant global currency, while the euro continues to serve as the regional currency used mainly in transactions involving Eurozone agents.

The third row in Table 6 provides more detailed patterns of currency usage in the baseline scenario. Notably, the US dollar is used as the settlement currency in international trade between the Eurozone and Latin America, despite the fact that the U.S. is not directly involved in this trade. In this sense, the US dollar functions as a vehicle currency in the baseline economy, consistent with the findings by Gourinchas, Rey, and Sauzet (2019) and

Country	Description	Baseline	Counterfactual
US	USD deposit share	1	1
	USD share in US transactions	1	1
	USD share of EU import	1	1
	USD share of LM import	1	1
EU	Euro deposit share	1	1
	Euro share in EU transactions	1	1
	Euro share of US import	1	1
	Euro share of LM import	1	1
LM	USD deposit share	1	0
	USD share in LM transactions	1	0
	USD share of US import	1	0
	USD share of EU import	1	0
	Peso deposit share	0	1
	Peso share in LM transactions	0	1
	Peso share of US import	0	1
	Peso share of EU import	0	1

Table 6: Quantitative results

Boz et al. (2020).

These results highlight the global role of the US dollar as the primary international medium of exchange. Importantly, it is the superior insurance properties of the US dollar that drive this status, rather than its lower inflation or the coincidence of multiple equilibria through coordination. The model suggests that the observed insurance/hedging properties might be yet explain an yet ignored but crucial drivers of the dominance of the US dollar as the international medium of exchange in the global monetary system.

The fourth column in Table 7 compares the baseline scenario with a counterfactual scenario where agents in the Latin America region are restricted to using the Latin America peso in domestic DM transactions. This counterfactual de-dollarization would simultaneously lead to the de-dollarization of both deposits and international mediums of exchange.

Finally, Table 7 compares the welfare outcomes between the baseline and counterfactual scenarios, based on the welfare decomposition outlined in Section 4.2. Under the counterfactual scenario, global welfare would decline, U.S. agents would experience a welfare loss, and Latin American agents would enjoy welfare gains. U.S. agents would lose welfare due to the

Country	Description	BL: $\gamma = 10$ ($\gamma = 20$)	CF: $\gamma = 10$ ($\gamma = 20$)	C.E.: $\gamma = 10$ ($\gamma = 20$)
Global	Total	-1.1072	-1.1093	
US	Total	-0.4307	-0.4356	-0.31 %
	Gains from Trade	-1.1071	-1.1071	
	Foreign currency holding cost	0	0	
	Seigniorage revenue	0.0049	0	
EU	Total	-0.4537	-0.4537	0
	Gains from Trade	-0.4537	-0.4537	
	Foreign currency holding cost	0	0	
	Seigniorage revenue	0	0	
LM	Total	-1.1151	-1.1105	0.35 % (0.11%)
	Gains from Trade	-1.1002 (-1.0306)	-1.1102 (-1.0507)	
	Foreign currency holding cost	-0.0049	0	
	Seigniorage revenue	0	0	

Table 7: Welfare quantification

Notes: BL (CF) in the first row represent the baseline (counterfactual) results. The numbers in parentheses are results from the case with a higher risk aversion $\gamma = 20$. C.E. in the first row means the annual consumption equivalent compensation for the transition from the baseline equilibrium to the counterfactual equilibrium.

reduction in seigniorage revenue, as the U.S. dollar would no longer serve as the international medium of exchange and would not be held by Latin American buyers.

On the other hand, Latin American agents would benefit from the elimination of the inflation tax paid to U.S. agents due to de-dollarization. However, they would still bear the welfare cost of de-dollarization through worse terms of trade in the DM. Importantly, as agents become more risk-averse, this welfare cost would grow larger, limiting the welfare gains from the de-dollarization policy, as shown in the parenthesis, where the risk aversion parameter is increased from 10 in the baseline to 20.

Finally, I compute the annual equivalent consumption compensation for the transition from the baseline economy to the counterfactual economy with de-dollarization in Latin America, following the approach of Lucas (1982). Let $\mathcal{W}_j$ represent the annual utility of the buyers and sellers in region j in the baseline, and $\mathcal{W}_j^c(\Delta_j)$ represent the annual utility in the counterfactual economy, where consumption in both the CM and DM is multiplied by $\Delta_j > 0$. I report the value of $\Delta_j - 1\%$ in the last column of Table 7, such that the baseline utility level becomes equal to that in the counterfactual economy with consumption compensation, i.e., $\mathcal{W}_j = \mathcal{W}_j^c(\Delta_j)$. If $\Delta_j - 1 > 0$, the region's welfare improves by transitioning from the baseline to the counterfactual economy. Conversely, if $\Delta_j - 1 < 0$, the region experiences a welfare loss due to the transition. [9]

The results indicate that the U.S. incurs a welfare loss from the de-dollarization policy, while Latin America benefits from this transition. These welfare changes are similar in magnitude to those observed for long-run inflation welfare costs and benefits in monetary models, such as Zhang (2014). However, the welfare effects in this paper are derived from the second moment properties of currency returns, rather than the first moment. This suggests that the insurance/hedging channel may play a significant role when considering the welfare effects of inflation in monetary models.

Existing studies on the welfare implications of deposit de-dollarization policies have primarily focused on limited risk-sharing, seigniorage, and original sin via stores of value, often overlooking the terms of trade channel (see, e.g., Bocola and Lorenzoni (2020), Chirstiano, Dalgic, and Nurbekyan (2022), Oskolkov and Sorá (2023), Dalgic (2024)). This paper contributes to the discussion by highlighting the additional welfare costs affecting the availability of prefered medium of exchange , both locally and globally, in the context of de-dollarization policies.

7 Conclusion

In this paper, I examined both the positive and normative implications of the hedging properties of currencies for their role as international mediums of exchange and as assets held across countries. Theoretically, currencies with stronger hedging properties—i.e., safer currencies—are more likely to be preferred as mediums of exchange and dominate currency holdings, which aligns with novel empirical evidence. The quantitative version of the model, calibrated to exchange rate cyclicality, is able to account for observed empirical patterns in

[9]The derivation of the equivalent consumption compensation Δ_j partly relies on the assumption that $\theta_{t+1}^{s,i,t}$ is log-normally distributed.

both the international use of currencies and currency holdings in deposits.

Furthermore, the model predicts that the insurance properties of currencies play a significant role. This suggests that de-dollarization policies in emerging countries could lead to substantial welfare gains and losses. These outcomes would primarily arise from seigniorage changes and shifts in the terms of trade, as removing the preferred medium of exchange—such as the US dollar—has far-reaching consequences. The welfare cost from deteriorating terms of trade could be considerable, potentially offsetting the large welfare gains from a reduced inflation tax, particularly in cases where agents in emerging countries are highly risk-averse.

The framework developed here could be further extended by incorporating alternative financial assets, such as government bonds, as well as more general forms of preferences and uncertainty. Such extensions would allow for a joint analysis of risk and liquidity premia in both currency and bond markets, providing a framework for the analysis of various polices such as quantatitive easing and foreign exchange interventions. I leave these extensions for future work.

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Online Appendix Not For Publication

A Proofs

In the Appendix, I drop the time subscript of random variables since they are iid and the conditional information is not relevant.

A.1 Proof of Proposition 1

Proof. The proof relies on the standard Kuhn-Tucker condition. Since the obtaining an additional unit of the DM consumption by using either currencies is identical, the first-order necessary and sufficient optimality condition is given by

$$\epsilon_c \equiv \epsilon \geq u'(c) - \lambda_c, \tag{A.1}$$

coupled with the nonnegative currency transfer condition $d_c \geq 0$, the feasibility condition $d_c \leq q_c$ for all $c \in \{1,2\}$, and complementary slackness conditions. Define a consumption level $c(\epsilon)$ such that the marginal utility is equal to ϵ, i.e. $u'(c(\epsilon)) = \epsilon$. Consider two cases: (i) $c(\epsilon)$ is budget feasible and (ii) $c(\epsilon)$ is not budget feasible.

(i) $\underline{v_1^s q_1 + v_2^s q_2 \geq c(\epsilon)}$ The terms of trade involve the consumption $c = c(\epsilon)$ and the currency portfolio transfer $v_1^s d_1 + v_2^s d_2 \geq c(\epsilon)$. To see this, set the Lagrange multiplier $\lambda_c = 0$. The definition of $c(\epsilon)$ implies that the first-order condition in eq.(A.1) holds with equality. Moreover, the the currency portfolio transfer (d_1, d_2) satisfies the feasibility condition and can be structured to satisfy the nonnegative amount of each currency transfer.

(ii) $\underline{v_1^s q_1 + v_2^s q_2 < c(\epsilon)}$ The terms of trade in this case are given by the consumption $c = v_1^s d_1 + v_2^s d_2$ and $d_c = q_c$ for all $c \in \{1,2\}$. In words, the buyer gives all the currency holdings to the seller to get the maximum level of consumption. To see this, set λ_c's in eq.(A.1) such that the first-order condition in eq.(A.1) holds with equality. Moreover, the currency portfolio transfer satisfies the nonnegativity and feasibility. □

A.2 Proof of Proposition 2

Proof. The proof utilizes relies on the standard Kuhn-Tucker condition. Anticipating the extension of the model in section 6, I provide the proof of three-currency version of the model.

Without loss of generality, assume that $\epsilon_1 < \epsilon_2 < \epsilon_3$ so that the currency 1 is the best medium of exchange, followed by currency 2 and then currency 3. This assumption implies that $c(\epsilon_3) < c(\epsilon_2) < c(\epsilon_1)$ since each ϵ represents the marginal cost of obtaining an additional unit of consumption by trasferring a currency. Define $q_1(\epsilon_1)$ be the real balance of currency 1 such that $v_1^s q_1(\epsilon_1) = c(\epsilon_1)$, i.e. the real balance holding of currency 1 enough to cover the consumption expense at the level of $c(\epsilon_1)$. Analogously, define the real balance holding of currency 1 enough to expense $c(\epsilon_2)$, i.e. $v_1^s q_1(\epsilon_2) = c(\epsilon_2)$. Obviously, $q_1(\epsilon_2) < q_1(\epsilon_2)$ since $c(\epsilon_2) < c(\epsilon_1)$.

There are three different first-order conditions: for $c \in \{1, 2, 3\}$,

$$\epsilon_c \geq u'(c) - \lambda_c. \tag{A.2}$$

I consider three distinct cases depending on the buyer's currency holding.

(i) $\underline{q_1 \geq q_1(\epsilon_1)}$ This case describes the situation where the buyer holds an adequate amount of currency 1 to cover all the necessary consumption expenditure solely with that preferrd medium of exchange. The terms of trade are given by the consumption $c = c(\epsilon_1)$ and currency transfer $d_1 = c(\epsilon_1)/v_1^s$, $d_2 = d_3 = 0$, implying that the buyer only uses the most preferred medium of exchange, currency 1 to purchase the sufficient amount of consumption. By setting $\lambda_1 = 0$, the first-order condition (A.2) for currency 1 holds with equality. The first-order conditions for currency 2 and 3 hold with inequality by setting $\lambda_c = 0$ for $c \in \{2, 3\}$ since the marginal cost of transferring those currencies is strictly larger than the marginal utility of consumption. Moreover, the currency portfolio transfer is budget feasible and trivially nonnegative.

(ii) $\underline{q_1(\epsilon_2) < q_1 < q_1(\epsilon_1)}$ This subcase corresponds to the scenario where the buyer does not hold enough amount of currency 1 to buy $c(\epsilon_1)$ with that currency but still adequate to purchase $c(\epsilon)$ amount of consumption without using other currencies. In this case, the terms of trade involve the consumption level $c = v_1^s q_1$ and the currency portfolio transfer $d_1 = q_1$, $d_2 = d_3 = 0$. To see this, set $\lambda_1 = u'(c) - \epsilon_1 > 0$ and $\lambda_2 = \lambda_3 = 0$, implying that the currency 1 transfer is now constrained. Then the first order condition (A.2) for currency 1 holds with equality while those for currency 2 and 3 hold with inequality. The currency transfers are budget feasible and nonnegative.

(iii) $\underline{q_1 < q_1(\epsilon_2) \text{ and } v_1^s q_1 + v_2 q_2 \geq c(\epsilon_2)}$ This describes the case where the buyer holds enough amount of currency 1 and 2 to cover the consumption expense at the level of $c(\epsilon_2)$ without using currency 3, the worst medium of exchange. The solution is the consumption $c = c(\epsilon_2)$ and currency transfers $d_1 = q_1$, $d_2 = (c(\epsilon_2) - v_1^s q_1)/v_2^s$, and $d_3 = 0$. To confirm this is actually the solution, set $\lambda_1 = u'(c) - \epsilon_1 > 0$ and $\lambda_2 = \lambda_3 = 0$. Then the first-order conditions (A.2) hold with equality for currency 1 and 2, and with inequality for currency 3. The currency transfer is nonnegative and budget-feasible.

(iv) $\underline{c(\epsilon_3) \leq v_1^s q_1 + v_2^s q_2 < c(\epsilon_2)}$ This corresponds to the situtation where the currency 1 and 2 holdings are not enough to obtain $c(\epsilon_2)$ units of consumption but adequate to expense $c(\epsilon_3)$ units of consumption. The terms of trade in this case are characterized by the consumption $c = v_1^s q_1 + v_2^s q_2$ and the currency transfer $d_1 = q_1$, $d_2 = q_2$, and $d_3 = 0$. To check if this is actually the solution, set $\lambda_c = u'(c) - \epsilon_c$ for currency $c \in \{1, 2\}$ and $\lambda_3 = 0$. Then the first order conditions (A.2) hold for currency 1 and 2 with equality while it holds for currency 3 with inequality.

(v) $\underline{v_1^s q_1 + v_2^s q_2 < c(\epsilon_3) \text{ and } c(\epsilon_3) \leq v_1^s q_1 + v_2^s q_2 + v_3 q_3}$ This is the case where the holdings of currency 1 and 2 are not sufficient to obtain $c(\epsilon_3)$ but the currency 3 holding can cover the rest of the expense. The terms of trade are provided by $c = c(\epsilon_3)$ and the currency transfer $d_1 = q_1$, $d_2 = q_2$, and d_3 such that $c(\epsilon_3) = v_1^s q_1 + v_2^s q_2 + v_3 d_3$. This is nonnegative and budget-feasible. To check if this satisfies the first-order condition (A.2), set $\lambda_c = u'(c) - \epsilon_c$ for currency $c \in \{1, 2\}$ and $\lambda_3 = 0$. Then the first order conditions hold for currency 1 and 2 with equality while it holds for currency 3 with inequality.

(vi) $\underline{v_1^s q_1 + v_2^s q_2 + v_3 q_3 < c(\epsilon_3)}$ The terms of trade in this case corresponds to $c = v_1^s q_1 + v_2^s q_2 + v_3 q_3$ and currency transfers $d_c = q_c$ for all $c \in \{1, 2\}$. The first-order conditions (A.2) hold with equality for all currencies by setting $\lambda_c = u'(c) - \epsilon_c$. □

A.3 Proof of Proposition 4

Proof. Following the proof of proposition 2 in appendix A.2, consider six different cases to compute $\partial_{q_{c,t}^{b,j,t}} V_{t+1}^{b,j,t}$. I denote $\partial_{q_{c,t}^{b,j,t}} V_{t+1}^{b,j,t} = \partial_c V^b$.

(i) $\underline{v_1^s q_1 + v_2^s q_2 \geq c(\epsilon)}$ In this case, the buyer never spend the additional unit of currency 1 in the DM and carry over to the next CM. Consequently, $\partial_1 V^b = v_1^b = (v_1^b / v_1^s) v_1^s = u'(c) v_1^s$, where the last equality comes from the first-order condition with respect to currency 1 (A.2). Moreover, $\partial_c V^b = v_c^b$, since those currencies will never be used in this current case and be carried over to the next CM.

(ii) $\underline{q_1(\epsilon_2) < q_1 < q_1(\epsilon_1)}$ The buyer uses the additional holding of currency 1 to obtain v_1^s units of consumption goods, yielding $\partial_1 V^b = u'(c) v_1^s$. Moreover, $\partial_c V^b = v_c^b$ for currency $c \in \{2, 3\}$ since those currencies will never be used in this current case and be carried over to the next CM.

(iii) $\underline{q_1 < q_1(\epsilon_2) \text{ and } v_1^s q_1 + v_2 q_2 \geq c(\epsilon_2)}$ In this case, the buyer uses the additional holding of currency 1 and reduce the transfer of currency 2 by v_1^s / v_2^s, which is the seller's exchange rate between currency 1 and 2, while maintaining the DM consumption at the same level. This yields $\partial_1 V^b = v_2^b (v_1^s / v_2^s) = u'(c) v_1^s$, where the last equality follows from the first-order condition with respect to currency 2 (A.2). Since the additional holding of currency 2 and 3 will never be used in this case, $\partial_c V^b = v_c^b$ for currency $c \in \{2, 3\}$.

(iv) $\underline{c(\epsilon_3) \leq v_1^s q_1 + v_2^s q_2 < c(\epsilon_2)}$ The buyer spends the additional holding of currency 1 to obtain additional v_1^s units of DM goods, yielding $\partial_1 V^b = u'(c)v_1^s$. Moreover, the buyer uses the additional holding of currency 2 to obtain the additional v_2^s units of DM goods, yielding $\partial_2 V^b = u'(c)v_2^s$. On the other hand, he will never spend the additional holding of currency 3 so that $\partial_3 V^b = v_3^b$.

(v) $\underline{v_1^s q_1 + v_2^s q_2 < c(\epsilon_3) \text{ and } c(\epsilon_3) \leq v_1^s q_1 + v_2^s q_2 + v_3 q_3}$ In this case, the buyer uses the additional holding of currency 1 or 2 and reduce the transfer of currency 3 by v_1^s/v_3^s or v_2^s/v_3^s, which is the seller's exchange rate between currency 1 (or 2) and 3, while maintaining the DM consumption at the same level. This yields $\partial_c V^b = v_3^b(vc1^s/v_3^s) = u'(c)v_c^s$ for currency $c \in \{1, 2\}$, where the last equality follows from the first order condition with respect to currency 3 in (A.2). Finally, the buyer will never spend the additional holding of currency 3 so that $\partial_3 V^b = v_3^b$.

(vi) $\underline{v_1^s q_1 + v_2^s q_2 + v_3 q_3 < c(\epsilon_3)}$ The buyer spends the additional holding of currency 1, 2, or 3 to obtain the additional v_c^s units of DM goods, yielding $\partial_c V^b = u'(c)v_c^s$, $c \in \{1, 2, 3\}$.

Using these results, I can rewrite the first-order condition with respect to currency $c \in \{1, 2, 3\}$ in (23) as

$$\theta^{b,j} \geq \beta \left[\sum_i \lambda_{j,i} \partial_{q_1} V_i^b(\mathbf{q}) + \left(1 - \sum_i \lambda_{j,i}\right) v_1^b \right] = \beta v_1^b \sum_i \lambda_{j,i} \left[1 + \max\left\{0, \frac{1}{\epsilon_1^{j,i}} u'(c) - 1\right\}\right]$$
$$= \beta v_1^b \left[1 + \sum_i \lambda_{j,i} l_1^{j,i}(\mathbf{q}) \right]. \tag{A.3}$$

□

A.4 Proof of Proposition 5

Proof. I start with expressing the government lump-sum transfer in the first line as

$$T_{j,t} = (1-\gamma_{j,t}^{-1})\phi_{j,t-1}M_{j,t-1} = (1-\gamma_{j,t}^{-1}).\sum_c q_{j,t-1}^{b,c,t-1}, \tag{A.4}$$

where the last equality uses the market clearing condition in period $t-1$ for the real balance of currency c: $\phi_{j,t-1}M_{j,t-1} = \sum_c q_{j,t-1}^{b,c,t-1}$. I am interested in how the period-t inflation rate $\gamma_{c,t}$ affects the country-j's total welfare. Since the buyer makes TIOLI offer in the DM, the sellers' welfare does not change in the inflation rates.

The young buyer's lifetime value in period t can be decomposed as

$$\begin{aligned}
W_t^{b,j,t} &= U(C^{**}(\theta^{b,j})) - \theta^{b,j}C^{**}(\theta^{b,j}) + \theta^{b,j}T_{j,t} - \sum_c \theta^{b,j}q_{c,t}^{b,j,t} + \beta\sum_i \lambda_{j,i}V_{i,t+1}^{b,j,t}(\mathbf{q}_t^{b,j,t}) \\
&+ \beta\left(1-\sum_i \lambda_{j,i}\right)W_{t+1}^{b,j,t}(\mathbf{q}_t^{b,j,t}) \\
&= \overline{W}_t^{b,j,t} + \theta^{b,j}(1-\gamma_{j,t}^{-1})\sum_c q_{j,t-1}^{b,c,t-1} + \left(1-\sum_i \lambda_{j,i}\right)\left[\sum_c(\beta v_c^{b,j} - \theta^{b,j})q_{c,t}^{b,j,t}\right] \\
&+ \sum_i \lambda_{j,i}\left[\beta u(c_{t+1}^{j,i}) + \sum_c \beta v_c^{b,j}(q_{c,t}^{b,jt} - d_{c,t}^{j,i}) - \theta^{b,j}q_{c,t}^{b,j,t}\right] \\
&= \overline{W}_t^{b,j,t} + \underbrace{\theta^{b,j}(1-\gamma_{j,t}^{-1})\sum_c q_{j,t-1}^{b,c,t-1}}_{\textit{seigniorage revenue}} + S(\mathbf{q}_t^{b,j,t}) + \underbrace{\sum_c(\beta v_c^{b,j} - \theta^{b,j})q_{c,t}^{b,j,t}}_{\textit{Inflation tax}}.
\end{aligned} \tag{A.5}$$

where

$$\overline{W}_t^{b,j,t} = U(C^{**}(\theta^{b,j})) - \theta^{b,j}C^{**}(\theta^{b,j}) + \beta W^{b,j,t}(\mathbf{0}) \tag{A.6}$$

is independent of the period-t inflation rate $\gamma_{c,t}$. Note that only the second term (seigniorage revenue) in eq.(A.5) depends on the period-t inflation rate $\gamma_{c,t}$. The last two terms only depend on the period-$t+1$ inflation rate $\gamma_{c,t}$. Consequently, the young buyer's lifetime value in period $t+1$ depends on the period-t inflation rate $\gamma_{c,t}$ through $S(\mathbf{q}_{t-1}^{b,j,t-1}) + \sum_c(\beta v_c^{b,j} - \theta^{b,j})q_{c,t-1}^{b,j,t-1}$.

Collecting all these terms depending on the period-t inflation rate $\gamma_{c,t}$, the relevant terms in total welfare of country j are given by

$$\mathcal{W}_j = S(\mathbf{q}_{t-1}^{b,j,t-1}) + \sum_c (\beta v_c^{b,j} - \theta^{b,j}) q_{c,t-1}^{b,j,t-1} + \beta^{gov}\theta^{b,j}(1-\gamma_{j,t}^{-1}) \sum_c q_{j,t-1}^{b,c,t-1}, \tag{A.7}$$

where β^{gov} is the government's discount factor. Letting $\beta = \beta^{gov} \to 1$, I obtain the desired result in eq. (25). Moreover, the global welfare (26) can be obtained by summing up the local welfare (25).

□

A.5 Proof of Proposition 6

Proof. The global welfare does not directly depend on the variance of inflation rates but only through the restrictions on the values that the covariance terms can take $Cov(\theta_{t+1}^{s,i}, \gamma_{j,t+1}^{-1})$. Moreover, the global welfare is nondecreasing in those covariances through the hedging and hence, affecting the terms of trade in DM. Consequently, the global planner chooses the largest variances and covariances of the inflation rates with the labor earning risks $\theta_{t+1}^{s,i,t}$. Moreover, setting the average currency return equal to the inverse discount factor $\mathbb{E}[\gamma_{j,t+1}^{-1}] = 1/\beta$, the currency j holdings become satiated to approach to infinity for all agents, which weakly increases the global gains from trade and hence, the global welfare.

□

A.6 Proof of Proposition 7

Proof. Notice that the local welfare in (25) does not directly depend on the national inflation rate but only through the restrictions on the values that the covariance terms can take $Cov(\theta_{t+1}^{s,i}, \gamma_{j,t+1}^{-1})$. Consequently, it is optimal for the local planner to maximize the variance and covariances. The local planner might not choose the Friedman rule on average, $\mathbb{E}[\gamma_{j,t+1}^{-1}] < 1/\beta$ since the local welfare might be improved by raising the inflation tax and increasing the seignirage collected from foreign agents. □