LABOR WEDGE AND JOB RATIONING

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ABSTRACT. We demonstrate that the Diamond–Mortensen–Pissarides model fails to reconcile the pronounced volatility of the labor wedge with the evidence that its countercyclical fluctuations are predominantly driven by the household-side gap between the real wage and the marginal rate of substitution. To address this issue, we develop a job-rationing framework that underscores the pivotal role of wage rigidity. Analytically, we show that sticky real wages induce labor rationing, pushing households off their labor supply curves, and this generates a substantial deviation between the wage and the MRS on the household side. Our numerical analysis shows that labor rationing helps explain the U.S. evidence on the decomposition. In the Great Recession, the job rationing-related shock (technology) accounts for 65.72% of unemployment fluctuations and 66.67% of labor-wedge fluctuations. By contrast, matching-friction–related shocks (job separation and matching efficiency) play a relatively minor role, together contributing 34.28% to unemployment and 33.33% to the labor wedge.

JEL Classification: E24, E32, J64

Keywords: Labor Wedge, Unemployment, Job Rationing, Search and Matching Model, Rationing Unemployment, Frictional Unemployment

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1. Introduction

The labor wedge—the gap between the marginal product of labor (MPL) and the marginal rate of substitution between leisure and consumption (MRS)—explains a significant portion of output fluctuations. In the U.S., Chari et al. (2002, 2007) find that the labor wedge played an important role in accounting for output fluctuations during the Great Depression and from 1959 to 2004. In Japan, Kobayashi and Inaba (2006) show that the labor wedge was a major contributor to the recession in the 1920s and the lost decade of the 1990s. Several studies recognize the role of taxes in labor supply decisions and the labor wedge (e.g., Prescott 2004; Chari et al. 2007), but taxes alone cannot explain the rising labor wedge at the business cycle frequency, as pointed out by Shimer (2009) and Ohanian (2010).

Like unemployment, the U.S. labor wedge is volatile and countercyclical. Accordingly to Chari et al. (2007) and Karabarbounis (2014), the labor wedge exhibits pronounced fluctuations over the business cycle, sometimes even more than output. Figure 1 plots unemployment, the labor wedge, and real GDP from 1959 to 2023 (see Appendix A for the data description). Unemployment and the labor wedge move in the opposite direction of real GDP. Karabarbounis (2014) shows that fluctuations in the labor wedge are largely driven by the household-side gap between the real wage and the MRS, rather than the firm-side gap between the MPL and the real wage. In this paper, we show that the standard Diamond–Mortensen–Pissarides (DMP; Diamond 1982; Mortensen 1982; Pissarides 1985) model fails to replicate both the observed volatility of the labor wedge and the empirical finding of Karabarbounis (2014) that its countercyclical movements are primarily driven by the gap between the real wage and the MRS. To overcome this deficiency, we then incorporate wage rigidity into the DMP framework and demonstrate that the resulting job rationing mechanism enables the model to capture the observed cyclicality of the labor wedge, as highlighted above.

Our analysis, following Hagedorn and Manovskii (2008), begins by examining the roles of bargaining power and the value of leisure (i.e., the steady-state MRS)

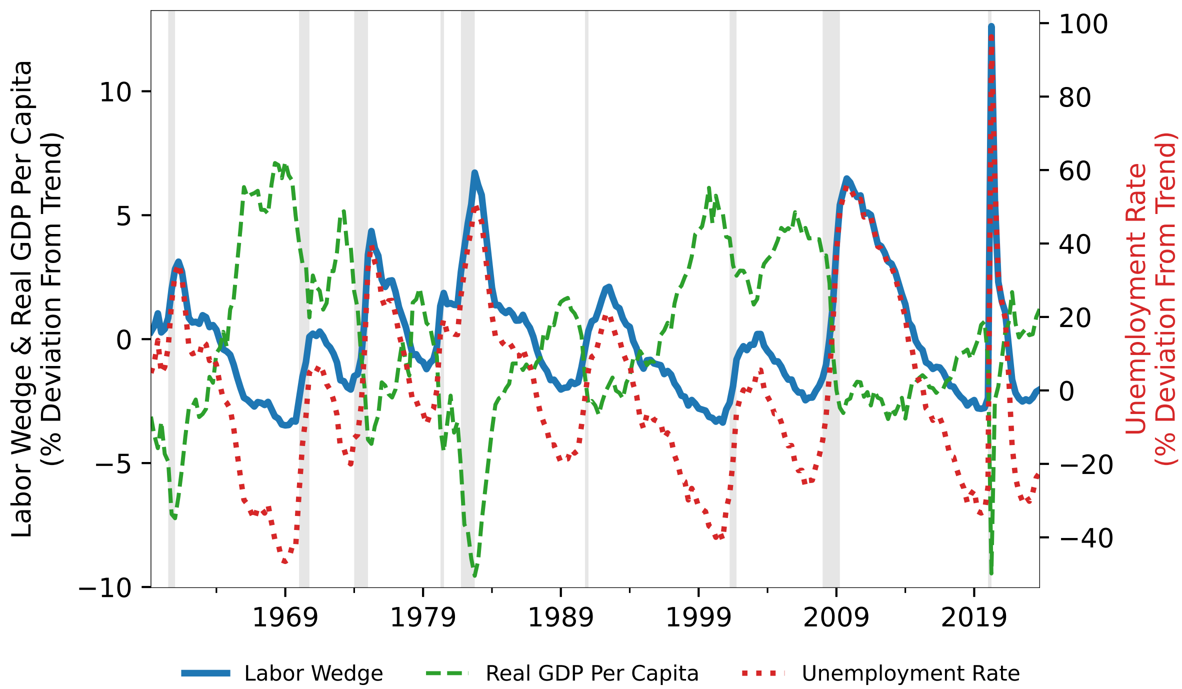


FIGURE 1. **Labor Wedge, Unemployment and Real GDP: 1959–2023**. The labor wedge is the ratio of MPL to MRS, calculated using the baseline model in Section 2. The left (right) y-axis shows the percent deviation from the trend paths of real GDP per capita and the labor wedge (the unemployment rate).

in accounting for the observed cyclical properties of the labor wedge: the volatility and fluctuations that are mainly driven by the wage–MRS gap. Hagedorn and Manovskii (2008) demonstrate that by adjusting these two factors, the DMP model can generate the observed high volatility of labor market variables without the need for wage rigidity. Instead, we find that the DMP model inherently embodies a trade-off in capturing the cyclicality of the labor wedge. Capturing the volatility of the labor wedge requires substantially low bargaining power and a low MRS for workers. By contrast, replicating the labor wedge fluctuations that are primarily driven by the wage–MRS gap necessitates substantially high bargaining power and a high MRS. This trade-off arises because, given the observed volatility of the labor wedge, lower worker bargaining power causes the equilibrium wage to track the MRS more closely, thereby narrowing the wage–MRS gap. In addition, a lower initial MRS reduces its procyclicality and thus also renders the wage–MRS gap largely acyclical.

To address the deficiency of the DMP model, we highlight the pivotal role of wage rigidity and develop a job-rationing framework following Michaillat (2012). Unlike Michaillat's (2012) simplified specification of a fixed degree of wage rigidity, we adopt a generalized specification as in Leduc and Liu (2020), which enables us to examine how varying degrees of wage rigidity influence the magnitude of job rationing and, in turn, govern the cyclicality of the labor wedge. We analytically and numerically show that the job-rationing mechanism enables the model to capture both the observed volatility of the labor wedge and its countercyclical fluctuations, which are primarily driven by the household-side wage-MRS gap.

Analytically, we establish the procyclical nature of both the MRS and the MPL in the job rationing model. A decrease in technology leads to increased unemployment and, consequently, more leisure time. Given convex indifference curves between consumption and leisure, households value leisure less, reducing the MRS of leisure for consumption. A decrease in technology also lowers factor productivity, reducing the MPL. Importantly, sticky real wages induce labor rationing, pushing households off their labor supply curves, while on the firm side the MPL continues to equal the wage in the absence of matching frictions during recessions. This generates a substantial deviation between the wage and the MRS on the household side. As a result, a higher degree of wage rigidity amplifies the procyclicality of the MRS relative to that of the MPL, rendering the labor wedge strongly countercyclical over the business cycle. Simply put, during downturns, intensified job rationing ——arising from real wage rigidity—— decreases the MRS relative to the MPL, leading to a countercyclical labor wedge that is predominantly driven by the wage–MRS gap.[1]

In our numerical analysis, labor rationing provides an explanation for the U.S. evidence in Karabarbounis's (2014) decomposition. In a standard DMP

[1]Alternative explanations have also been proposed for a countercyclical labor wedge. For example, Gourio and Rudanko (2014) argue that intangible capital contributes to a countercyclical and volatile labor wedge. Atesagaoglu and Elgin (2015) argue that the presence of an informal sector is crucial for a countercyclical and volatile labor wedge.

model with flexible wages, fluctuations in the labor wedge are attributed solely to matching frictions, which exert only limited influence on the household-side MRS. Consequently, given the volatility of the labor wedge, the model fails to replicate the markedly high procyclicality of the MRS, resulting in a countercyclical labor wedge that is primarily driven by the gap between the real wage and the MRS. As emphasized by Karabarbounis (2014), "business cycle theories of the labor wedge must focus on improving the household side of the neoclassical growth model." Our analysis demonstrates that when the real wage is sticky enough, then during downturns, intensified job rationing amplifies the procyclicality of the MRS, constituting a primary driver of the countercyclical labor wedge (the household-side wage–MRS gap accounts for 89% of labor wedge fluctuations).

In the U.S., the countercyclical movements of both unemployment and the labor wedge during downturns are largely accounted for by job rationing. In the Great Recession, the job rationing-related shock (technology) accounts for 65.72% of unemployment fluctuations and 66.67% of labor-wedge fluctuations. In contrast, the matching friction-related shock (job separation and matching efficiency) plays a relatively minor role: separation contributes 12.87% to unemployment and 17.63% to the labor wedge, while matching efficiency contributes only 21.41% and 15.70%, respectively. These results contradict those in Cheremukhin and Restrepo-Echavarria (2014a). They argue that theories emphasizing wage rigidity and bargaining processes—commonly considered in the search literature—are not helpful in explaining the behavior of the labor wedge. Instead, they conclude that the labor wedge is largely explained by matching efficiency, and unemployment is mainly accounted for by job separation, both related to matching frictions. In their model, job rationing is overlooked, and matching frictions exclusively account for fluctuations in unemployment and the labor wedge. In the absence of job rationing, time-varying separation and matching efficiency drive most fluctuations in unemployment and the labor wedge.

This paper is also related to other studies of the labor wedge at business cycle frequency. In addition to the aforementioned papers, Hall (2009) studies the cyclical fluctuations in the marginal value of time (related to MRS) and

MPL and finds that rising inefficiency during recessions is mainly the result of an employment adjustment failure. Motivated by Shimer's (2009) conclusion that, "...by arguing for a more promising, if still preliminary, explanation—search frictions, combined with real wage rigidities, create an endogenous cyclical wedge between the MRS and MPL (p.281)," we highlight the relative importance of job rationing in the labor wedge, compared to matching frictions as inCheremukhin and Restrepo-Echavarria (2014b). Our analysis also aligns with the findings of Shimer (2010), which emphasize that the positive correlation between the labor wedge and unemployment is central to generating labor wedge fluctuations and to understanding cyclical labor market dynamics. We also confirm its importance in generating cyclical unemployment, as emphasized in Shimer (2009), Shimer (2010), and Blanchard and Galí (2010).

2. The Model

To thoroughly examine the cyclicality of the labor wedge, we build a standard DMP model with constant marginal returns and flexible wages, following Hagedorn and Manovskii (2008), and then extend it into a job-rationing framework by incorporating diminishing marginal returns and wage rigidity, as in Michaillat (2012). To highlight its role, wage rigidity is modeled following Leduc and Liu (2020).

2.1. **The Labor Market.** We normalize the labor force to unity, with workers moving only between employment and unemployment. Thus, unemployment is determined according to

$$u_t = 1 - n_t, \tag{1}$$

where u_t denotes unemployment (which, under labor force normalization, can also be interpreted as the unemployment rate) and n_t is the employment level.

The representative firm posts vacancies, v_t, to hire workers. The number of hires, h_t, is determined by the following Cobb-Douglas matching function:

$$h_t(u_t, v_t) = \mu_t \cdot u_t^{\xi} \cdot v_t^{1-\xi}, \tag{2}$$

where μ_t represents the matching efficiency in period t. Moreover, $0 < \xi < 1$ is the matching elasticity with respect to unemployment.

Let s_t denote the separation rate in period t. As in Fujita and Ramey (2007), newly hired workers in period $t-1$ remain productive if they do not experience separation, joining the existing workforce $(1-s_{t-1})n_{t-1}$. Hence, employment evolves according to

$$n_t = (1-s_{t-1}) \cdot (n_{t-1} + h_{t-1}). \tag{3}$$

Drawing from equation (2), we define the job finding rate as $f_t \equiv \frac{h_t}{u_t}$ and the vacancy filling rate as $q_t \equiv \frac{h_t}{v_t}$. The labor market condition can be summarized by the job market tightness, $\theta_t \equiv \frac{v_t}{u_t}$. A higher θ_t indicates a tighter labor market, that is, more vacancies (a bigger v_t) or less unemployment (a smaller u_t). This makes it easier for job seekers to find jobs and more difficult for firms to fill vacancies than under a smaller θ_t.

2.2. **Firms.** The production function is given by

$$y_t = a_t \cdot n_t^{\alpha}, \tag{4}$$

where y_t is output, a_t is technology (or total factor productivity, TFP), and α is the output elasticity with respect to labor. Thus, the marginal product of labor is

$$MPL_t = \frac{\alpha y_t}{n_t}. \tag{5}$$

When $\alpha = 1$, the production function exhibits constant returns to labor, consistent with the standard DMP model (Hagedorn and Manovskii 2008). When $0 < \alpha < 1$, the production function features diminishing marginal returns to labor, giving rise to a downward-sloping labor demand curve, as in Michaillat (2012).

The present value of the representative firm's life-time profit is

$$\sum_{t=0}^{\infty} \beta_{0,t} \underbrace{(y_t - w_t n_t - \nu_t v_t)}_{\equiv d_t}, \tag{6}$$

where $\beta_{0,t}$ denotes the stochastic discount factor (defined in the household problem), w_t the real wage, d_t instantaneous profit, and $\nu_t = \mathbf{c} a_t$ the unit vacancy posting cost, with $\mathbf{c}$ representing the vacancy cost parameter. As in Pissarides (2000) and Michaillat (2012), the unit vacancy cost is positively and linearly related to technology a_t.

The firm maximizes life-time profit, subject to equations (1)–(3). Let J_t be the marginal asset value when the firm hires one additional worker. The optimal hiring condition is

$$J_t = MPL_t - w_t + (1 - s_t)E_t[\beta_{t,t+1}J_{t+1}]. \tag{7}$$

At time t, the firm's marginal asset value of filling a vacancy (J_t) equals the marginal benefit, $MPL_t - w_t$, plus the discounted continuation value when a hired worker does not separate from the job at $t+1$: $(1 - s_t)E_t[\beta_{t,t+1}J_{t+1}]$. Conditional on the vacancy filling rate, q_t, the free entry condition is

$$q_t(1 - s_t)E_t[\beta_{t,t+1}J_{t+1}] = \mathbf{c}a_t, \tag{8}$$

which implies that the expected marginal value of vacancy filling equals the unit cost of vacancy posting.

2.3. **Households.** The representative household's utility depends on consumption, c_t, and labor supply, n_t. Following Karabarbounis (2014) and Cheremukhin and Restrepo-Echavarria (2014a), the household's life-time utility takes the form of

$$\sum_{t=0}^{\infty} \beta^t \underbrace{\left(\ln c_t - \chi \frac{n_t^{1+\phi}}{1+\phi}\right)}_{\equiv U_t}, \tag{9}$$

where $\beta > 0$ is a constant discount factor, $\phi > 0$ is the inverse of the Frisch elasticity of labor supply, and $\chi > 0$ is the disutility weight of labor.

Following Merz (1995) and Andolfatoo (1996), all workers, employed and unemployed, belong to the same family. The "big family" assumption implies that each household has a unified preference capturing the utility of all household members and faces the pooled budget constraint:

$$c_t = w_t n_t + d_t. \tag{10}$$

The representative household maximizes lifetime utility, equation (9), by choosing consumption (c_t) and labor supply (n_t), subject to equations (1)–(3), and (10). Let V_t be the net asset value when an unemployed worker finds a job and λ_t be the Lagrange multiplier associated with the budget constraint. The household's optimality conditions are given by:

$$\lambda_t = \frac{1}{c_t} \tag{11}$$

and

$$V_t = \underbrace{w_t - MRS_t}_{\text{net return of working}} + \underbrace{E_t \beta_{t,t+1} \left[(1-s_t)(1-f_{t+1})V_{t+1}\right]}_{\text{expected continuation value of being employed}}, \tag{12}$$

where $\beta_{t,t+1} \equiv \beta \frac{\lambda_{t+1}}{\lambda_t}$ is the stochastic discount factor.

In equilibrium, the net asset value of finding a job is equal to the sum of the net return from working at time t and the expected value of continuing to be employed at time $t+1$. The net return of working consists of the real wage received minus the disutility from working, captured by the marginal rate of substitution between leisure and consumption, calculated as

$$MRS_t = -\frac{\partial U_t / \partial n_t}{\partial U_t / \partial c_t} = \chi \cdot c_t n_t^{\phi}, \tag{13}$$

where U_t is defined in equation (9). Moreover, the goods market clearing condition is given by $y_t = c_t + \mathbf{c}a_t \cdot v_t$.

2.4. **Labor Wedge.** Following Leduc and Liu (2020), the real wage, w_t, is determined by

$$w_t = (1-\varphi) w_t^N + \varphi w_{t-1}, \tag{14}$$

where w_t^N denotes the Nash bargaining wage, and φ captures the degree of wage rigidity. As in the standard DMP model, the bargaining wage is derived by solving the Nash bargaining problem

$$\max_{w_t^N} \; (V_t)^{\eta} \cdot (J_t)^{1-\eta}, \tag{15}$$

where η represents the bargaining power of workers. Solving equation (16) yields the bargaining wage w_t^N as follows:

$$w_t^N = \eta \cdot (MPL_t + \theta_t \mathbf{c}a_t) + (1-\eta) \cdot MRS_t. \tag{16}$$

This expression shows that the bargained wage is a weighted average of $(MPL_t + \theta_t \mathbf{c} a_t)$ and MRS_t, with the weights determined by the bargaining power parameter η.

In line with Karabarbounis (2014), with the real wage level, the labor wedge, τ_t, is given by

$$\tau_t = \frac{MPL_t}{MRS_t} = \underbrace{\frac{MPL_t}{w_t}}_{\tau_t^F} \cdot \underbrace{\frac{w_t}{MRS_t}}_{\tau_t^W}. \tag{17}$$

Henceforth, we use τ_t^F to denote the firm-side component: $\frac{MPL_t}{w_t}$, the gap between the MPL and the real wage. Moreover, we use τ_t^W to denote the household-side component: $\frac{w_t}{MRS_t}$, the gap between the real wage and the MRS.

2.5. **Shocks.** There are three shocks: technology, a_t, separation rates, s_t, and matching efficiency, μ_t in the model. These shocks following the AR(1) process specified as

$$\begin{aligned} \ln a_t &= (1 - \phi^a) \cdot a + \phi^a \cdot \ln a_{t-1} + e_t^a, \\ \ln s_t &= (1 - \phi^s) \cdot s + \phi^s \cdot \ln s_{t-1} + e_t^s, \quad \text{and} \\ \ln \mu_t &= (1 - \phi^\mu) \cdot \mu + \phi^\mu \cdot \ln \mu_{t-1} + e_t^\mu. \end{aligned} \tag{18}$$

where ϕ^i denotes the AR(1) coefficient, e_t^i the corresponding innovation, given $i \in \{a, s, \mu\}$. Moreover, a, s, and μ represent the corresponding initial steady-state values. The innovations (e_t^a, e_t^s, e_t^μ) are drawn from the joint normal distribution $N(0, \Sigma)$, where Σ represents the covariance matrix.

3. The Cyclicality of Labor Wedge in the DMP Model

3.1. **Limits of the DMP model in Capturing Labor Wedge Cyclicality.** The labor wedge is characterized by two salient cyclical features: volatility and fluctuations that are mainly driven by the wage–MRS gap. In this section, we show both numerically and analytically that the standard DMP model—with constant marginal returns ($\alpha = 1$) and flexible wages ($\varphi = 0$)—cannot simultaneously replicate the two empirical features of the labor wedge. Instead, the DMP model inherently entails a trade-off in capturing its cyclicality.

3.2. **Numerical Analysis.** For clarity, we first present the numerical analysis.

3.2.1. *Calibration.* To maintain the standard characteristics of the DMP model, we adopt a conventional calibration strategy, following Coles and Kelishomi (2018) and Leduc and Liu (2020). The model frequency is monthly. We set $\beta = 0.9967$ to match an annual discount rate of 4 percent. Following Fujita and Ramey (2007), we set the matching function elasticity, ξ, to 0.6. Based on the average unemployment rate from January 1959 to December 2019 (Bureau of Labor Statistics, 2024, (BLS)), the steady-state unemployment rate is $u = 0.06$. The steady-state vacancy-filling rate is $q = 0.6415$, consistent with Davis et al. (2013). Steady-state technology is normalized to $a = 1$, and the steady-state job separation rate is set to $s = 0.036$, based on Job Openings and Labor Turnover Survey (Bureau of labor Statistics, 2024, JOLTS,) data from December 2000 to December 2019. Following Cheremukhin and Restrepo-Echavarria (2014a), we set the labor supply elasticity to $\phi = 0.5$. In the DMP framework, the parameterization $\alpha = 1$ and $\varphi = 0$ entails constant returns to scale in production and fully flexible wages, as in Hagedorn and Manovskii (2008).

The analysis accounts for three primary sources of shocks: technology, job separation, and matching efficiency. Their AR(1) coefficients and innovations are identified using the following data. First, we take output per worker from the Nonfarm Business Sector (BLS) as the quarterly technology shock series from January 1959 to December 2019. Second, following Shimer (2005), we construct monthly job separation and job-finding rates and take their quarterly averages. Third, using the job-finding rate equation, $f_t = \mu_t \cdot \theta_t^{1-\xi}$, we derive the observed matching efficiency, μ_t, from the observed job-finding rate and labor market tightness, measured as the ratio of job vacancies to unemployment. To proxy job vacancies, we use the composite Help-Wanted Index from Barnichon (2010) and job postings from JOLTS. We also use the quarterly average of the seasonally adjusted monthly unemployment level from the BLS.

Table 1. **Calibration**

Parameters	Value	Source
β: Discount Factor	0.9967	4 Percent Annual Rate
ξ: Matching Elasticity on Unemployment	0.5	Fujita and Ramey (2007)
u: Steady-State Unemployment Rate	0.06	BLS, 1959 – 2019
q: Steady-State Vacancy Filling Rate	0.6415	Leduc and Liu (2020)
a: Technology	1	Normalization
s: Job Separation Rate	0.036	JOLTS, 2001–2019
ϕ: Labor Supply Elasticity	0.5	Cheremukhin and Restrepo-Echavarria (2014a)
ϕ^a : Technology Autocorrelation	0.965	BLS, 1959 – 2019
ϕ^s: Separation Autocorrelation	0.924	BLS, 1959 – 2019
ϕ^μ: Matching Autocorrelation	0.854	BLS, 1959 – 2019
σ^a: Technology Standard deviation	0.0052	BLS, 1959 – 2019
σ^s: Separation Standard deviation	0.031	BLS, 1959 – 2019
σ^μ: Separation Standard deviation	0.041	BLS, 1959 – 2019
Parameters: Specific to DMP	Value	Source
α: Labor Elasticity	1	Constant Marginal Returns
φ: Degree of Wage Rigidity	0	No Wage Rigidity
Parameters: Specific to Job Rationing	Value	Source
α: Labor Elasticity	$\frac{2}{3}$	Michaillat (2014)

Finally, we use the Hodrick-Prescott (HP) filter with a smoothing parameter of 10^5 to detrend the log of quarterly technology, job separation rates, and matching efficiency.[2] Using the quarterly detrended shock series, we derive $\phi^a = 0.965$, $\phi^s = 0.924$, and $\phi^\mu = 0.854$; and $\sigma^a = 0.0052$, $\sigma^s = 0.031$, and $\sigma^\mu = 0.041$, at a monthly frequency. The calibration is summarized in Table 1.

3.2.2. *The Trade-Off in Capturing the Labor Wedge Cyclicality.* Hagedorn and Manovskii (2008) demonstrate that variations in bargaining power, η, and the worker's outside option (captured by the MRS in our model) enable the DMP model to replicate the observed standard deviations of labor market variables without relying on wage rigidity. Drawing on equation (19), we extend their analysis to further investigate the roles of these two parameters in shaping the cyclicality of the labor wedge. Specifically, we assess whether the standard DMP model

[2]The smoothing parameter is set to 10^5, in line with Michaillat (2012) and Coles and Kelishomi (2018).

can reproduce a volatile labor wedge over the business cycle (standard deviation of 0.022), and whether its fluctuations are primarily driven by the household-side wage–MRS gap (with a contribution of $\tau^W = 79.6\%$).

We begin by constructing the observed labor wedge for the subsequent quantitative analysis.[3] First, we use observed real GDP and employment to construct MPL_t from equation (5) and use observed real consumption and employment to construct MRS_t from equation (13). To compute the contribution of the firm-side component, τ_t^F, and the household-side component, τ_t^W, to labor wedge fluctuation, we use (17) and log-linearize τ_t, τ_t^F, and τ_t^W around their HP-filter trends.[4] We thus have

$$\hat{\tau}_t = \hat{\tau}_t^F + \hat{\tau}_t^W. \tag{19}$$

Here, $\hat{\tau}_t = \ln \tau_t - \ln \tau_t^{Tr}$, $\hat{\tau}_t^F = \ln \tau_t^F - \ln \tau_t^{F,Tr}$, and $\hat{\tau}_t^W = \ln \tau_t^W - \ln \tau_t^{W,Tr}$, where τ_t^{Tr} represents the HP-filter trend. Equation (19) indicates that fluctuations in the labor wedge, $\hat{\tau}_t$, are the sum of fluctuations from the firm-side component, $\hat{\tau}_t^F$, and the household-side component, $\hat{\tau}_t^W$. Given the constructed MRS_t and MPL_t, along with the observed wage w_t, we derive $\hat{\tau}_t$, $\hat{\tau}_t^F$, and $\hat{\tau}_t^W$.

Accordingly, we can compute the contributions of τ_t^F and τ_t^W to labor wedge fluctuations by using the following formulas:

$$\begin{aligned} &\text{Contribution of } \tau_t^F\text{: } \frac{cov(\hat{\tau}_t^F, \hat{\tau}_t)}{V(\hat{\tau}_t)}, \\ &\text{Contribution of } \tau_t^W\text{: } \frac{cov(\hat{\tau}_t^W, \hat{\tau}_t)}{V(\hat{\tau}_t)}. \end{aligned} \tag{20}$$

Here, $cov(x, y)$ is the covariance of x and y, and $V(x)$ is the variance of x.

In Table 2, the upper panel presents the simulated standard deviation of the labor wedge, while the lower panel shows the contribution of the household-side component to fluctuations in the labor wedge for various values of η and

[3]The data used to construct the labor wedge is seasonally adjusted and quarterly. Appendix A provides details on these data sources.

[4]We use the HP filter with smoothing parameter 10^5 to obtain these trend series.

TABLE 2. **Simulation Results: the DMP Model**

σ^τ: Standard Deviation of τ (Data: $\sigma^\tau = 0.022$)	$\eta = 0.25$	$\eta = 0.5$	$\eta = 0.75$	$\eta = 0.9$
$\overline{MRS}/\overline{MPL} = 0.1$	0.025	0.016	0.012	0.011
$\overline{MRS}/\overline{MPL} = 0.3$	0.022	0.015	0.012	0.011
$\overline{MRS}/\overline{MPL} = 0.5$	0.019	0.013	0.011	0.011
$\overline{MRS}/\overline{MPL} = 0.7$	0.015	0.012	0.010	0.010
$\overline{MRS}/\overline{MPL} = 0.9$	0.010	0.008	0.008	0.007
Contribution: τ^W (Household Component) (Data: Contribution of $\tau^W = 79.6\%$)	$\eta = 0.25$	$\eta = 0.5$	$\eta = 0.75$	$\eta = 0.9$
$\overline{MRS}/\overline{MPL} = 0.1$	−30.2%	17.0%	61.4%	85.3%
$\overline{MRS}/\overline{MPL} = 0.3$	−17.8%	28.1%	68.0%	88.0%
$\overline{MRS}/\overline{MPL} = 0.5$	−2.8%	41.0%	75.0%	90.9%
$\overline{MRS}/\overline{MPL} = 0.7$	16.6%	56.1%	82.5%	93.8%
$\overline{MRS}/\overline{MPL} = 0.9$	44.2%	74.3%	90.5%	96.8%

the (initial) steady-state ratio of MRS to MPL, $\overline{MRS}/\overline{MPL}$.[5] As shown in the upper panel, a reduction in either η or the ratio $\overline{MRS}/\overline{MPL}$ is associated with an increase in the standard deviation of the labor wedge. When $\eta = 0.25$ and $\overline{MRS}/\overline{MPL} = 0.3$, the DMP model replicates the observed standard deviation of the labor wedge (0.022). This finding is generally consistent with the literature: Hagedorn and Manovskii (2008) show that a smaller bargaining power η amplifies the standard deviation of labor market variables. Moreover, the result that a lower $\overline{MRS}/\overline{MPL}$ raises the standard deviations of labor market variables is consistent with Chodorow-Reich and Karabarbounis (2016), who show that a lower steady-state outside option increases the model-generated standard deviation when the outside option is procyclical, as in our model.

However, the combination of lower values of η and $\overline{MRS}/\overline{MPL}$ does not capture the empirical fact that labor wedge fluctuations are primarily driven by the household-side wage–MRS gap. The lower panel of Table 2 shows that when $\eta = 0.25$ and $\overline{MRS}/\overline{MPL} = 0.3$, the DMP model reproduces the observed standard deviation of 0.022, but with a contribution of τ^W equal to −17.8%,

[5]We convert monthly data to quarterly series using quarterly averages. We then detrend both the observed and simulated quarterly series with the HP filter (smoothing parameter 10^5) and compute the moments.

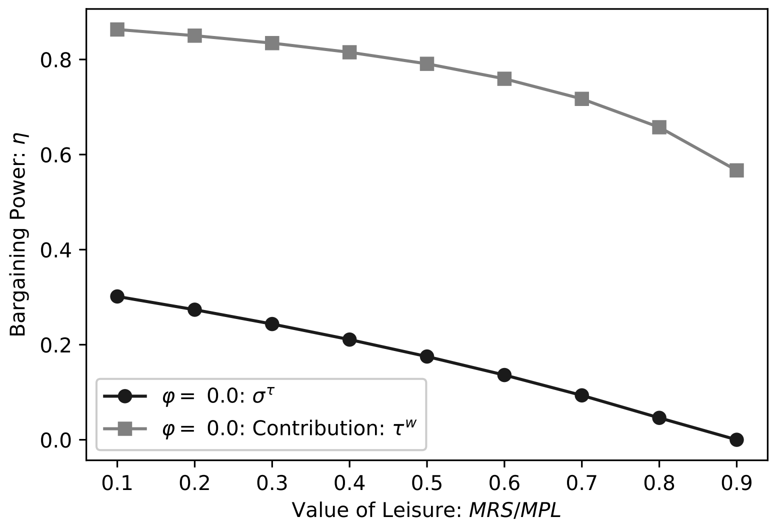


FIGURE 2. **Values of η and $\overline{MRS}/\overline{MPL}$ for the DMP Model to Match the Observed σ^{τ} and τ^{W} Contribution**

indicating that labor wedge fluctuations are accounted for entirely by the firm-side MPL–wage gap rather than the household-side wage–MRS gap.

In contrast to the result for the standard deviation, a greater contribution of the household-side component to labor wedge fluctuations requires a higher value of either η or $\overline{MRS}/\overline{MPL}$. Equations (12) and (16) imply that stronger worker bargaining power η amplifies the gap between the bargained wage and the MRS, thereby rendering τ^{W} more countercyclical. Moreover, equations (7), (8), and (16) imply that a higher value of $\overline{MRS}/\overline{MPL}$ increases the weight of $\ln MRS_t$ in τ^{W}, likewise enhancing its countercyclicality. In both cases, fluctuations in the overall labor wedge τ, which is countercyclical, are primarily driven by the household-side wage–MRS gap, τ^{W}. The underlying intuition is elaborated in the analytical interpretation of the next subsection.

Table 2 highlights the trade-off of the standard DMP framework in accounting for the cyclicality of the labor wedge. Capturing the observed volatility of the labor wedge requires a substantially low bargaining power parameter η and a low steady-state ratio $\overline{MRS}/\overline{MPL}$. In contrast, replicating labor wedge fluctuations that are predominantly driven by the wage–MRS gap requires a significantly high η and a high $\overline{MRS}/\overline{MPL}$.

Figure 2 illustrates our argument with greater clarity. The line with circles depicts the combinations of η and $\overline{MRS}/\overline{MPL}$ that allow the model to replicate the observed standard deviation of the labor wedge ($\sigma^{\tau} = 0.022$), while the line with squares depicts the combinations that reproduce the observed contribution of τ^{W} to labor wedge fluctuations (79.6%). Given the feasible ranges of $0 < \eta < 1$ and $0 < \overline{MRS}/\overline{MPL} < 1$, the absence of an intersection between the two lines underscores the failure of the standard DMP model in reproducing the two empirical observations under consideration.

3.3. **Analytical Interpretation.** The DMP model discussed above entails an inherent trade-off in replicating two empirical regularities: the volatility of the labor wedge and the dominant role of the wage–MRS gap in driving its fluctuations. This subsection provides an analytical interpretation based on steady-state analysis.

In the steady state, dropping the time subscript and applying equations (7), (8), and (16) yields the following relationship:

$$MPL - MRS = \frac{1}{1-\eta}\left[\frac{1-\beta(1-s)}{\beta(1-s)\mu}\theta^{\xi} + \eta\theta\right] \cdot \mathbf{c}a. \tag{21}$$

Assume a set of steady-state values $\overline{\theta}, \overline{a}, \overline{MRS}$, and $\overline{MPL}$ satisfy equation (21). We log-linearize (21) around these values and obtain:

$$\hat{\tau} = \hat{MPL} - \hat{MRS} = \frac{1}{(1-\eta)\overline{MPL}} \cdot \mathbf{G}(\hat{\theta}, \hat{a}) - (1 - \frac{\overline{MRS}}{\overline{MPL}}) \cdot \hat{MRS}(\hat{c}, \hat{n}). \tag{22}$$

Here, $\hat{x} \equiv \ln x - \ln \overline{x}$ represents the log-deviation of x from $\overline{x}$ and it measures the fluctuations in variable x. The term $\mathbf{G}(\hat{\theta}, \hat{a}) = \overline{a}\mathbf{c}\{\left[\frac{1-\beta(1-s)}{\beta(1-s)\mu}\xi\overline{\theta}^{\xi} + \eta\overline{\theta}\right] \cdot \hat{\theta} + \left[\frac{1-\beta(1-s)}{\beta(1-s)\mu}\overline{\theta}^{\xi} + \eta\overline{\theta}\right] \cdot \hat{a}\}$ is monotonically increasing in labor market tightness $\hat{\theta}$ and technology $\hat{a}$. Moreover, $\hat{MRS} = \hat{c} + \phi\hat{n}$ is monotonically increasing in consumption $\hat{c}$ and employment $\hat{n}$.

Equation (22) indicates that labor-wedge fluctuations, $\hat{\tau}$, are driven by two components: $\frac{1}{(1-\eta)\overline{MPL}} \cdot \mathbf{G}(\hat{\theta}, \hat{a})$ and $-(1 - \frac{\overline{MRS}}{\overline{MPL}}) \cdot M\hat{R}S(\hat{c}, \hat{n})$. The first component is positively related to fluctuations in tightness $\hat{\theta}$ and technology $\hat{a}$. When labor market tightness and technology display procyclical behavior (i.e., both $\hat{\theta}$ and $\hat{a}$ increase during expansions) as observed in the data, this channel generates a corresponding procyclicality in the labor wedge. By contrast, the second component is negatively related to fluctuations in the marginal rate of substitution $M\hat{R}S$. Since $M\hat{R}S$ is monotonically increasing in both $\hat{c}$ and $\hat{n}$, and given that consumption and employment are procyclical over the business cycle, as documented in the data, $M\hat{R}S$ also exhibits procyclical behavior. Consequently, this second channel induces countercyclical movements in the labor wedge. As the labor wedge is empirically countercyclical (as shown in Figure 1), it follows that the second channel is likely to dominate in practice.

Following Hagedorn and Manovskii (2008), we examine how bargaining power η and the initial steady-state ratio $\overline{MRS}/\overline{MPL}$ affect labor wedge fluctuations. It is clear from equation (22) that a smaller η attenuates the effect of the first channel, while a lower $\overline{MRS}/\overline{MPL}$ amplifies the effect of the second channel. Both make the model-implied labor wedge more countercyclical and increase its volatility, as shown in the upper panel of Table 2.

To further evaluate the contribution of the household-side MRS to the labor wedge, we employ equation (22) to derive the covariance between the MRS and the labor wedge:

$$cov(\hat{\tau}, M\hat{R}S) = \frac{1}{(1-\eta)\overline{MPL}} \cdot cov(\mathbf{G}(\hat{\theta}, \hat{a}), M\hat{R}S) - (1 - \frac{\overline{MRS}}{\overline{MPL}}) \cdot V(M\hat{R}S), \quad (23)$$

where $V(M\hat{R}S)$ is the variance of $M\hat{R}S$. Note that since the model generates procyclical $\hat{\theta}$, $\hat{a}$, and $M\hat{R}S$, $cov(\mathbf{G}(\hat{\theta}, \hat{a}), M\hat{R}S) > 0$ is true. From (23), it follows that a smaller η reduces the weight on the positive first term, while a lower $\overline{MRS}/\overline{MPL}$ increases the weight on the negative second term. Thus, both a small η and a small $\overline{MRS}/\overline{MPL}$ reduce the covariance between the MRS and the labor wedge $cov(\hat{\tau}, M\hat{R}S)$. As reported in the lower panel of Table 2, this

Table 3. **Simulation Results: the Job Rationing Model ($\eta = 0.25$ and $\overline{MRS}/\overline{MPL} = 0.3$)**

Panel (A): Moments	$\varphi = 0.5$	$\varphi = 0.9$	$\varphi = 0.95$	$\varphi = 0.98$	Observed
σ^τ: Standard Deviation of τ	0.018	0.019	0.019	0.020	0.022
Panel (B): Contributions	$\varphi = 0.5$	$\varphi = 0.9$	$\varphi = 0.95$	$\varphi = 0.98$	Observed
Firm Component: τ^F	127.3%	78.5%	57.2%	35.2%	20.4%
Household Component: τ^W	−27.3%	21.5%	42.8%	64.8%	79.6%

result implies that fluctuations in the labor wedge are primarily driven by the firm-side MPL–wage gap rather than the household-side wage–MRS gap. Building on equations (22) and (23), we provide an analytical explanation for why the DMP model entails an inherent trade-off in replicating two key empirical regularities of the labor wedge.

The analytical results above do not rely on a specific value of α. It implies that the DMP model's limitation in accounting for the cyclicality of the labor wedge persists even when the assumption of constant marginal returns ($\alpha = 1$) is relaxed to allow for diminishing returns ($0 < \alpha < 1$), as in previous labor wedge studies such as Cheremukhin and Restrepo-Echavarria (2014a).[6] This inability of the traditional DMP model appears to echoe Karabarbounis's (2014) argument, "business cycle theories of the labor wedge must focus on improving the household side of the neoclassical growth model."

4. The Cyclicality of Labor Wedge in the Job Rationing Model

In this section, we underscore the pivotal role of wage rigidity in explaining labor wedge cyclicality by incorporating a job-rationing framework into our analysis. To isolate the role of wage rigidity, we follow Cheremukhin and Restrepo-Echavarria (2014a) and impose decreasing marginal returns, setting $\alpha = 2/3$.

4.1. The Significance of Wage Rigidity.

Table 2 indicates that under $\eta = 0.25$ and $\overline{MRS}/\overline{MPL} = 0.3$, the DMP model replicates the observed standard

[6]In Appendix B, we show that the results in Table 2 still hold under $\alpha = 2/3$.

deviation of 0.022, but attributes labor wedge fluctuations exclusively to the firm-side MPL–wage gap, at odds with the decomposition of Karabarbounis (2014). Table 3 indicates that when $\eta = 0.25$ and $\overline{MRS}/\overline{MPL} = 0.3$, greater wage rigidity φ increases both the standard deviation of τ and the contribution of τ^W to labor wedge fluctuations, thereby aligning the job-rationing model with the empirical observations. Specifically, when $\varphi = 0.98$—the value employed by Shimer (2010)—the job-rationing model yields a standard deviation of τ equal to 0.020 and a τ^W contribution of 64.8%, both of which are close to the empirical observations.

Figure 3 provides further support for our argument. As in Figure 2, the line with circles depicts the combinations of η and $\overline{MRS}/\overline{MPL}$ that allow the model to replicate the observed standard deviation of the labor wedge ($\sigma^\tau =$ 0.022), while the line with squares depicts the combinations that reproduce the observed contribution of τ^W to labor wedge fluctuations (79.6%). Accordingly, the upper panel of Figure 3 reproduces the findings of Figure 2. With fully flexible wages ($\varphi = 0$), the two lines never intersect, implying that the DMP model cannot replicate the observed standard deviation of the labor wedge together with the dominant contribution of the household-side component to its fluctuations. However, the bottom panel of Figure 3 shows that as wage rigidity φ increases, the two lines converge, and at $\varphi = 0.98$ they intersect when both η is low and $\overline{MRS}/\overline{MPL}$ is high enough. In other words, the job-rationing model is capable of simultaneously reproducing both empirical observations.

4.2. **Insights from the Analytical Steady State.** This subsection extends Michaillat's (2012) approach to examine the role of job rationing, under variable degrees of wage rigidity, in shaping the labor wedge within a steady-state framework. The steady-state analysis provides a transparent framework for interpreting the empirical evidence.

4.2.1. *Job Rationing and Unemployment.* First, from equation (2), we use $f_t \equiv \frac{h_t}{u_t}$ and $\theta_t \equiv \frac{v_t}{u_t}$ to write the steady-state job finding rate as a function of job tightness:

$$f(\theta) = \mu \cdot \theta^{1-\xi}. \tag{24}$$

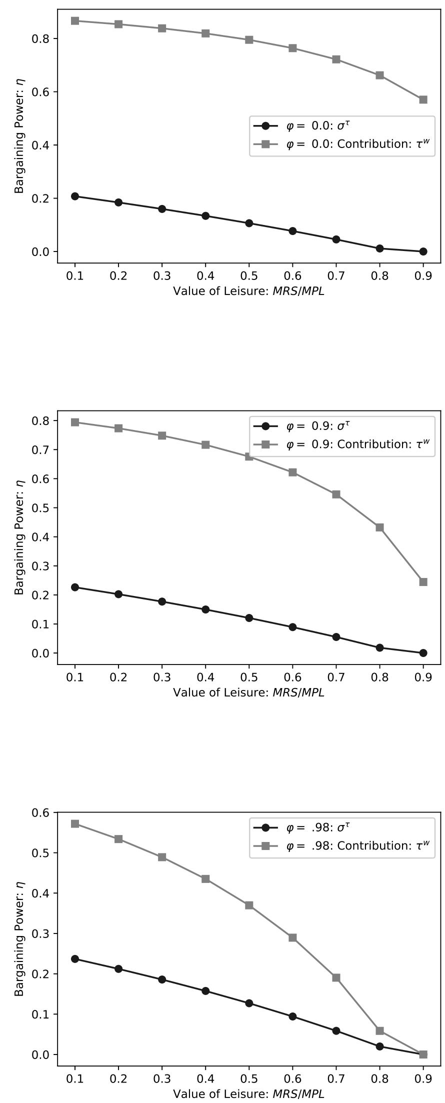


FIGURE 3. **Values of η and $\overline{MRS}/\overline{MPL}$ for the Job Rationing Model to Match the Observed σ^{τ} and τ^{W} Contribution**

Thus, combining equations (1) and (3), the steady-state level of employment can be expressed as a function of the job-finding rate,

$$n = \frac{(1-s)f(\theta)}{(1-s)f(\theta)+s}, \tag{25}$$

which mirrors the Beveridge curve, linking unemployment to job vacancies. Equation (25) further implies that an increase in market tightness (θ) raises employment, since a higher θ enhances the job-finding rate, as indicated by $\frac{\partial f(\theta)}{\partial \theta} = (1-\xi)\frac{f(\theta)}{\theta} > 0$ in equation (24).

From equations (7), and (8), (14), and (16), the steady-state job creation condition can be derived as

$$\alpha n^{\alpha-1} - \frac{(1-\varphi)w^N + \varphi\overline{w}}{a} = \frac{1-\beta(1-s)}{\beta(1-s)}\frac{\mathbf{c}}{q(\theta)}, \tag{26}$$

where $q(\theta) = \mu\theta^{-\xi}$ is the vacancy filling rate and $w^N = \eta(MPL + \theta a\mathbf{c}) + (1-\eta)MRS$ is the bargained wage in the steady state.[7] To simplify the analytical derivation, we impose two conditions in the following discussion. First, since the proportion of vacancy costs in aggregate output is negligible (1% in the baseline calibration), the goods market clearing condition simplifies to $y = c$. Second, in the steady-state analysis, the lagged wage w_{t-1} is treated as a constant, fixed at the initial steady-state level $w = \overline{w}$. These restrictions are adopted solely for the analytical derivation in this section and are not applied in the numerical analyses throughout the paper.

Accordingly, using equations (24) and the steady-state bargained wage $w^N = \eta(MPL + \theta a\mathbf{c}) + (1-\eta)MRS$, we can rewrite equation (26) as

$$\underbrace{\left[\alpha(1-(1-\varphi)\eta) - (1-\varphi)(1-\eta)\psi n^{1+\phi}\right](n^{\alpha-1}) - \varphi\frac{\overline{w}}{a}}_{MGP} = \underbrace{\Phi(n)\mathbf{c}}_{MRC}, \tag{27}$$

where $\Phi(n) = \frac{1-\beta(1-s)}{\beta(1-s)}\frac{1}{\mu^{\frac{1-2\xi}{1-\xi}}}(\frac{s}{1-s}\frac{n}{1-n})^{\frac{\xi}{1-\xi}} + (1-\varphi)\eta(\frac{1}{\mu}\frac{s}{1-s}\frac{n}{1-n})^{\frac{1}{1-\xi}}$. Analogous to Michaillat (2012), the left-hand side represents the firm's marginal gross profits (MGP), derived from the MPL minus the wage cost. The right-hand side represents the marginal recruiting costs (MRC), which are used to measure the firm's amortized recruiting costs. It is straightforward to show that marginal gross profits decrease with n (i.e., $MGP'(n) < 0$), whereas marginal recruiting costs increase with n (i.e., $MRC'(n) = \mathbf{c}\Phi'(n) > 0$). Equation (27) determines the steady-state level of employment n, which plays a central role in the analysis.

[7] Equation (26) is normalized by a.

Following Michaillat (2012), rationing employment, $n^{\mathcal{R}}$, is the employment level when matching frictions vanish (i.e., $\mathbf{c} = 0$). Setting $\mathbf{c} = 0$ in (27) yields

$$\left[\alpha(1-(1-\varphi)\eta) - (1-\varphi)(1-\eta)\psi(n^{\mathcal{R}})^{1+\phi}\right](n^{\mathcal{R}})^{\alpha-1}) = \varphi\frac{\overline{w}}{a}, \tag{28}$$

which determines $n^{\mathcal{R}}$. Thus, the gap between 1 (representing the total labor force) and $n^{\mathcal{R}}$ is the rationing unemployment, denoted as $u^{\mathcal{R}} = 1 - n^{\mathcal{R}}$. Moreover, frictional unemployment, $u^{\mathcal{F}}$, is defined as the residual arising from matching frictions: $u^{\mathcal{F}} = u - u^{\mathcal{R}}$. Given that $u = 1 - n$ and $u^{\mathcal{R}} = 1 - n^{\mathcal{R}}$, it follows that $u^{\mathcal{F}} = n^{\mathcal{R}} - n$.

Michaillat (2012) indicates that job rationing primarily drives unemployment in recessions. Intuitively, in bad times, lower employment makes matching easier for firms, reducing marginal recruiting costs and, in turn, lowering frictional unemployment. In addition, firms are less willing to hire, increasing the severity of job rationing in downturns. Combining these two factors, rationing unemployment dominates in bad times. It is worth noting that this result holds even though, unlike Michaillat's (2012) specification of fixed wage rigidity, our model incorporates a more generalized specification of wage rigidity. The corresponding proof is provided in Appendix C.

4.2.2. *Job Rationing and Labor Wedge.* In the steady state, the labor wedge is given by

$$\tau = \frac{MPL}{MRS}, \tag{29}$$

where the steady-state MPL and MRS are computed using equations (5) and (13). In the following analysis, we denote the elasticity of variable x with respect to variable z as $\varepsilon_z^x \equiv \frac{\partial \ln x}{\partial \ln z}$.

LEMMA 1. *Given equations* (4)–(16) *and* (9)*, MPL and MRS are increasing functions in technology; i.e., $\varepsilon_a^{MPL} > 0$ and $\varepsilon_a^{MRS} > 0$.*

Proof. All proofs are relegated to Appendix C. □

Lemma 1 implies that both the marginal product of labor and the marginal rate of substitution between consumption and labor are procyclical. In bad times, a decline in technology a lowers the MPL and output, and it also decreases employment and consumption. Lower employment decreases the marginal disutility of labor, $-U_n$, and lower consumption increases the marginal utility for consumption, U_c, resulting in a decline in the MRS.

With Lemma 1, Proposition 1 further describes the cyclical nature of the labor wedge.

PROPOSITION 1. *The labor wedge, τ, is decreasing in technology; i.e., $\varepsilon_a^\tau < 0$.*

Despite the procyclical MRS and MPL, Proposition 1 states that the labor wedge is countercyclical. This implies that the degree of procyclicality of the MRS in response to a technology change is greater than that of the MPL. Thus, the effect of the MRS on the labor wedge dominates that of the MPL, as shown in Appendix C. This result is consistent with the empirical findings in Karabarbounis (2014): for many countries, and most notably for the U.S., fluctuations in the labor wedge predominantly reflect fluctuations in the gap between the real wage and the MRS, rather than the gap between the real wage and the MPL.

Intuitively, sticky real wages lead to labor rationing, pushing households off their labor supply curves. This creates an additional deviation between the MRS and the wage on the household side, while on the firm side, the MPL still equals the wage in the absence of matching frictions. Consequently, labor rationing widens the gap between the MRS and the wage. Thus, labor rationing provides a compelling explanation for Karabarbounis's (2014) decomposition. This also provides the theoretical foundation underlying the findings in Figure 2 and Table 3.

4.3. **Rationing and Frictional Labor wedge.** To further examine the roles of job rationing and matching frictions in the labor wedge, we define the portion of the labor wedge attributed to job rationing as the *rationing labor wedge*, denoted by $\tau^{\mathcal{R}}$, defined as the ratio of the MPL to the MRS evaluated at the rationing employment, $n^{\mathcal{R}}$ (see equation (28)). The rationing labor wedge is then calculated as

$$\tau^{\mathcal{R}} = \frac{MPL(n^{\mathcal{R}})}{MRS(n^{\mathcal{R}})}. \tag{30}$$

Also, the *frictional labor wedge* is $\tau^{\mathcal{F}} = \tau - \tau^{\mathcal{R}}$.

Proposition 2 states the cyclical properties of the rationing labor wedge:

PROPOSITION 2. *A lower technology level increases the rationing labor wedge ($\varepsilon_a^{\tau^{\mathcal{R}}} < 0$) and its proportion in the overall labor wedge ($\frac{\partial\left(\frac{\tau^{\mathcal{R}}}{\tau}\right)}{\partial a} < 0$).*

Proposition 2 indicates that the rationing labor wedge is countercyclical. Moreover, job rationing contributes more to the rising overall labor wedge than matching frictions in bad times, captured by a lower a.

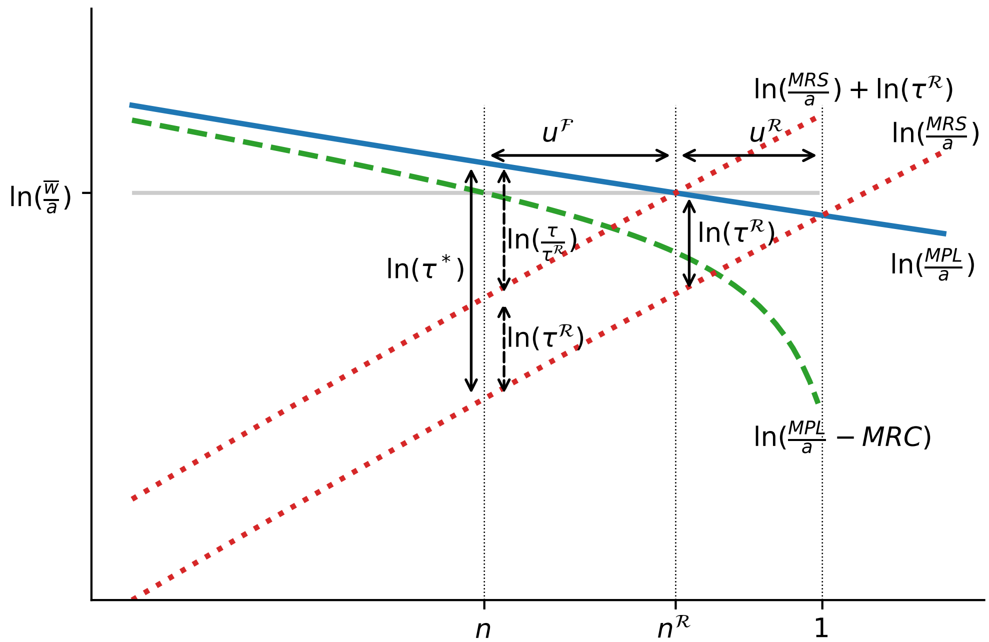
4.3.1. *Graphical Analysis.* Figure 4 graphically illustrates the result established in Proposition 2. Although Proposition 2 holds whenever wage rigidity exists, for ease of exposition we conduct the graphical analysis under the simplifying assumption that $\varphi = 1$, which mirrors the setting of Michaillat (2012).

As rationing and fractional labor wedges fluctuate over the cycle, Figure 4 shows the expansion case with higher a in Panel (A) and the recession case with lower a in Panel (B). To develop the intuition, consider equation (26) in logarithmic form:

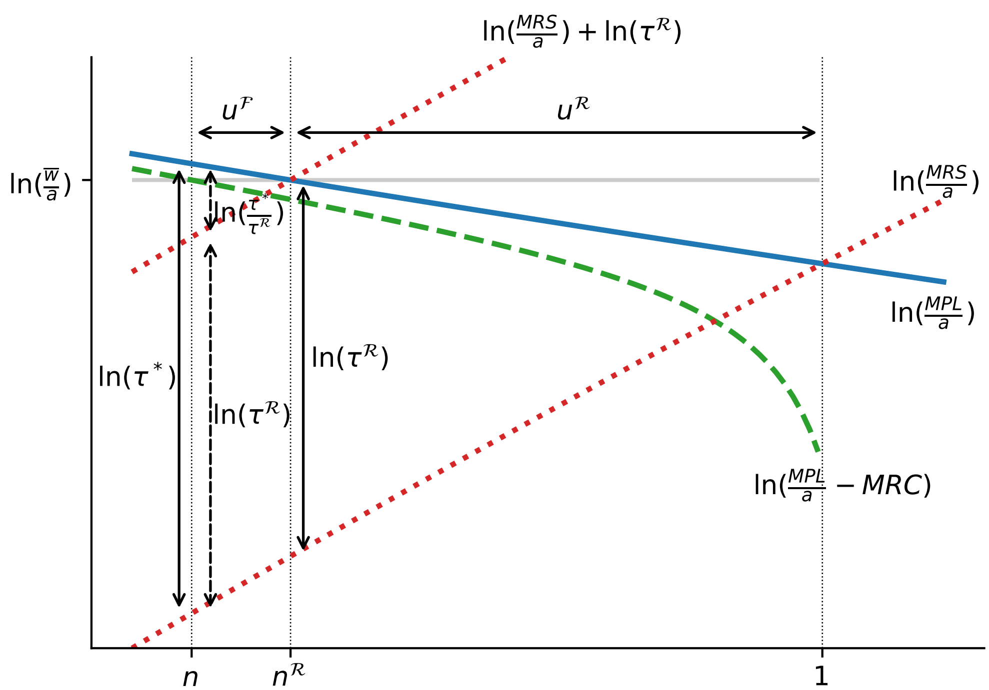
$$\ln\Big(\underbrace{\alpha n^{\alpha-1}}_{\frac{MPL}{a}} - \underbrace{\frac{1-\beta(1-s)}{\beta(1-s)}\frac{\mathbf{c}}{\mu^{\left(\frac{1-2\xi}{1-\xi}\right)}}\left[\frac{sn}{(1-s)(1-n)}\right]^{\frac{\xi}{1-\xi}}}_{MRC}\Big) = \ln\Big(\frac{\overline{w}}{a}\Big). \tag{31}$$

Because of the diminishing marginal product of labor and increasing marginal recruiting cost with respect to employment, $\frac{MPL}{a} - MRC$ on the left-hand side is a decreasing function of n, thus the dashed line, $\ln\left(\frac{MPL}{a} - MRC\right)$, is downward sloping.[8] On the right-hand side, $\frac{w}{a}$ is independent of employment so its logarithm does not change when n moves.

[8]From equation (31), we have $\frac{\partial(\frac{MPL}{a} - MRC)}{\partial n} = -\left\{\alpha(1-\alpha)n^{\alpha-2} + \frac{MRC}{n(1-n)}\right\} < 0$.



Panel (A): High Technology



Panel (B): Low Technology

FIGURE 4. **Labor Wedge Decomposition: Rationing vs. Frictional Labor Wedge**

With a positive unit vacancy cost ($\mathbf{c} > 0$), the equilibrium employment, n, is determined by the intersection of $\ln\left(\frac{MPL}{a} - MRC\right)$ and $\ln(\frac{w}{a})$. With no vacancy cost ($\mathbf{c} = 0$), equation (31) reduces to $\ln(\frac{MPL}{a}) = \ln(\frac{w}{a})$, which determines the rationing employment and unemployment ($n^{\mathcal{R}}$ and $u^{\mathcal{R}}$). As shown in Figure 4, the distance between n and $n^{\mathcal{R}}$ is frictional unemployment ($u^{\mathcal{F}}$).

Next, we derive the MRS between consumption and leisure. From equations (11)–(13), we have $\frac{MRS}{a} = \chi \cdot n^{(\phi+\alpha)}$, implying that

$$\ln\left(\frac{MRS}{a}\right) = \ln\chi + (\phi+\alpha)\ln n, \tag{32}$$

which is an increasing function of n. Using equation (32), we depict the rationing labor wedge ($\ln\tau^{\mathcal{R}}$), the overall labor wedge ($\ln\tau$), and $\ln\left(\frac{\tau^{\mathcal{R}}}{\tau}\right)$ in Figure 4. Under normalization (scaled by a), the definition of the labor wedge remains the same: $\tau = \frac{MPL}{MRS} = \frac{\frac{MPL}{a}}{\frac{MRS}{a}}$.

From equations (31) and (32), it can be seen that technology, a, has no impact on either $\frac{MPL}{a} = \alpha n^{\alpha-1}$ or $\frac{MRS}{a} = \chi n^{\phi+\alpha}$, while it has a negative effect on $\frac{\overline{w}}{a}$ in the presence of wage rigidity ($\varphi = 1$). Thus, relative to the high technology scenario, low technology causes the locus of $\ln(\frac{\overline{w}}{a})$ to shift upwards, while leaving the loci of $\ln(\frac{MPL}{a})$ and $\ln(\frac{MPL}{a} - MRC)$ unchanged, as shown in Panel (B) of Figure 4. As a result, lower technology decreases frictional unemployment, increases rationing unemployment, and raises the proportion of the rationing labor wedge in the overall labor wedge, $\ln\left(\frac{\tau^{\mathcal{R}}}{\tau}\right)$.

4.3.2. *Wage Rigidity and the Countercyclical Labor Wedge.* The results of Proposition 2 contradict those in Cheremukhin and Restrepo-Echavarria (2014a). They argue that theories emphasizing wage rigidity and bargaining processes—commonly considered in the search literature—are not helpful in explaining the behavior of the labor wedge. Instead, they conclude that the labor wedge is largely explained by matching efficiency, and unemployment is mainly accounted for by job separation, both related to matching frictions. In contrast, we find that wage rigidity is crucial for generating countercyclical fluctuations in the labor wedge. More importantly, labor wedge fluctuations are largely driven by technology, with job rationing, rather than matching frictions, serving as the central explanatory mechanism, as will be elaborated in the subsequent section.

One pitfall of neglecting wage rigidity is that technological changes no longer affect labor wedge fluctuations. This contradicts the countercyclical property of the labor wedge observed in the data, as previously discussed. With wage rigidity, the labor wedge exhibits cyclicality that parallels unemployment. To see this, note that equations (1), (5), and (13) imply that

$$\tau = \frac{MPL}{MRS} = \frac{\alpha}{\chi} \cdot n^{-(1+\phi)}, \tag{33}$$

which suggests that the labor wedge is a decreasing function of employment (n), or an increasing function of unemployment (u) as shown by equation (1). Once equation (27) determines employment, the labor wedge (τ) and unemployment can also be determined accordingly. It can then be seen that any disturbance affecting unemployment must simultaneously affect the labor wedge in our model.

Instead, when wage rigidity is missing ($\varphi = 0$), equations (27) and (33), tell us that technology fluctuations do not matter for employment, unemployment, and the labor wedge. In other words, the cyclical property of the labor wedge vanishes without wage rigidity.

Our results align with Shimer (2010) (see Chapters 1 and 4) on two points. First, the positive correlation between the labor wedge and unemployment is central to generating labor wedge fluctuations and understanding cyclical labor-market dynamics. Second, wage rigidity is essential for labor-search models to reproduce the observed countercyclical behavior of the labor wedge.

5. Unemployment and Labor Wedge: Contribution of Job rationing vs. Matching Friction

This section calibrates the job-rationing model to the U.S. economy to validate our theoretical results and to assess, via numerical analysis, the roles of job-rationing shocks (technology) and matching-friction shocks (job separation and matching efficiency) in shaping unemployment and the labor wedge. As in Sections 3 and 4, the analysis is conducted within the log-linearized framework.

For the U.S. calibration, we adopt standard parameter values from the literature: $\varphi = 0.95$ for wage rigidity (Leduc and Liu, 2020), $\eta = 0.5$ for bargaining power (Fujita and Ramey, 2007), and $\overline{MRS}/\overline{MPL} = 0.7$ for the outside option (Coles and Kelishomi, 2018). All remaining parameters are set as reported in Table 1. Accordingly, the household-side wage–MRS gap accounts for 89% of labor wedge fluctuations. This share exceeds our estimate of 76.9% but is appropriately aligned with the range documented in the decomposition of Karabarbounis (2014). The job rationing model yields a labor wedge standard deviation of 0.013, which is below the observed value of 0.022. Moreover, the model yields a standard deviation of unemployment of 0.127, which is comparable to the observed value of 0.194.

5.1. **Job Rationing as a Driver of Unemployment and Labor Wedge.** We begin by examining the role of job rationing in shaping unemployment and the labor wedge during recessions. To this end, we feed the observed unemployment series back into the model and, following the definitions in Section 4, compute rationing unemployment $u_t^{\mathcal{R}}$, frictional unemployment $u_t^{\mathcal{F}}$, the rationing labor wedge $\tau^{\mathcal{R}}$, and the frictional labor wedge $\tau^{\mathcal{F}}$.

Figure 5 depicts the decomposition results. The left panel reproduces result similar to Michaillat (2012): increased unemployment in bad times is primarily driven by job rationing, with rationing unemployment rising and frictional unemployment declining. The core argument of Michaillat (2012) remains valid, even under a more generalized specification of wage rigidity. The right panel illustrates the main messages of the paper: first, the labor wedge is countercyclical; second, job rationing accounts for most of its fluctuations, mirroring unemployment. These results are consistent with the qualitative implications of Proposition 2.

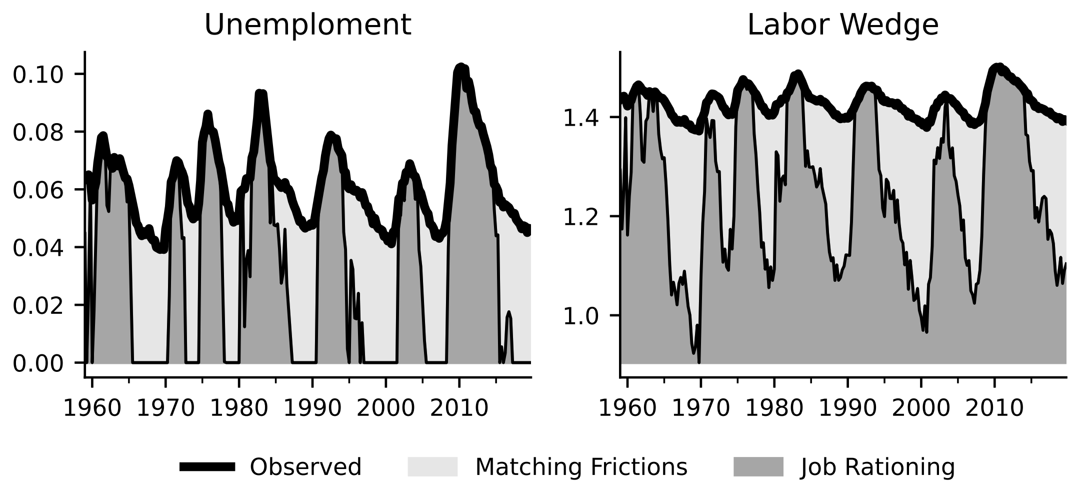


FIGURE 5. **Rationing v.s. Frictions: Unemployment and Labor Wedge**

5.2. **Job Rationing-related vs. matching friction-related fluctuations.** We next quantify the effects of job rationing- and matching friction-related fluctuations on unemployment and the labor wedge by classifying the three shocks considered in the model. As shown in equation (28), rationing employment depends exclusively on technology and is unaffected by either the separation rate or matching efficiency. Accordingly, technology a_t is identified as the job-rationing driver, whereas separation s_t and matching efficiency μ_t are classified as matching-friction drivers. To account for the three model shocks, we feed the observed unemployment series, along with the observed separation and tightness series, back into the job-rationing model and evaluate the respective contributions of technology, separation, and matching efficiency to fluctuations in unemployment and the labor wedge.

We assess the contributions of technology a_t, separation s_t, and matching efficiency μ_t during the Great Recession, via shock-isolation counterfactuals. Specifically, for each shock in turn, we fix the remaining shocks at their pre-recession levels and simulate the implied trajectories of unemployment and the labor wedge. The proximity of each counterfactual to the full-shock benchmark provides a measure of that shock's contribution. Thus, we can construct counterfactual series

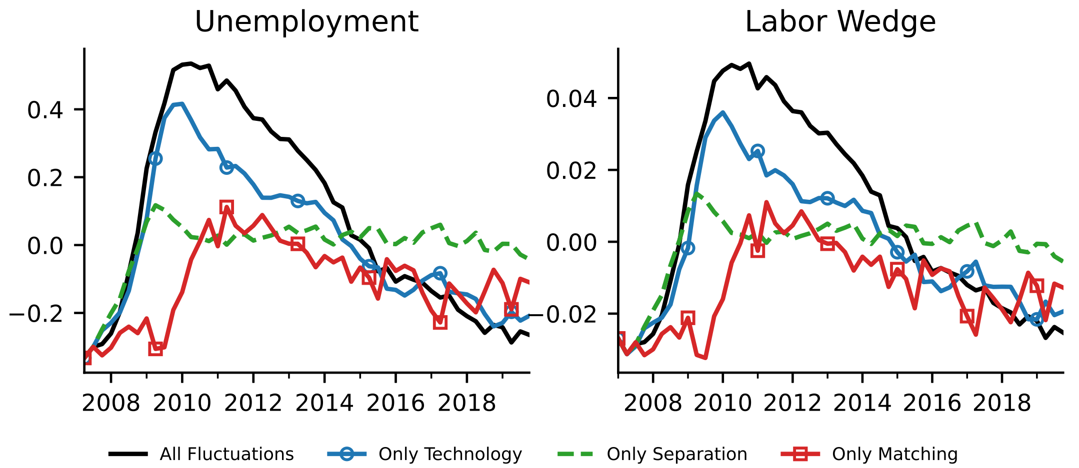


FIGURE 6. **Counterfactual Decomposition of technology, Separation, and Matching Efficiency – Great Recession**

for technology, separation, and matching efficiency. Moreover, counterfactual unemployment and labor wedge series are constructed by shutting down each fluctuation in turn, fixing the corresponding process at its pre-recession levels.

Figure 6 compares the baseline series (with all fluctuations active) to the corresponding counterfactuals. Quantitative results are summarized in Table 4. Figure 6 demonstrated that the counterfactual path with technology as the sole varying shock tracks the observed series closely, whereas the counterfactuals with only separation or only matching efficiency deviate substantially from the data. These results identify technology a_t as the primary source of variation in both unemployment and the labor wedge. Table 4 reports the numerical contributions of technology, separation, and matching to unemployment and the labor wedge. Technology (a_t) accounts for 65.72% of unemployment fluctuations and 66.67% of labor-wedge fluctuations. By contrast, fluctuations in separation (s_t) and matching efficiency (μ_t) play relatively minor roles: separation contributes 12.87% to unemployment and 17.63% to the labor wedge, while matching efficiency contributes 21.41% and 15.70%, respectively.

These counterfactual experiments indicate that during downturns, the job rationing-related shock, rather than the matching friction-related shock, is the primary driver of fluctuations in unemployment and the labor wedge. This evidence not only corroborates Michaillat's (2012) argument concerning unemployment but also reinforces our theoretical result regarding the labor wedge. This finding—that job rationing plays a central role in both unemployment and the labor wedge, underscoring the critical importance of wage rigidity—stands in sharp contrast to Cheremukhin and Restrepo-Echavarria (2014a), who ignores the role of job rationing during downturns.

TABLE 4. **Contribution of Technology, Separation, and Matching Efficiency–Great Recession**

	Unemployment	Labor Wedge
Technology fluctuations: a_t	65.72%	66.67%
Separation fluctuations: s_t	12.87%	17.63%
Matching fluctuations: μ_t	21.41%	15.70%

While Cheremukhin and Restrepo-Echavarria (2014a) emphasize matching frictions as the primary source of unemployment and the labor wedge, our results offer a contrasting perspective. In their model, job rationing is overlooked, and matching frictions exclusively account for fluctuations in unemployment and the labor wedge in the absence of job rationing. Consequently, the time-varying separation rate drives the unemployment fluctuations, and the time-varying matching efficiency explains the labor wedge fluctuations. Moreover, they argue that wage rigidity and bargaining processes commonly considered in the search literature are not useful for explaining the labor wedge. By contrast, when wage rigidity is incorporated and job rationing arises, technology —the job rationing-related shock— becomes the primary driver of fluctuations in both unemployment and the labor wedge, overshadowing separation and matching efficiency.

5.3. **Time-Varying Labor Force Participation.** The preceding analysis assumes a fixed labor force participation rate, whereas in reality, labor force participation is procyclical. We now relax this assumption to examine the robustness of the main results presented in this section.

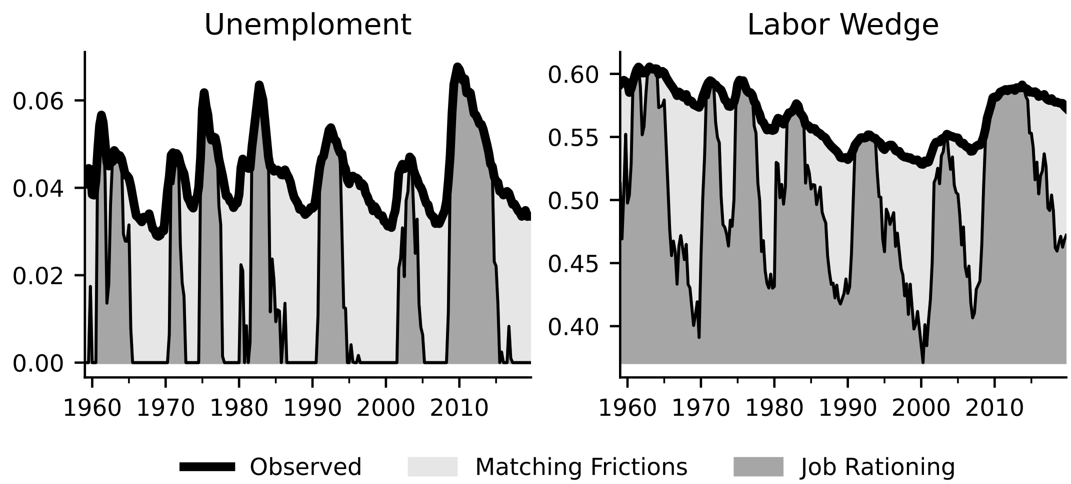


Figure 7. **Rationing v.s. Frictions: Unemployment and Labor Wedge in the Model with Time-Varying Labor Force**

Let LF_t denote the labor force at time t. Unemployment in equation (1) is thus redefined as

$$u_t = LF_t - n_t. \tag{34}$$

With a time-varying labor force participation rate, rationing unemployment is modified as

$$u_t^{\mathcal{R}} = \max\left\{0, 1 - \frac{n_t^{\mathcal{R}}}{LF_t}\right\}, \tag{35}$$

and frictional unemployment is $u_t^{\mathcal{F}} = u_t - u_t^{\mathcal{R}}$. The definitions of employment and the labor wedge remain unchanged.

To account for time-varying labor force participation, we follow Clymo (2020) and match the model's participation rate to its empirical counterpart. For comparability with the benchmark model with a fixed unitary labor force, we employ the same parameterization as in the preceding analysis.

Figure 7 presents the decomposition results, which are consistent with the benchmark findings in Figure 5. Job rationing continues to be the primary driver of the increase in unemployment and the labor wedge during recessions. This conclusion remains robust when allowing for time-varying labor force participation.

Table 5. **Contribution of Technology, Separation, and Matching Efficiency in the Model with Time-Varying Labor Force – Great Recession-**

	Unemployment	Labor Wedge
Technology fluctuations: a_t	58.08%	53.67%
Separation fluctuations: s_t	17.34%	23.29%
Matching fluctuations: μ_t	24.58%	23.04%

The relative contributions of job rationing and matching friction shocks remain consistent across the fixed and time-varying labor force models. Table 5 shows that although the contribution of technology declines, it remains the primary driver, accounting for more than half of the fluctuations in both unemployment and the labor wedge. Interestingly, the role of matching efficiency becomes more pronounced when labor force participation varies over the business cycle.

6. Conclusion

To thoroughly examine the cyclicality of the labor wedge, we construct a standard DMP model with constant marginal returns and flexible wages, following Hagedorn and Manovskii (2008), and then extend it into a job-rationing framework by incorporating diminishing marginal returns and wage rigidity, as in Michaillat (2012). We show that the standard DMP model fails to simultaneously replicate the observed volatility of the labor wedge and the empirical finding of Karabarbounis (2014) that its countercyclical movements are primarily driven by the gap between the real wage and the MRS. By contrast, the extended job-rationing model is able to successfully capture the two cyclical features of the labor wedge.

In the job rationing model, sticky real wages induce labor rationing, pushing households off their labor supply curves, while on the firm side the MPL continues to equal the wage in the absence of matching frictions during downturns. This creates a pronounced gap between the wage and the MRS on the household side. A higher degree of wage rigidity amplifies the procyclicality of the MRS relative to the MPL, rendering the labor wedge strongly countercyclical over the business cycle. By contrast, in the standard DMP model with flexible wages, fluctuations in the labor wedge stem solely from matching frictions, which exert only limited influence on the household-side MRS. As a result, the model fails to replicate the observed high procyclicality of the MRS, and the countercyclical labor wedge is primarily driven by the wage–MRS gap. Since fluctuations in the labor wedge primarily reflect variations in the gap between the real wage and the MRS, Karabarbounis (2014) argues that business cycle theories of the labor wedge should concentrate on the household side of the labor market. Our analysis, in both analytical and numerical respects, provides a theoretical foundation for this perspective.

We also show that job rationing accounts for most of the fluctuations not only in unemployment but also in the labor wedge. Our finding that job rationing is a dominant driver of countercyclical labor wedge fluctuations is at odds with the existing literature that emphasizes the importance of matching frictions in explaining labor wedge fluctuations. However, it aligns with studies highlighting the role of wage rigidity in generating countercyclical movements in both unemployment and the labor wedge, as observed in the data.

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Appendix A. Data Description and Some Parameter Calibration

This appendix describes the data plotted in Figure 1 and the data series used in the steady-state approximation of Section 3.

- Unemployment, u_t: The unemployment rate is the seasonally adjusted series (LNS14000000) from the U.S. Bureau of Labor Statistics. We convert the monthly unemployment rate to quarterly data by averaging the monthly figures. The quarterly unemployment rate is detrended using the HP filter with a smoothing parameter of 10^5, following Michaillat (2012).

- Real GDP per capita, y_t: Real GDP is the seasonally adjusted quarterly series of real GDP in 2017 chain dollars, line 1 of the NIPA Table 1.1.6 from the U.S. Bureau of Economic Analysis. The quarterly population level is the quarterly average of series LNS10000000 from the U.S. Bureau of Labor Statistics. The quarterly series is detrended using the HP filter with a smoothing parameter of 10^5.

- Marginal Product of Labor, MPL_t, Marginal Rate of Substitution, MRS_t, and Labor wedge, τ_t: We use equations (4) and (5) to construct the marginal product of labor and equation (13) to construct the marginal rate of substitution. Following equation (17), the labor wedge is computed as $\tau_t = \frac{MPL_t}{MRS_t}$.

- Technology, a_t: We follow Fujita and Ramey (2007) and use Nonfarm Business Sector: Output per Worker for All Workers from the U.S. Bureau of Labor Statistics as the measure of technology.

- Job Separation Rate, s_t: We construct the monthly separation rate following Shimer (2005) using data on the seasonally adjusted employment level (LNS12000000), the unemployment level (LNS13000000), and the number unemployed for less than 5 weeks (LNS13008396) from the U.S. Bureau of Labor Statistics.

- Matching Efficiency, μ_t: Equation (24) implies that matching efficiency is given by $\mu_t = \frac{f_t}{\theta_t^{1-\xi}}$. We construct tightness $\theta_t = v_t/u_t$ using the observed unemployment u_t and vacancies v_t. We construct the monthly job-finding rate, f_t, following Shimer (2005) using the same data in constructing s_t. Thus, given $\xi = 0.6$ based on our calibration, we derive matching efficiency, μ_t.

Appendix B. Diminishing Marginal Returns

We repeat the numerical exercise in Table 2 under diminishing marginal returns ($\alpha = 2/3$). As in the DMP model, the wage is the bargaining wage given by (16). From Table B.1, the DMP continues to exhibit a trade-off in capturing the volatility of the labor wedge and it fluctuations that are mainly driven by the wage–MRS gap.

Table B.1. **Simulation Results:** $\alpha = 2/3$

σ^τ: Standard Deviation of τ (Data: $\sigma^\tau = 0.022$)	$\eta = 0.25$	$\eta = 0.5$	$\eta = 0.75$	$\eta = 0.9$
$\overline{MRS}/\overline{MPL} = 0.1$	0.018	0.013	0.0106	0.0098
$\overline{MRS}/\overline{MPL} = 0.3$	0.016	0.012	0.0102	0.0096
$\overline{MRS}/\overline{MPL} = 0.5$	0.014	0.011	0.0098	0.0093
$\overline{MRS}/\overline{MPL} = 0.7$	0.012	0.010	0.0091	0.0088
$\overline{MRS}/\overline{MPL} = 0.9$	0.009	0.008	0.0070	0.0068
Contribution: τ^W (Household Component) (Data: Contribution of $\tau^W = 79.6\%$)	$\eta = 0.25$	$\eta = 0.5$	$\eta = 0.75$	$\eta = 0.9$
$\overline{MRS}/\overline{MPL} = 0.1$	−65.76%	0.57%	56.19%	83.71%
$\overline{MRS}/\overline{MPL} = 0.3$	−44.38%	16.66%	64.33%	86.92%
$\overline{MRS}/\overline{MPL} = 0.5$	−19.91%	34.24%	72.78%	90.17%
$\overline{MRS}/\overline{MPL} = 0.7$	9.14%	53.56%	81.56%	93.46%
$\overline{MRS}/\overline{MPL} = 0.9$	44.76%	74.86%	90.67%	96.79%

Appendix C. Proofs of Lemmas and Propositions

Proof. **LEMMA 1**

To prove that $\varepsilon_a^{MPL} > 0$ and $\varepsilon_a^{MRS} > 0$, we first need to establish $\varepsilon_a^n > 0$.

Using equation (26), we derive that

$$\varepsilon_a^n = \frac{\varphi\frac{\overline{w}}{a}}{(1-\alpha)(MRC+\varphi\frac{\overline{w}}{a}) + (1+\phi)(1-\varphi)(1-\eta)\psi n^{\alpha+\phi} + \Phi'(n)n\mathbf{c}}. \tag{C.1}$$

Because $n/(1-n)$ increases in n, $\Phi(n) = \frac{1-\beta(1-s)}{\beta(1-s)}\frac{1}{\mu^{\frac{1-2\xi}{1-\xi}}}(\frac{s}{1-s}\frac{n}{1-n})^{\frac{\xi}{1-\xi}} + (1-\varphi)\eta(\frac{1}{\mu}\frac{s}{1-s}\frac{n}{1-n})^{\frac{1}{1-\xi}}$ is an increasing function of n and $\Phi'(n)$. Thus, we have $\varepsilon_a^n > 0$.

Due to $MPL = \alpha a n^{\alpha-1}$, we rewrite equation (26) as follows:

$$\Psi(n)MPL - \varphi\overline{w} = \Phi(n)\mathbf{c}a, \tag{C.2}$$

where $\Psi(n) = (1-(1-\varphi)\eta) - (1-\varphi)(1-\eta)\frac{\psi}{\alpha}n^{1+\phi} = (\Phi(n)\mathbf{c}a + \varphi\overline{w})/MPL > 0$. Thus, using equation (C.2), we derive that

$$\varepsilon_a^{MPL} = \frac{\left[\Phi'(n)n\mathbf{c} - \Psi'(n)MPL\frac{n}{a}\right]\varepsilon_a^n + \Phi(n)\mathbf{c}}{\frac{MPL}{a}\Phi(n)}. \tag{C.3}$$

Because $\Psi'(n) = -(1-\varphi)(1-\eta)(1+\phi)\frac{\psi}{\alpha}n^{\phi} < 0$, we have $\varepsilon_a^{MPL} > 0$.

To prove $\varepsilon_a^{MRS} > 0$, we need to show that $\varepsilon_a^y > 0$ first. Because $MPL = \alpha y/n$, we have that

$$\varepsilon_a^y = \varepsilon_a^{MPL} + \varepsilon_a^n > 0. \tag{C.4}$$

Then, we derive $MRS = -\frac{U_n}{U_c} = \chi n^{\phi}c$ from $U(c,n) = \ln c - \chi\frac{n^{1+\phi}}{1+\phi}$. The elasticity of MRS with respect to technology is

$$\varepsilon_a^{MRS} = \frac{\partial \ln MRS}{\partial \ln a} = \phi\cdot\varepsilon_a^n + \varepsilon_a^c.$$

Since the goods market-clearing condition implies $y = c$, we have $\varepsilon_a^c = \varepsilon_a^y$. With $\phi > 0$, equations (C.1) and (C.4) imply

$$\varepsilon_a^{MRS} = \phi\cdot\varepsilon_a^n + \varepsilon_a^y > 0. \tag{C.5}$$

□

Proof. **PROPOSITION 1**

Lemma 1 immediately yields Proposition 1. Given equation (C.1) and $\phi > 0$, $\varepsilon_a^{MPL} = \varepsilon_a^y - \varepsilon_a^n$ and $\varepsilon_a^{MRS} = \phi\varepsilon_a^n + \varepsilon_a^y$, the elasticity of labor wedge with respect to technology is

$$\varepsilon_a^\tau = \varepsilon_a^{MPL} - \varepsilon_a^{MRS} = \varepsilon_a^y - \varepsilon_a^n - (\phi \cdot \varepsilon_a^n + \varepsilon_a^y) = -\big(1+\phi\big) \cdot \varepsilon_a^n < 0. \tag{C.6}$$

□

Proof. **PROPOSITION 2**

To prove $\varepsilon_a^{\tau,\mathcal{R}} < 0$, we first need to show that $\varepsilon_a^{n,\mathcal{R}} > 0$. From Proposition 1, we have $\varepsilon_a^\tau = \varepsilon_a^{MPL} - \varepsilon_a^{MRS} = -(1+\phi)\,\varepsilon_a^n < 0$. With the definition of the rationing labor wedge, equation (30), $\varepsilon_a^{\tau,\mathcal{R}} = -(1+\phi)\varepsilon_a^{n,\mathcal{R}}$. From equation (28), we derive that

$$\varepsilon_a^{n,\mathcal{R}} = \frac{\varphi\frac{\overline{w}}{a}}{(1-\alpha)\varphi\frac{\overline{w}}{a} + (1+\phi)(1-\eta)\psi(n^R)^{1+\phi}} > 0. \tag{C.7}$$

Thus, we have $\varepsilon_a^{\tau,\mathcal{R}} = -(1+\phi)\varepsilon_a^{n,\mathcal{R}} > 0$.

To prove $\frac{\partial(\frac{n^{\mathcal{R}}}{n})}{\partial a} > 0$, we use z to denote $\frac{n^{\mathcal{R}}}{n}$ and obtain the following expression from equations (27) and (28):

$$\frac{A\frac{1}{n^{1+\phi}} - B}{A\frac{1}{n^{1+\phi}} - Bz^{1+\phi}} z^{1-\alpha} = 1 + \frac{\Phi(n)\mathbf{c}}{\Lambda(n^{\mathcal{R}})}. \tag{C.8}$$

Here, $A = \alpha(1-(1-\varphi)\eta)$ and $B = (1-\varphi)(1-\eta)\psi$. Moreover, $\Lambda(n^{\mathcal{R}}) = \big[A - B(n^{\mathcal{R}})^{1+\phi}\big]\,(n^{\mathcal{R}})^{\alpha-1} > 0$ is a decreasing function in $n^{\mathcal{R}}$, i.e., $\Lambda'(n^{\mathcal{R}}) < 0$.

Using equation (C.8), we derive that

$$\frac{\partial(\frac{n^{\mathcal{R}}}{n})}{\partial a} = \frac{\partial z}{\partial a} = \frac{\frac{\Phi'(n)\mathbf{c}}{\Lambda(n^{\mathcal{R}})} - \frac{\Phi(n)\mathbf{c}}{\Lambda(n^{\mathcal{R}})^2}\Lambda'(n^{\mathcal{R}}) + \Xi\frac{\partial n}{\partial a}}{(1-\alpha)\left[\frac{A\frac{1}{n^{1+\phi}} - B}{A\frac{1}{n^{1+\phi}} - Bz^{1+\phi}}\right] z^{-\alpha}}. \tag{C.9}$$

Here, $\Xi = \frac{\frac{(1+\phi)A}{n^{2+\phi}}B(1-z^{1+\phi})}{(A\frac{1}{n^{1+\phi}} - Bz^{1+\phi})^2}$. Because $z = \frac{n^{\mathcal{R}}}{n} \leq 1$, we know that $\Xi \geq 0$. From the proof of Proposition 1, we know that $\frac{\partial n}{\partial a} > 0$. In addition, equation (27) implies that $A\frac{1}{n^{1+\phi}} - B = \frac{\Phi(n)\mathbf{c} + \varphi\frac{\overline{w}}{a}}{n^{\alpha+\phi}} > 0$ and equation (28) implies that $A\frac{1}{n^{1+\phi}} - Bz^{1+\phi} = \frac{\varphi\frac{\overline{w}}{a}}{(n^{\mathcal{R}})^{\alpha-1}n^{1+\phi}} > 0$. Thus, equation (C.9) indicates that $\frac{\partial(\frac{n^{\mathcal{R}}}{n})}{\partial a} > 0$.

Thus, we immediately have

$$\frac{\partial(\frac{\tau^{\mathcal{R}}}{\tau})}{\partial a} = -(1+\phi)\left(\frac{n^{\mathcal{R}}}{n}\right)^{-(2+\phi)} \frac{\partial(\frac{n^{\mathcal{R}}}{n})}{\partial a} < 0.$$

$\square$