Foreign Exchange Intervention under Dominant Currency Paradigm: A New-Monetarist Approach

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Abstract

This paper studies the real effects of foreign exchange intervention (FXI) under the dominant currency paradigm. I develop a central-periphery monetary search model in which domestic transactions are settled in local currency while international transactions are settled in dominant currency, and prices are flexible. In this environment, FXI works by changing the relative purchasing power of currencies rather than through the standard sticky-price expenditure-switching channel. A depreciation implemented through reserve purchases redistributes purchasing power away from local-currency holders in the intervening country and toward dominant-currency holders at home and abroad. When intervention is anticipated, this redistribution feeds back into production decisions: sectors earning dominant currency expand, while the domestic sector in the intervening country contracts. Under asymmetric information, FXI also creates expenditure switching across exporters and may generate incentives for competitive devaluation.

Keywords: exchange rate intervention, international reserves, monetary policy pass-through, competitive devaluation

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1 Introduction

Foreign exchange intervention (FXI) is widely used in practice, especially by emerging and small open economies, yet its real effects remain theoretically unsettled. Recent empirical work shows that FXI is pervasive, often effective at smoothing exchange rates, and disproportionately used through foreign-exchange purchases in spot markets, particularly by emerging economies (Fratzscher et al. (2019)). At the same time, a large share of world trade is invoiced in a dominant currency like the U.S. dollar rather than in the producer's or importer's currency (Gopinath et al. (2020)). These two facts together pose a puzzle. If international trade is largely settled in dominant currency, and if exchange-rate movements therefore have limited pass-through to export prices in the short run, through what mechanism can FXI still generate real effects?

The conventional answer comes from standard open-economy models in which nominal rigidities and expenditure switching are central. In the classic producer-currency-pricing environment, a depreciation improves competitiveness by lowering export prices in foreign-currency terms. In local-currency-pricing environments, pass-through is gone and FXI works against exporting sector. Under the dominant currency paradigm (DCP), however, these standard channels are attenuated: export prices of non-dominant-currency countries become relatively insensitive to bilateral exchange-rate movements against the dominant currency, and the expansionary effect of depreciation works mainly through reducing imports and redirecting expenditure toward domestic goods. This view is incomplete when imported goods are not close substitutes for domestic goods, and it is also hard to reconcile with the prevalence of spot-market FXI in practice.

This paper proposes an alternative mechanism for the real effects of FXI under DCP. I develop a central-periphery monetary search model in the spirit of Lagos and Wright (2005) and Rocheteau and Wright (2005), in which domestic transactions are settled in local currency while international transactions are settled in dominant currency. In this environment, FXI affects the economy not by changing preset trade prices in certain currency terms, but by changing the relative purchasing power of currencies. An intervention that depreciates the domestic currency against the dominant currency lowers the real value of domestic-currency claims and raises the real value of dominant-currency claims. The key implication is a wealth redistribution effect: FXI redistributes purchasing

power away from agents who are paid in domestic currency and toward agents who receive dominant-currency income. Because tradable-sector producers earn dominant currency while domestic-sector producers earn local currency, the intervention alters sectoral incentives and generates real effects even under flexible prices. This is the central mechanism of the paper.

The model is motivated by several empirical patterns, which I illustrate using Taiwan as a case study. Taiwan is a natural example because its trade is predominantly invoiced in U.S. dollars, and available proxy measures suggest that foreign exchange intervention is tilted toward depreciation purchases of foreign reserves. Moreover, an increasing share of import payments is made directly in foreign currency rather than through contemporaneous conversion from domestic currency and more share of export receipts are retained in the foreign currency instead of direct conversion to domestic currency. This pattern indicates that importers accumulate foreign currency in advance for transaction purposes, providing empirical support for modeling importers as choosing currency portfolios in anticipation of cross-border payments. At the same time, the pattern also suggests that exporters have incentive to keep revenue in the foreign currency form. Finally, the data show substantial variation in import and export price indices, which is consistent with treating prices as flexible rather than fixed by long-lived nominal rigidities. Taken together, these facts support the three key ingredients of the model: dominant-currency settlement in international trade, transaction-driven currency holding decisions by private agents, and different currencies of revenue across sectors.

The paper delivers three main results. First, FXI generates a wealth redistribution effect from local-currency holders in the intervening country to dominant-currency holders both at home and abroad. In the pure-shock case, output does not move because production decisions are made before the intervention is realized, but purchasing power is redistributed immediately: domestic producers in the intervening country lose, while dominant-currency earners benefit. Second, when FXI is anticipated, the redistribution effect feeds back into production decisions. Producers who expect to be paid in dominant currency increase labor supply and output, whereas those who expect to be paid in depreciating local currency reduce them. Consequently, sectors earning dominant currency expand, while the domestic sector in the intervening country contracts. Third, under asymmetric information, a scenario which only domestic agents foresee the intervention, FXI also creates an

expenditure-switching effect across exporters, because only producers in the intervening country adjust ex ante. This gives the other peripheral country an incentive to intervene as well, making competitive devaluation a natural equilibrium outcome when policy transparency is limited.

These results carry several policy implications. A first implication is that FXI need not harm foreign exporters when trade is denominated in dominant currency. In the anticipated case, exporters in both foreign countries can expand because they are paid in the currency whose purchasing power rises. A second implication is that the domestic non-tradable sector of the intervening country bears the cost of intervention: it is the only sector systematically harmed across scenarios because it is paid in the depreciating currency. A third implication is that the widespread use of spot-market intervention, rather than simple inflationary finance, is easier to understand in a framework where policy works by altering the relative purchasing power of currencies. Pure money creation depreciates local currency, but reserve purchases simultaneously raise the scarcity and purchasing power of dominant-currency claims held by the private sector. A final implication is that greater transparency about intervention can reduce the likelihood of competitive devaluation. When intervention is publicly observed, agents in other countries can adjust their decisions accordingly, eliminating the misperceptions that give rise to the expenditure-switching effect in the asymmetric-information case.

Literature review. This paper relates to four strands of literature. The first is the literature on exchange-rate regimes and FXI. Friedman (1953) argued for flexible exchange rates as a mechanism for relative-price adjustment, while Johnson (1969) emphasized that smaller and less developed economies may prefer exchange-rate arrangements tied to major trading partners. More recent work studies FXI as an optimal policy instrument in environments with financial frictions, segmented markets, or occasionally binding constraints. Fanelli and Straub (2021) show that FXI can be an optimal stabilization tool when asset-market segmentation generates a pecuniary externality. Amador et al. (2020) study exchange-rate policy at the zero lower bound and provide a framework in which exchange-rate management can be desirable even for advanced economies. Hassan et al. (2023) develop a risk-based theory of exchange-rate stabilization in which countries stabilize against the currency of the largest economy, helping explain why anchor currencies emerge endogenously.

Relative to this literature, my paper studies FXI through a different channel: not segmented financial markets or asset-pricing risk, but the redistribution induced by changes in the real return to currencies that are used asymmetrically in domestic and international transactions.

The second strand is the literature on dominant currency pricing, invoicing, and pass-through. Gopinath et al. (2010) show that invoicing currency is central for exchange-rate pass-through. Gopinath et al. (2020) formalize the dominant currency paradigm and document that the U.S. dollar plays a disproportionate role in global trade pricing and transmission. Amiti et al. (2022) study how firms choose invoicing currencies and why those choices matter for international adjustment, while Egorov and Mukhin (2023) characterize optimal monetary policy under dollar pricing. Gopinath and Stein (2021) further emphasize that dominant-currency status is tied not only to trade invoicing but also to the broader financial role of the dominant currency. My paper complements this literature by showing that, even under flexible prices and DCP, exchange-rate policy can still have real effects through a currency-returns channel. In that sense, the paper does not reject the DCP view; rather, it adds a distinct transmission mechanism that becomes operative when the dominant currency is not only the invoicing unit, but also the settlement asset used in international transactions.

This emphasis on settlement, rather than invoicing alone, also connects the paper to the literature on reserve accumulation and international liquidity. Once cross-border payments require dominant-currency balances, official reserve operations matter not only for public balance sheets but also for the private sector's access to the means of international payment. International reserve holdings expanded dramatically over the past two decades, with the U.S. dollar remaining the dominant reserve currency. Existing work has studied reserve accumulation as a response to sovereign risk, capital controls, credit frictions, and macroprudential concerns. Aizenman and Marion (2004) emphasize the role of reserves in the presence of sovereign risk and costly taxation. Bacchetta et al. (2013) show how reserve accumulation can interact with capital controls in fast-growing economies with constrained households. Arce et al. (2019) interpret reserves as a macroprudential tool when private agents fail to internalize future borrowing constraints. My paper is complementary to this literature. While reserve purchases appear in the model as the operational counterpart of FXI, the main object of interest is not the optimal stock of reserves per se, but how FXI generates real

effects once reserve operations alter the scarcity and purchasing power of the dominant currency in circulation.

The fourth strand is the monetary-search literature on the use and circulation of currencies. Lagos and Wright (2005) and Rocheteau and Wright (2005) provide the basic framework used here. Zhang (2014) studies how international currency status can arise from informational advantages. My contribution is to embed DCP and FXI into a monetary-search environment in which transaction needs determine currency portfolios. This modeling choice is not incidental. It allows the paper to connect the invoicing-currency facts emphasized by the DCP literature with the reserve-accumulation facts emphasized by the FXI literature, while delivering a sharp prediction about who gains and who loses from intervention.

Relative to the existing literature, the paper's main contribution is therefore twofold. Substantively, it identifies a new mechanism through which FXI affects real activity under DCP: by changing the relative purchasing power of currencies and thereby redistributing wealth across sectors and countries. Methodologically, it brings together dominant-currency trade, currency portfolio choice, and FX intervention in a unified Lagos-Wright framework. This combination allows the paper to rationalize three empirical regularities at once: the pervasive use of FXI, the preference for spot-market operations, and the distributional tension between tradable and non-tradable sectors in intervening economies.

The remainder of the paper proceeds as follows. Section 2 presents empirical patterns that motivate the model. Section 3 lays out the environment and characterizes the monetary equilibrium. Section 4 studies FXI under three information structures: pure shock, anticipated intervention, and asymmetric information. Section 5 concludes. All derivations and proofs are provided in the Appendix.

2 Empirical Motivation

This section documents several empirical patterns from Taiwan that motivate the key ingredients of the model. The purpose is not to identify a causal effect of foreign exchange intervention (FXI), but rather to show that the institutional and behavioral assumptions embedded in the model are empirically relevant in an economy where FXI has been a recurring policy concern. In particular,

I document four facts. First, Taiwan's international trade is overwhelmingly invoiced in U.S. dollars, consistent with the dominant currency paradigm (DCP). Second, an increasing share of import payments is made directly in foreign currency, suggesting that importers accumulate foreign currency in advance to settle cross-border transactions. At the same time, an increasing share of export revenue is retained in foreign currency, suggesting that exporters may also find it optimal to hold foreign currency rather than immediately convert it into local currency. Third, import and export prices display substantial variation over time, supporting the assumption of flexible prices. Fourth, available proxy measures suggest that FXI in Taiwan is tilted toward net purchases of foreign reserves associated with depreciation pressure. Taken together, these facts support modeling international trade as settled in dominant currency and treating private currency holding decisions as an economically meaningful margin.

Data. I focus on Taiwan because it is a natural empirical counterpart to the environment studied in the model. Taiwan's trade is heavily invoiced in U.S. dollars, its central bank has frequently been discussed in the policy debate on foreign exchange intervention, and the available data allow me to connect trade invoicing, currency-use behavior, price movements, and intervention proxies in a unified way. The following Table 1 summarizes the length, frequency, and source of data.

Name	Length	Frequency	Source
Invoicing Currency	2019-2025	Annual	Ministry of Finance of Taiwan
Currency Converting Rate	1987-2025	Annual	Central Bank of Taiwan
Import/Export Price Index	1981/01-2026/02	Monthly	National Statistics of Taiwan
FXI/GDP	2000/01-2025/05	Monthly	Adler et al. (2025)

Table 1: Data

Dominant-currency pricing. Figure 1 indicates that the external trade of Taiwan follows dominant currency paradigm. Specifically, according to data from the Ministry of Finance of Taiwan, the U.S. dollar plays a dominant role. Over the period 2019–2025, the ratio of trade invoiced in USD keeps increasing for Taiwan's imports and exports. In 2025, over 80% of imports and 90% of exports are invoiced in USD. This pattern makes Taiwan a natural case for studying exchange rate policy under DCP. In particular, when most cross-border transactions are priced and settled

in a dominant currency, bilateral exchange rate movements between the domestic currency and the dominant currency become central for firms' transaction needs and for the transmission of FXI.

Currency holding for trade settlement. Figure 2 reports the share of export revenue that is not immediately converted into New Taiwan dollars (NTD) upon receipt and the share of import payments that is not made in NTD upon settlement. Both series rise over time.

The import-side pattern directly supports the currency-portfolio decision of final-goods producers in the model. If a growing share of import payments is made in foreign currency, importers must accumulate foreign currency in advance to settle cross-border transactions. The exporter-side pattern is also informative: a growing share of export revenue is retained in foreign currency, suggesting that exporters may have incentives to hold foreign currency rather than immediately convert it into local currency. Taken together, these facts suggest that foreign-currency holdings serve an active economic role in international transactions, rather than simply appearing as passive balance-sheet positions. This is the key behavioral margin emphasized in the model.

Import/Export price index. Figure 3 plots the inflation rates of Taiwan's import and export price indices. Both series vary substantially over time rather than remaining fixed for long periods. A simple t-test rejects the null hypothesis that average import or export inflation is equal to zero at the 1 percent significance level. These patterns are consistent with treating trade prices as flexible, rather than relying on long-lived nominal rigidities to generate real effects of exchange rate policy.[1] This empirical feature is important for the model because the mechanism emphasized in the paper does not depend on sticky prices. Instead, it relies on changes in the relative purchasing power of currencies and on the resulting redistribution across agents.

FXI proxy. Although the Central Bank of Taiwan does not publish a comprehensive official time series of intervention, Adler et al. (2025) construct proxy measures of FXI from central-bank balance-sheet data for countries that do not disclose intervention records. Figure 4 plots the ratio of this FXI proxy to GDP for Taiwan. A simple t-test rejects the null hypothesis that the average

[1] In fact, most of international trade is B2B and relies on contracts. In such a relationship, it is both price and quantity are either fixed or flexible. See Barro (1977) for details.

level of intervention is non-positive at the 1 percent significance level. This finding suggests that intervention in Taiwan has been biased toward net purchases of foreign reserves, which is consistent with depreciation-oriented intervention.

This pattern is useful for the analysis that follows. The policy experiment studied in the model involves a peripheral country that purchases dominant-currency reserves through open market operations in order to depreciate its own currency. The Taiwan evidence suggests that this type of intervention is not merely hypothetical, but closely related to observed policy behavior in practice.

Taken together, the empirical patterns in this section motivate the two central ingredients of the model. First, Taiwan's trade structure strongly supports modeling international transactions under the dominant currency paradigm. Second, the evidence on foreign-currency import payments suggests that private agents hold foreign currency in advance for transaction purposes, making currency-portfolio decisions an economically meaningful object. Meanwhile, the nearly one-hundred percent export revenues retained in foreign currency implies private agents might benefit from this decision. Combined with the observed flexibility of trade prices and the evidence of depreciation-oriented reserve purchases, these facts provide a natural empirical foundation for the model developed in the next section.

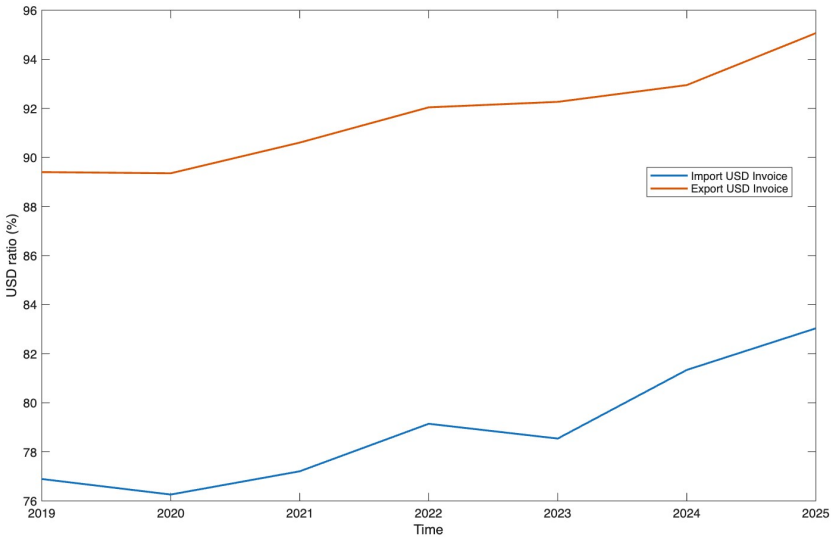

Figure 1: Trade Invoiced in USD

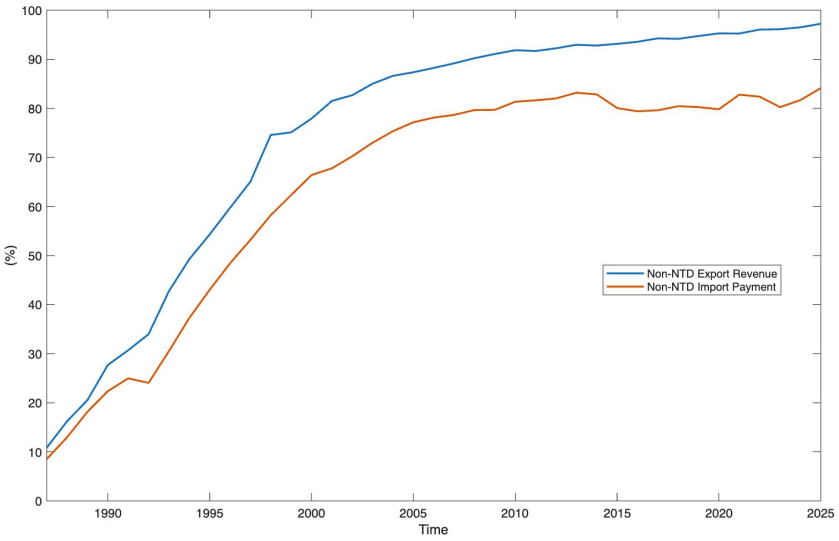

Figure 2: Currency Holding Behavior

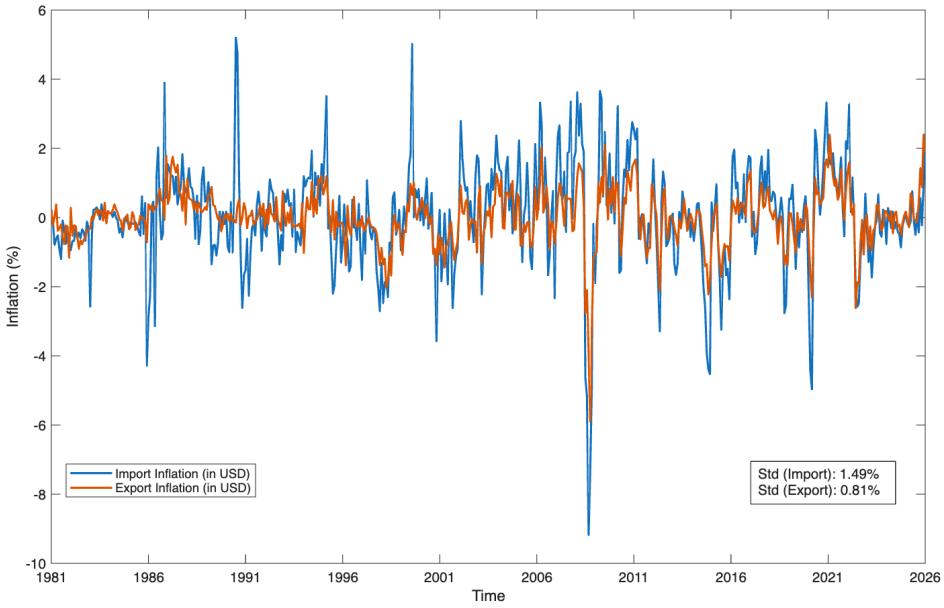

Figure 3: Trade Price Inflation Rate

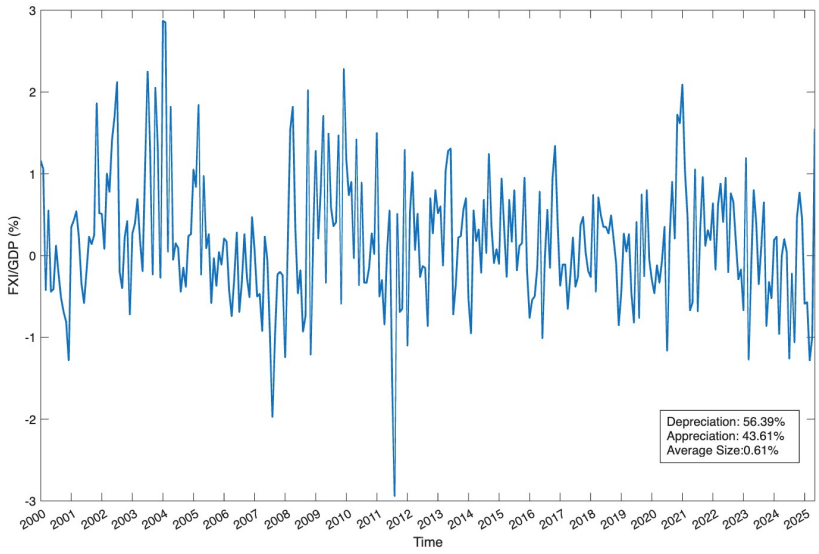

Figure 4: FXI/GDP

3 The model

The model follows the monetary search-theoretic framework developed by Lagos and Wright (2005) and Rocheteau and Wright (2005). The DM-CM structure allows us to see the redistributional effect of monetary policy between the exporting sector and domestic sector more clearly. Additionally, to address the potential competitive devaluation issue, I adopt central-peripheral model as in Corsetti et al. (2000) with non-tradable sector (domestic sector). There are three countries, denoted by A, B, and C, in an imagined economy. Hereinafter, country C will be referred to as the *Center*, while countries A and B together constitute the *Periphery*.

Agent, time, production, and preferences. Each country has three types of agents with equal share within a country: domestic producers, denoted by N, exporters, denoted by T, and final goods producers, denoted by Y. Without loss of generality, I assume the population size is equal across the three countries and is normalized as 1 in each country. Additionally, there is a

competitive global tradable-input aggregator, denoted by F.[2] Throughout the paper, the variable is denoted by X_{ijt}^k, which the superscript k represents the origin country and the subscript i represents the sector, j represents the owner's country, and t represents the time period. The subscript t is used only when necessary. The superscript is only used on currency holding.

Time is discrete and indexed by $t = 0, 1, 2, \ldots$ to infinity, and there are two stages within each period. In stage 1, the final goods sectors produce final goods by combining the domestic intermediate goods and the foreign intermediate goods. The production functions are characterized by standard CES functions:

$$Y_j = \left[\omega_D I_{Nj}^{\rho_D} + \omega_F I_{Fj}^{\rho_D}\right]^{\frac{1}{\rho_D}},\ j \in \{A, B, C\},\ \rho_D = \frac{\sigma_D - 1}{\sigma_D},\ \sigma_D > 1. \tag{1}$$

The final goods producers use domestic intermediate goods I_{Nj} (non-tradable) and foreign intermediate goods I_{Fj} (tradable) as input to produce final goods and carry these final goods into stage 2 to trade or consume in a centralized market.

At the same time, there is a competitive global intermediate goods sectors with infinite many producers who produce foreign intermediate goods for the whole world by combining goods from the exporting sector from each country. The production function is characterized by a standard CES function:

$$I_F = \left[\sum_j \omega_{Fj} I_{Tj}^{\rho_F}\right]^{\frac{1}{\rho_F}},\ j \in \{A, B, C\},\ \rho_F = \frac{\sigma_F - 1}{\sigma_F},\ \sigma_F > 1. \tag{2}$$

Finally, the domestic sector and exporting sector in each country produce intermediate goods in a linear production function with labor as the only input. The production functions are characterized as follows.

$$I_{ij} = A_{ij}\ell_{ij},\ i \in \{N, T\},\ j \in \{A, B, C\}. \tag{3}$$

Meanwhile, exerting labor effort generates disutility, which is captured by the following functions.

$$L(\ell_{ij}) = \frac{\ell_{ij}^2}{2},\ i \in \{N, T\},\ j \in \{A, B, C\}. \tag{4}$$

[2] As it will be clear later, this is equivalent to assume the contribution of tradable goods on final goods production is the same across countries.

In stage 2, agents can produce final goods one-for-one using effort h, and consume final goods c. Period utility is

$$U(c,h) = c - h$$

Transaction arrangement. In stage 1, final goods producers purchase non-tradable intermediate goods from the domestic sector and tradable intermediate goods from the global intermediate goods producer. The transaction must be paid upfront with specific means of payment since the trade credit is not allowed in this paper.[3] I assume only currencies issued by governments can be used as means of payment. Motivated by the empirical patterns, domestic currency is used when the transaction occurs between agents from the same country while dominant currency, which is issued by country C, is used when the transaction occurs between agents from different countries.[4] In stage 2, all agents can trade in a centralized market for final goods and asset holding. Only assets across periods are storable while goods perish by the end of each period.

Government, asset, and policy. The government in each country issues a perfectly recognizable, non-counterfeitable asset called currency. Let M_t^k denote the nominal money supply **per final goods producer** of the issuer's country. The nominal money supplies grow at rates $\mu_t^k = \frac{M_t^k}{M_{t-1}^k}$ in country $k \in \{A, B, C\}$. The fiscal authority in each country passively makes a lump-sum nominal transfer $T_t^k,\ k \in \{A, B, C\}$ (or a lump-sum nominal tax if negative) to domestic final goods producers at the beginning of stage 2 each period to satisfy budget constraint.

Finally, the monetary authority in peripheral countries can conduct a foreign exchange intervention (FXI) to depreciate the exchange rate against country C through open market operations at the beginning of stage 2. Although in principle, all countries can implement FXI, I limit my analysis by focusing on FXI from country A and discuss the possibility of competitive devaluation

[3] This is a simplified and ad-hoc assumption. Alternatively, trade credit between agents could be allowed, but due to imperfect enforcement of debt settlement, such as the huge cost of international litigation, sellers would require collateral in the form of reserves, leading to the same result.

[4] A richer monetary-search model could endogenize which currency becomes the medium of exchange in international transactions. This is one of the appealing features of the search-theoretic approach: acceptance and liquidity can be derived from decentralized trading frictions rather than imposed exogenously. See Zhang (2014) for an example in an international-currency setting. I abstract from that margin here and instead take dominant-currency use as given, so as to focus on how foreign exchange intervention affects allocations once international trade is organized around a dominant currency.

from the perspective of country B.[5] As a result, the government in peripheral countries needs to finance the transfer and FXI through money issuance and satisfies the budget constraint every period.

$$(\mu_{t+1}^k - 1)M_t^k = T_t^k + e_t^k \triangle X_{t+1}^k \tag{5}$$

where $\triangle X_{t+1}^k = X_{t+1}^k - X_t^k$ is the change of government's dominant currency reserve and e_t^k is the exchange rate of dominant currency in terms of domestic currency. If e_t^k goes up, it implies the domestic currency depreciates against dominant currency. Since the central country does not conduct FXI, $X_t^C = 0$ for all t.

I derive the equilibrium conditions in the rest of this section. Since the agents are set up almost symmetrically, I start from final goods producer and then proceed to exporting and domestic intermediate goods producer in each country. To keep the derivation concise, the time subscribe is suppressed. Variables for the next period are denoted with a prime.

3.1 Final goods producer

Stage 2 problem. Consider a final good producer in country $j \in \{A, B, C\}$ at the beginning of stage 2 with currency portfolio $\mathbf{M}_{Yj} = (M_{Yj}^A,\ M_{Yj}^B,\ M_{Yj}^C)$, which represent currency holdings of country A, B, and C, and final goods produced in stage 1, Y_j. The value function of the agent $W_{Yj}(\mathbf{M}_{Yj},\ Y_j)$ satisfies

$$W_{Yj}(\mathbf{M}_{Yj},\ Y_j) = \max_{\{\mathbf{M}'_{Yj}\}} c_{Yj} - h_{Yj} + \beta V(\mathbf{M}'_{Yj})$$

$$s.t. \qquad c_{Yj} = Y_j + h_{Yj} + \sum_k \frac{M_{Yj}^k - M_{Yj}^{\prime k}}{e^k P_C} + \frac{T^j}{e^j P_C},\ M_{Yj}^{\prime k} \geq 0,\ k \in \{A, B, C\}$$

where P_C is the nominal price of the final goods in terms of dominant currency, $V(\mathbf{M}'_{Yj})$ is the agent's value function at the beginning of the next period, and $\beta \in (0, 1)$ is the discount rate. The law of one price holds, and hence the nominal exchange rates are defined as $e^j = \frac{P_j}{P_C}$ which P_j is the the nominal price of the final goods in terms of country j currency. Substituting the budget

[5] Theoretically, country C can do FXI as well. However, dominant currency issuers are mostly advanced economies and relatively large in terms of economy size. According to Hassan et al. (2023), the cost of FXI is increasing in economic size. Therefore, I focus on FXI from periphery countries.

constraint into value function, the maximization problem is rewritten as

$$W_{Yj}(\mathbf{M}_{Yj},\ Y_j) = \sum_k \frac{M^k_{Yj}}{e^k P_C} + Y_j + \underbrace{\max_{M'^k_{Yj}} \left\{ \frac{T^j}{e^j P_C} - \sum_k \frac{M'^k_{Yj}}{e^k P_C} + \beta V(\mathbf{M}'_{Yj}) \right\}}_{=W_{Yj}(0,0)} \tag{6}$$

which suggests that the optimal asset holding decision for the next period is independent of the current asset holdings. Also, the value function is linear in current asset holding, a convenient property that will be used later.

Stage 1 problem. In stage 1, final goods producers with predetermined currency portfolio, $\mathbf{M}_{Yj}$, purchase intermediate goods from domestic intermediate goods producer and global intermediate goods producer to produce final goods and pay upfront corresponding currency according DCP arrangement. Therefore, the final goods producer solves the following maximizing problem

$$V_{Yj}(\mathbf{M}_{Yj}) = \max_{\{I_{Nj}, I_{Fj}\}} W_{Yj}(\mathbf{M}_{Yj} - \triangle \mathbf{M}_{Yj},\ Y_j)$$
$$s.t.\ \frac{M^k_{Yj} - \triangle M^k_{Yj}}{e^k P_C} \geq 0,\ k \in \{A, B, C\}$$
$$\triangle M^A_{Yj} = P_{NA} I_{NA},\ \triangle M^B_{Yj} = P_{NB} I_{NB},\ \triangle M^C_{Yj} = P_F I_{Fj} + P_{NC} I_{NC},\ Y_j \text{ follows (1)},$$
$$P_{ij},\ P_F \text{ are the corresponding prices of goods.}$$

Because of the linearity in stage 2 value function, the value function is rewritten as

$$V_{Yj}(\mathbf{M}_{Yj}) = \max_{\{I_{Nj}, I_{Fj}\}} \sum_k \frac{M^k_{Yj} - \triangle M^k_{Yj}}{e^k P_C} + Y_j + W_{Yj}(0,0), \tag{7}$$
$$s.t.\ \frac{M^k_{Yj} - \triangle M^k_{Yj}}{e^k P_C} \geq 0,\ k \in \{A, B, C\}$$
$$\triangle M^A_{Yj} = P_{NA} I_{NA},\ \triangle M^B_{Yj} = P_{NB} I_{NB},\ \triangle M^C_{Yj} = P_F I_{Fj} + P_{NC} I_{NC}$$

The optimal conditions in stage 2 suggest that the currency holdings should satisfy the following conditions.

$$\frac{1}{e^k P_C} \leq \beta \frac{\partial V(\mathbf{M}'_{Yj})}{\partial M'^k_{Yj}},\ \text{binding if } M'^k_{Yj} > 0,\ k \in \{A, B, C\}.$$

With the envelope conditions from equation (7), we can further rewrite the condition as follows.

$$\frac{\Pi^k}{\beta} \leq 1 + \lambda^k_{Yj},\ \text{binding if } M'^k_{Yj} > 0,\ k \in \{A, B, C\}$$

where $\Pi^k = \frac{P'_k}{P_k}$ and λ^k_{Yj} is the corresponding Lagrangian multiplier for DCP constraint.

For the rest of this paper, we will focus on the case which $\frac{\Pi^k}{\beta} > 1$ for all k. This assumption makes holding currency across periods costly. As a result, spending currency in the period 1 must generate enough benefit to incentivize final goods producers to hold currency. Such a benefit comes from mitigating the DCP constraint, which implies $\lambda^k_{Yj} > 0$ and $\frac{\Pi^k}{\beta} = 1 + \lambda^k_{Yj}$. This result also implies the final goods producer would spend all the currency on intermediate goods purchase due to KKT condition. The result is summarized in the following Lemma.

Lemma 1. *If $\frac{\Pi^k}{\beta} > 1$ for all k, then the final goods producer would hold positive amount of currency and spend all the currency on intermediate goods purchase in the next period.*

A quick observation from the final goods production function is that final goods producers in peripheral countries need to hold domestic currency and dominant currency while the final goods producer in central country only needs to hold dominant currency.

Lemma 1 suggests that the domestic and global intermediate goods demands are as follow.

$$I_{Nj} = \frac{M^j_{Yj}}{P_{Nj}},\ I_{Fj} = \frac{M^C_{Yj}}{P_F},\ j \in \{A, B\},\ I_{NC} = \frac{S_C M^C_{YC}}{P_{NC}},\ I_{FC} = \frac{(1-S_C)M^C_{YC}}{P_F}, \tag{8}$$

$$S_C = \omega_D^{\sigma_D}\left(\frac{P_{NC}}{P_{YC}}\right)^{1-\sigma_D},\ P_{YC} = \left[\omega_D^{\sigma_D} P_{NC}^{1-\sigma_D} + \omega_F^{\sigma_D} P_F^{1-\sigma_D}\right]^{\frac{1}{1-\sigma_D}}. \tag{9}$$

The maximization problem for final goods producer can be further rewritten as follows.

$$V_{Yj}(\mathbf{M}_{Yj}) = \max_{\mathbf{M}'_{Yj}} \left[\omega_D\left(\frac{M^j_{Yj}}{P_{Nj}}\right)^{\rho_D} + \omega_F\left(\frac{M^C_{Yj}}{P_F}\right)^{\rho_D}\right]^{\frac{1}{\rho_D}} + \frac{T^j}{P_j} - \sum_k \frac{M'^k_{Yj}}{P_k} + \beta V_{Yj}(\mathbf{M}'_{Yj}),\ j = \{A, B\}$$

$$V_{YC}(\mathbf{M}_{YC}) = \max_{\mathbf{M}'_{YC}} \left[\omega_D\left(\frac{S_C}{P_{NC}}\right)^{\rho_D} + \omega_F\left(\frac{1-S_C}{P_F}\right)^{\rho_D}\right]^{\frac{1}{\rho_D}} M^C_{YC} + \frac{T^C}{P_C} - \sum_k \frac{M'^k_{YC}}{P_k} + \beta V_{YC}(\mathbf{M}'_{YC}),$$

The first-order conditions imply the currency demands satisfy the following conditions.

$$m^C_{YA} = \left(\frac{\omega_F}{\omega_D}\frac{\Pi^A}{\Pi^C}\right)^{\sigma_D}\left(\frac{P_{NA}/P_A}{P_F/P_C}\right)^{\sigma_D - 1} m^A_{YA}, \tag{10}$$

$$m^C_{YB} = \left(\frac{\omega_F}{\omega_D}\frac{\Pi^B}{\Pi^C}\right)^{\sigma_D}\left(\frac{P_{NB}/P_B}{P_F/P_C}\right)^{\sigma_D - 1} m^B_{YB}, \tag{11}$$

$$\frac{P_C}{P_{YC}} = \frac{\Pi^C}{\beta} \tag{12}$$

where $m_{Yj}^k = \frac{M_{Yj}^k}{P_j}$ is the real money balance.

The first-order conditions (10) and (11) also suggest that for each unit of final goods in peripheral countries is used to exchange for money holding, they allocate S_j unit for local currency and $(1-S_j)$ for dominant currency, which

$$S_j = \frac{(\omega_D/\Pi^j)^{\sigma_D}(P_{Nj}/P_j)^{1-\sigma_D}}{(\omega_D/\Pi^j)^{\sigma_D}(P_{Nj}/P_j)^{1-\sigma_D} + (\omega_F/\Pi^C)^{\sigma_D}(P_F/P_C)^{1-\sigma_D}},\ j \in \{A, B\}. \tag{13}$$

As a result, the marginal costs of holding one unit of real balance are $\frac{S_A\Pi^A+(1-S_A)\Pi^C}{\beta}$, $\frac{S_B\Pi^B+(1-S_B)\Pi^C}{\beta}$, $\frac{\Pi^C}{\beta}$ for the final goods producers in country A. B, and C, respectively.

3.2 Global intermediate goods producer

The global intermediate goods producer plays a passive role in this model. It is equivalent to assume that the final goods producer use the same CES production function to produce I_{Fj} with exported intermediate goods across the world. The representative global intermediate goods producer takes prices as given and maximizes its profit

$$\max_{\{I_{Tj}\}} P_F \left[\sum_j \omega_{Fj} I_{Tj}^{\rho_F}\right]^{\frac{1}{\rho_F}} - \sum_j P_{Tj} I_{Tj},\ j \in \{A, B, C\}.$$

The standard first-order conditions show that

$$P_F = \left[\sum_j \omega_{Fj}^{\sigma_F} P_{Tj}^{1-\sigma_F}\right]^{\frac{1}{1-\sigma_F}},\ S_{Fj} = \frac{P_{Tj}I_{Tj}}{P_F I_F} = \left[\sum_i \frac{P_{Ti}}{P_{Tj}}\frac{I_{Ti}}{I_{Tj}}\right]^{-1} = \left(\frac{P_{Tj}}{P_F}\right)^{1-\sigma_F} \omega_{Fj}^{\sigma_F} \tag{14}$$

where S_{Fj} is the expenditure share on I_{Tj} and the demand function for each exporting intermediate goods producer is

$$I_{Tj} = \left(\frac{P_{Tj}}{P_F}\right)^{-\sigma_F} \omega_{Fj}^{\sigma_F} I_F. \tag{15}$$

3.3 Domestic and exporting intermediate goods producer

Stage 2 problem. Consider a intermediate good producer in the sector $i \in \{N, T\}$ of country $j \in \{A, B, C\}$ at the beginning of stage 2 with currency portfolio $\mathbf{M}_{ij} = (M_{ij}^A,\ M_{ij}^B,\ M_{ij}^C)$. The

value function of the agent $W_{ij}(\mathbf{M}_{ij},\ 0)$ satisfies

$$W_{ij}(\mathbf{M}_{ij},\ 0) = \max_{\{\mathbf{M}'_{ij}\}} c_{ij} - h_{ij} + \beta V(\mathbf{M}'_{ij})$$
$$s.t. \qquad c_{ij} = h_{ij} + \sum_k \frac{M^k_{ij} - M'^k_{ij}}{e^k P_C},\ M'^k_{ij} \geq 0,\ k \in \{A, B, C\}.$$

Substituting the budget constraint into the value function, the maximization problem is rewritten as

$$W_{ij}(\mathbf{M}_{ij},\ 0) = \sum_k \frac{M^k_{ij}}{e^k P_C} + \underbrace{\max_{M'^k_{ij}} \left\{ -\sum_k \frac{M'^k_{ij}}{e^k P_C} + \beta V(\mathbf{M}'_{ij}) \right\}}_{=W_{ij}(0,0)} \tag{16}$$

which suggests that the optimal asset holding decision for the next period is independent of the current asset holdings and the value function is linear in current asset holding.

Stage 1 problem. In stage 1, the intermediate goods producers with predetermined currency portfolio, $\mathbf{M}_{ij}$, decide how much intermediate goods to produce and receive the payment upfront with corresponding currency according DCP arrangement. Therefore, the intermediate goods producer solves the following maximizing problem

$$V_{ij}(\mathbf{M}_{ij}) = \max_{\{\ell_{ij}\}} -\frac{\ell^2_{ij}}{2} + W_{ij}(\mathbf{M}_{ij} + \triangle\mathbf{M}_{ij},\ 0)$$
$$\triangle M^A_{ij} = P_{NA} I_{NA},\ \triangle M^B_{ij} = P_{NB} I_{NB},\ \triangle M^C_{ij} = \sum_k P_{Tk} I_{Tk} + P_{NC} I_{NC},\ I_{ij} = A_{ij}\ell_{ij}.$$

Because of the linearity in stage 2 value function, the value function is rewritten as

$$V_{ij}(\mathbf{M}_{ij}) = \max_{\{\ell_{ij}\}} \sum_k \frac{M^k_{ij} + \triangle M^k_{ij}}{e^k P_C} - \frac{\ell^2_{ij}}{2} + W_{ij}(0,0), \tag{17}$$

The optimal conditions in stage 2 suggest that the currency holdings should satisfy the following conditions.

$$\frac{1}{e^k P_C} \leq \beta \frac{\partial V(\mathbf{M}'_{ij})}{\partial M'^k_{ij}},\ \text{binding if } M'^k_{ij} > 0,\ k \in \{A, B, C\}.$$

With the envelope conditions from equation (7), we can further rewrite the condition as follows.

$$\frac{\Pi^k}{\beta} \leq 1 + \lambda^k_{ij},\ \text{binding if } M'^k_{ij} > 0,\ k \in \{A, B, C\}.$$

Since intermediate goods producers have no use to hold currency in the stage 1, their money demand is zero in the stage 2 if $\frac{\Pi^k}{\beta} > 1$ for all k. The result is summarized in the following Lemma.

Lemma 2. *If $\frac{\Pi^k}{\beta} > 1$ for all k, then the intermediate goods producer would hold zero amount of currency in stage 2 and spend all they earn in the stage 1.*

Lemma 2 implies the intermediate producers only need to solve intra-period labor supply decisions. The optimal labor supply satisfies the follow first-order condition.

$$\ell_{Nj} = \frac{P_{Nj}A_{Nj}}{P_j},\ \ell_{Tj} = \frac{P_{Tj}A_{Tj}}{P_C},\ j \in \{A, B, C\} \tag{18}$$

which implies the intermediate goods supply is as follows.

$$I_{Nj} = \frac{P_{Nj}A_{Nj}^2}{P_j},\ I_{Tj} = \frac{P_{Tj}A_{Tj}^2}{P_C},\ j \in \{A, B, C\} \tag{19}$$

3.4 Government policy and budget

As mentioned in the previous paragraph, the government in each country needs to satisfy its budget constraint period by period and only peripheral countries are allowed to conduct FXI against dominant currency. Therefore, the government budget constraint in each country is as follows.

$$(\mu_{t+1}^A - 1)M_t^A = T^A + e_t^A \triangle X_{t+1}^A, \tag{20}$$

$$(\mu_{t+1}^B - 1)M_t^B = T^B + e_t^B \triangle X_{t,t+1}^B, \tag{21}$$

$$(\mu_{t+1}^C - 1)M_t^C = T^C. \tag{22}$$

The FXI is implemented through open market operation at the beginning of stage 2. Specifically, the monetary authority issues additional domestic currency $\triangle M_{t+1}^k$ to exchange for dominant currency $\triangle X_{t+1}^k$ at desired exchange rate e_t^k. The exporting intermediate goods producers in the same country is the transaction counterpart as they are agents who hold dominant currency and are willing to accept domestic currency.

3.5 Market clearing

Define the relative prices vector as

$$\mathbf{P} = \left(\frac{P_{Nj}}{P_j}, \frac{P_{Tj}}{P_C}\right),\ j \in \{A, B, C\}.$$

In the period t, money supply equals money demand for each currency.

$$M_{t+1}^A = M_{YA,t+1}^A,\ M_{t+1}^B = M_{YB,t+1}^B,$$
$$\overline{M}_{t+1}^C = M_{t+1}^C - X_{t+1}^A - X_{t+1}^B = \sum_j M_{Yj,t+1}^C,\ j \in \{A,B,C\}. \tag{23}$$

Notice that part of the dominant currency is held within central banks in peripheral countries X_{t+1}^A and X_{t+1}^B. Therefore, only the rest, $\overline{M}_{t+1}^C$, is privately circulating. The intermediate goods supply (19) equals intermediate goods demand for each sector (9) and (15).

$$I_{ij}(\mathbf{P}) = I_{ij}(\mathbf{P}, \mathbf{M}) \tag{24}$$

where the supply is a function of relative prices, $\mathbf{P}$, and demand is a function of relative prices, $\mathbf{P}$, and buyer's currency portfolio, $\mathbf{M}$.

The competitive equilibrium is defined as follows.

Definition 1. *A competitive equilibrium is a policy parameter sequence* $\{\mu_{t+1}^k,\ T_t^k,\ \triangle X_{t+1}^A,\ \triangle X_{t+1}^B\}$, *a price sequence* $\{\mathbf{P}_t\}$, *and a allocation sequence* $\{\ell_{ijt},\ \mathbf{M}_t\}$ *which* $i \in \{N,T\}$, $j \in \{A,B,C\}$ *and* $k \in \{A,B,C\}$, *such that the following conditions are satisfied*

- *final goods producers' optimal currency holdings decisions (10) - (12);*
- *optimal labor supply conditions (18);*
- *government budget constraints (20) - (22);*
- *money market clearing conditions (23);*
- *goods market clearing conditions (24).*

In the equilibrium, the proportion of circulating dominant currency received by an exporting intermediate goods sector, S_{Fj},is increasing in its productivity A_{Tj} and preference ω_{Fj}. To see this, the equilibrium condition in global intermediate goods implies that

$$S_{Fj}[M_{YA}^C + M_{YB}^C + (1-S_C)M_{YC}^C] = P_{Tj}I_{Tj} = \frac{P_{Tj}^2 A_{Tj}^2}{P_C},\ j \in \{A,B,C\};$$

Combining this equation with implies equation (14), we can get

$$\frac{S_{Fi}}{S_{Fj}} = \left(\frac{P_{Ti}A_{Ti}}{P_{Tj}A_{Tj}}\right)^2 = \left(\frac{P_{Ti}}{P_{Tj}}\right)^{1-\sigma_F}\left(\frac{\omega_{Fi}}{\omega_{Fj}}\right)^{\sigma_F},\ i,j \in \{A,B,C\}$$

which suggests the relative prices

$$\frac{P_{Ti}}{P_{Tj}} = \left[\frac{A_{Tj}}{A_{Ti}}\left(\frac{\omega_{Fi}}{\omega_{Fj}}\right)^{\sigma_F}\right]^{\frac{1}{1+\sigma_F}},\ i,j \in \{A,B,C\}.$$

Substituting this expression into the definition of S_{Fj}, (14), we get

$$S_{Fj} = \left\{\sum_i \left[\left(\frac{\omega_{Fi}}{\omega_{Fj}}\right)^{\sigma_F}\left(\frac{A_{Ti}}{A_{Tj}}\right)^{\sigma_F-1}\right]^{\frac{2}{1+\sigma_F}}\right\}^{-1},\ j \in \{A,B,C\}. \tag{25}$$

which suggests that S_{Fj} is increasing in its productivity A_{Tj} and preference ω_{Fj} given other things being equal.

On the other hand, the expenditure of intermediate goods purchase from final goods producers in the country C shows that

$$S_C M_{YC}^C = P_{NC}I_{NC} = \frac{P_{NC}^2 A_{NC}^2}{P_C}; \tag{26}$$

$$(1-S_C)M_{YC}^C = \frac{P_F I_F}{1+(M_A^C+M_B^C)/(1-S_C)} = \frac{P_F\left[\sum_j \omega_{Fj}\left(\frac{P_{Tj}}{P_C}A_{Tj}\right)^{\rho_F}\right]^{\frac{1}{\rho_F}}}{1+(M_A^C+M_B^C)/(1-S_C)}. \tag{27}$$

Dividing these two equations, it shows that

$$\left(\frac{P_{NC}}{P_F}\right)^2 \frac{A_{NC}^2[1+(M_A^C+M_B^C)/(1-S_C)]}{\left[\sum_j \omega_{Fj}\left(\frac{P_{Tj}A_{Tj}}{P_F}\right)^{\rho_F}\right]^{\frac{1}{\rho_F}}} = \left(\frac{P_{NC}}{P_F}\right)^{1-\sigma_D}\left(\frac{\omega_D}{\omega_F}\right)^{\sigma_D}.$$

We can further rewrite it as

$$\left[\left(\frac{P_{NC}}{P_F}\right)^{\sigma_D-1} + \frac{M_A^C+M_B^C}{\omega_F^{\sigma_D}}\right]\left(\frac{P_{NC}}{P_F}\right)^2 = \frac{\left(\frac{\omega_D}{\omega_F}\right)^{\sigma_D}\left[\sum_j \omega_{Fj}\left(\frac{P_{Tj}A_{Tj}}{P_F}\right)^{\rho_F}\right]^{\frac{1}{\rho_F}}}{A_{NC}^2}$$

which suggests that the relative price $\frac{P_{NC}}{P_F}$ is decreasing in productivity A_{NC}^2 given other things being equal. The expenditure ratio S_C can be expressed as

$$S_C = \left[1+\omega_F^{\sigma_D}\left(\frac{P_{NC}}{P_F}\right)\right]^{-1}$$

which is decreasing in relative price $\frac{P_{NC}}{P_F}$. As a result, the expenditure ratio S_C is increasing in productivity A_{NC}^2 given other things being equal.

In this paper, I focus on the analysis of stationary equilibrium. This implies the following policy parameters are fixed.

$$\left\{\mu_{t+1}^k,\ \frac{T_t^k}{P_{kt}},\ \frac{\triangle X_{t+1}^A}{P_{Ct}},\ \frac{\triangle X_{t+1}^B}{P_{Ct}}\right\} = \left\{\mu^k,\ \frac{T^k}{P_k},\ \frac{\triangle X^A}{P_C},\ \frac{\triangle X^B}{P_C}\right\},\ k \in \{A, B, C\}.$$

As a result, the money growth rate of each country is constant, which implies the inflation rate is constant as well. Therefore, we have

$$\mu^k = \Pi^k. \tag{28}$$

In the stationary equilibrium, the currency holding decisions for final goods producers imply the net profit rate should satisfy the following conditions.

$$\frac{1}{\beta} = \left[\omega_D^{\sigma_D}\left(\frac{P_{NA}}{P_A}\mu^A\right)^{1-\sigma_D} + \omega_F^{\sigma_D}\left(\frac{P_F}{P_C}\mu^C\right)^{1-\sigma_D}\right]^{\frac{-1}{1-\sigma_D}} \tag{29}$$

$$\frac{1}{\beta} = \left[\omega_D^{\sigma_D}\left(\frac{P_{NB}}{P_B}\mu^B\right)^{1-\sigma_D} + \omega_F^{\sigma_D}\left(\frac{P_F}{P_C}\mu^C\right)^{1-\sigma_D}\right]^{\frac{-1}{1-\sigma_D}} \tag{30}$$

$$\frac{1}{\beta} = \left[\omega_D^{\sigma_D}\left(\frac{P_{NC}}{P_C}\mu^C\right)^{1-\sigma_D} + \omega_F^{\sigma_D}\left(\frac{P_F}{P_C}\mu^C\right)^{1-\sigma_D}\right]^{\frac{-1}{1-\sigma_D}} \tag{31}$$

Equation (29) to (31) represents the net profit rate requirement from country A to country C. The interpretation is that for each unit of final goods that a final producer gives up in the stage 2, the return after considering the inflation cost is on the right hand side and this rate should equal the time discount rate. Meanwhile, the standard Friedman's rule is globally optimal in this model with one subtle difference. Take (31) as an example. $\frac{P_{NC}}{P_C}$ and $\frac{P_F}{P_C}$ are decreasing in μ^C. As μ^C keeps decreasing, $\frac{P_{NC}}{P_C}$ and $\frac{P_F}{P_C}$ increase and approach to 1. From (18), the intermediate goods output is increasing in the relative price like $\frac{P_{NC}}{P_C}$ and $\frac{P_F}{P_C}$, which implies the final goods output increase as well. Since there is a lower bound such that $\mu^C \geq \beta$, the optimal allocation in country C is achieved when $\mu^C = \beta$. However, for peripheral countries, once they set their money growth rate equal to time discount rate is not enough, as the imported intermediate goods need to be paid in

the dominant currency. As a result, the optimal allocation in peripheral country $k \in \{A, B\}$ is achieved when $\mu^k = \beta$ and $\mu^C = \beta$. The following proposition summarizes the result.

Proposition 1. *In a stationary monetary equilibrium, the central country can achieve optimal allocation by setting its inflation rate equals time discount rate. On the other hand, the optimal allocation is not guaranteed even if the periphery countries set their domestic inflation equals time discount rate. The optimal allocation in periphery countries is achieved only when domestic inflation rate and central country's inflation rate equal time discount rate at the same time.*

3.6 Exchange rate dynamic

Due to the flexible pricing assumption, the exchange rate variation, by itself, has no real impact in this model. In fact, what really matters is the real return on each currency and the exchange rate variation is just a by-product, a mechanism which I will explain more details in the next policy thought experiment section. Nevertheless, in reality, the exchange rate is still at the heart of policy debate because it is a variable that is easier to observe comparing to money growth rate.[6] As a result, I show the exchange rate dynamic in a stationary equilibrium and its relationship with relative money growth rate.

The exchange rate in this model is defined as the price of final goods in different currencies and law of one price holds. The exchange rate is as follows.

$$e_t^k = \frac{P_{kt}}{P_{Ct}},\ k \in \{A, B, C\}.$$

Rearrange the first-order conditions (10) and (11), we get

$$e_{t+1}^k = \frac{P_{kt+1}}{P_{Ct+1}} = \frac{\omega_F}{\omega_D} \left(\frac{\Pi_{t+1}^k}{\Pi_{t+1}^C} \right)^{\sigma_D} \left(\frac{P_{Nkt}/P_{kt}}{P_{Ft}/P_{Ct}} \right)^{\sigma_D - 1} \frac{M_{t+1}^k}{M_{t+1}^C},\ k \in \{A, B\}.$$

Therefore, in a stationary equilibrium, the growth rate of exchange rate follows

$$\frac{e_{t+1}^k}{e_t^k} = \frac{\mu^k}{\mu^C},\ k \in \{A, B\}.$$

The exchange rate dynamic in a stationary equilibrium suggests that the periphery countries can create a long-term depreciation trend against dominant currency simply by raising relative money

[6] Although we do have money supply statistic, it has multiple definitions such as M1 and M2 and is updated less frequently.

growth (inflation) rate. However, there will be a negative welfare impact at the same time. The result is summarized in the following Proposition.

Proposition 2. *The periphery countries can create a long-term depreciation trend against dominant currency simply by raising inflation rate. However, such a long-term depreciation trend caused by a pure high inflation only reduces its welfare as Proposition 1 suggests.*

Different from the conventional theoretic prediction, the depreciation purely caused by money creation only hurts its own economy. This might be the reason why most of the countries choose other ways instead of pure inflation to conduct FXI policy as shown in Adler et al. (2025).

4 Foreign exchange rate intervention (FXI)

In this section, I investigate the effect of one-time FXI conducted by country A to depreciate its own currency against dominant currency. Specifically, the government in country A conducts FXI in the beginning of stage 2 through open market operation from a stationary equilibrium. For the simplicity, I assume the country B remains the same and the money growth rate of country C is fixed relative to the circulated $\overline{M}_t^C = M_t^C - X_t^A - X_t^B$ for the whole section. The first assumption eliminate the possibility of competitive devaluation, a topic I will explore later. The second assumption makes the inflation rate Π^C equal to money growth rate μ^C except for the period of intervention. The analysis can be applied to country B symmetrically. The government buys dominant currency from domestic exporters with newly issued domestic currency at the targeted exchange rate. I consider three cases depending on whether agents know the intervention or not in stage 1. The first case is the **pure-shock** case. That is, all agents are unaware of the intervention until it happens. The second case is the **anticipated** case. That is, all agents, including agents in other countries, know the intervention in the beginning of stage 1. The last case is the **asymmetric** case. In this case, only domestic agents know the intervention in advance while foreign agents treat it like a shock. I analyze these three cases in order and discuss the potential competitive devaluation issue.

4.1 Pure-shock case

When FXI arrives as a pure shock in period t, all agents around the world make the same decisions as in the stationary equilibrium. The reason is intuitive. Stage 1 decision, which is intermediate goods production, happens before the shock hits. Without knowing the intervention, agents remain at the stationary allocation. Stage 2 decision, which is currency portfolio choice for final good producers, happens after the shock hits. However, since the future inflation rate remains fixed, the decision stays the same as equation (10) to equation (12) suggest.

Although the FXI as a pure shock has no impact on agents' decision, it does cause wealth redistribution across agents. To see this clearly, I compare purchasing power before and after the intervention. Since the stage 1 decision remains fixed, the purchasing power is the same as the one in the stationary equilibrium for all agents when stage 2 begins. Therefore, the purchasing power before shock is

Country A:

$$Domestic: \frac{M_t^A}{P_{At}}, \quad Exporting: \frac{S_{FA}(\overline{M}_t^C - S_C M_{Ct}^C)}{P_{Ct}}.$$

Country B:

$$Domestic: \frac{M_t^B}{P_{Bt}}, \quad Exporting: \frac{S_{FB}(\overline{M}_t^C - S_C M_{Ct}^C)}{P_{Ct}}.$$

Country C:

$$Domestic: \frac{S_C M_{Ct}^C}{P_{Ct}}, \quad Exporting: \frac{S_{FC}(\overline{M}_t^C - S_C M_{Ct}^C)}{P_{Ct}}.$$

Please refer to equation (9) and (14) for the definition of S_C and $S_{Fj},\ j \in \{A, B, C\}$. After shock, the supply of dominant currency decreases as the government in country A keeps part of them from being circulated. The result is the price of final goods in terms of dominant currency decreases from P_{Ct} to P'_{Ct} which $P_{Ct} > P'_{Ct}$.

For country C, both domestic and exporting intermediate goods producers benefit from FXI as the real consumption goes up when the final goods price drops. In country B, exporting producers benefit from FXI for the same reason and the domestic producers are immune to FXI as the circulated quantity of currency B remains the same. Nevertheless, currency B depreciates against dominant currency due to the price drops in P_{Ct}.

For the country A, the currency portfolio of exporting producers becomes

$$\frac{S_{FA}(\overline{M}_t^C - S_C M_{Ct}^C)}{P_{Ct}} \rightarrow \frac{S_{FA}(\overline{M}_t^C - S_C M_{Ct}^C) - \triangle X_{t+1}^A}{P'_{Ct}} + \frac{\triangle M_t^A}{P'_{At}}$$

which $\triangle M_t^A$ is newly issued currency for FXI and $e_t^{A'}$ is the targeted exchange rate such that $\frac{P'_{At}}{P'_{Ct}} = e_t^{A'} > e_t^A = \frac{P_{At}}{P_{Ct}}$. Notice that FXI is carried out through open market operation and hence there is no gain or loss for such currency transaction within a centralized market. As a result, $e_t^{A'} \triangle X_{t+1}^A = \triangle M_t^A$ and the real consumption for exporting producers in country A is

$$\frac{S_{FA}(\overline{M}_t^C - S_C M_{Ct}^C) - \triangle X_{t+1}^A}{P'_{Ct}} + \frac{\triangle M_t^A}{P'_{At}} = \frac{S_{FA}(\overline{M}_t^C - S_C M_{Ct}^C)}{P'_{Ct}}$$

which suggests the exporting producers in country A also benefit from FXI.

Finally, the currency holding of domestic intermediate goods producers in country A remains the same. Nevertheless, the total supply of currency A increases as the consequence of FXI, which drives up the price of final goods in terms of currency A from P_{At} to P'_{At} which $P_{At} < P'_{At}$. Such a nominal change reduces domestic producers' consumption as their currency holdings fail to increase proportionally. In sum, the purchasing power after the shock is

Country A:

$$Domestic: \ \frac{M_t^A}{P_{At}} > \frac{M_t^A}{P'_{At}}, \quad Exporting: \ \frac{S_{FA}(\overline{M}_t^C - S_C M_{Ct}^C)}{P_{Ct}} < \frac{S_{FA}(\overline{M}_t^C - S_C M_{Ct}^C)}{P'_{Ct}}.$$

Country B:

$$Domestic: \ \frac{M_t^B}{P_{Bt}} = \frac{M_t^B}{P_{Bt}}, \quad Exporting: \ \frac{S_{FB}(\overline{M}_t^C - S_C M_{Ct}^C)}{P_{Ct}} < \frac{S_{FB}(\overline{M}_t^C - S_C M_{Ct}^C)}{P'_{Ct}}.$$

Country C:

$$Domestic: \ \frac{S_C M_{Ct}^C}{P_{Ct}} < \frac{S_C \overline{M}_t^C}{P'_{Ct}}, \quad Exporting: \ \frac{S_{FC}(\overline{M}_t^C - S_C M_{Ct}^C)}{P_{Ct}} < \frac{S_{FC}(\overline{M}_t^C - S_C M_{Ct}^C)}{P'_{Ct}}.$$

Since the labor supply decision in stage 1 is determined before the shock is realized, the welfare is positively correlated with the purchasing power. Therefore, the welfare change is in the same direction as the purchasing power. The result is summarized below.

Proposition 3. *When FXI is implemented as a pure shock through open market operation by country A to depreciate its own currency against dominant currency, the purchasing power increases for dominant currency holders, remains the same for currency B holders, and decreases for currency A holders. The welfare change is in the same direction as the purchasing power. The effect lasts only for the period in which the intervention is conducted. The equilibrium allocation after the intervention remains the same.*

4.2 Anticipated case

In the anticipated case, the government A announces targeted exchange rate $e_t^{A'} > e_t^A$ in the beginning of the period t and commit to implement such an intervention through open market operation. For simplicity, I assume the other governments do not react to the FXI.

Since future inflation is unaffected, the final goods producers' currency holding decision in stage 2 remain the same. The price of final goods in terms of each currency reacts the same way as we have seen in the pure shock case because the nominal money supply change after FXI,. The difference between the pure-shock case and the anticipated case is that the intermediate goods producers anticipate the change in the price level and adjust their production decision in the stage 1. Recall their optimal labor supply conditions are:

$$\ell_{Nj} = \frac{P_{Nj}A_{Nj}}{P_j},\ \ell_{Tj} = \frac{P_{Tj}A_{Tj}}{P_C},\ j = \{A, B, C\}.$$

Therefore, the labor supply increases for $\{\ell_{TA},\ \ell_{TB},\ \ell_{TC},\ \ell_{NC}\}$, remains the same for ℓ_{NB}, and decreases for ℓ_{NA} corresponding to the price decreases in P_C, remains fixed in P_B, and increases in P_A for a given selling price $\{P_{Nj},\ P_{Tj}\}$.

When the FXI is implemented in the anticipated way, it works like a purchasing power increase for dominant currency holders proportionally to their nominal holding. However, the nominal holding of currency is predetermined when the intervention is announced, and the final goods producers from each country are not allowed to readjust their portfolio. The equilibrium prices of intermediate goods which are priced in dominant currency are determined in the same way as before. Therefore, the relative prices of intermediate goods priced in dominant currency remain the same because the relative productivity is unaffected. The unchanged relative prices gives the

same expenditure share.[7] As a result, the nominal amount of currency that each sector, including sectors use non-dominant currency, receive is unaffected. The direct policy implication is that such an anticipated intervention would not improve trade balance.

The second implication is that the equilibrium quantity increases for intermediate goods priced in the dominant currency as the corresponding producers are more willing to produce for a given amount of revenue, a condition which implies the new equilibrium relative prices $\{\frac{P_{NC}}{P_C}, \frac{P_{Tj}}{P_C}\}$, $j \in \{A, B, C\}$ increase from the optimal labor supply condition. Meanwhile, the equilibrium result in domestic intermediate goods sector in country B remains unaffected while output in the domestic intermediate goods sector in country A declines.

The third implication is that the wealth redistribution effect after intervention still exists and the net effect is the same as the one in the pure shock case. Furthermore, all intermediate goods producers using dominant currency benefit due to positive wealth redistribution effect while the domestic sector in A suffers from the negative counterpart and the domestic sector in B is unaffected. All results are summarized in the following Proposition.

Proposition 4. *When FXI is implemented through open market operation by country A to depreciate its currency against the dominant currency and the intervention is announced in advance, the purchasing power increases for dominant currency holders, remains the same for currency B holders, and decreases for currency A holders. As a result, the intermediate goods production increases for producers who accept dominant currency, decreases for producers who accept currency A, and remains fixed for producers who accept currency B in stage 1. The net effect is the equilibrium output increases for all sectors which use dominant currency as the means of payment, decreases for the domestic intermediate goods sector in country A, and remains unaffected for the domestic intermediate goods sector in country B. The wealth redistribution effect remains the same as the one in the pure shock case. Additionally, all intermediate goods producers using dominant currency benefit due to positive wealth redistribution effect while the domestic sector in A suffers from the negative counterpart and the domestic sector in B is unaffected. The effect lasts only for the period in which the intervention is conducted. The equilibrium allocation after the intervention remains the same.*

[7] Please refer to Section 3.5.

4.3 Asymmetric case

Finally, I reconsider the assumption that information is fully available across all countries and agents. In reality, agents in peripheral countries often get information easily from central country as transparency is one of the precondition for issuing dominant currency. The policy transparency of a country's own government depends on institutional quality and it often increases with economic development. As a result, in the asymmetric case, I assume that only agents in country A know the intervention in advance while the rest of the world treats the intervention as a pure shock.

Once again, the future inflation in stage 2 is unaffected. The price of final goods in terms of each currency reacts the same way as we have seen in the pure shock case. However, since the intermediate goods producers in country A anticipate the change in the price level and adjust their production decision in the stage 1, a scenario which leads to labor supply increases for ℓ_{TA}, and decreases for ℓ_{NA} corresponding to the price decreases in P_C and increases in P_A. For the rest of the world, the supply of intermediate goods remains unchanged as the intermediate goods producers fail to perceive the correct final goods price. As a result, the labor supply become

$$\begin{aligned}
&\ell_{NA} = \frac{P_{NA}A_{NA}}{P'_A},\ \ell_{TA} = \frac{P_{TA}A_{TA}}{P'_C},\\
&\ell_{NB} = \frac{P_{NB}A_{NB}}{P_B},\ \ell_{TB} = \frac{P_{TB}A_{TB}}{P_C},\\
&\ell_{NC} = \frac{P_{NC}A_{NC}}{P_C},\ \ell_{TC} = \frac{P_{TC}A_{TC}}{P_C},\\
&\text{which } \frac{P_A}{P'_A} = x \in (0,1) \text{ and } \frac{P_C}{P'_C} = z > 1.
\end{aligned}$$

We can rewrite the labor supply in country A as

$$\begin{aligned}
&\ell_{NA} = \frac{P_{NA}(xA_{NA})}{P_A} = \frac{P_{NA}A'_{NA}}{P_A},\ \ell_{TA} = \frac{P_{TA}(zA_{TA})}{P_C} = \frac{P_{TA}A'_{TA}}{P_C},\\
&\text{which } \frac{A'_{NA}}{A_{NA}} = x \in (0,1) \text{ and } \frac{A'_{TA}}{A_{TA}} = z > 1.
\end{aligned}$$

This result shows that the real effect of asymmetric FXI is mathematically equivalent to a temporary negative shock in domestic sector productivity and a temporary positive shock in exporting sector within the country A. Such increase in exporting sector productivity reduces the relative price $\frac{P_{TA}}{P_{TB}}$ and $\frac{P_{TA}}{P_{TC}}$, a condition which leads to an expenditure switching effect within the global intermediate

goods towards I_{TA} globally. Moreover, a lower P_{TA} reduces the price of imported intermediate goods P_F, which leads to another expenditure switching effect within the country C towards I_F.

In terms of real impact, the exporting sector in country A expands its production while the rest of the world, except for the domestic sector in country B, shrinks their production. The domestic sector in country B is immune to this intervention because the purchasing power of currency B is unaffected.

I further investigate the impact of FXI on trade conditions. First, define exporting revenue as $S_{Fj}(\overline{M}^C - S_C M_C^C)$ and trade balance as $S_{Fj}(\overline{M}^C - S_C M_C^C) - M_j^C$ for $j \in \{A, B\}$ and $S_{FC}(M_A^C + M_B^C) - (1 - S_{FC})(1 - S_C)M_C^C$ for country C. The exporting value and trade balance in country A increases and the opposite occurs in country C. For the exporting value and trade balance in country B, the net effect is ambiguous because it depends on negative intensive margin (global expenditure switching effect) and positive extensive margin (country C expenditure switching effect). Finally, for the impact on welfare, all the sectors using dominant currency as means of payment benefit from FXI due to the wealth redistribution effect. Nevertheless, only the exporting sector in country A optimizes its production decision as the producers perceive the price correctly. The domestic sector in country A hurts as before while the domestic sector in country B remains unaffected. The result is summarized in the Proposition below.

Proposition 5. *When FXI is implemented through open market operation by country A to depreciate its currency against the dominant currency and the intervention is only perceived among country A in advance, the purchasing power increases for dominant currency holders, remains the same for currency B holders, and decreases for currency A holders. As a result, the labor supply increases for exporting producers in country A, decreases for domestic producers in country A, and remains fixed for the rest of the world in stage 1. The net effect is the equilibrium output increases for the exporting sector in country A while the rest of the world, except for the domestic sector in country B, shrinks their production. The equilibrium quantity of domestic sector in country B is unaffected. The wealth redistribution effect remains the same as the one in the pure shock case. All the sectors using dominant currency as means of payment benefit from FXI due to the wealth redistribution effect. Nevertheless, only the exporting sector in country A optimizes its production decision as the producers perceive the price correctly. Meanwhile, the domestic sector in country A*

hurts as before while the domestic sector in country B remains unaffected. The effect lasts only for the period in which the intervention is conducted. The equilibrium allocation after the intervention remains the same.

4.4 Competitive devaluation

Is there any incentive for the government in country B to conduct similar FXI against dominant currency, a scenario called competitive devaluation, within this model? In the pure-shock case and anticipated case, there is no incentive for government B to conduct FXI because the exporting sector in country B benefits from the FXI by country A and the domestic sector in country B is unaffected.

However, it is unrealistic to assume that agents can freely obtain information from other countries. If there are information frictions for agents to perceive the shock correctly, then there might be incentives for country B to compete in devaluation against dominant currency for the following two reasons.

First, let's assume the government's objective function is the overall welfare of intermediate goods producers. As shown above, the FXI can increase exporting sector's welfare by sacrificing domestic sector's welfare. In the anticipated case, the loss is not compensated by the gain if the government assigns equal weight to the two sectors. Therefore, there is no incentives to compete on devaluation for country B. However, in the asymmetric case, there exists additional benefit from the expenditure switching effect at global intermediate goods level. If this positive expenditure switching effect is large enough, then such a policy effect provides sufficient incentive for government to do FXI no matter what other government does. As a result, it leads to competitive devaluation.

Second, if the government cares about exporting value or trade balance, then the government B also has incentives to do FXI. The reason is that exporting sector in country B suffers from a negative expenditure switching effect when country A conducts FXI. Although the same sector benefit a positive expenditure switching effect due to country C raises expenditure ratio on imported intermediate goods, it might not be enough to cover the loss. In this scenario, it is country B's best response no matter there is a FXI in country A or not.

The key policy implication of the model prediction is that countries compete on devaluation is

due to the lack of information access. Once the information of intervention can be freely accessed by agents around the world, there would be no incentives to compete on devaluation anymore.

4.5 Counterparty of intervention

So far, the counterpart of intervention is only the exporting sector in the intervening country. Does FXI work differently if the counterparty becomes the importers, which is equivalent to final goods producers in the stage 1? When the inflation remains the same, the domestic sector in country A would not move its supply curve. Meanwhile, the flexible prices imply that the incremental currency A, which are created for FXI, raises the domestic price P_{NA} proportionally. As a result, more nominal holding in currency A does not allow final goods producer in country A to buy more domestic intermediate goods.

On the other hand, FXI reduces dominant currency holding for final goods producers of country A. When the inflation remains the same, the exporting sectors across countries would not move their supply curve. However, less holding of dominant currency decreases the input purchased by the final goods producers in country A. As a result, this FXI is not incentive compatible and hence cannot be carried out through open market operation.

5 Numerical exercise

In this section, I use a numerical example to demonstrate the quantitative aspect of this model and the effect of different types of FXI.

Table 2 summarized the parameters used as a baseline model.

β	σ_D	σ_F	ω_D	ω_F	ω_{FA}	ω_{FB}	ω_{FC}	
0.98	2.5	2.5	0.3	0.7	0.4	0.4	0.2	
A_{NA}	A_{NB}	A_{NC}	A_{TA}	A_{TB}	A_{TC}	μ_A	μ_B	μ_C
1	1	1	1.5	1.2	1	1.02	1.03	1.05

Table 2: Baseline parameter

First, we examine some key variables in the stationary equilibrium. Specifically, I keep country B unchanged and vary either μ_A or μ_C to see how key variables such as trade balance, exporting

quantity, real GDP, and welfare react. The definition of each variables are as follows.

$$\begin{aligned}
\text{trade balance:} \quad TB_j &= P_{Tj} I_{Tj} - P_F I_{Fj} \\
\text{real GDP:} \quad Y_j &= \frac{P_{Nj}}{P_j} I_{Nj} + \frac{P_{Tj}}{P_C} I_{Tj} \\
\text{welfare:} \quad W_j &= \frac{P_{Nj}}{P_j} I_{Nj} - \frac{\ell_{Nj}^2}{2} + \frac{P_{Tj}}{P_C} I_{Tj} - \frac{\ell_{Tj}^2}{2}
\end{aligned}$$

In each plot, we measure the change of the variable relative to the one when either μ_A or μ_C equals 0.98. Figure 5 and 6 represent the result.

First, we focus on Figure 5, which the inflation rate in country A keeps growing. The long-run expansionary monetary policy does not improve the trade balance of a peripheral country, which contradicts to expenditure switch channel proposed in Gopinath et al. (2020). As you can see, TB_A keeps decreasing as μ_A keeps increasing. However, this does not implies country A is running a trade balance deficit. On the other hand, country B improves its trade balance significantly while the country C's trade balance keeps shrinking. Nevertheless, the trade balance is not equivalent to output and welfare in this model. The result of real GDP and suggests that country A is the one pays for most of his own inflation cost.

Second, when the inflation originates in dominant currency, there is a spillover across countries. This is not surprising as all final goods producers need to hold dominant currency to purchase imported intermediate goods. The result shows that all three countries suffer from higher inflation cost together while country C improves its trade balance at the same time.

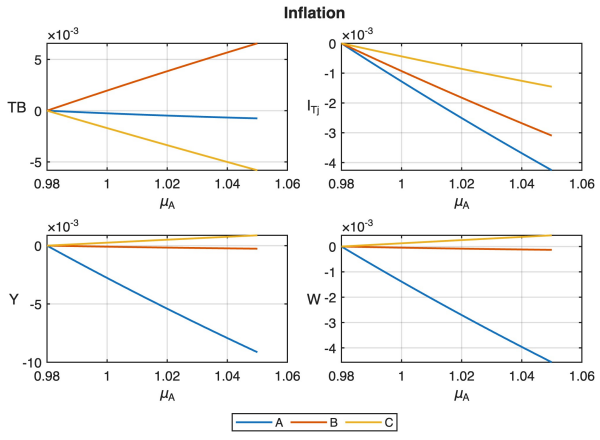

Figure 5: Country A

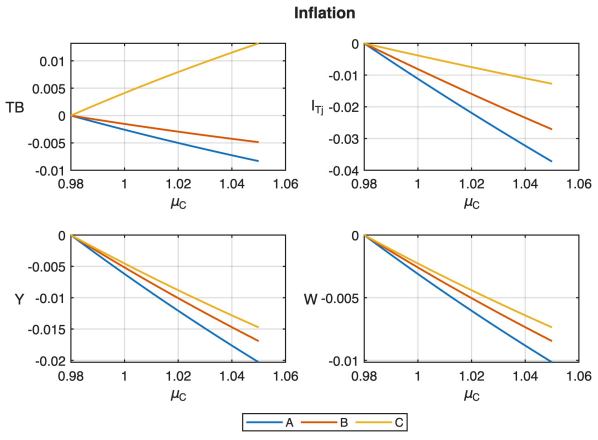

Figure 6: Country C

Next, we examine the effect of FXI in a pure shock case. The welfare gain, which is equivalent to purchase power redistribution after intervention in stage 2, is summarized in the Figure 7. In the left and middle panels, I examine the effect of 5% depreciation FXI. In the right panel, I examine the the effect of FXI by increasing the level of depreciation θ in the baseline parameter setting. These three scenarios show that the exporting sector in the intervening country benefits the most relative to the exporting sector in another peripheral country and country C. The level of redistribution is increasing in the level of intervention and decreasing in the inflation. One thing is noteworthy that all welfare gain here comes from the domestic sector in the intervening country. Hence, pure shock FXI is not a welfare improving policy for the intervening country.

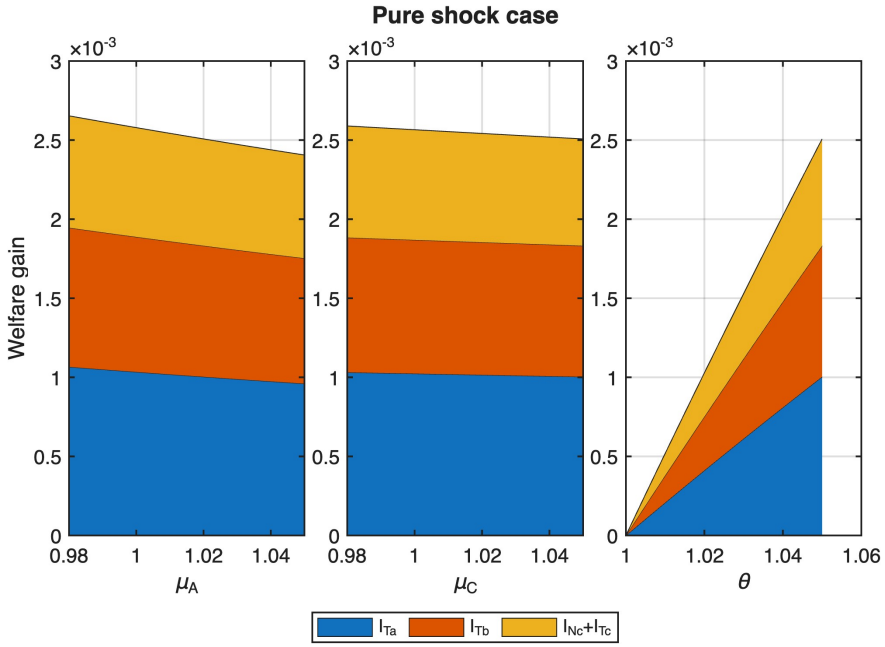

Figure 7: Pure Shock

For the anticipated case (Figure 8 and 9), since all agents expect the intervention and take it into consideration when making production decision, it works just like reducing the inflation in dominant currency by increasing the inflation in the intervening country's currency. The result implies all countries benefit from this intervention with central country benefits the most and intervening country benefits the least. The level of effect is increasing in the level of intervention. Meanwhile,

the other peripheral country improves its trade balance the most while the trade balance of central country keeps shrinking.

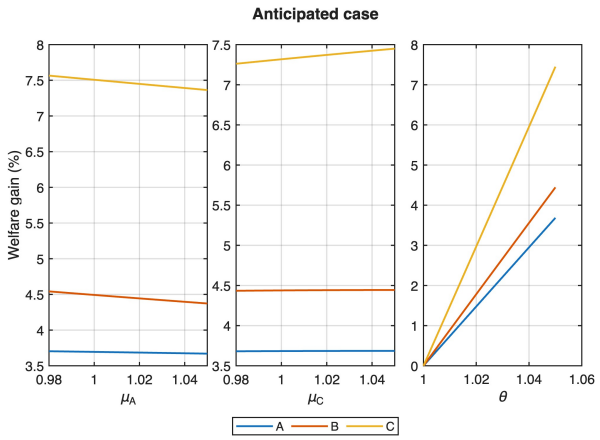

Figure 8: Welfare

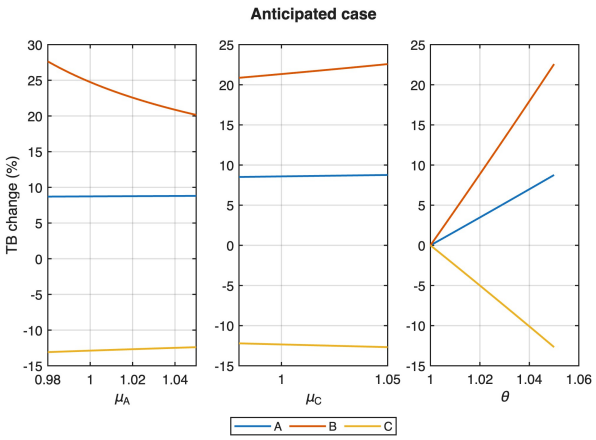

Figure 9: Trade balance

Finally, for the asymmetric case, it is obvious that the intervening country benefits the most in this scenario. What draws our attention is that the other two countries also benefit from the export expansion originated from intervening country. Nevertheless, only the intervening country experiences an improvement in the trade balance while the other two countries have a lower trade balance. In a richer model, this effect might have welfare implication such as balance payment crisis in the country B and hence lead to the competitive devaluation.

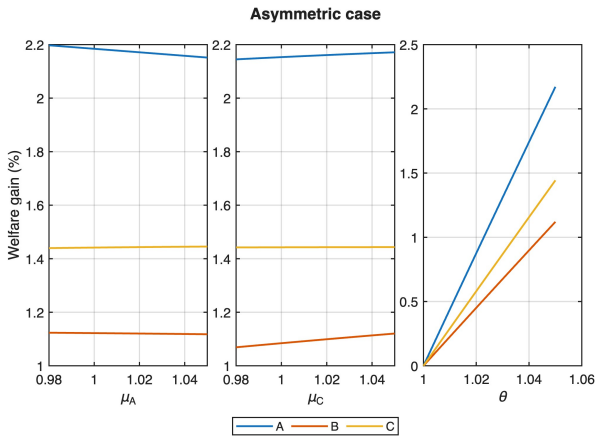

Figure 10: Welfare

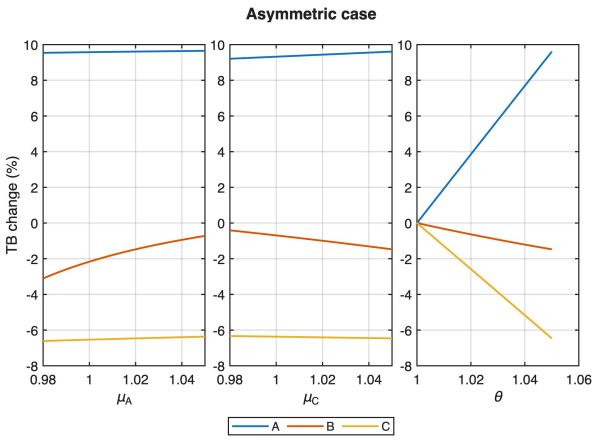

Figure 11: Trade balance

6 Conclusion

This paper develops a new mechanism through which foreign exchange intervention affects real activity under the dominant currency paradigm. I build a central-periphery monetary search model in which domestic transactions are settled in local currency while international transactions are settled in dominant currency. In this environment, the real effect of FXI does not rely on nominal rigidity. Instead, intervention changes the relative purchasing power of currencies and thereby redistributes purchasing power across sectors and countries. This redistributional channel is the central force behind the paper's results.

The analysis delivers three main findings. First, when FXI arrives as a pure shock, production decisions remain unchanged because they are made before the intervention is realized. Even so, intervention immediately redistributes purchasing power away from local-currency holders in the intervening country and toward dominant-currency holders both at home and abroad. Second, when FXI is anticipated, the redistribution effect changes production incentives. Producers who expect to be paid in dominant currency expand production, while producers who expect to be paid in depreciating local currency contract. As a result, sectors earning dominant currency expand, whereas the domestic sector in the intervening country shrinks. Third, when only domestic agents anticipate the intervention, FXI also generates an expenditure-switching effect across exporters. In that case, the intervening country gains market share in tradable production, and the policy may induce the other peripheral country to respond with its own intervention.

These results have several implications. A first implication is that, under dominant-currency trade, FXI need not hurt foreign exporters. In the anticipated case, exporters outside the intervening country may also benefit because they are paid in the currency whose purchasing power rises. A second implication is that the domestic non-tradable sector of the intervening country is the main loser from FXI. Since it earns revenue in the depreciating currency, it bears the cost of intervention without sharing in the gain from stronger dominant-currency claims. A third implication is that the widespread use of spot-market reserve purchases is easier to understand in a framework where reserve operations affect both the supply of domestic currency and the scarcity of dominant currency in private circulation. A final implication is that greater transparency about intervention can

reduce the likelihood of competitive devaluation. When intervention is publicly observed, agents in other countries can adjust their decisions accordingly, eliminating the misperceptions that generate the expenditure-switching effect in the asymmetric-information case.

More broadly, the paper shows that the dominant currency matters not only as an invoicing unit, but also as a settlement asset. Once international transactions require dominant-currency balances, changes in the relative return to currencies can alter private behavior even when prices are flexible. Embedding this mechanism in a monetary-search framework makes it possible to connect dominant-currency trade, private currency portfolio choice, and foreign exchange intervention in a unified environment. In this sense, the paper contributes to the literature on FXI, dominant currency pricing, and monetary search by highlighting a transmission channel that is distinct from the conventional sticky-price view.

Several extensions would be useful for future research. One is to endogenize the dominant-currency role rather than taking it as given. Another is to formalize the government's policy problem and characterize optimal intervention under alternative objectives, such as sectoral welfare or external competitiveness. A third is to examine repeated or persistent interventions in a richer dynamic environment. These extensions would help clarify the broader role of currency settlement and information frictions in shaping the international transmission of FXI.

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Appendix A - Derivation

This Appendix provides detailed derivations for all equations and results.

Notation

A variable is denoted as I_{ijt} which $i \in \{N, T\}$ represents non-tradable or tradable sector, $j \in \{A, B, C\}$ represents country, and t represents the period of time. Most of time, the time subscript is dropped and I'_{ij} is used to represent the variable in the next period while I_{ij} is the variable in the current period. Additionally, there are two superscripts are used. M^k_{ij} is nominal currency issued by country k and held by sector i in country j. Meanwhile, $e^j = \frac{P_j}{P_C}$ is the nominal exchange rate between currency j and dominant currency. Finally, $\mathbf{M}_{ij} = (M^A_{ij}, M^B_{ij}, M^C_{ij})$ is the currency portfolio held by sector i in country j.

Assumptions

- DCP arrangement: domestic goods are paid in domestic currency and tradable goods are paid in dominat currency.
- $\frac{\Pi_j}{\beta} \geq 1$ for all $j \in \{A, B, C\}$. With this assumption, the monetary equilibrium exists.
- $\sigma_D > 1$ and $\sigma_F > 1$. With this assumption, the expenditure share is decreasing in relative prices.

Stage 2 problem

For an agent in sector i in country j with currency portfolio $\mathbf{M}_{ij}$ and final goods production Y_{ij}, the value function is

$$W_{ij}(\mathbf{M}_{ij}, Y_{ij}) = \max_{\mathbf{M}'_{ij}} C_{ij} - H_{ij} + \beta V_{ij}(\mathbf{M}'_{ij})$$
$$s.t.\ C_{ij} = H_{ij} + \sum_k \frac{M^k_{ij} - M^{k'}_{ij}}{P_C e^k} + Y_{ij} + \frac{T_{ij}}{P_j}.$$

In stage 2, everyone can produce one unit of final goods with one unit of labor H_{ij}. Furthermore, agents might receive lump-sum transfer or pay lump-sum tax $\frac{T_{ij}}{P_j}$.

After substituting budget constraint into value function, we get

$$W_{ij}(\mathbf{M}_{ij}, Y_{ij}) = \sum_k \frac{M_{ij}^k}{P_C e^k} + Y_{ij} + \underbrace{\max_{\mathbf{M}'_{ij}} \left\{ \frac{T_{ij}}{P_j} - \sum_k \frac{M_{ij}^{k'}}{P_C e^k} + \beta V_{ij}(\mathbf{M}'_{ij}) \right\}}_{\equiv W_{ij}(0,0)}.$$

Here, we can see the optimal currency portfolio is independent of currency portfolio and production. The foc suggests that

$$\beta \frac{\partial V_{ij}(\mathbf{M}'_{ij})}{\partial M_{ij}^{k'}} \geq \frac{1}{e^k P_C}. \tag{32}$$

Stage 1 problem - Final goods producer

The production function for final goods producer is

$$Y_{Yj} = \left[\omega_D I_{Nj}^{\rho_D} + \omega_F I_F^{\rho_D} \right]^{\frac{1}{\rho_D}},\ \rho_D = \frac{\sigma_D - 1}{\sigma_D},\ \omega_D + \omega_F = 1.$$

The final goods producer with currency portfolio $\mathbf{M}_{Yj}$ decides how much domestic intermediate goods I_{Nj} and imported intermediate goods I_F to buy. The value function is

$$V_{Yj}(\mathbf{M}_{ij}) = \max_{I_{Nj}, I_F} W_{Yj}(\mathbf{M}_{ij} - \triangle \mathbf{M}_{ij}, Y_{Yj})$$
$$s.t.\ \frac{M_{Yj}^k - \triangle M_{Yj}^k}{e^k P_C} \geq 0,\ \triangle M_{Yj}^A = P_{NA} I_{NA},\ \triangle M_{Yj}^B = P_{NB} I_{NB},$$
$$\triangle M_{Yj}^C = P_{NC} I_{NC} + P_F I_F.$$

With the linearity in the W_{ij}, we rewrite the value function as follows.

$$V_{Yj}(\mathbf{M}_{Yj}) = \max_{I_{Nj}, I_F} \sum_k \frac{M_{Yj}^k - \triangle M_{Yj}^k}{e^k P_C} + Y_{Yj} + W_{Yj}(0,0) + \sum_k \lambda_{Yj}^k \frac{M_{Yj}^k - \triangle M_{Yj}^k}{e^k P_C}$$

which λ_{Yj}^k is the Lagrange multiplier. The envelop conditions implies

$$\beta \frac{\partial V_{ij}(\mathbf{M}'_{ij})}{\partial M_{ij}^{k'}} = \beta \frac{1 + \lambda_{Yj}^{k'}}{e^{k'} P'_C} \geq \frac{1}{e^k P_C} \Rightarrow 1 + \lambda_{Yj}^{k'} \geq \frac{\Pi_k}{\beta}$$

which $\Pi_k = \frac{P'_k}{P_k}$ is the inflation rate. By assumption, we know $1 + \lambda_{Yj}^{k'} \geq \frac{\Pi_k}{\beta} > 1$ for all $k \in \{A, B, C\}$ which means $\lambda_{Yj}^{k'} > 0$. From KKT conditions, we know final goods producers will spends all currency on intermediate goods purchases, which is the Lemma 1.

Since the domestic intermediate goods can only be used by final goods producer in the same country, there is no reason for producer in country A to hold currency B and vice versa. Therefore, $\mathbf{M}_{YA} = (M_{YA}^A, 0, M_{YA}^C)$, $\mathbf{M}_{YB} = (0, M_{YB}^B, M_{YB}^C)$, and $\mathbf{M}_{YC} = (0, 0, M_{YC}^C)$. Meanwhile, with Lemma 1, we can rewrite the maximization problem for final goods producers in country A as follows.

$$V_{YA}(\mathbf{M}_{YA}) = \max_{\mathbf{M}'_{YA}} \left[\omega_D \left(\frac{M_{YA}^A}{P_{NA}}\right)^{\rho_D} + \omega_F \left(\frac{M_{YA}^C}{P_F}\right)^{\rho_D}\right]^{\frac{1}{\rho_D}} + \frac{T_{YA}}{P_A} - \frac{M_{YA}^{A'}}{P_A} - \frac{M_{YA}^{C'}}{P_C} + \beta V_{YA}(\mathbf{M}'_{YA}).$$

Take first order derivative with respect to $M_{YA}^{A'}$ and $M_{YA}^{C'}$, we get.

$$\begin{aligned}
\frac{\Pi_A\, m_{YA}^{A'}}{\beta} &= \left[\omega_D \left(\frac{m_{YA}^{A'}}{P'_{NA}/P'_A}\right)^{\rho_D} + \omega_F \left(\frac{m_{YA}^{C'}}{P'_F/P'_C}\right)^{\rho_D}\right]^{\frac{1-\rho_D}{\rho_D}} \omega_D \left(\frac{m_{YA}^{A'}}{P'_{NA}/P'_A}\right)^{\rho_D} \\
\frac{\Pi_C\, m_{YA}^{C'}}{\beta} &= \left[\omega_D \left(\frac{m_{YA}^{A'}}{P'_{NA}/P'_A}\right)^{\rho_D} + \omega_F \left(\frac{m_{YA}^{C'}}{P'_F/P'_C}\right)^{\rho_D}\right]^{\frac{1-\rho_D}{\rho_D}} (1-\omega_D) \left(\frac{m_{YA}^{C'}}{P'_F/P'_C}\right)^{\rho_D}
\end{aligned}$$

By summing up and dividing these two equations, we get

$$\begin{aligned}
\frac{\Pi_A\, m_{YA}^{A'} + \Pi_C\, m_{YA}^{C'}}{\beta} &= \left[\omega_D \left(\frac{m_{YA}^{A'}}{P'_{NA}/P'_A}\right)^{\rho_D} + \omega_F \left(\frac{m_{YA}^{C'}}{P'_F/P'_C}\right)^{\rho_D}\right]^{\frac{1}{\rho_D}} \\
m_{YA}^{C'} &= \left[\frac{\omega_F}{\omega_D} \left(\frac{P'_{NA}/P'_A}{P'_F/P'_C}\right)^{\rho_D} \frac{\Pi_A}{\Pi_C}\right]^{\frac{1}{1-\rho_D}} m_{YA}^{A'}
\end{aligned}$$

The second equation corresponds to (10) and substitutes it into the first equation to get

$$\frac{1}{\beta} = \left[\omega_D^{\sigma_D} \left(\frac{P'_{NA}}{P'_A}\Pi_A\right)^{1-\sigma_D} + \omega_F^{\sigma_D} \left(\frac{P'_F}{P'_C}\Pi_C\right)^{1-\sigma_D}\right]^{\frac{-1}{1-\sigma_D}}$$

which is the general version of (29). We apply the same procedure to final goods producers in country B and C and get

$$\begin{aligned}
\frac{1}{\beta} &= \left[\omega_D^{\sigma_D} \left(\frac{P'_{NB}}{P'_B}\Pi_B\right)^{1-\sigma_D} + \omega_F^{\sigma_D} \left(\frac{P'_F}{P'_C}\Pi_C\right)^{1-\sigma_D}\right]^{\frac{-1}{1-\sigma_D}} \\
\frac{1}{\beta} &= \left[\omega_D^{\sigma_D} \left(\frac{P'_{NC}}{P'_C}\Pi_C\right)^{1-\sigma_D} + \omega_F^{\sigma_D} \left(\frac{P'_F}{P'_C}\Pi_C\right)^{1-\sigma_D}\right]^{\frac{-1}{1-\sigma_D}} \\
m_{YB}^{C'} &= \left[\frac{\omega_F}{\omega_D} \left(\frac{P'_{NB}/P'_B}{P'_F/P'_C}\right)^{\rho_D} \frac{\Pi_A}{\Pi_C}\right]^{\frac{1}{1-\rho_D}} m_{YB}^{B'}
\end{aligned}$$

Stage 1 problem - Intermediate goods producer

For the intermediate goods producer with currency holding $\mathbf{M}_{ij}$ in sector $i \in \{N, T\}$ country $j \in \{A, B, C\}$, the value function at the beginning of each period is

$$\begin{aligned} V_{ij}(\mathbf{M}_{ij}) &= \max_{\ell_{ij}} -\frac{\ell_{ij}^2}{2} + W_{ij}(\mathbf{M}_{ij} + \triangle\mathbf{M}_{ij}, 0)\triangle \\ &= \max_{\ell_{ij}} \left[-\frac{\ell_{ij}^2}{2} + \sum_k \frac{M_{ij}^k + \triangle M_{ij}^k}{e^k P_C} \right] + \max_{\mathbf{M}'_{ij}} \left[\frac{T_{ij}}{P_j} + \beta V(\mathbf{M}'_{ij}) \right] \end{aligned}$$

Focusing on the currency holding choice $\mathbf{M}'_{ij}$ first. The envelope condition tells us that

$$\frac{\partial V_{ij}(\mathbf{M}'_{ij})}{\partial M_{ij}^{k'}} = \frac{1}{e^{k'} P'_C}$$

and the intermediate goods producer holds positive amount of currency if

$$\beta \frac{\partial V_{ij}(\mathbf{M}'_{ij})}{\partial M_{ij}^{k'}} = \frac{\beta}{e^{k'} P'_C} \geq \frac{1}{e^k P_C} \Leftrightarrow 1 \geq \frac{\Pi_k}{\beta}$$

which violates the assumption that $\frac{\Pi_k}{\beta} > 1$ for all k. This leads to the result that intermediate good producers spend all revenue in stage 2 and carry nothing to the next period, which is Lemma 2.

Therefore, the intermediate goods producers only need to focus on intra-period labor supply decision. For $i = N$, they receive domestic currency as payment. The profit maximization problem is

$$\max_{\ell_{Nj}} \left[-\frac{\ell_{Nj}^2}{2} + \frac{P_{Nj} A_{Nj} \ell_{Nj}}{P_j} \right] \Rightarrow \ \ell_{Nj} = \frac{P_{Nj} A_{Nj}}{P_J}, \ I_{Nj} = \frac{P_{Nj} A_{Nj}^2}{P_J}$$

For $i = T$, they receive dominant currency as payment. The profit maximization problem is

$$\max_{\ell_{Tj}} \left[-\frac{\ell_{Tj}^2}{2} + \frac{P_{Tj} A_{Tj} \ell_{Tj}}{P_j} \right] \Rightarrow \ \ell_{Tj} = \frac{P_{Tj} A_{Tj}}{P_C}, \ I_{Tj} = \frac{P_{Tj} A_{Tj}^2}{P_C}$$

Stage 1 problem - Global intermediate goods producer

Global intermediate goods producer is passive in this model. The producer solves the following problem in each stage 1.

$$\max_{I_{Tj}} \left[\sum_j \omega_{Fj} I_{Tj}^{\rho_F}\right]^{\frac{1}{\rho_F}} - \sum_j P_{Tj} I_{Tj},$$

$$s.t.\ \sum_j P_{Tj} I_{Tj} \leq M_{YA}^C + M_{YB}^C + (1 - S_D) M_{YC}^C,\ S_D = \left(\frac{P_{NC}}{P_{YC}}\right)^{1-\sigma_D} \omega_D^{\sigma_D}$$

$$P_{YC} = \left[\omega_D^{\sigma_D} P_{NC}^{1-\sigma_D} + \omega_F^{\sigma_D} P_F^{1-\sigma_D}\right]^{\frac{1}{1-\sigma_D}},\ P_F = \left[\sum_j \omega_{Fj}^{\sigma_F} P_{Tj}^{1-\sigma_F}\right]^{\frac{1}{1-\sigma_F}}$$

The optimal conditions imply the expenditure share is decreasing in relative price, which is

$$S_{Fj} = \frac{P_{Tj} I_{Tj}}{\sum_k P_{Tk} I_{Tk}} = \left(\frac{P_{Tj}}{P_F}\right)^{1-\sigma_F} \omega_{Fj}^{\sigma_F}.$$

Stage 1 - Market clearing conditions

According to agents' optimal conditions, the relative price adjust to clean the market. The market clearing conditions are following

$$\begin{cases} \frac{P_{NA}}{P_A} A_{NA}^2 = \frac{M_{YA}^A}{P_{NA}} \\ \frac{P_{NB}}{P_B} A_{NB}^2 = \frac{M_{YB}^B}{P_{NB}} \\ \frac{P_{NC}}{P_C} A_{NC}^2 = \frac{S_D M_{YC}^C}{P_{NC}} \\ \frac{P_{TA}}{P_C} A_{TA}^2 = \frac{S_{FA} M_{YA}^C}{P_{TA}} \\ \frac{P_{TB}}{P_C} A_{TB}^2 = \frac{S_{FB} M_{YA}^C}{P_{TB}} \\ \frac{P_{TC}}{P_C} A_{TC}^2 = \frac{S_{FC} M_{YA}^C}{P_{TC}} \end{cases} \quad \text{which implies that} \quad \begin{cases} \frac{P_{NA}}{P_A} = \frac{\sqrt{m_{YA}^A}}{A_{NA}} \\ \frac{P_{NB}}{P_B} = \frac{\sqrt{m_{YB}^B}}{A_{NB}} \\ \frac{P_{NC}}{P_C} = \frac{\sqrt{S_D m_{YC}^C}}{A_{NC}} \\ \sum_j \left(\frac{P_{Tj} A_{Tj}}{P_C}\right)^2 = m_{YA}^C + m_{YB}^C + (1 - S_D) m_{YC}^C \\ \frac{P_{Ti}}{P_{Tj}} = \left[\left(\frac{A_{Tj}}{A_{Ti}}\right)^2 \left(\frac{\omega_{Fi}}{\omega_{Fj}}\right)^{\sigma_F}\right]^{\frac{1}{1+\sigma_F}} \end{cases}$$

The relative price of tradable intermediate goods implies the expenditure share is increasing in relative TFP because

$$S_{Fj} = \left[\sum_k \left[\left(\frac{\omega_{Fk}}{\omega_{Fj}}\right)^{\sigma_F} \left(\frac{A_{Tj}}{A_{Tk}}\right)^{1-\sigma_F}\right]^{\frac{2}{1+\sigma_F}}\right]^{-1} \quad \text{and} \quad \frac{\partial S_{Fj}}{\partial A_{Tj}/A_{Tk}} > 0.$$

Computation algorithm

The whole economy can be summarized into following 9 conditions.

$$m_{YA}^{C'} = \left[\frac{\omega_F}{\omega_D}\left(\frac{P'_{NA}/P'_A}{P'_F/P'_C}\right)^{\rho_D}\frac{\Pi_A}{\Pi_C}\right]^{\frac{1}{1-\rho_D}} m_{YA}^{A'}$$

$$m_{YB}^{C'} = \left[\frac{\omega_F}{\omega_D}\left(\frac{P'_{NB}/P'_B}{P'_F/P'_C}\right)^{\rho_D}\frac{\Pi_A}{\Pi_C}\right]^{\frac{1}{1-\rho_D}} m_{YB}^{B'}$$

$$m_{YA}^{A'} = \left(\frac{P'_{NA}A_{NA}}{P'_A}\right)^2$$

$$m_{YB}^{B'} = \left(\frac{P'_{NB}A_{NB}}{P'_B}\right)^2$$

$$S_D m_{YC}^{C'} = \left(\frac{P'_{NC}A_{NC}}{P'_C}\right)^2$$

$$m_{YA}^{C'} + m_{YB}^{C'} + (1-S_D)m_{YA}^{C'} = \left(\frac{P'_{TA}}{P'_C}\right)^2\left[A_{TA}^2 + \left(\frac{P'_{TB}A_{TB}}{P'_{TA}}\right)^2 + \left(\frac{P'_{TC}A_{TC}}{P'_{TA}}\right)^2\right]$$

$$\frac{1}{\beta} = \left[\omega_D^{\sigma_D}\left(\frac{P'_{NA}}{P'_A}\Pi_A\right)^{1-\sigma_D} + \omega_F^{\sigma_D}\left(\frac{P'_F}{P'_C}\Pi_C\right)^{1-\sigma_D}\right]^{\frac{-1}{1-\sigma_D}}$$

$$\frac{1}{\beta} = \left[\omega_D^{\sigma_D}\left(\frac{P'_{NB}}{P'_B}\Pi_B\right)^{1-\sigma_D} + \omega_F^{\sigma_D}\left(\frac{P'_F}{P'_C}\Pi_C\right)^{1-\sigma_D}\right]^{\frac{-1}{1-\sigma_D}}$$

$$\frac{1}{\beta} = \left[\omega_D^{\sigma_D}\left(\frac{P'_{NC}}{P'_C}\Pi_C\right)^{1-\sigma_D} + \omega_F^{\sigma_D}\left(\frac{P'_F}{P'_C}\Pi_C\right)^{1-\sigma_D}\right]^{\frac{-1}{1-\sigma_D}}$$

Here are the steps to calculate the equilibrium.

- Start from guessing $\frac{P'_F}{P'_C} \in (0,1)$
- Calculate $\frac{P'_{Nj}}{P'_j}$ for $j \in \{A,B,C\}$ from equations 7 - 9;
- Calculate m_{Yj}^j for $j \in \{A,B,C\}$ from equations 3 - 5;
- Calculate m_{YA}^C and m_{YB}^C from equations 1 - 2;
- Calculate $\frac{P'_{Tj}}{P'_C}$ for $j \in \{A,B,C\}$ from equation 6;
- Calculate updated guess $\widehat{\frac{P'_F}{P'_C}} = \left[\sum_j \omega_{Fj}^{\sigma_F}\left(\frac{P'_{Tj}}{P'_C}\right)^{1-\sigma_F}\right]^{\frac{1}{1-\sigma_F}}$

- Check the distance between updated guess and initial guess and if $|\frac{P'_F}{P'_C} - \widehat{\frac{P'_F}{P'_C}}| < \varepsilon$, for example 10^{-6}, then we are done.
- Otherwise, replace the initial guess with $\frac{1}{2}(\frac{P'_F}{P'_C} + \widehat{\frac{P'_F}{P'_C}})$ and repeat until convergence.

Optimal allocation

From the equations

$$\frac{1}{\beta} = \left[\omega_D^{\sigma_D}\left(\frac{P'_{NA}}{P'_A}\Pi_A\right)^{1-\sigma_D} + \omega_F^{\sigma_D}\left(\frac{P'_F}{P'_C}\Pi_C\right)^{1-\sigma_D}\right]^{\frac{-1}{1-\sigma_D}}$$
$$\frac{1}{\beta} = \left[\omega_D^{\sigma_D}\left(\frac{P'_{NB}}{P'_B}\Pi_B\right)^{1-\sigma_D} + \omega_F^{\sigma_D}\left(\frac{P'_F}{P'_C}\Pi_C\right)^{1-\sigma_D}\right]^{\frac{-1}{1-\sigma_D}}$$
$$\frac{1}{\beta} = \left[\omega_D^{\sigma_D}\left(\frac{P'_{NC}}{P'_C}\Pi_C\right)^{1-\sigma_D} + \omega_F^{\sigma_D}\left(\frac{P'_F}{P'_C}\Pi_C\right)^{1-\sigma_D}\right]^{\frac{-1}{1-\sigma_D}}$$

we know that the relative prices $(\frac{P'_{Nj}}{P_j},\ \frac{P'_F}{P'_C})$ are decreasing in inflation rates. At the same time, the relative prices are real wage for intermediate goods producers. Hence, the intermediate goods production is increasing in relative prices and decreasing in inflation rates. Consequently, the production is higher if inflation is lower. For country C, only Π_C is relevant. On the other hand, peripheral countries are affected by domestic inflation and Π_C. This result is Proposition 1.